Area Between Curves

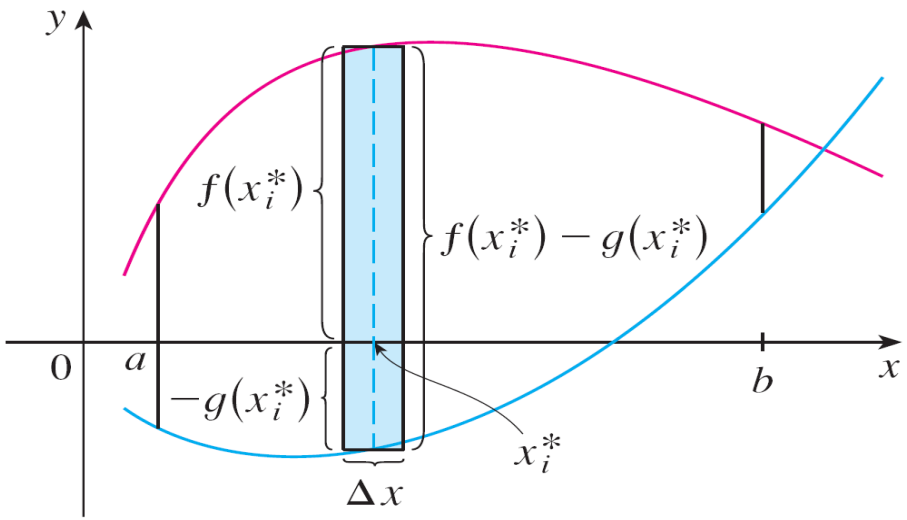
Section 6.1

Outline

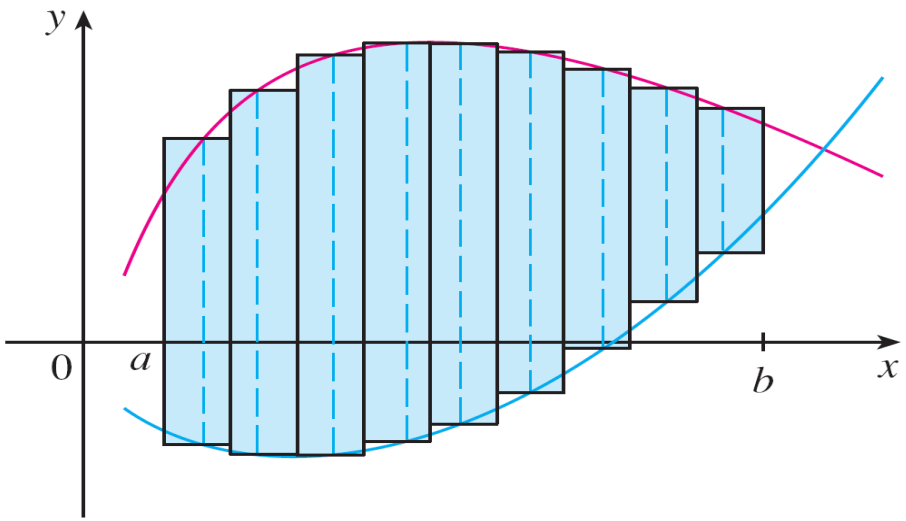
- Areas Between Curves
  - Case 1
  - Case 2
  - Case 3

Areas Between Curves

- Compute the area between $y = f(x)$ and $y = g(x)$.
- Case 1:



(a) Typical rectangle



(b) Approximating rectangles

Areas Between Curves

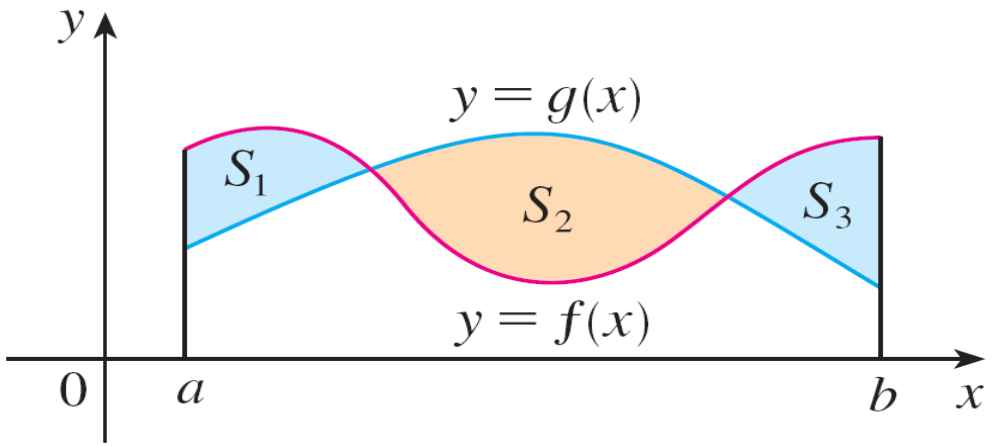
2 The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$, $x = b$, where f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$, is

