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Accuracy Analysis of 3-RSS Delta Parallel Manipulator

Mansoor Ghazi, Qasim Nazir, Sajid Ullah Butt, Aamer Ahmed Baqai

National University of Sciences and Technology(NUST), Islamabad, Pakistan

Abstract

Accuracy analysis of parallel manipulators is the first step in selecting an appropriate error model for further design. In this paper, the kinematic accuracy of a Delta parallel manipulator is evaluated by Jacobian and geometric error models. The Jacobian (or condition number) based error models have been widely used for analysis and optimal design of parallel manipulators. However, as it is highlighted in this study, these models are dependent on the choice of particular matrix norm and do not capture the directional nature of accuracy. The geometric error model, derived for the Delta parallel manipulator, computes the exact value of positioning errors in task space that arise due to errors in joint space. The proposed model is used to compute overall as well as individual positioning errors along each DOF. It is revealed that the kinematic accuracy exhibits a highly directional nature over the reachable workspace. Moreover, individual errors along each DOF should be analyzed for complete evaluation of the accuracy of the Delta parallel manipulator.

Keywords: accuracy analysis; error modeling; condition number; positioning errors; delta parallel manipulator; geometric error model

1. Introduction

Parallel manipulators are characterized by intrinsically higher accuracies, stiffness and enhanced force/velocity transmission characteristics [1]. For most applications, positioning accuracy is one of the most important design requirements and can be classified into:

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1. kinematic accuracy that is attributed to uncertainties/errors in active joint inputs and geometric tolerances in joints and links
2. dynamic accuracy that is defined in terms of positioning errors due to finite stiffness of links and joints

Kinematic accuracy analysis aims to evaluate the positioning errors over the reachable workspace of a manipulator. Jacobian error models, which include condition number based [2] and linear first order approximate evaluation of errors [3] have been widely used for accuracy analysis of both spatial and planar parallel manipulators. The condition number based error model involves computing the condition number of the Jacobian matrix of the manipulator. According to its classical definition [4], the condition number denotes an amplification of errors in the inputs of a system on its output given there are no truncation or floating-point errors in the solution. In case of parallel manipulators, the condition number serves as a performance index for assessing the accuracy or dexterity of the manipulator at a given configuration. Moreover, the condition number also gives a "sense" of the distance of a configuration from a singular configuration. For manipulators with mixed translational and rotational degree of freedoms (DOFs), the Jacobian matrix must be homogenized by dividing it with a characteristic length [5]. The condition number model is used in several studies [6-12] for optimal design of parallel manipulators. As an extension of the condition number based approach, an error amplification index based on the condition number of the Error Transformation Matrix (ETM) was defined and used in [13] to assess and maximize the accuracy of a 3-PUS (prismatic-universal-spherical) parallel manipulator. Xu. et al. [14] proposed a composite error index based on the minimum eigenvalue and condition number of the ETM, for evaluating the accuracy of a spatial parallel manipulator. Recently, Briot and Bonev [15] employed a geometric error model to perform the accuracy analysis of 3-DOF planar parallel robots. Then Liu and Bonev [16] employed a geometric error model for dimensional optimization of two parallel kinematic tool heads. Contrary to the Jacobian based condition number model, the geometric error model uses inverse and forward kinematic formulations to compute the exact value of positioning error about a nominal position for a known error in active joint inputs.

In this study, kinematic accuracy analysis of a 3-RSS (Revolute-Spherical-Spherical) Delta parallel manipulator is performed with both Jacobian and geometric error models. It was shown by Wu et al. [17] that errors in active joint inputs (or actuated joints) are the most significant sources of kinematic inaccuracies, therefore, bounded errors in active joint inputs are considered in this study. The mainstay of this work is an in-depth analysis of the correlation between the condition number and exact values of positioning errors at the end-effector of 3-RSS Delta parallel manipulator. Moreover, both overall positioning error at a nominal position and individual positioning errors along translation in each DOF are evaluated. As it will be shown later, the geometric error reveals a highly directional aspect of the kinematic accuracy of a Delta parallel manipulator, whereas results of the condition number based model are governed by the choice of our matrix norm.

