The Indefinite Integral and the Substitution Rule

Section 5.4-5.5

Outline

- 1. Indefinite Integrals
  - Notation
  - Application
- 2. The Substitution Rule
  - For Indefinite Integrals
  - For Definite Integrals
  - Application

Indefinite Integrals

- In section 5.4 we introduce a notation for antiderivatives, review the formulas for antiderivatives, and use them to evaluate definite integrals.

Indefinite Integrals

- Notation for Antiderivative:

The notation $\int f(x)dx$ is traditionally used for an antiderivative of f and is called an **indefinite integral.**

$$\boxed{\int f(x)\,dx = F(x) \qquad \text{means} \qquad F'(x) = f(x)}$$

Indefinite Integrals

- The most general antiderivative on a *given interval* is obtained by adding a constant to a particular antiderivative.
- Hence, an indefinite integral is an entire family of functions.

Indefinite Integrals

1 Table of Indefinite Integrals

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

Indefinite Integrals

- Application:
- We can reformulate FTC2 as follows.

  If $f(x)$ is continuous on $[a,b]$, then

$$\int_a^b f(x)dx = \int f(x)dx \,\Big|_a^b$$

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Ex: $\displaystyle\int x^{\sqrt{2}} + \frac{\sin x}{\cos^2 x} + \sqrt{\frac{2}{1-x^2}}\, dx$

Ex: $\displaystyle\int \tan^2 x \, dx$

Ex: $\int \left(\frac{x+1}{2x} \right)^2 dx$

Ex: $\int_0^2 2|x-1| + \frac{1}{1+x^2} \, dx$

Ex: $S(t)$: position function. $S'(t) = v(t)$: velocity function.

$$\int_a^b v(t)\,dt =$$

Ex: $P(t)$: population function, $P'(t)$: rate of growth of the population.

$$\int_a^b P'(t)\,dt =$$