Differential Rules (Part 1)

Section 3.1-3.2

Outline

- Derivatives of Basic Functions
  - Power Functions
  - Exponential Functions
- Differential Rules:
  - The Constant Multiple Rule
  - The Sum Rule
  - The Product Rule
  - The Quotient Rule

Derivatives of Basic Functions: Power Function

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Ex: Suppose that $n \in \mathbb{N}$. Compute $\frac{d}{dx}(x^n)$

Ex: $\frac{d}{dx}\left(\frac{1}{x}\right) =$

$\frac{d}{dx}\left(\sqrt[3]{x}\right) =$

$\frac{d}{dx}\left(x^{\sqrt{2}}\right) =$

Ex: Compute $f'(x)$, where $f(x) = c$.

Derivatives of Exponential Functions

- For an exponential function $f(x) = a^x$,

$$f'(x) = f'(0)a^x = f'(0)f(x) \quad .$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Ex: Find $f'(x)$ where $f(x) = a^x$.

Differential Rules

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Ex: Prove the Sum Rule: If $f(x)$ and $g(x)$ are both differentiable, then $\frac{d}{dx}[f(x)+g(x)] = f'(x) + g'(x)$.

Ex: $P(x) = \sum_{k=0}^{n} a_k x^k$. Find $P'(x)$.

sol:

Ex: $f(x) = \sqrt{x}\left(\frac{1}{x} + 2x^2\right) - e^{x+2} + 1$. Find $f'(x)$.

sol:

Differential Rules

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\dfrac{d}{dx}[f(x)] - f(x)\dfrac{d}{dx}[g(x)]}{[g(x)]^2}$$

Ex: Prove the Product Rule: If $f(x)$ and $g(x)$ are both differentiable, then $\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\cdot g'(x) + f'(x)\cdot g(x)$.

Sol: $$(f(x)\,g(x))' = \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

Ex: $f(x) = \left(\frac{1}{x} - \frac{1}{x^2}\right)\left(x^3 + 2\sqrt{x}\right)$