The rest of this article is organized as follows: Section 2 presents the kinematic architecture and Jacobian analysis of the Delta parallel manipulator. The Jacobian based condition number and geometric error models are derived in section 3. Section 4 discusses the accuracy analysis viz a comparison of the two error models. Finally, the study is concluded in Section 5.

2. 3-RSS Delta Parallel Manipulator

The Delta parallel manipulator is a spatial parallel manipulator with three translational DOFs. It consists of a fixed and a moving platform connected through three identical kinematic chains as illustrated in Fig. 1(a). The fixed platform houses the actuation sub-system, whereas the moving platform serves as the end-effector. Each kinematic chain consists of a proximal link and two distal links. The two distal links form a parallelogram that constrains the moving platform to three translational DOFs. Four passive spherical joints (S) form the corners of the parallelogram. An active (actuated) revolute joint (R) connects the proximal link to the fixed platform.

Fig. 1(b) illustrates the i^{th}kinematic chain of the Delta parallel manipulator. Geometric centers of the fixed and moving platforms are denoted by points O and P respectively. For complete kinematic configuration of i^{th} kinematic chain, two coordinate axes are adopted. The coordinate system $O(xyz)$ is assumed to be at the geometric center of the fixed platform while coordinate system, $A_i(x_i y_i z_i)$ is taken at the center A_i of the revolute joint R_i $(i = 1, 2, 3)$. The $x_i -$ axis is aligned with vector OA_i and is perpendicular to the center of the revolute joint. $\varphi_i (i = 1,2,3)$ is the angle between x and x_i axis, and represents the constant orientation of the three kinematic chains with respect to $O - xyz$

frame. As evident from Fig. 1(b), θ_{1i}, θ_{2i} and θ_{3i}describe the configuration or posture of the manipulator in the joint space

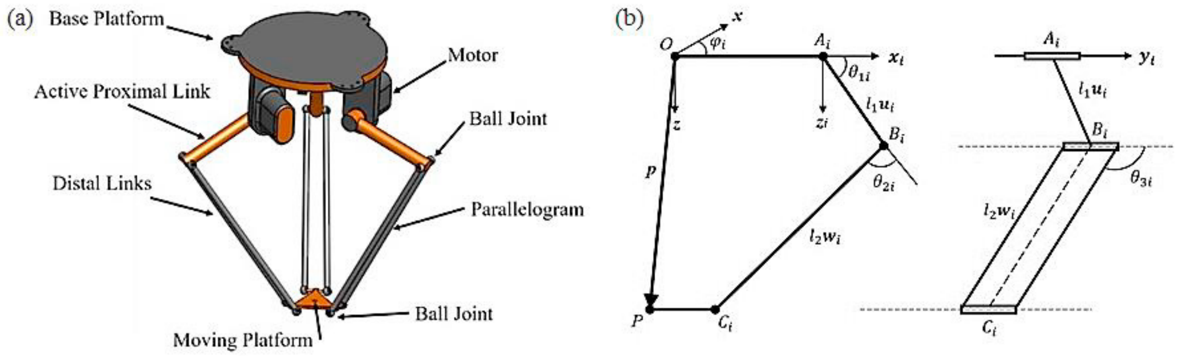

Fig. 1: (a) 3-RSS Delta Parallel Manipulator; (b) Kinematic description of the i^{th} kinematic chain

2.1. Inverse and Forward Kinematics

$\boldsymbol{\theta} = [\theta_{11}\theta_{12}\theta_{13}]^T$is the vector of active joint inputs, whereas the vector $\mathbf{P} = [xyz]^T$ specifies the position, in task space, of point P relative to the $O - xyz$ frame of reference. Based on above convention, inverse kinematics is a mapping from the end-effector position to the active joint inputs expressed as $f : \boldsymbol{P} \to \boldsymbol{\theta}$. Similarly, forward kinematics is the mapping from active joint inputs to end-effector position expressed as $g : \boldsymbol{\theta} \to \boldsymbol{P}$. Readers should refer to [18] for the in-depth analytical formulation of the inverse and forward kinematics of the 3-RSS Delta parallel manipulator.

