1) $T(n) = \begin{cases} C_1 & khi\ n = 1 \\ T(n-1) + C_2 & khi\ n > 0 \end{cases}$

$T(n) = T(n-1) + C_2$

$T(n) = T(n-2) + 2C_2$

$T(n) = T(n-i) + iC_2$

Quá trình dừng lại khi: $n - i = 1 \rightarrow\ i = n - 1$

Lần thay cuối: $T(n) = T(1) + 5(n-1) = 5n - 5$

2) $T(n) = \begin{cases} C_1 & khi\ n = 1 \\ T(n-1) + n & khi\ n > 0 \end{cases}$

$T(n) =\ T(n-1) + n$

$T(n) = T(n-2) + 2n - 1$

$T(n) = T(n-i) + in - \sum_{k=0}^{i-1} k$

$T(n) = T(n-i) + in - \ \frac{i(i-1)}{2}$

Quá trình dừng lại khi: $n - i = 1 \rightarrow i = n - 1$

Lần thay cuối:

$T(n) = T(1) + n(n-1) - \ \frac{(n-1)(n-2)}{2}$

$= 1 + \ \frac{n^2+n-2}{2} = \frac{n^2+n}{2}$

3) $T(n) = \begin{cases} C_1 & khi\ n = 1 \\ 3T(n-1) + C_2 & khi\ n > 0 \end{cases}$

$T(n) = 3T(n-1) + C_2$

$T(n) = 9T(n-2) + 4C_2$

$T(n) = 3^i T(n-i) + C_2 \sum_{k=0}^{i-1} 3^k$

$T(n) = 3^i T(n-i) + C_2 . \frac{3^i - 1}{2}$

Quá trình dừng lại khi $n - i = 1 \rightarrow\ i = n - 1$

Lần thay cuối: $T(n) = 3^{n-1} T(1) + \frac{3^{n-1} - 1}{2}$

$$= \frac{3^{n+1} - 1}{2}$$

4) $T(n) = \begin{cases} C_1 & khi\ n = 1 \\ 2T\left(\frac{n}{2}\right) + C_2 & khi\ n > 1 \end{cases}$

$T(n) = 2T\left(\frac{n}{2}\right) + C_2$

$T(n) = 4T\left(\frac{n}{4}\right) + 3C_2$

$T(n) = 2^i T\left(\frac{n}{2^i}\right) + C_2 . \sum_{k=0}^{i-1} 2^k$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + C_2(2^i - 1)$$

Quá trình dừng lại khi: $\frac{n}{2^i} = 1 \to \ i = \log_2 n$

Lần thay cuối: $T(n) = 2^{\log_2 n} T(1) + (2^{\log_2 n} - 1)$

$$= \ n + n - 1 = 2n - 1$$

5) $T(n) = \begin{cases} C_1 & khi\ n = 1 \\ 2T\left(\frac{n}{2}\right) + n & khi\ n > 1 \end{cases}$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$T(n) = 4T\left(\frac{n}{4}\right) + 2n$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + in$$

Quá trình dừng lại khi : $\frac{n}{2^i} = 1 \to i = \log_2 n$

Lần thay cuối: $T(n) = 2^{\log_2 n} T(1) + n \log_2 n$

$$= \ n + n \log_2 n$$

6) $T(n) = \begin{cases} C_1 & khi\ n = 1 \\ 2T\left(\frac{n}{2}\right) + n^2 & khi\ n > 1 \end{cases}$

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2$$

$$T(n) = 4T\left(\frac{n}{4}\right) + \frac{3n^2}{2}$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + n^2 \ \sum_{k=0}^{i-1} \frac{1}{2^k}$$

$$T(n) = 2^i T\left(\frac{n}{2^i}\right) + n^2 . \frac{1-(\frac{1}{2})^i}{1-\frac{1}{2}} = 2^i T\left(\frac{n}{2^i}\right) + 2n^2[1 - (\frac{1}{2})^i]$$

Quá trình dừng lại khi: $\frac{n}{2^i} = 1 \rightarrow i = \log_2 n$

Lần thay cuối:

$$T(n) = 2^{\log_2 n} T(1) + 2n^2[1 - (\frac{1}{2})^{\log_2 n}]$$

$$= n + 2n^2(1 - n^{-1})$$

$$= 2n^2 - n$$

7) $T(n) = \begin{cases} C_1 & khi\ n = 1 \\ 2T\left(\frac{n}{2}\right) + logn & khi\ n > 1 \end{cases}$

$$T(n) = 2T\left(\frac{n}{2}\right) + logn$$

$$T(n) = 4T\left(\frac{n}{4}\right) + logn + log(\frac{n}{2})^2$$

$$T(n) = 2^iT\left(\frac{n}{2^i}\right) + logn + \sum_{k=1}^{i-1} 2^k.log\frac{n}{2^k}$$

$$T(n) = 2^iT\left(\frac{n}{2^i}\right) + logn + \sum_{k=1}^{i-1} 2^k logn - \log 2 \sum_{k=1}^{i-1} k.2^k$$

$$T(n) = 2^iT\left(\frac{n}{2^i}\right) + logn + (2^i - 2)\log n -$$
$$[(\mathrm{i} - 1)2^i - 2^i + 2]\log 2$$

Quá trình kết thúc khi $\frac{n}{2^i} = 1 \rightarrow i = \log_2 n$

Thay lần cuối:

$$T(n) = 2^{\log_2 n}T(1) + logn + (2^{\log_2 n} - 2)\log n -$$
$$(2^{\log_2 n}\log_2 n - 2.2^{\log_2 n} + 2)\log 2$$
$$= n + logn + (n-2)logn - (n\log_2 n - 2n + 2)\log 2$$
$$= n + (n-1)logn - nlogn + (2n-2)\log 2$$
$$= n - logn + (2n-2)\log 2$$