$f'(x) =$

Ex: $f(x) = e^{2x} = e^x \cdot e^x$, $f'(x) =$

Ex: $f(x) = \dfrac{e^{2x}}{\sqrt{x}}$, $f'(x) =$

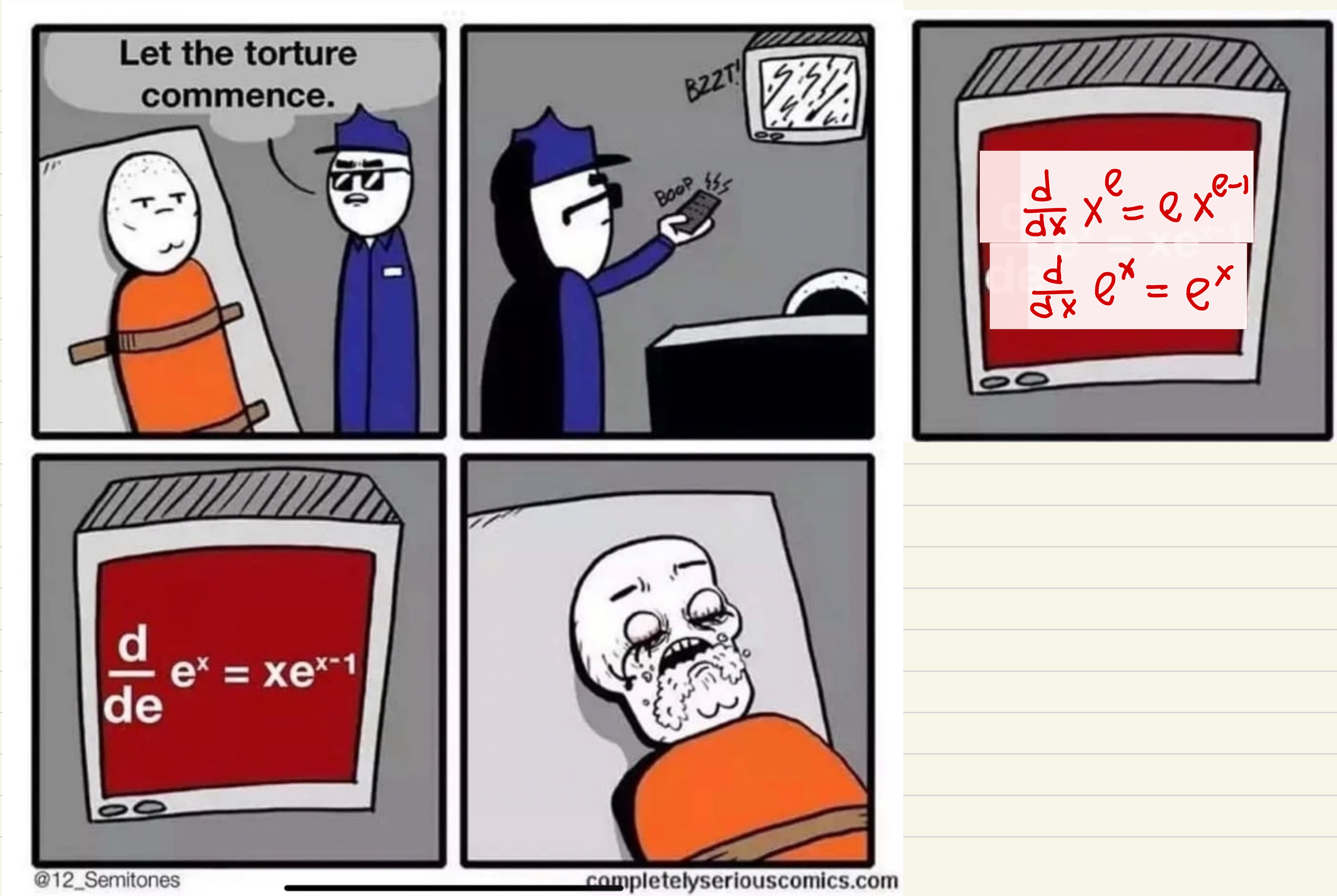
Ex: $\frac{d}{dx}\left(f(x)\, g(x)\, h(x)\right) =$

Ex: $\frac{d}{dx}\left[(f(x))^2\right] =$

Ex: $n \in \mathbb{N}$, $\frac{d}{dx}\left[(f(x))^n\right] =$

Ex: $f(x) = \left(x^e - \frac{1}{x^3} + e^x\right)^{2023}$. Find $f'(x)$.

Sol:


Let the torture commence.
BZZT!
BOOP
$\frac{d}{dx} x^e = e x^{e-1}$
$\frac{d}{dx} e^x = e^x$
$\frac{d}{de} e^x = xe^{x-1}$
@12_Semitones
completelyseriouscomics.com

Ex: Prove the Quotient Rule.

Sol: $$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \lim_{h\to 0} \frac{\dfrac{f(x+h)}{g(x+h)} - \dfrac{f(x)}{g(x)}}{h}$$

Ex: Compute $\frac{d}{dx}\left(\frac{1}{g(x)}\right)$ where $g(x)$ is differentiable.

Ex: Compute $\frac{d}{dx}\left[(f(x))^{-n}\right]$, where $n \in \mathbb{N}$ and $f(x)$ is differentiable.

Ex: $f(x) = \dfrac{x-1}{x+2}$, $f'(x) =$

Ex: $f(x) = \dfrac{2x+1}{(x^2-1)^3}$, $f'(x) =$

Ex: $f(x) = \dfrac{x^4 - 3x^3 + 5x}{x^6 - 2x^3 + 6x - 4}$. Compute $f'(0)$.

Review

- What is the derivative of a power function ?
- What is the derivative of an exponential function ?
- State the algebraic differential rules.