

Functions of Several variables

Section 14.1-14.3

Outline

- ▶ Functions of Several Variables
 - ▶ Graphs
 - ▶ Level Curves
- ▶ Limits and Continuity
- ▶ Partial Derivatives
 - ▶ Definition
 - ▶ Geometric Interpretation
 - ▶ Higher Derivatives and Clairaut's Theorem

Functions of Several Variables

Definition A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

A **function of n variables** is a rule that assigns a number $z = f(x_1, x_2, \dots, x_n)$ to an n -tuple of real numbers (x_1, x_2, \dots, x_n) .

Domains of Multivariable Functions

Ex: Find the domain of $f(x, y) = \frac{\sqrt{y - x^2 + 1}}{\ln(y - x)}$

sol:

Functions of Several Variables: Graphs

Definition If f is a function of two variables with domain D , then the **graph** of f is the set of all points (x, y, z) in \mathbb{R}^3 such that $z = f(x, y)$ and (x, y) is in D .

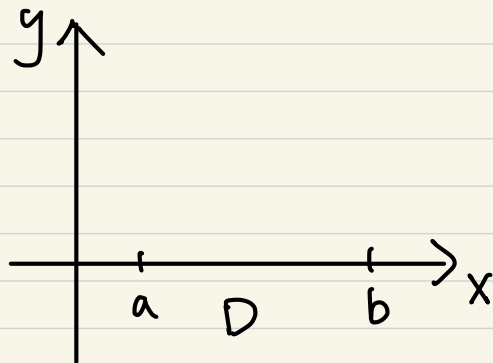
Just like the graph of $f(x)$ is a curve C satisfying equation $y = f(x)$, the graph of a function f of two variables is a surface S with equation $z = f(x, y)$.

- ▶ The graph of a function $f(x_1, x_2, \dots, x_n)$ is a hyper surface S in R^{n+1} space satisfying the equation $x_{n+1} = f(x_1, x_2, \dots, x_n)$.

Graphs of Multivariable Functions.

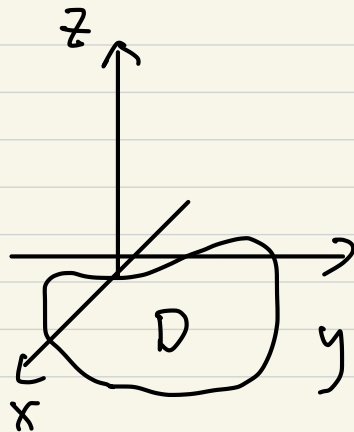
$f(x)$

graph of $f(x)$ is a



$f(x, y)$

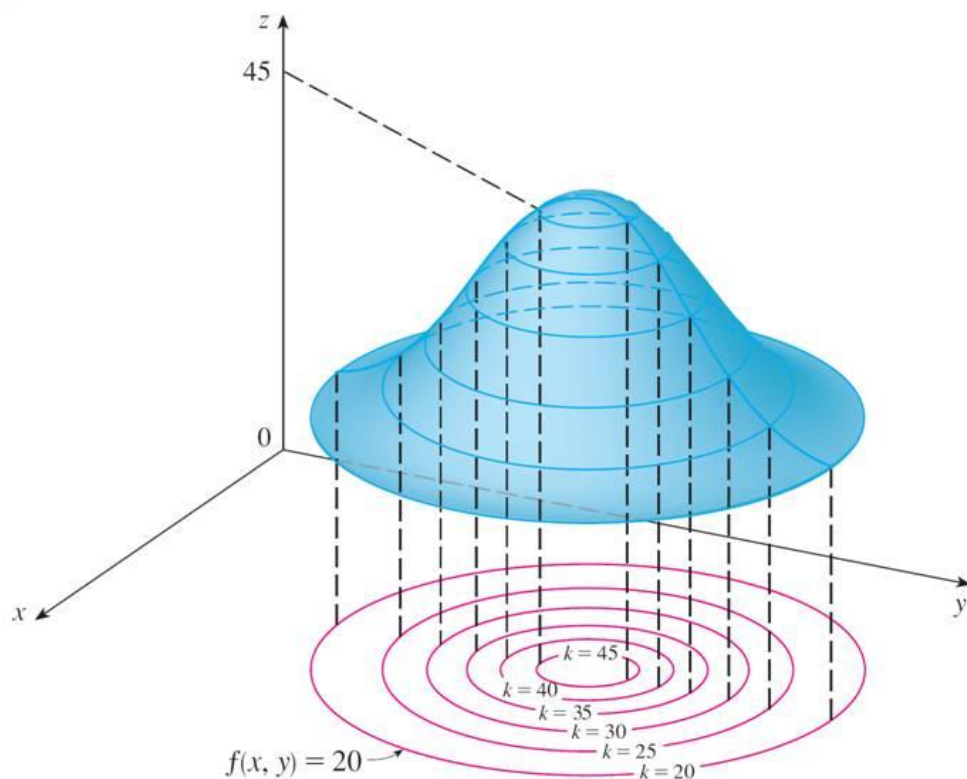
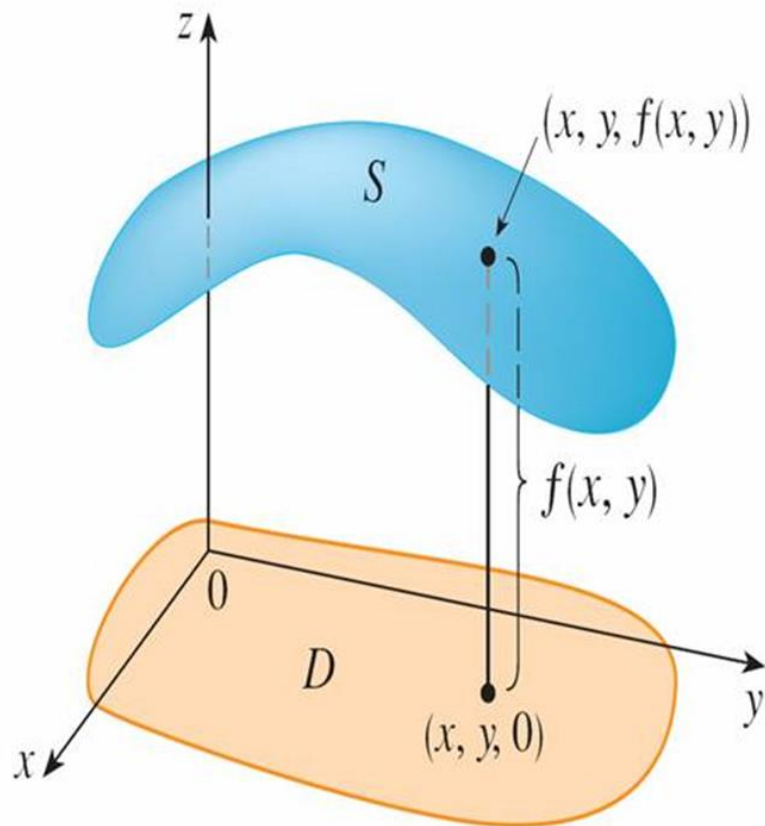
graph of $f(x, y)$ is a



