More Techniques of Integration

Section 7.4-7.5

Outline

- Integration of Rational Functions by Partial Fractions
  - Partial Fractions
  - Rationalizing Substitution
- Strategy for Integration

Review of Basic Integrals

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int a^x\,dx = \frac{a^x}{\ln a}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
15. $\int \sinh x\,dx = \cosh x$
16. $\int \cosh x\,dx = \sinh x$
17. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

*19. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

*20. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

Strategy for Integration

- 1. Simplify the Integrand if Possible
- 2. Look for an Obvious Substitution
- 3. Classify the Integrand According to Its Form
  - Trigonometric functions.
  - Rational functions.
  - Radicals. i) $\sqrt{\pm x^2 \pm a^2}$ ii) $\sqrt[n]{ax+b}$

Strategy for Integration

- 3. Classify the Integrand According to Its Form
  - Integration by Parts. $f(x)$ is a product of a power of x and a transcendental function (such as a trigonometric, exponential, or logarithmic function).

Strategy for Integration

- 4. Try Again.
  - Try Substitution.
  - Try Integration by Parts
  - Manipulate the integrand.
  - Relate the problem to previous problems.
  - Use several methods.

Simplify the Integrand

Ex: $\displaystyle\int \frac{1}{1+e^x}\,dx$

Ex: $\displaystyle\int \sqrt{\frac{1-x}{1+x}}\,dx$

Integration by Part

Ex: $\int \frac{x \ln x}{\sqrt{x^2-1}} \, dx$

Ex: $\int \sqrt{x} (\ln x)^2 \, dx$

Substitution

Ex: $\displaystyle\int \frac{1}{\sqrt{\sqrt{x}+1}}\,dx$

Ex: $$\int \frac{dx}{(x+2)^{2/3} + x}$$

Can We Integrate All Continuous Functions?

- The functions that we have been studied here are called **elementary functions.**
  These are the polynomials, rational functions, power functions , exponential functions, logarithmic functions, trigonometric and inverse trigonometric functions, hyperbolic and inverse hyperbolic functions, and all functions that can be obtained from these by the five operations of addition, subtraction, multiplication, division, and composition.

Can We Integrate All Continuous Functions?

- If f is an elementary function, then f' is an elementary function but the integral of f need not be an elementary function.
- For example, $\int e^{t^2}\ dt$

$$\int \frac{1}{\ln x} dx \qquad \int \frac{e^x}{x} dx \qquad \int \frac{\sin x}{x} dx$$

$$\int \sqrt{x^3+1} dx \quad \int \cos(e^x) dx \quad \int \sin(x^2) dx$$

are not elementary functions.

Review

- How do we decompose a rational function as partial fractions?
- What is the rationalizing substitution?
- What are our strategies for integration?
- Could we always express the integral of an elementary function as an elementary function?