The inverse and forward kinematics are required for developing a geometric error model to compute the exact positioning errors. However, obtaining a closed form analytical formulation and solution of forward kinematics can be challenging for certain parallel manipulator architectures. In such cases Eq. (2) can be obtained via numerical optimization methods such as back propagation trained neural networks [19].

2.2. Design Parameters

The kinematic accuracy of the Delta parallel manipulator depends upon the four design parameters (Fig. 2):

1. Length of the proximal link; denoted by a.
2. Length of the distal link; denoted by b.
3. Distance from the center of the base platform to the center of revolute joint; denoted by R.
4. Distance from the center of moving platform to the mid-point of the line $S_{2i}S_{2i+1}$; denoted by r.

The above-stated design parameters can be assembled into a design vector $\mathbf{G} = [a\ b\ r\ R]^T$. Note that the, in this article, centimeters (cm) is used as the unit of length.

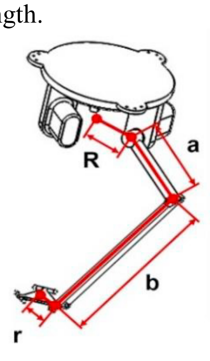

Fig. 2. Design parameters of the Delta parallel manipulator

2.3. Jacobian Analysis

The Jacobian matrix $\mathbf{J}$ relates active joint input velocities $\dot{\boldsymbol{\theta}}$ to end-effector velocities $\dot{\mathbf{P}}$ as:

$$\dot{\boldsymbol{\theta}} = \boldsymbol{J}\dot{\boldsymbol{P}} \qquad (1)$$

In case of parallel manipulators, the inverse of the Jacobian matrix can usually be more readily computed, therefore Eq. (1) is remodeled as:

$$\dot{\boldsymbol{P}} = \boldsymbol{J}^{-1}\dot{\boldsymbol{\theta}} \qquad (2)$$

$\mathbf{J}^{-1}$ is the inverse Jacobian matrix that can be computed by differentiating the inverse kinematics equation.

$$\frac{\partial f(\boldsymbol{\theta},\boldsymbol{P})}{\partial \boldsymbol{\theta}}\dot{\boldsymbol{\theta}} + \frac{\partial f(\boldsymbol{\theta},\boldsymbol{P})}{\partial \boldsymbol{P}}\dot{\boldsymbol{P}} = \boldsymbol{J}_{\boldsymbol{\theta}}\dot{\boldsymbol{\theta}} + \boldsymbol{J}_{\boldsymbol{P}}\dot{\boldsymbol{P}}$$

$$\dot{\boldsymbol{P}} = {\boldsymbol{J}_{\boldsymbol{P}}}^{-1}\boldsymbol{J}_{\boldsymbol{\theta}}\dot{\boldsymbol{\theta}} = \boldsymbol{J}^{-1}\dot{\boldsymbol{\theta}} \qquad (3)$$

Eq. (3) implies that the inverse Jacobian matrix is a product of two matrices; an inverse kinematic Jacobian $\mathbf{J}_{\boldsymbol{\theta}}$ matrix and the inverse of a forward kinematic Jacobian matrix ${\mathbf{J}_{\boldsymbol{P}}}^{-1}$. The inverse, forward kinematic and full inverse Jacobian matrices for the 3-RSS Delta parallel manipulator were computed by Lopez et al. [20].

It should be noted that in case of the Delta parallel manipulator the Jacobian matrix is a dimensionally homogenous 3×3 square matrix. This is because the Delta parallel manipulator possesses solely translational DOFs. Eq. (3) also signifies the role of Jacobian matrix in determining the amplification effect of errors in active joint inputs on positioning errors. Therefore, various Jacobian based performance indices have been developed to assess the kinematic accuracy of parallel manipulators.

3. Error Models

In literature, various error models are developed for analyzing the kinematic accuracy of manipulators across its reachable workspace. These error models can be broadly classified into (a) Jacobian based error model and (b) Geometric error model. Jacobian based error model gives a qualitative value to the kinematic accuracy in terms of the condition number of the Jacobian matrix. Contrary to this, the geometric error model allows for computing the exact value of positioning error at a point.

3.1. Jacobian Based Error Model

The condition number κ of the Jacobian matrix is used as an index for measuring the kinematic accuracy and dexterity of parallel manipulators at a nominal position. A condition number of 1 denotes an isotropic pose, that is no amplification of positioning error for a given input error. A condition number approaching infinity implies that a bounded error in active joint inputs will lead to an infinite amplification of positioning error. Moreover, the condition number also gives a sense of the distance of a given pose from singular configurations. The condition number of the Delta parallel manipulator at a nominal position can be found by analyzing Eq. (2). Given an error in active joint velocities $\Delta\dot{\boldsymbol{\theta}}$, there must be an error $\Delta\dot{\mathbf{P}}$ in the positioning velocities. The two errors can be related as:

$$\Delta\dot{\boldsymbol{P}} = \boldsymbol{J}^{-1}\Delta\dot{\boldsymbol{\theta}} \qquad (4)$$

Eq. (4) can be obtained by adding the positioning and input errors to their respective nominal values and subtracting the resulting equation from Eq. (2). The condition number can now be defined by considering the norm:

$$\frac{\|\Delta\dot{P}\|}{\|\dot{P}\|} \leq \|\boldsymbol{J}\|.\|\boldsymbol{J}^{-1}\|\frac{\|\Delta\dot{\theta}\|}{\|\dot{\theta}\|} \tag{5}$$

The factor $\|\mathbf{J}\|.\|\mathbf{J}^{-1}\|$ is essentially the condition number of the Jacobian matrix and, as per the classical definition, gives the bounds on error in computing the solution of the linear system in Eq. (5) for known errors in the inputs. By analogy and as suggested by Gosselin [2], the condition number also linearly relates the errors in active joint inputs to errors in positioning. It is prudent to mention that the Jacobian error model does not require homogenization of the Jacobian matrix in case of the Delta parallel manipulator, as the manipulator only possess translational DOFs.

The value of the condition number is dependent on the choice of norm $\|.\|$ being used. 2-norm and Euclidean norm are the two most commonly used norms for condition number evaluation. However, the minimum value of condition number is 1 regardless of the norm being used. It is prudent to mention that most authors have used the inverse of the condition number $\kappa^{-1} \in [0,1]$, also known as the isentropy or dexterity index [21]. The closer κ^{-1} is to 1 the more accurate the manipulator about a nominal position. In this paper the isentropy index is used in place of the condition number for evaluating the kinematic accuracy of the Delta parallel manipulator. Furthermore, both 2-norm and Euclidean norm are used to highlight the dependence of condition number on the selection of matrix norm.

3.2. Geometric Error Models

The geometric error model derived in the following discussion maps the error $\pm\varepsilon$ in the three active joint inputs $\boldsymbol{\theta_0} = \left[\theta_{11_0}\theta_{12_0}\theta_{13_0}\right]^T$ to positioning errors $\Delta\mathbf{P} = [\Delta x\ \Delta y\ \Delta z]^T$ about a nominal position $\mathbf{P_0} = [x_0\ y_0 z_0]^T$. The error in active joint inputs $\pm\varepsilon$ is result of the finite resolution of the three encoders that provide position feedback to the robot controller. Due to this error or uncertainty, the three active joint inputs can lie in the intervals $\theta_{1i} \in \left[\theta_{1i_0} - \varepsilon, \theta_{1i_0} + \varepsilon\right]$ for$(i = 1,2,3)$. Consequently, the actual position of the end-effector can lie anywhere in the intervals $x \in [x_0 - \Delta x, x_0 + \Delta x], y \in [y_0 - \Delta y, y_0 + \Delta y]$ and $z \in [z_0 - \Delta z, z_0 + \Delta z]$. Geometrically, the three active joint inputs are bounded by a cuboid $\mathcal{R}$ having eight corner points. Fig. 3 illustrates the input bounding region (cuboid $\mathcal{R}$) of a Delta parallel manipulator with design parameters $\mathbf{G} = [30\ 70\ 10\ 20]^T$, about a nominal position $\mathbf{P_0} = [0067]^T$ for $\varepsilon = \pm 0.00062$ rad.

