Bài 3:

T(1) = 1

1) T(n) = $\begin{cases} C1 & khi\ n = 1 \\ 3T\left(\frac{n}{2}\right) + n^2 & khi\ n > 1 \end{cases}$

T(n) = $3T\left(\frac{n}{2}\right) + n^2$

T(n) = $9T\left(\frac{n}{4}\right) + \frac{3n^2}{4} + n^2$ = $9T\left(\frac{n}{4}\right) + \frac{7n^2}{4}$

T(n) = $27T\left(\frac{n}{8}\right) + \frac{9n^2}{16} + \frac{7n^2}{4}$ = $27T\left(\frac{n}{8}\right) + \frac{37n^2}{16}$

T(n) = $3^iT\left(\frac{n}{2^i}\right) + n^2\sum_{k=0}^{i-1}\frac{3^k}{(2^k)^2}$ =$3^iT\left(\frac{n}{2^i}\right) + n^2\sum_{k=0}^{i-1}(\frac{3}{4})^k$

T(n) =$3^iT\left(\frac{n}{2^i}\right) + \frac{1-(\frac{3}{4})^i}{1-\frac{3}{4}}.n^2$=$3^iT\left(\frac{n}{2^i}\right) + 4[1-(\frac{3}{4})^i]\,n^2$

Quá trình kết thúc khi: $\frac{n}{2^i} = 1 \rightarrow\ i = \log_2 n$

Lần thay cuối:

T(n) = $3^{\log_2 n}.T(1) + 4[1-(\frac{3}{4})^{\log_2 n}]\,n^2$

= $n^{\log_2 3} + 4n^2 - 4n^{\log_2 3 - 2}.n^2$

= $4n^2 - 3n^{\log_2 3}$

2)T(n) = $\begin{cases} C1 & khi\ n = 1 \\ 8T\left(\frac{n}{2}\right) + n^3 & khi\ n > 1 \end{cases}$

T(n) = $8T\left(\frac{n}{2}\right) + n^3$

T(n) = $64T\left(\frac{n}{4}\right) + 2n^3$

T(n) = $512T\left(\frac{n}{8}\right) + 3n^3$

T(n) = 8^iT($\frac{n}{2^i}$) $+ in^3$

Quá trình kết thúc khi: $\frac{n}{2^i} = 1 \rightarrow i = \log_2 n$

Lần thay cuối:

T(n) = $8^{\log_2 n}.T(1) + n^3 \log_2 n$

= $n^3 + n^3 \log_2 n$ = $n^3(\log_2 n + 1)$

3)T(n) = $\begin{cases} C1 & khi\ n = 1 \\ 4T\left(\frac{n}{3}\right) + n & khi\ n > 1 \end{cases}$

T(n) = $4T\left(\frac{n}{3}\right) + n$

T(n) = $16T\left(\frac{n}{9}\right) + \frac{4n}{3} + n = 16T\left(\frac{n}{9}\right) + \frac{7n}{3}$

T(n) = $4^iT\left(\frac{n}{3^i}\right) + n\sum_{k=0}^{i-1}(\frac{4}{3})^k$ = $4^iT\left(\frac{n}{3^i}\right) + 3n[\left(\frac{4}{3}\right)^i - 1]$

Quá trình kết thúc khi: $\frac{n}{3^i} = 1 \rightarrow i = \log_3 n$

Lần thay cuối:

T(n) = $4^{\log_3 n} T(1) + 3n[\left(\frac{4}{3}\right)^{\log_3 n} - 1]$

= $n^{\log_3 4} + 3n. n^{\log_3 4 - 1} - 3n$

= $4n^{\log_3 4} - 3n$

4) T(n) = $\begin{cases} C1 & khi\ n = 1 \\ 9T\left(\frac{n}{3}\right) + n^2 & khi\ n > 1 \end{cases}$

T(n) = $9T\left(\frac{n}{3}\right) + n^2$

T(n) = $81T\left(\frac{n}{9}\right) + 2n^2$

T(n) = $9^i T\left(\frac{n}{3^i}\right) + in^2$

Quá trình kết thúc khi: $\frac{n}{3^i} = 1 \rightarrow i = \log_3 n$

Lần thay cuối:

T(n) = $9^{\log_3 n} T(1) + n^2 \log_3 n$

= $(\log_3 n + 1)n^2$

5) T(n)= $\begin{cases} C1 & khi\ n = 2 \\ 2T(\sqrt{n}) + C2 & khi\ n > 2 \end{cases}$

T(n) = $2T(\sqrt{n}) + C2$ = $2T(n^{\frac{1}{2}}) + C2$

T(n) = $4T\left(n^{\frac{1}{4}}\right) + 3C2$

T(n) = $2^i T\left(n^{\frac{1}{2^i}}\right) + C2.\sum_{k=0}^{i-1} 2^k$

T(n) = $2^i T\left(n^{\frac{1}{2^i}}\right) + C2(2^i - 1)$

Quá trình kết thúc khi: $n^{\frac{1}{2^i}} = 2 \rightarrow$ i =$-\log_2(\log_n 2)$

Lần thay cuối:

T(n) = $2^{-\log_2(\log_n 2)}.T(2) + 2^{-\log_2(\log_n 2)} - 1$

= $(\log_n 2)^{-1} - 1 = \log_2 n - 1$