

A Body-oriented Method for Finding a Linear Form of the Dynamic Equation of Fully Parallel Robots

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Abstract

In order to identify the dynamic parameters in nonlinear adaptive control the robot's dynamic equation has to be written in a linear form. Many methods have been proposed for serial robots, but for parallel robots, the few solutions proposed so far lead to complicated equations that are not readily usable for real-time implementation. In this paper we propose a new method based on the virtual work principle to find a linear form of the dynamic equation of robots. Compared to other methods, it has the advantage that it does not need to open the closed loop structure into a tree-structure robot. It considers rather each body separately using its Jacobian matrix to project the forces into the joint space of the robot. Thus, simplification can be made at the very beginning of the modeling. This is very efficient when used to model fully parallel robots. As an illustration, the proposed method is applied to the 3dof DELTA parallel robot.

1 Introduction

The dynamic equation of robots is linear in its dynamic parameters [15, 17]. This property has been exploited to identify the dynamic parameters of robots using Least-Square techniques [1, 18] or in nonlinear adaptive control [6, 20]. Many solutions have been proposed to find a linear form of the dynamic equation of serial [9, 17] and tree-structure robots [14]. For manipulators with closed loop chains, the dynamic equation is generally difficult to establish. It is thus not easy to write it in a linear form. Solutions have been proposed [10, 13] in which the model is usually obtained by opening the loops in order to form a tree-structure robot. The dynamics equation is then derived using classical techniques [14] developed for serial robots and a loop closure constraint equation to eliminate the non-actuated joints from the tree-structure dynamic model. The method has been

applied to planar parallel mechanisms, but becomes very cumbersome for spacial parallel robots.

In this paper we propose a new method for finding the dynamic equation in an explicit linear function of the the dynamic parameters. It is based on the virtual work principle and uses Jacobian matrices to "project" the forces of each body in the space of the active joints. It has the following advantages:

- The robot does not need to be opened in an equivalent tree-structure mechanism.
- Simplifying hypothesis can be used at the very beginning of the modeling already.
- It leads to simple equations that can be used for real time implementation of nonlinear adaptive control.

The paper is organized as follows. The virtual work principle is firstly presented along with some definitions in section 2. The proposed method is then developed (sec. 3) and finally illustrated with a 3 dof fully parallel robot, the DELTA (sec. 4).

2 Mathematical framework

2.1 Definitions

A robot is a system of N rigid bodies connected together in order to form a kinematic chain (figure 1). Let q be the vector of active joint variables and \mathcal{X} the vector of operational space variables describing the pose of the end-effector in a reference frame \mathcal{O} . A frame \mathcal{C}_i is attached to the centre of mass of body i . Let m_i and I_i be the mass and inertia tensor of body i , and x_i the position of its centre of mass in the reference frame \mathcal{O} . \dot{x}_i and ω_i are its linear and angular velocity respectively. The rotation matrix between body frame \mathcal{C}_i and the inertial reference frame \mathcal{O} is noted ${}^{\mathcal{O}}_i R$. For each body, the linear and angular velocity of the center

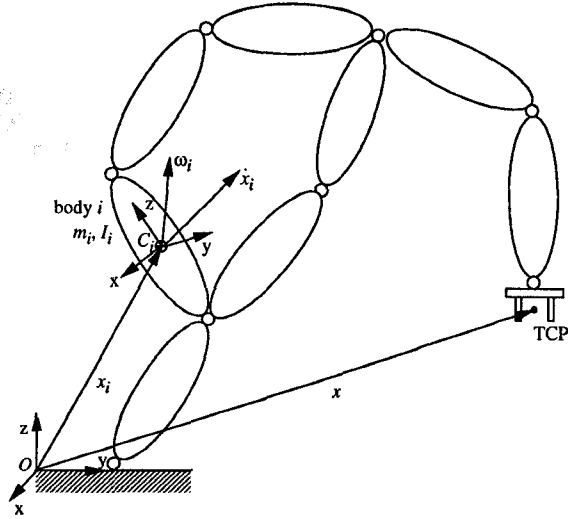


Figure 1: A system of rigid bodies forming a kinematic chain

of mass can be expressed as a function of the joint velocity vector \dot{q} by the mean of the *basic Jacobian* [16] defined by:

$$\dot{\mathcal{X}}_i = \begin{bmatrix} \dot{x}_i \\ \omega_i \end{bmatrix} \equiv J_{0i} \dot{q} \quad (1)$$

This basic Jacobian can also be used to transform a virtual displacement of the joint space vector into a virtual displacement of body i .