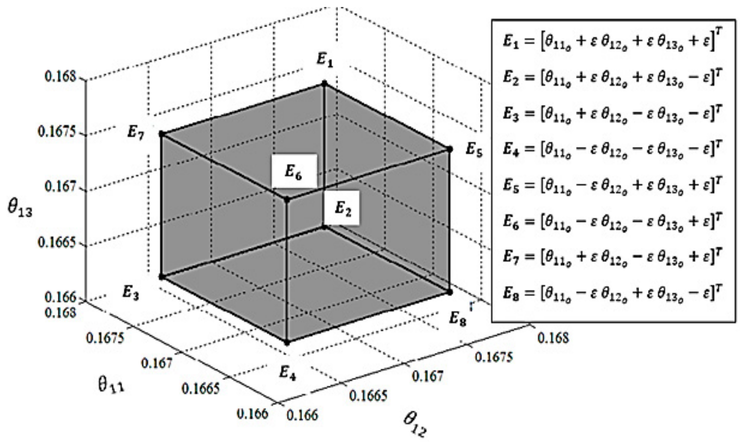

Fig. 3. Input bounding region $\mathcal{R}$.

Corresponding to the input bounding region there is an output bounding region $\mathcal{O}$, which is generated by sweeping the active joint inputs in the interval $\theta_{1i} \in \left[\theta_{1i_0} - \varepsilon, \theta_{1i_0} + \varepsilon\right]$ for$(i = 1,2,3)$ and computing the positions of the end effector at these active joint inputs. Distances between the resulting points that lie on the boundary and inside of the output bounding region and the nominal position $\mathbf{P_0}$ are called local positioning errors about the nominal position. Briot et al. [15] showed that maximum local positioning error $\Delta X_{max(\mathbf{P_0})}$ occurs with the one of the corner points of the input bounding region used as active joint inputs:

1. the end-effector is not at a Type 1 or Type 2 singularity

2. the input bounding region is significantly small in comparison to the reachable workspace.

Since the Delta parallel manipulator is typically employed in industrial environments, their controllers are designed to explicitly avoid singular configurations. Therefore, the maximum local positioning error is simply equal to the largest value of local positioning error computed with the eight corner points of the input bounding region as active joint inputs.

Let $\Delta X_{\max(\mathbf{P}_0)} = g(\mathbf{P}_0, \mathbf{G}, \varepsilon)$ be the maximum local positioning error about the nominal position $\mathbf{P}_0$ for the Delta parallel manipulator having design parameters $\mathbf{G}$ and error of $\pm\varepsilon$ in active joint inputs. The mapping $g(.)$ is found by first resolving the inverse kinematics to compute $\boldsymbol{\theta}_0$ for a known $\mathbf{P}_0$. In the second step, forward kinematics is solved for each of the eight corner points of $\mathcal{R}$ by setting $\boldsymbol{\theta}_{0i} \in [\mathbf{E}_1, \dots, \mathbf{E}_i]^T$ for $i = 1, 2, \dots, 8$. This creates an array of positions $\{\mathbf{P}\}$. The maximum distance between the nominal position and the positions $\{\mathbf{P}\}$ is the maximum overall local positioning error about $\mathbf{P}_0$. Algorithmic formulation of $g(.)$ is presented in Algorithm 1.

Algorithm 1: Maximum local output error at $\mathbf{P_0}$

Inputs:
- Vector of design variables, $\mathbf{G} = [abrR]^{\boldsymbol{T}}$
- Uncertainty in active joint inputs $\pm\varepsilon$
- Nominal position, $\mathbf{P_0} = [x_0\ y_0 z_0]^T$

Find:
- Maximum local output error about $\mathbf{P_0}$, $\Delta X_{\max(\mathbf{P}_0)}$