$f(x_1, \dots, x_n)$

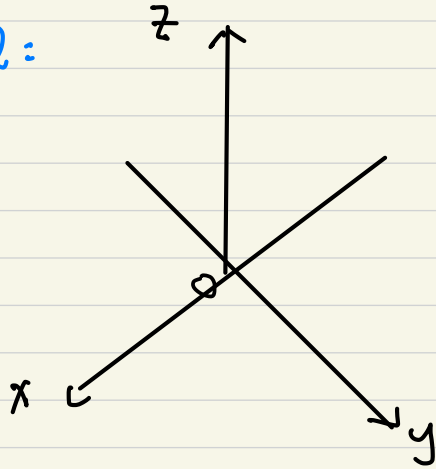
graph of $f(x_1, \dots, x_n)$
is a

Functions of Several Variables: Graphs



Ex: Sketch the graph of $f(x,y) = 2+x-2y$.

sol:



Ex: Find the graph of $f(x,y) = \sqrt{4-x^2-y^2}$.

Ex: Sketch the graph of $f(x,y) = x^2 + ay^2$, where $a \in \mathbb{R}$ is a constant.

Functions of Several Variables: Level Curves

- ▶ Definition: A **level curve** $f(x, y) = k$ is the set of all points *in the domain of* f at which f takes on a given value k .
- ▶ The level curves $f(x, y) = k$ are just the traces of the graph of f in the horizontal plane $z = k$ projected down to the xy -plane.

► Examples of level curves:



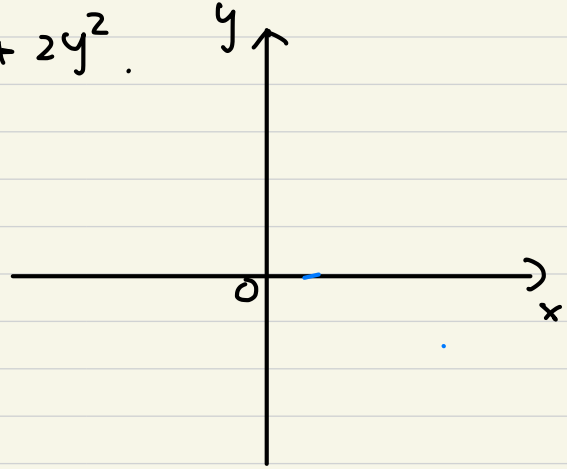
Functions of Several Variables: Level Curves

- ▶ For functions of n variables, $f(x_1, x_2, \dots, x_n)$, we can examine its **level surface**, which are the (hyper) surfaces satisfying equations $f(x_1, x_2, \dots, x_n) = k$, where k is a constant.

Level Curves

Ex: Find level curves for $f(x,y) = x^2 + 2y^2$.

sol:



Ex: Describe the level surfaces of $f(x,y,z) = x + 2y - z$

sol:

Ex: Find level surfaces of $f(x,y,z) = x^2 - y^2 + 4z^2$.

Sol: