Optimization Problems and Antiderivatives

Section 4.7, 4.9

Outline

Optimization Problems

- Strategies of solving the problems:
- 1. Understand the problem.
- 2. Draw a diagram.
- 3. Introduce notations.
- 4. Express the concerning quantity, Q, in terms of other quantities.

Optimization Problems

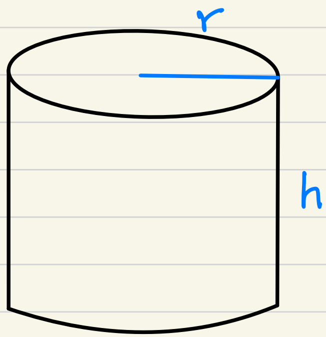
- 5. Eliminate quantities if necessary so that Q is a function, of one variable, say $f(x)$. Determine the domain of this function.
- 6. Find the absolute extreme values of $f(x)$.

Optimization Problems

- Example:
- A cylindrical can is to be made to hold $V\,m^3$ of liquid. Find the dimensions that will minimize the area of the surface of the can.

Which shape is more economical ?

Sol:



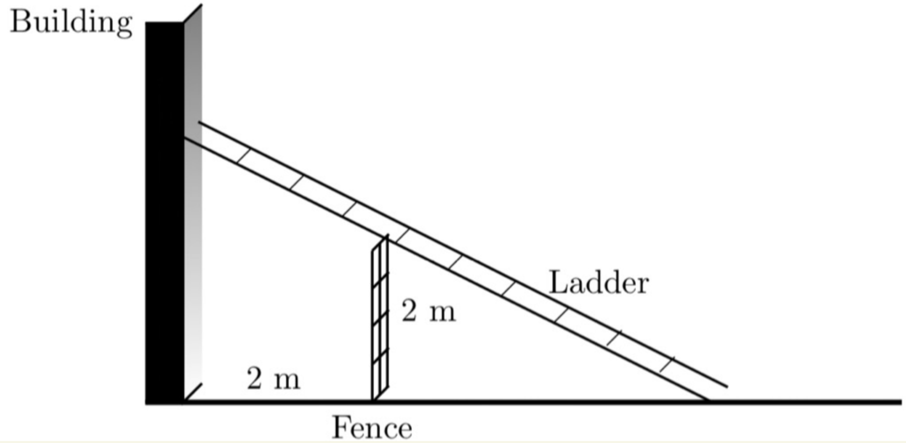
Optimization Problems

First Derivative Test for Absolute Extreme Values Suppose that c is a critical number of a continuous function f defined on an interval.

(a) If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f.

(b) If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f.

(12 pts) A fence 2 m tall is parallel to a tall building at a distance of 2 m from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?



sol:

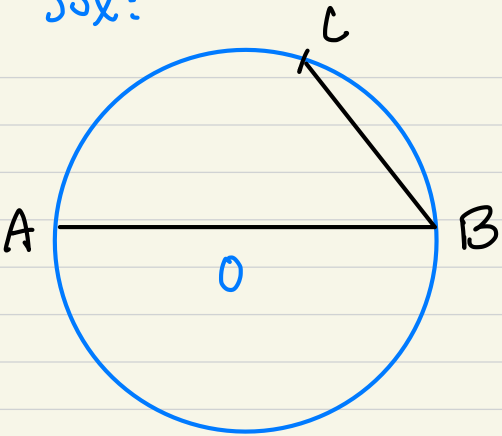
Ex : Consider an isosceles triangle whose legs (the equal sides) have length ℓ and whose vertex angle is θ. As ℓ and θ vary, the area of the triangle stays the same. At which θ does ℓ attain its extreme value? Is this extreme value the maximum length or minimum length?

sol:

Optimization Problems

- Example :
- There is a circular pool and A, B are two points on the edge of the pool so that $\overline{AB}$ is the diameter. A man runs twice faster then he swims. He wants to run along the pool from A to a point C and then swim from C to B in the shortest time. How should he choose the point C ?

Sol:



Antiderivatives

Definition A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

$\boxed{1}$ **Theorem** If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Antiderivatives

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sec^2 x$	$\tan x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec x \tan x$	$\sec x$
$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
$\dfrac{1}{x}$	$\ln \lvert x \rvert$	$\dfrac{1}{1+x^2}$	$\tan^{-1} x$
e^x	e^x	$\cosh x$	$\sinh x$
$\cos x$	$\sin x$	$\sinh x$	$\cosh x$
$\sin x$	$-\cos x$		

Ex: Find $f(x)$ such that $f''(x) = \sin x + x^{\frac{4}{3}} + 2^{x}$

and $f'(0) = \frac{1}{\ln 2} - \frac{1}{7}$, $f(0) = \frac{1}{(\ln 2)^2}$.

Review

- What is an optimization problem? What is your strategy for solving such problems?
- What is the antiderivative of a function?
- Review the table of antiderivatives of special functions.