1. **function** $g(\mathbf{P}_0, \mathbf{G}, \varepsilon)$
2. $\boldsymbol{\theta}_0 = inverseKinematics(\mathbf{P_0}, \mathbf{G})$
3. $\boldsymbol{\theta} = cornerPoints(\boldsymbol{\theta}_0, \varepsilon)$
4. for $i = 1{:}1{:}8$
5. $\mathbf{P}[i] = forwardKinematics(\boldsymbol{\theta}[i], \mathbf{G})$
6. $\{\Delta X_{(\mathbf{P}_0)}[i]\} = |\mathbf{P}_0 - \mathbf{P}[i]|$
7. end
8. $\Delta X_{\max(\mathbf{P}_0)} = \max(\Delta X_{(\mathbf{P}_0)})$
9. **return** $\Delta X_{\max(\mathbf{P}_0)}$
10. **end function**

The local positioning errors are computed as:

$$\Delta X_{(\boldsymbol{P}_0)}[i] = |\boldsymbol{P}_0 - \boldsymbol{P}[i]| = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2} \qquad (6)$$

It should be noted that $g(.)$ returns the resultant of positioning error about a nominal position. This mapping can be modified to return individual positioning errors $\Delta x_{\max(\mathbf{P}_0)}$, $\Delta y_{\max(\boldsymbol{P}_0)}$ and $\Delta z_{\max(\mathbf{P}_0)}$ in translation along x, y and z-axes respectively. Furthermore, unlike the Jacobian based error models, the mapping $g(.)$ computes the exact positioning error about a nominal position under known uncertainties in active joint inputs.

4. Accuracy Analysis: Results and Discussion

Accuracy analysis is done by visualizing κ^{-1}, $\Delta X_{\max(\mathbf{P}_0)}$, $\Delta x_{\max(\mathbf{P}_0)}$, $\Delta y_{\max(\boldsymbol{P}_0)}$ and $\Delta z_{\max(\mathbf{P}_0)}$ across a plane inside the reachable workspace of the Delta parallel manipulator. The two condition indices κ^{-1}, one based on 2-norm and another one based on Euclidean norm, are computed via the Jacobian based error model, whereas the maximum overall local positioning error $\Delta X_{\max(\mathbf{P}_0)}$, maximum local positioning error along x-axis $\Delta x_{\max(\mathbf{P}_0)}$, maximum local positioning error along y-axis $\Delta y_{\max(\mathbf{P}_0)}$ and maximum local positioning error along z-axis $\Delta z_{\max(\mathbf{P}_0)}$ are computed using the geometric error model.

For both error models, a pre-specified plane inside the workspace is discretized into 50,000 points and the condition indices and local positioning errors computed at each point. For our analysis, a Delta parallel manipulator is considered with design vector $\mathbf{G} = [30\ 70\ 10\ 20]^T$ and known uncertainty of $\varepsilon = \pm 0.00109\ rad$ in the active joint inputs.