2.2 Virtual work principle

Using the virtual work (or d'Alembert) principle [2], the dynamics of a system of N rigid bodies can be described by:

$$\sum_{i=1}^N \delta \mathcal{X}_i^T \begin{bmatrix} m_i \ddot{x}_i - F_i \\ I_i \dot{\omega}_i + \omega_i \times I_i \omega_i - T_i \end{bmatrix} = 0, \quad (2)$$

where

$$\delta \mathcal{X}_i \equiv \begin{bmatrix} \delta x_i \\ \delta \phi_i \end{bmatrix}. \quad (3)$$

In this equation, F_i and T_i are the applied forces and torques (e.g. motor torques, gravity, friction), and δx_i , $\delta \phi_i$ are position and angular virtual displacements. Expressing the virtual displacements (equ. (3)) as a function of joint variable displacements using the basic Jacobian J_{0i} (1) we obtain for equation (2):

$$\delta q^T \sum_{i=1}^N J_{0i}^T \begin{bmatrix} m_i \ddot{x}_i - F_i \\ I_i \dot{\omega}_i + \omega_i \times I_i \omega_i - T_i \end{bmatrix} = 0 \quad (4)$$

As this equation must yield for every δq^T , it follows that:

$$\sum_{i=1}^N J_{0i}^T \begin{bmatrix} m_i \ddot{x}_i - F_i \\ I_i \dot{\omega}_i + \omega_i \times I_i \omega_i - T_i \end{bmatrix} = 0 \quad (5)$$

To simplify the notation we will use the wrench notation and decompose equation (5) in an inertia wrench $\mathcal{F}_i^{\text{inertia}}$ and a wrench $\mathcal{F}_i^{\text{applied}}$ of applied forces defined as:

$$\mathcal{F}_i^{\text{inertia}} \equiv \begin{bmatrix} m_i \ddot{x}_i \\ I_i \dot{\omega}_i + \omega_i \times I_i \omega_i \end{bmatrix}, \quad (6)$$

$$\mathcal{F}_i^{\text{applied}} \equiv \begin{bmatrix} F_i \\ T_i \end{bmatrix}. \quad (7)$$

Equation (5) can thus be rewritten as:

$$\sum_{i=1}^N J_{0i}^T (\mathcal{F}_i^{\text{inertia}} - \mathcal{F}_i^{\text{applied}}) = 0 \quad (8)$$

This equation describes the dynamics of the system composed of N rigid bodies and is similar to Kane's formulation [12]. Its physical meaning is that the sum of all inertial forces must be equal to all non-inertial forces, and thus corresponds also to Newton's equation of motion.

2.3 Force transmission on a rigid body

In equation (8), wrenches are given with respect to the center of mass C of the body¹. If the position of the center of mass is not known, we can choose an arbitrary reference frame \mathcal{A} attached to point A of body i and express the wrenches relatively to this frame. Let ${}^{\mathcal{O}}_A R$ be the rotation matrix between \mathcal{A} and the inertial frame \mathcal{O} (figure 2) and ${}^{\mathcal{A}}_C R$ be the rotation matrix between frames \mathcal{A} and \mathcal{C} .