$$A = \int_a^b [f(x) - g(x)]\, dx$$

Ex: Find the area of the region enclosed by $y = x^2$ and $y = 2x - x^2$

Areas Between Curves

- Case 2:



3 The area between the curves $y = f(x)$ and $y = g(x)$ and between $x = a$ and $x = b$ is

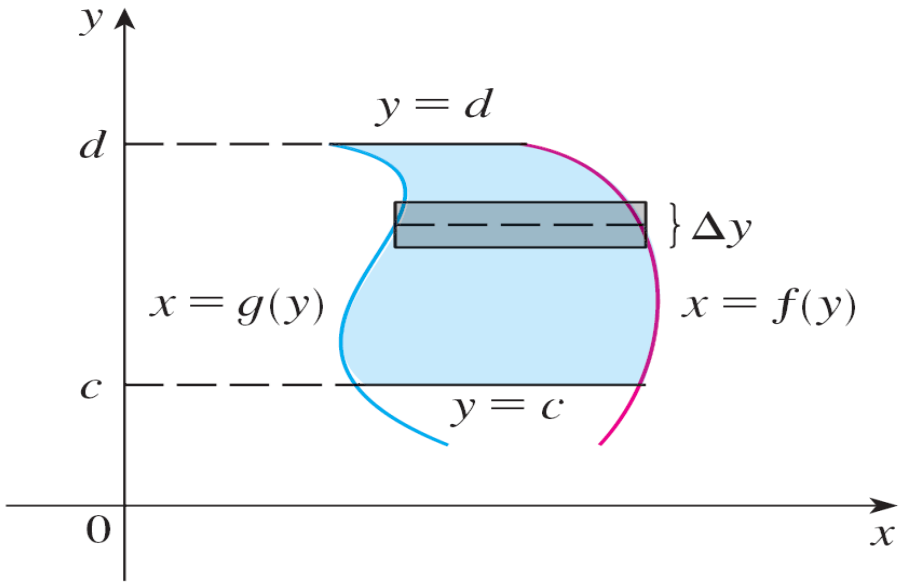
$$A = \int_a^b |f(x) - g(x)|\, dx$$

Ex: Sketch the region enclosed by curves $y = \cos x$, $y = \sin 2x$, $0 \le x \le \frac{\pi}{2}$ and find its area.

Areas Between Curves

- Case 3:
- Sometimes a region is best described as bounded by curves with equations $x = f(y)$, $x = g(y)$ (left and right boundary curves), $y = c$ and $y = d$, where f and g are continuous and $f(y) \geq g(y)$ for $c \leq y \leq d$.

Areas Between Curves

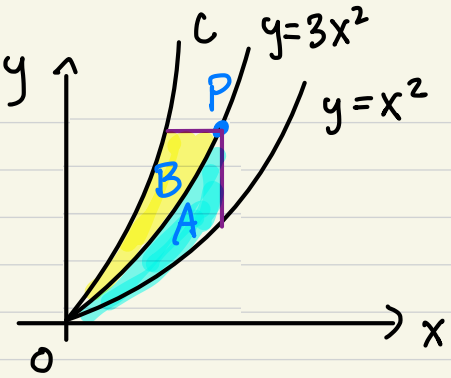


- Then the area of the region is

$$A = \int_c^d [f(y) - g(y)]\, dy$$

Ex: Find the area of the region bounded by $y^2 = 4x + 5$ and $y = x$.

Ex:



Find the curve C such that for every point of $y=2x^2$, the area A and B are equal.

Review

- How do we compute area between curves?
  - Case 1
  - Case 2
  - Case 3