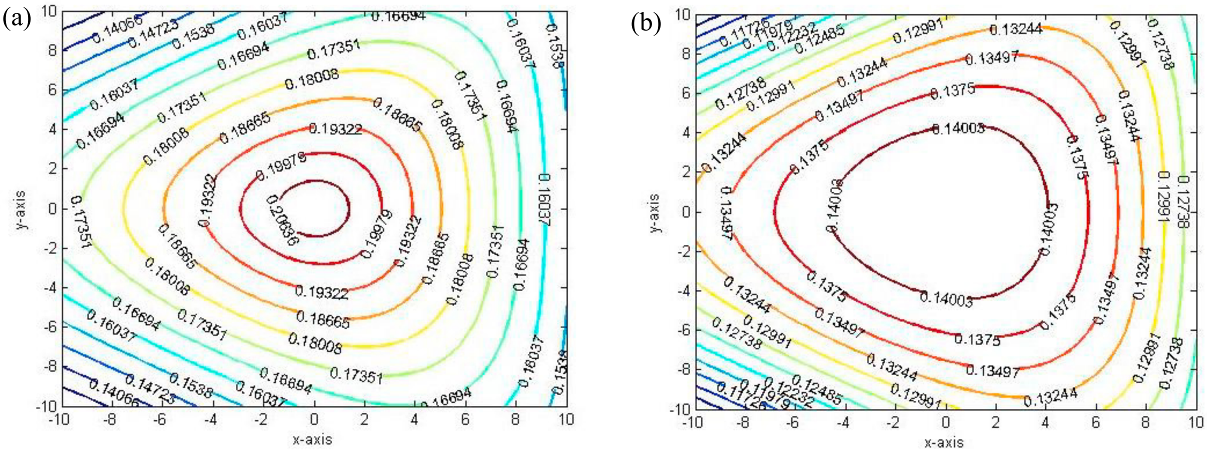

Figure 4: (a) 2-norm condition index κ^{-1} evaluated at $z = 67$; (b) Euclidean condition index κ^{-1} evaluated at $z = 67$.