A wrench \mathcal{F}_C applied to C on a rigid body and described with respect to frame \mathcal{C} gives the following contribution at point A with respect to frame \mathcal{A} [7]:

$${}^{\mathcal{A}}\mathcal{F}_A = J_r^T {}^{\mathcal{C}}\mathcal{F}_C, \quad (9)$$

where J_r^T is a wrench transmission matrix defined as:

$$J_r^T \equiv \begin{bmatrix} {}^{\mathcal{A}}_C R & 0 \\ {}^{\mathcal{A}}_{\hat{r}} {}^{\mathcal{A}}_C R & {}^{\mathcal{A}}_C R \end{bmatrix} \quad (10)$$

and

$$\hat{r} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \quad (11)$$

is a skew symmetric matrix corresponding to the cross-product with the vector r linking point A to C .

¹We omit here the index i in order to simplify the notation.

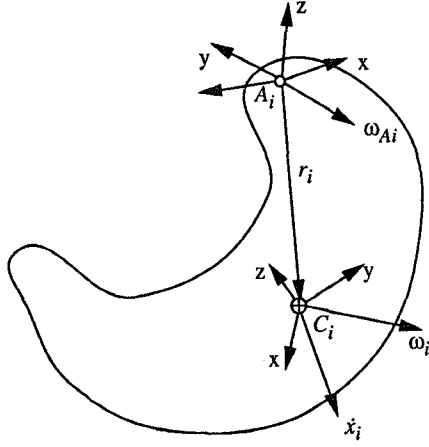


Figure 2: Changing the reference frame on a body

3 Linear form of the dynamic equation

Assuming that the kinematics of the system of N rigid bodies is known, its dynamic equation can be written in a linear form of the dynamic parameters:

$$\tau = \Psi(q, \dot{q}, \ddot{q}) p, \quad (12)$$

where Ψ is the *dynamic matrix* and p the *dynamic parameter vector* containing functions of the lengths, masses and inertias.

3.1 Extracting the actuator forces

In order to write equation (8) in a linear form of the parameters vector p , the actuator forces τ must firstly be extracted. By decomposing the wrench of applied forces and torques $\mathcal{F}_i^{applied}$ in an actuator wrench \mathcal{F}_i^a and an external wrench \mathcal{F}_i^{ext} , we can rewrite equation (8) as follows:

$$\sum_{i=1}^N J_{0i}^T \mathcal{F}_i^a = \sum_{i=1}^N J_{0i}^T (\mathcal{F}_i^{inertial} - \mathcal{F}_i^{ext}). \quad (13)$$

If the wrench \mathcal{F}_i^a is not directly applied to the centre of mass of the body, a wrench transmission matrix has to be used to compute the contribution of the motor at the center of mass of body i :

$${}^C_i \mathcal{F}_i^a = J_{ri}^{-T} {}^{A_i} \mathcal{F}_{A_i}^a, \quad (14)$$

where the wrench transmission matrix J_{ri} is defined by equation (10), and $\mathcal{F}_{A_i}^a$ is the wrench applied by the motor in its reference frame A_i . Supposing that the frame attached to the joint has its z -axis aligned

with the motor axis, as in the Denavit-Hartenberg formulation, we can then write:

$${}^{A_i} \mathcal{F}_{A_i}^a = \begin{bmatrix} 0 \\ 0 \\ \epsilon_i \\ 0 \\ 0 \\ \bar{\epsilon}_i \end{bmatrix} \tau_i \equiv \Upsilon_i \tau_i, \quad (15)$$

with:

$$\epsilon_i \equiv \begin{cases} 0 & \text{for revolute joint} \\ 1 & \text{for prismatic joint} \end{cases}, \quad \bar{\epsilon}_i = 1 - \epsilon_i,$$

and τ_i is the scalar torque contribution of joint i .

Replacing (14) and (15) into the left hand side of equation (13), we can rewrite it in a vector form of the actuator torques vector τ , i.e.:

$$\sum_{i=1}^N J_{0i}^T \mathcal{F}_i^a \equiv J_\tau \tau, \quad (16)$$

with

$$J_\tau \equiv [J_{A1}^T \Upsilon_1 \quad J_{A2}^T \Upsilon_2 \quad \dots \quad J_{Am}^T \Upsilon_m], \quad (17)$$

where m is the number of actuators and $J_{Ai} \equiv J_{ri}^{-1} J_{0i}$ is the Jacobian matrix linking the velocity of frame A_i attached to the motor and the joint velocity vector \dot{q} .

