

Name: _____

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Quiz 10

Consider the phylogenetic tree shown in Figure 1. Along each branch, a trait X_t (e.g., body size) evolves according to the stochastic differential equation:

$$dX_t = -0.5 X_t dt + 0.2 dW_t, \quad 0 \leq t \leq 3,$$

where W_t is standard Brownian motion. The speciation and extinction rates are trait-dependent and given by:

$$\lambda(X_t) = 1 + 0.3 X_t, \quad \mu(X_t) = 0.2.$$

You observe the full tree topology with branching times $\{1, 2, 2.5\}$, and the continuous trait path along every branch. Assume the trait value at the root is known and fixed at $X_0 = 0$.

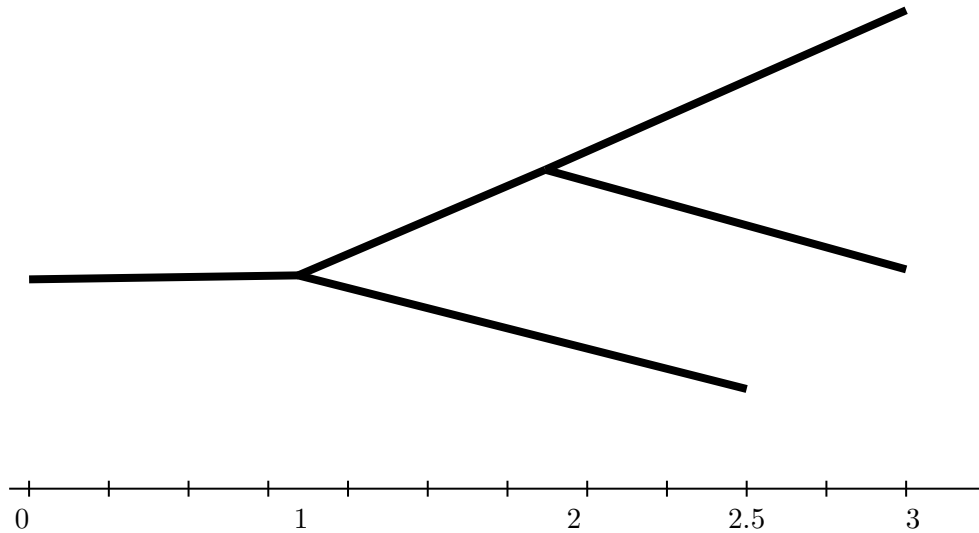


Figure 1: Trait-dependent birth-death tree.

- (1) Using a computer, compute a Monte Carlo estimator \hat{L}_N for the likelihood of the observed tree and trait paths under the given model. Simulate N independent realizations of the tree and trait dynamics (starting from $X_0 = 0$), and compute the likelihood of the observed data under these simulations. Report your estimate \hat{L}_N and provide a convergence plot of N vs. \hat{L}_N .