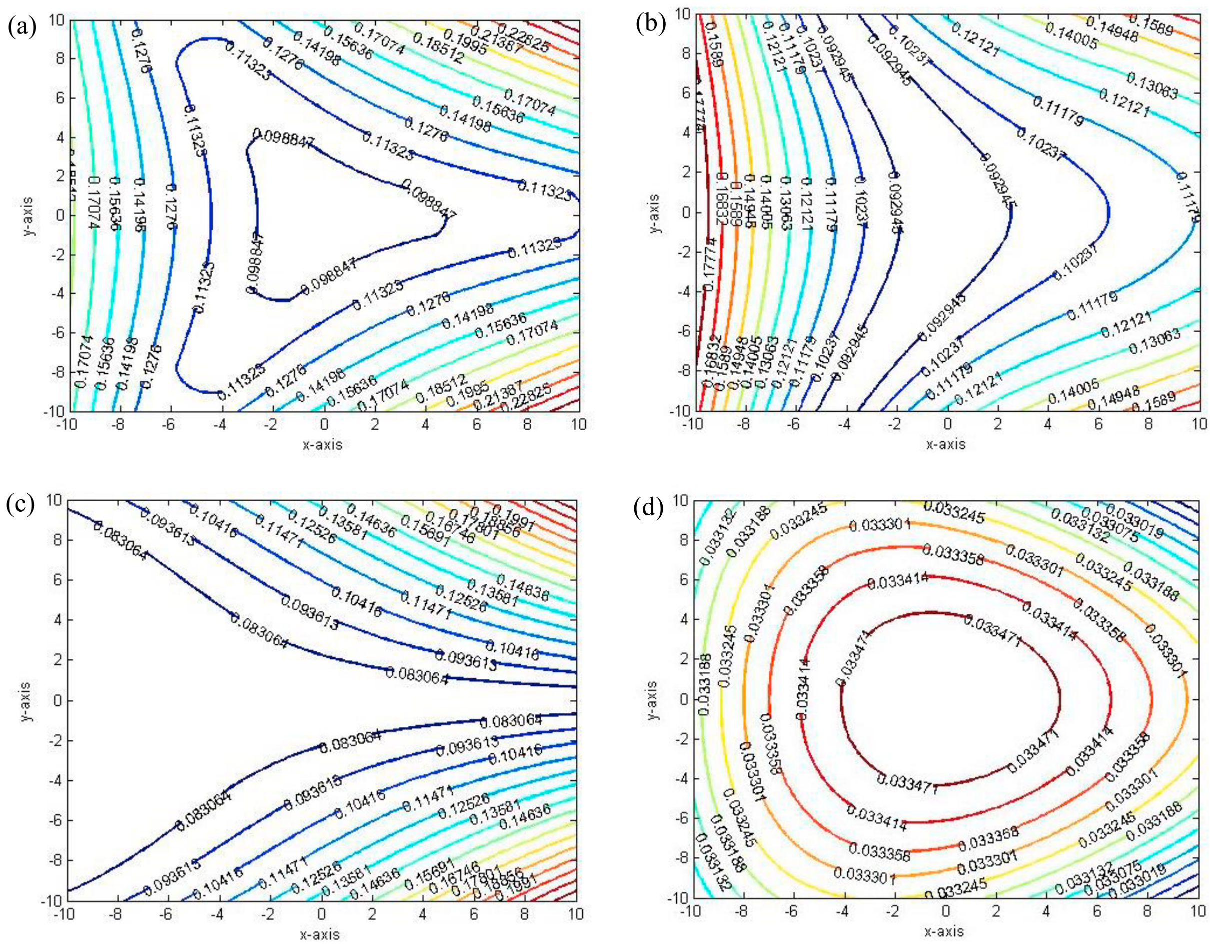

Figure 5: (a) Maximum local positioning errors $\Delta X_{\max(\mathbf{P_0})}$ evaluated at $z = 67$; (b) Maximum local positioning errors along x-axis, $\Delta x_{\max(\mathbf{P_0})}$; (c) Maximum local positioning errors along y-axis, $\Delta y_{\max(\mathbf{P_0})}$; (d) Maximum local positioning errors along z-axis $\Delta z_{\max(\mathbf{P_0})}$,.

The value of condition index, based on the 2-norm and Euclidean norm, across a plane defined at $z = 67$ for the test case Delta parallel manipulator are illustrated in Fig. 4(a) and 4(b) respectively. It is evident that both the value and distribution condition indices are different. This result is in close agreement with the observations made by Merlet [22] who suggested that the choice of a norm may alter the observed distribution of dexterity across the workspace of a planar parallel manipulator. However, it is also noticed that, while moving away from the workspace center, both 2-norm and Euclidean norm based condition indices point at an increase in error amplification, or alternatively a decrease in kinematic accuracy. Interestingly, this also implies that the Delta parallel manipulator tends towards a singular configuration as the end-effector reaches the workspace boundary.

As illustrated in Fig. 5(a), $\Delta X_{\max(\mathbf{P}_0)}$ reveals the highly directional nature of kinematic accuracy across the workspace. It is evident from the figure that iso-contours of errors are orientated in the same way as the three active joints around the point O. Further understanding of the directional nature of accuracy is made by visualizing the individual positioning errors along each DOF. In Fig. 5(b), $\Delta x_{\max(\mathbf{P}_0)}$ is plotted which shows highly uniform behavior along the negative x axis. This points to an interesting correlation between the kinematic accuracy along x-axis and the condition index. The condition index is skewed towards the negative x axis whereas the positioning errors along x direction become more uniformly distributed in the negative x axis. This fact can be leveraged in practice by installing the Delta parallel manipulator such that the most end-effector movements are carried out along the negative x axis. $\Delta y_{\max(\boldsymbol{P}_0)}$ is illustrated in Fig. 5(c). The errors are found to be symmetric about the x axis and are increasingly uniform parallel and perpendicular to the positive x axis. The relative uniformity of errors in translation along the z-axis DOF is evident from a visual analysis of $\Delta z_{\max(\boldsymbol{P}_0)}$ illustrated in Fig. 5(d).

By analyzing the results of both Jacobian or condition index based and geometric error models, similarities and differences between the models in context of the kinematic accuracy can be drawn. While both models highlight a decrease in kinematic accuracy at the workspace boundary, the two models paint a significantly different picture of the distribution of errors across the workspace. The Jacobian based error models, while faster to evaluate, do not give the value of actual positioning errors. The geometric error model, on the other hand, requires formulation of both inverse and forward kinematics to yield the actual (or exact) value of positioning errors for a known uncertainty in active joint inputs. Furthermore, the geometric error model can be used to evaluate the overall as well as individual positioning errors along translation in the x, y and z axes. This can be useful for formulating accuracy centric design criteria that are more "functional" than the traditional dexterity or condition number based objective functions.

5. Conclusion

In this work, kinematic accuracy analysis of a 3-DOF translational Delta parallel manipulator has been performed via both Jacobian and geometric error models. The geometric error model computes the exact value of positioning errors for a known error in active joint inputs. It is concluded, in this study, that the geometric error model should be used for effectively analyzing the kinematic accuracy of a Delta parallel manipulator since the Jacobian error model does not give the exact value and directionality of accuracy. In future, an experimental study will be conducted for the accuracy of DELTA parallel manipulator under the effects of uncertainties in active joint inputs and the deflection of links under inertial loading. A multi-objective design approach would be proposed to maximize both the kinematic and dynamic accuracy of parallel manipulators

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