Using this definition, the dynamic equation (8) can be rewritten as:

$$\tau = J_\tau^{-1} \left[\sum_{i=1}^N J_{0i}^T (\mathcal{F}_i^{inertial} - \mathcal{F}_i^{ext}) \right]. \quad (18)$$

3.2 Inertial forces on a rigid body

Inertial forces are usually described relatively to the center of mass of a body. If the body is described relatively to an arbitrary frame \mathcal{A} attached to body i , a transformation has to be realized².

We assume that the Jacobian matrix J_A linking the joint velocity \dot{q} to the linear and angular velocities of frame \mathcal{A} is known, as well as its velocities \dot{x}_A , ω_A and accelerations.

According to equation (6), inertial forces ${}^C \mathcal{F}_C$ acting at the centre of mass of the body are calculated using the following equation:

$${}^C \mathcal{F}_C = \begin{bmatrix} m {}^C \ddot{x}_C \\ {}^C I_C {}^C \dot{\omega}_C + {}^C \omega_C \times {}^C I_C {}^C \omega_C \end{bmatrix} \quad (19)$$

²again the index i is omitted in the following

Using the wrench transmission matrix defined in equation (9), \mathcal{F}_C gives the following contribution at the reference frame \mathcal{A} :

$${}^A\mathcal{F}_A = J_r^T {}^C\mathcal{F}_C. \quad (20)$$

Given the acceleration \ddot{x}_A of point A on body i , the acceleration \ddot{x}_C of the center of mass C is [7]:

$${}^A\ddot{x}_C = {}^A\ddot{x}_A + {}^A\omega_A \times ({}^A\omega_A \times {}^Ar) + {}^A\dot{\omega}_A \times {}^Ar \quad (21)$$

and

$${}^C\ddot{x}_C = {}^C_R {}^A\ddot{x}_C \quad (22)$$

We have also:

$${}^C\omega_C = {}^C_R {}^A\omega_A \quad (23)$$

and :

$${}^C\dot{\omega}_C = {}^C_R {}^A\dot{\omega}_A. \quad (24)$$

Replacing equations (21) to (24) into equation (9) and using:

$$a \times (b \times (b \times a)) = b \times (a^T a \mathbf{1}_{3 \times 3} - a a^T) b, \quad (25)$$

$$a \times (b \times a) = (a^T a \mathbf{1}_{3 \times 3} - a a^T) b, \quad (26)$$

where $\mathbf{1}_{3 \times 3}$ is the 3×3 identity matrix, we obtain:

$${}^A\mathcal{F}_A = \begin{bmatrix} m({}^A\ddot{x}_A + {}^A\hat{\omega}_A {}^A\hat{\omega}_A {}^Ar) + {}^A\hat{\omega}_A {}^Ar \\ m {}^A\hat{r} {}^A\ddot{x}_A + {}^A I_A {}^A\dot{\omega}_A + {}^A\hat{\omega}_A {}^A I_A {}^A\omega_A \end{bmatrix} \quad (27)$$

with ${}^A I_A \equiv {}^A I_C + m({}^Ar^T {}^Ar \mathbf{1}_{3 \times 3} - {}^Ar {}^Ar^T)$. This expression represents actually the inertia with respect to the reference frame \mathcal{A} (instead of \mathcal{C}). Equation (27) can be written in a linear form of the dynamic parameters of body i :

$${}^A\mathcal{F}_A = \begin{bmatrix} {}^A\ddot{x}_A & \Lambda & 0 \\ 0 & -{}^A\hat{\ddot{x}}_A & \psi_\omega \end{bmatrix} \begin{bmatrix} m \\ m r \\ I_p \end{bmatrix}, \quad (28)$$

where

$$\Lambda = {}^A\hat{\omega}_A {}^A\hat{\omega}_A + {}^A\hat{\omega}_A, \quad (29)$$

