The Indefinite Integral and the Substitution Rule

Section 5.4-5.5

Outline

- ▶ 1. Indefinite Integrals
 - Notation
 - Application
- ▶ 2. The Substitution Rule
 - For Indefinite Integrals
 - For Definite Integrals
 - Application

- For indefinite integrals:
- The Substitution Rule If u = g(x) is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x) dx = \int f(u) du$$

pf of the substitution Rule:

- ▶ Note 1: The Substitution Rule for integration is proved using the Chain Rule for differentiation.
- Note 2: If u = g(x), then du = g'(x)dx, so a way to remember the Substitution Rule is to think of dx and du in the formula as differentials.

Ex: Compute
$$\int 2x \sqrt{3+x^2} dx$$

Ex: Compute
$$\int \frac{x}{\sqrt{1+4x^2}} dx$$

$$\int \frac{f(x)}{f(x)} \, qx$$

$$Ex: \int \frac{f(x)}{f(x)} dx$$

$$\exists x : \int \frac{2^x}{2^x + 1} dx$$

$$Ex: \int \frac{1}{\alpha^2 + x^2} dx$$

Ex:
$$\int \frac{1}{\sqrt{\alpha^2 - x^2}} dx$$
, where $\alpha > 0$.

$$Ex: \int \frac{dx}{x^2+2x+10}$$

Complicated Integrals

Ex:
$$\int (1+e^x)^e e^{2x} dx$$

$$\exists x : \int \frac{x^{\frac{1}{2}}}{1+x^{\frac{3}{4}}} dx$$

Ex: Eing
$$\int \frac{x+x}{x+x} dx$$

- For definite integrals:
- **The Substitution Rule for Definite Integrals** If g' is continuous on [a, b] and f is continuous on the range of u = g(x), then

$$\int_{a}^{b} f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

Note: The formula is true even if g(b) < g(a).

Ex: Prove that $\int_{a}^{b} f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$ if f(x) and g'(x) are continuous.

Ex: compute $\int_{e}^{e^{4}} \frac{1}{x \int \ln x} dx$.

Ex: By the substitution $u=\frac{1}{x}$, show that for any $\alpha>1$,

$$\int_{a}^{\alpha} \frac{\ln x}{1+x+x^{2}} dx = 0$$

Sin X + GS X

Ex:

Ex: Compute
$$\int_{0}^{\frac{\pi}{2}} \frac{\sin x}{\sin x + 2\cos x} dx \text{ and } \int_{0}^{\frac{\pi}{2}} \frac{\cos x}{\sin x + 2\cos x} dx$$

$$\int_{0}^{1} x^{a} (1-x)^{b} dx = \int_{0}^{1} x^{b} (1-x)^{a} dx$$

Ex:
$$\frac{dx}{dx} \int_{x^2}^{x} f(x+t) dt$$

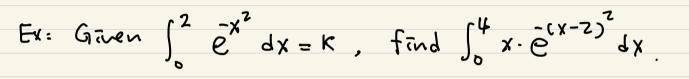
Application:

- 7 Integrals of Symmetric Functions Suppose f is continuous on [-a, a].
- (a) If f is even [f(-x) = f(x)], then $\int_{-a}^{a} f(x) dx = 2 \int_{0}^{a} f(x) dx$.
- (b) If f is odd [f(-x) = -f(x)], then $\int_{-a}^{a} f(x) dx = 0$.



Ex: compute
$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \chi^4 \sin \chi \, d\chi$$
.





Review

- What is an indefinite integral? What is the notation for indefinite integrals?
- Write down the formula for the substitution rule (for both indefinite integral and definite integral).