

# Derivatives of Trigonometric Functions

Section 3.3

# Outline

- ▶ Derivatives of Trigonometric Functions
  - ▶ Special Limits
  - ▶ Derivatives of Trigonometric Functions.

Ex: Try to compute  $\frac{d}{dx} \sin x$  from the definition.

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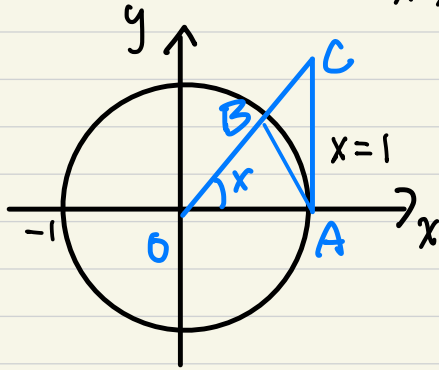
$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

Ex: Compute  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$



Sol: For  $x > 0$ , Consider  $\triangle OAB$ ,  $\triangle OAB$ ,  $\triangle OAC$ .

Ex: Compute  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$

sol:

Ex: Compute  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$

sol:

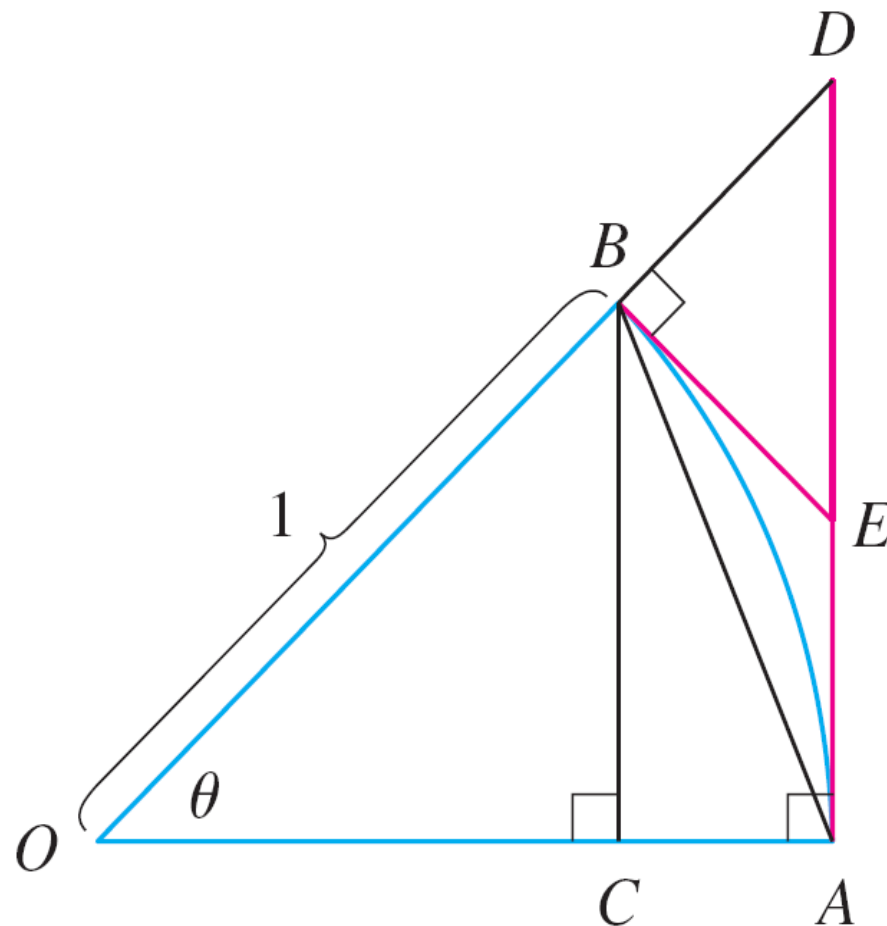
# Derivatives of Trigonometric Functions

## ► Special Limits:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2} = -\frac{1}{2}$$



## Applications of $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Ex: Find  $\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$ , where  $a, b \neq 0$

Sol:

Ex: Find  $\lim_{x \rightarrow -\infty} \sqrt{4x^2 - 1} \sin\left(\frac{1}{x}\right)$

Sol:

Ex: Find  $\lim_{x \rightarrow 0} \frac{(\tan 3x)^2}{1 - \cos x}$

Sol:



Ex: Find  $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1 - \cos 2x}}$

sol:

# Derivatives of Trigonometric Functions

► Derivatives of trigonometric functions:

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Ex: Compute  $\frac{d}{dx} \sin x$ .

sol:

Ex: Compute  $\frac{d}{dx} \cos x$

sol:

Ex: Compute  $\frac{d}{dx} \tan x$

Sol:

Ex: Compute  $\frac{d}{dx} \sec x$

Sol:

Ex: compute  $\frac{d}{dx} \sin x$ ,  $\frac{d^2}{dx^2} \sin x$ ,  $\frac{d^3}{dx^3} \sin x$ ,  $\frac{d^n}{dx^n} \sin x$ .

Sol:

Ex: compute  $\frac{d^n}{dx^n} \cos x$ .

Sol:

Ex: compute  $(\sec x)''$ .

Sol:

Ex:  $f(x) = \frac{1 - \sec x}{\tan x}$ . Find  $f'(x)$ .

sol:

# Review

- ▶ Review special limits in this section.
- ▶ Derive derivatives of trigonometric functions.