$$\psi_\omega = {}^A\hat{\omega}_A {}^A\tilde{\omega}_A + {}^A\tilde{\omega}_A, \quad (30)$$

with

$$\tilde{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z & 0 & 0 & 0 \\ 0 & \omega_x & \omega_y & \omega_z & 0 & 0 \\ 0 & 0 & \omega_x & 0 & \omega_y & \omega_z \end{bmatrix} \quad (31)$$

and where ${}^A\hat{\ddot{x}}_A$ is the skew symmetric matrix notation of the acceleration vector ${}^A\ddot{x}_A$, m the mass

of the body, $m r$ the first order moment vector and $I_p \equiv [I_{xx}, I_{xy}, I_{xz}, I_{yy}, I_{yz}, I_{zz}]^T$ a vector composed of the second order moments (inertia) of the body. Expression (28) is similar to the one obtained by Goldenberg [10].

The wrench ${}^A\mathcal{F}_A$ can finally be calculated in reference frame \mathcal{O} as follows:

$${}^O\mathcal{F}_A = \begin{bmatrix} {}^O_R & 0 \\ 0 & {}^O_R \end{bmatrix} {}^A\mathcal{F}_A \quad (32)$$

3.3 Linear form

Assuming that the external forces are composed of gravitational forces only, equation (18) can be rewritten with respect to body points A_i , $i = 1, \dots, N$, as:

$$\tau = J_r^{-1} \left[\sum_{i=1}^N J_{A_i}^T {}^O\mathcal{F}_A - \sum_{i=1}^N J_{O_i}^T \begin{bmatrix} m_i G \\ 0 \end{bmatrix} \right], \quad (33)$$

where G is the vector of gravitational acceleration. By adding the acceleration of gravity to the acceleration of body i , this equation can be rearranged as follows:

$$\tau = J_r^{-1} \begin{bmatrix} J_{A_1}^T \Omega_1 & J_{A_2}^T \Omega_2 & \dots & J_{A_N}^T \Omega_N \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix},$$

where

$$\Omega_i \equiv \begin{bmatrix} {}^O_R & 0 \\ 0 & {}^O_R \end{bmatrix} \begin{bmatrix} {}^A_i a_i & \Lambda_i & 0 \\ 0 & -{}^A_i \hat{a}_i & \psi_{\omega_i} \end{bmatrix}, \quad (34)$$

with

$${}^A_i a_i \equiv {}^A_i \ddot{x}_{A_i} - {}^A_i G,$$

and

$$p_i = [m_i, m_i r_i^T, I_{p_i}^T]^T.$$

Equation (34) is a linear representation of the dynamic equation. If a column of the dynamic matrix is composed of zeros only it can be cancelled together with the corresponding parameter, since it has no influence on the dynamics. Furthermore, if columns of the dynamic matrix can be expressed as linear combinations of other columns, the corresponding parameters can be grouped and only one column kept in the dynamic matrix [8, 10, 13].

In nonlinear adaptive control, it is necessary to compute the inverse dynamic model in real-time. For fully parallel robots, an efficient solution is to neglect the inertias of the legs. It has been shown in [4] that for most fully parallel robots this assumption leads to a simple model without too much loss. In this case,

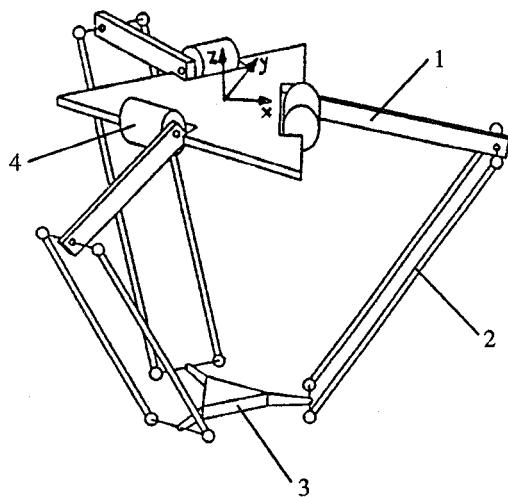


Figure 3: Sketch of the DELTA robot

finding a linear form of the dynamics is highly simplified by using the method proposed in this paragraph. Indeed, the complicated kinematics of the legs does not need to be computed, and the robot is reduced to a few bodies. For simplicity, this will be illustrated on a 3 dof parallel robot in next section but can be applied easily to 6 dof fully parallel robots also [11].

