Differential Rules (Part 2)

Section 3.4-3.5

Outline

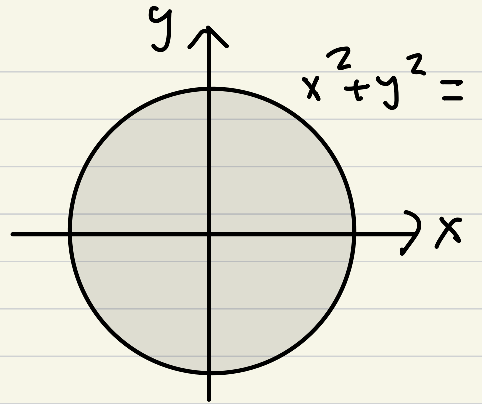
- The Chain Rule
- The Implicit Differentiation
  - The Implicit Functions
  - The Implicit Differentiation

Implicit Differentiation

- Implicit Functions:
- Two variables x and y may be related in an "implicit" way. For example, x and y satisfy the equation $f(x, y) = 0$. Locally, this relation defines y as an implicit function of x or x as an implicit function of y.

Ex: $x^2+y^2=1$

Sol:



Q: Explore

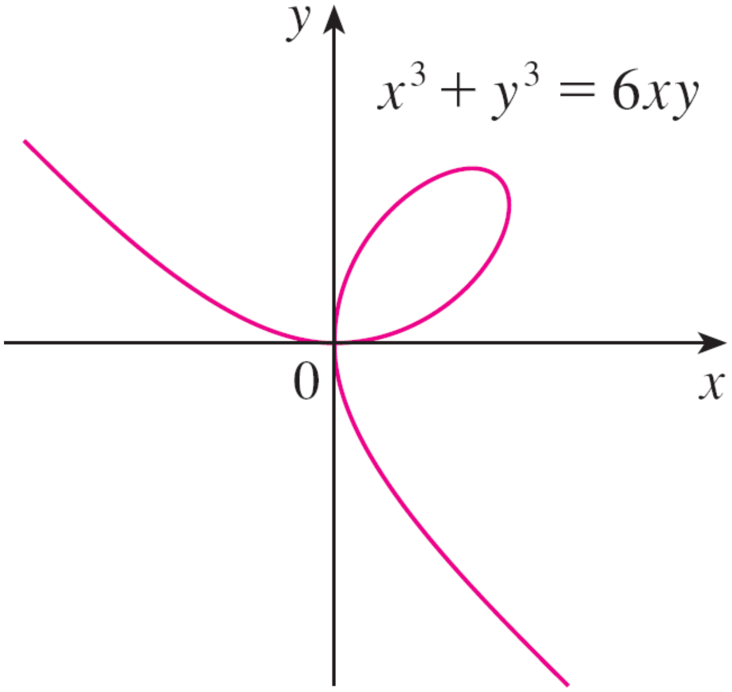
① $x^{2n}+y^{2n}=1$, $n \in \mathbb{N}$.

② $|x|^{\frac{1}{n}}+|y|^{\frac{1}{n}}=1$, $n \in \mathbb{N}$.

③ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

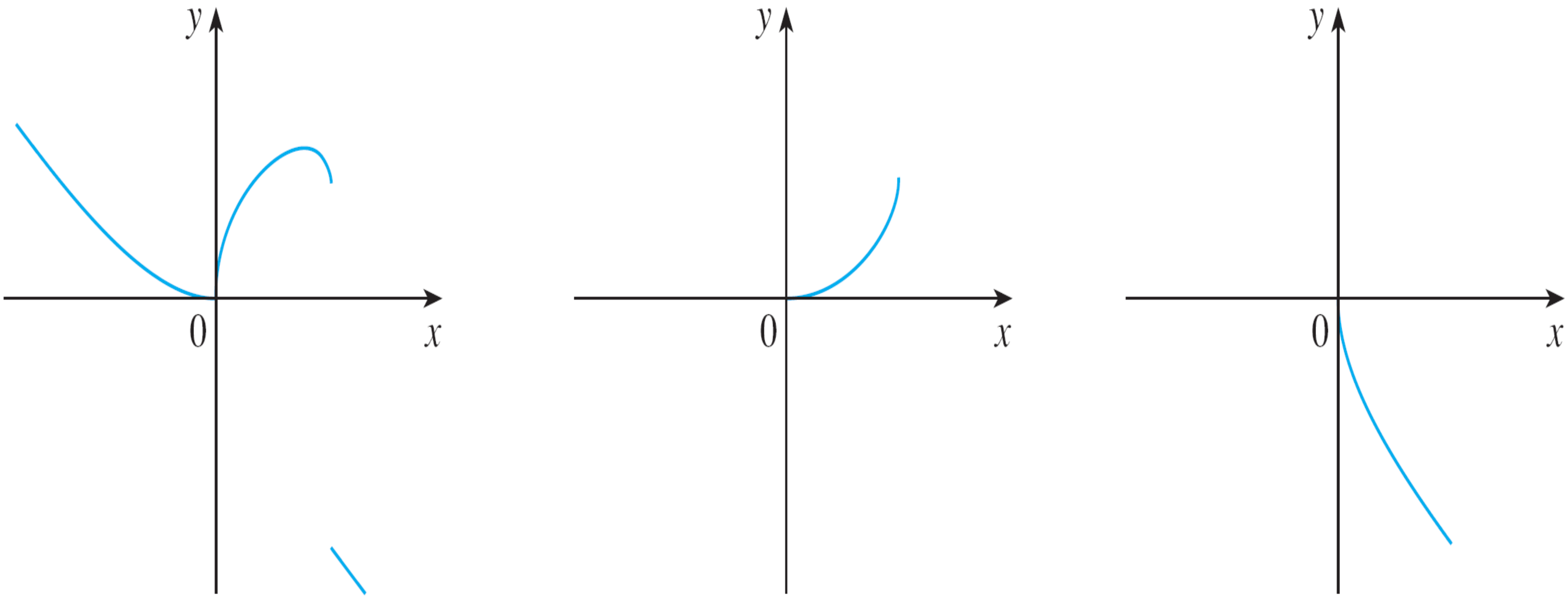
Implicit Differentiation

- Example: The **folium of Descartes**



Implicit Differentiation

- Example: The **folium of Descartes**



Ex: For $x^2 + y^2 = 4$, find $\frac{dy}{dx}$ at $(1, \sqrt{3})$.

Implicit Differentiation

- Compute $\dfrac{dy}{dx}$ given $f(x, y) = 0$:
- Differentiate both sides of the equation $f(x, y) = 0$ with respect to x. When you encounter terms involving y, use the chain rule to calculate the differentiation.
- Solve the resulting equation for $\dfrac{dy}{dx}$.

Ex: For $x^3+y^3=6xy$, find the tangent line at $(3,3)$.

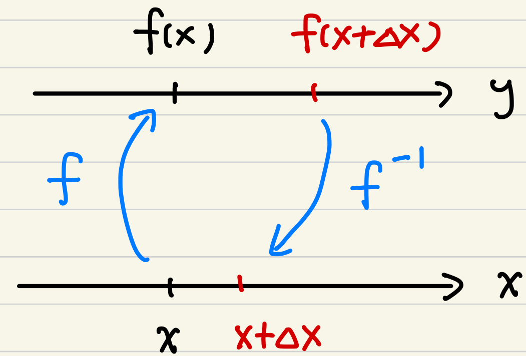
At what point in the first quadrant is the tangent line horizontal (vertical)?

Ex: Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $x^4 + y^4 = 1$.

Ex: Find an equation of the tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ at (x_0, y_0) where $c > 0$ is a constant. Show that the sum of the x- and y-intercept of any tangent line is equal to c.

Derivatives of Inverse Functions

Ex: $y = f^{-1}(x)$ if and only if $f(y) = x$. Find $\frac{dy}{dx}$.



Ex: $f(x) = x + e^{2(x-1)}$ is one-to-one. Find $\left.\frac{d}{dx} f^{-1}\right|_{x=2}$ and $\left.\frac{d^2}{dx^2} f^{-1}\right|_{x=2}$.

Review

- State the Chain Rule for differentiation.
- Describe the process of implicit differentiation.