4 Example: DELTA robot

A *fully parallel robot* is a closed loop mechanism with an n dof end-effector connected to the base by n independent chains which have at most two links and are actuated by a unique prismatic or rotary actuator [19]. The DELTA, a 3 degree-of-freedom parallel robot dedicated to fast and accurate pick and place operations [3], is a typical fully parallel robot. Its mechanical structure is illustrated in figure 3. It is made of 3 parallel kinematics chains linked at the travelling plate (3). Each chain is moved, driven by a direct drive motor (4) fixed to the robot base. Motions of the travelling plate are achieved by the combination of movements of the arms (1) which are transmitted to the plate by the system of parallel rods (2) through a pair of ball-and-socket passive joints. These parallel rods (forearms) assure that the travelling plate always remains parallel to the robot base.

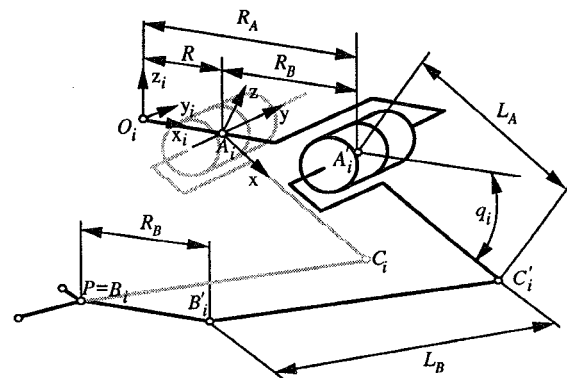


Figure 4: Geometric parameters of one kinematic chain of the DELTA robot

4.1 Kinematics of the DELTA

The absolute reference frame \mathcal{O} is chosen as shown in figure 3, i.e. at the centre of the triangle drawn by the axis of the 3 motors, z pointing upward, and x being perpendicular to the axis of motor 1. Due to the symmetry of the robot, each arm can be treated separately. Its geometric parameters are defined in figure 4. The arms are separated each by an angle of 120° . For each arm i ($i = 1, 2, 3$), a corresponding frame \mathcal{O}_i is chosen as being located at the same place as \mathcal{O} but rotated by an angle $\theta_i = 0^\circ, 120^\circ, 240^\circ$ for $i = 1, 2, 3$ respectively. Thus, the transformation matrix between frame \mathcal{O}_i and \mathcal{O} is given by:

$${}^{\mathcal{O}}_i R = \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 \\ \sin \theta_i & \cos \theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (35)$$

As the travelling plate can only be translated, a frame attached to it will always keep the same orientation as \mathcal{O} . This fact allows us to consider the distance from the reference frame \mathcal{O} to the motor as being $R = R_A - R_B$, and thus $P = B_1 = B_2 = B_3$, i.e. the travelling plate is reduced to a single point. This definition simplifies the derivation of the model without affecting the results.

4.2 Linear form of the dynamics

It has been shown in [5], how one can simplify the dynamics of the DELTA robot by neglecting the inertia of the forearms and placing $1/3$ of their mass on the travelling plate and $2/3$ at the elbow (extremity of the arm). This simplification does not lead to too much loss compared to a complete model of the dynamics [4]. The DELTA robot can thus be reduced to 4 bodies, namely the travelling plate and the 3 arms.

A linear form of the dynamic equation of the DELTA robot can be derived by a direct application of equation (34). For each of the 3 arms, a reference frame \mathcal{A}_i , $i = 1, 2, 3$, is attached at its upper extremity with the x axis along the arm and y axis along the motor axis. We get the following basic Jacobian matrices:

$$\begin{aligned} A_1 J_{A1} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 J_{A2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ A_3 J_{A3} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (36)$$

We have furthermore:

$$A_i \ddot{x}_{A_i} = 0, \quad (37)$$

$$A_i \dot{\omega}_{A_i} = [0, \dot{q}_i, 0]^T, \quad (38)$$

$$A_i \dot{\omega}_{A_i} = [0, \ddot{q}_i, 0]^T. \quad (39)$$

The rotation matrix from frame \mathcal{A}_i to the inertial frame \mathcal{O} is given by:

$${}^{\mathcal{O}}_i R = {}^{\mathcal{O}}_i R \quad {}^i_{A_i} R, \quad (40)$$

where

$${}^i_{A_i} R \equiv \begin{bmatrix} \cos q_i & 0 & -\sin q_i \\ 0 & 1 & 0 \\ \sin q_i & 0 & \cos q_i \end{bmatrix}, \quad i = 1, 2, 3. \quad (41)$$

The fourth body is the travelling plate. Its basic Jacobian matrix J_{04} is given by:

$$J_{04} = \begin{bmatrix} J \\ 0_{3 \times 3} \end{bmatrix} \quad (42)$$

where J is the Jacobian matrix of the robot which can be computed as shown in [5]. Because no rotation occurs on the travelling plate, the part of the matrix corresponding to rotations is equal to zero, as well as its angular velocity ω_4 and acceleration $\ddot{\omega}_4$. The linear acceleration of the travelling plate \ddot{x}_n can be calculated by numerical differentiation of its cartesian position.

For the arms, the components of equation (34) are:

$$J_{A1}^T \Omega_1 = \begin{bmatrix} 0 & g_1 & 0 & 0 & 0 & 0 & 0 & \ddot{q}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} J_{A2}^T \Omega_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 & 0 & 0 & 0 & \ddot{q}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ J_{A3}^T \Omega_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_3 & 0 & 0 & 0 & 0 & 0 & \ddot{q}_3 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (43)$$

with $g_i \equiv -g \cos q_i$ (g is the gravitational acceleration).

The matrix J_τ can be calculated using equation (17). For the DELTA robot this gives:

$$J_\tau = 1_{3 \times 3} \quad (44)$$

In equation (43), all of the columns composed of zeros only can be cancelled because they do not affect the dynamics. This corresponds to a reduction of the parameter vector. The linear form of the dynamic equation of the DELTA is then:

$$\tau = \Psi p, \quad (45)$$

$$\Psi \equiv \begin{bmatrix} g_1 & \ddot{q}_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_2 & \ddot{q}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_3 & \ddot{q}_3 \end{bmatrix} J^T(\ddot{x}_n - G),$$

$$p \equiv [m_1 r_{1,x}, I_{1,yy}, m_2 r_{2,x}, I_{2,yy}, m_3 r_{3,x}, I_{3,yy}, m_n]^T.$$

In this last equation, $r_{i,x}$ is the distance from the motor to the center of gravity of arm i . $G = [0, 0, -g]$ is the vector of gravitational acceleration.

This equation is simple and can thus be used in a real time implementation of a non-linear adaptive control scheme. The most time consuming part is the computation of the Jacobian matrix. It can be calculated numerically [4] and takes about 1 ms on a T800 transputer.

5 Conclusion

Finding a linear form of the dynamic equation of parallel robots is a difficult task, especially when a real-time implementation is needed. In this paper, we presented a new method based on the virtual work principle. It considers each body separately in a body-oriented approach. Jacobian matrices are used to project the forces and torques of each body onto the space of actuated joints. An advantage of the method is that simplifying hypothesis can be used at the very beginning already. As an example, the method was

applied to the DELTA, a 3 dof fully parallel robot. It led to simple equations which can be used in a real time nonlinear adaptive control scheme. The method is however not limited to the DELTA robot, but can be applied to almost all robots and especially to fully parallel robots with 6 dof [11].

Acknowledgements Many thanks to Marcel Honegger for his valuable contributions.

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