

Par4: very high speed parallel robot for pick-and-place

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Abstract – This paper introduces a four-degree-of-freedom parallel manipulator dedicated to pick-and-place. It has been developed with the goal of reaching very high speed. This paper shows that its architecture is particularly well adapted to high dynamics. Indeed, it is an evolution of Delta, H4 and I4 robots architectures: it keeps the advantages of these existing robots, while overcoming their drawbacks. In addition, an optimization method based on velocity using Adept Motion has been developed and applied to this high speed parallel robot. All these considerations led to experimentations that proved we can reach high accelerations (13 G) and obtain a cycle time of 0.28 s.

Index Terms – *Schonflies Motion, PKM, pick-and-place, Articulated traveling plate*

I. INTRODUCTION

The first parallel mechanism is attributed to Gough with its well-known platform [1] and Stewart [2] created the famous flight simulator few years later. Thanks to actuators located close to the frame, the dynamics of these mechanisms is high compared to serial robots. These machines have six degrees of freedom (*dof*) and the range of their angular motion is limited.

However, all robotized tasks do not need six *dof*. Brogardh proposed a classification [3] giving the necessary number of *dof* for different industrial tasks. Generally, pick-and-place needs four *dof*: three translations and one rotation around a vertical axis. These motions are named Schoenflies motions [4] or SCARA motions. The main industrial applications for these mechanisms are packaging, including picking, packing and palletizing tasks.

Delta robot [5], developed by Clavel at EPFL at the end of 80s, generates Schonflies motion and is well adapted to pick-and-place tasks because of its high dynamics. Indeed, actuators of this robot are fixed on the frame which minimizes moving parts masses. However, the rotational motion of this robot is obtained using a central RUPUR chain (R: Revolute, U: Universal, P: Prismatic, bold representing the actuated joint) which could suffer from a lack of stiffness at workspace extremities. In addition, this telescopic arm has a short service life.

Other lower mobility parallel mechanisms able to realize SCARA motions have been developed. For example, Angeles [6] proposed a four-*dof* parallel mechanism. In addition, EPFL developed Kanuk and Manta robots [7], and the machine tool HITA STT which has been derived to pick-and place manipulator [8]. At last, authors proposed H4 [9], I4L [10]

and I4R [11] introducing the articulated traveling plate concept (see Fig. 1).

This paper presents a new parallel manipulator based on H4 and I4 architectures: Par4. This mechanism has been developed with the aim of reaching very high speed and acceleration. Indeed, the second part of the paper presents the improvements of Par4 compared to I4 and H4. The third part exposes an optimization method dedicated to pick-and-place robots, applied to Par4. The last part presents experimental results showing that the prototype is able to reach accelerations up to 13 G.

II. DESCRIPTION OF PAR4

The particularity of Par4 compared to H4 and I4 is its articulated traveling plate. It is composed of four parts: two main parts (1,2) linked by two rods (3,4) thanks to revolute joints (see Fig. 2). The shape of this assembly is a planar parallelogram and the internal mobility of the traveling plate is a PI joint [12] (circular translation) which produces the rotational motion about the vertical axis of the global robot. The range of this rotation is $\pm\pi/4$. That's why, an amplification system has to be added in order to obtain a complete turn: $\pm\pi$. The amplification system can be made of gears or belt/pulleys. The chosen mechanism for the prototype is belt/pulleys with an amplification ratio $\rho = 4$ (see Fig. 2b).

The overall architecture is similar to H4 or I4R as described in [9] and [11]. Arms and forearms, made of carbon fiber, are taken from ABB Flexpicker robot. Par4 is a Delta-like mechanism. The key difference with the Delta robot is the use of four kinematic chains instead of three. In addition, it uses the concept of articulated traveling plate in order to avoid the central telescopic leg. The key difference with H4 and I4 robots will be explained in the following.

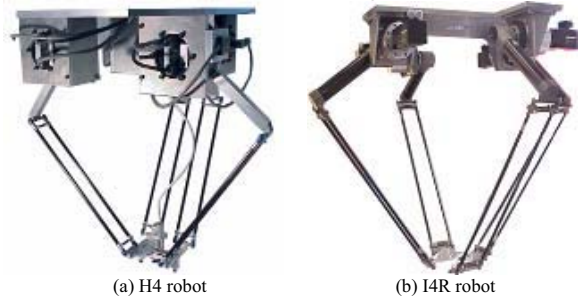


Fig. 1 Existing 4-*dof* prototypes

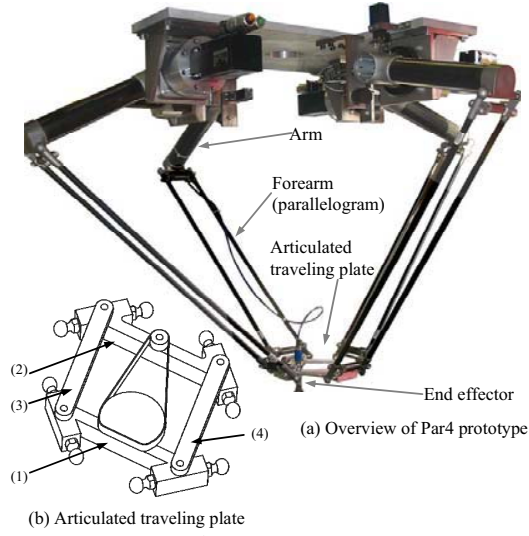


Fig. 2 Par4 prototype

Remark about models for kinematics solutions:

As mentioned previously, the articulated traveling plate of Par4 produces a rotational translation. From a modeling point of view, it can be assimilated to H4. Thus, models for kinematics solutions (position and velocity) can be found in [9] and will not be presented here.

III. WHY PAR4 INSTEAD OF H4 AND I4

This new prototype has been developed in order to reach very high speed and acceleration, and to obtain a homogeneous behavior and a good stiffness in the whole workspace and for every direction. All these constraints can not be respected at the same time neither by H4 nor I4 for different reasons that are now explained.

A. From I4 to Par4

An advantage of this architecture is the good arrangement of actuators unlike H4 (see § III.B). In addition, models for kinematics solution of this mechanism are simple. However, the main weak point of I4 [10][11] is the use of prismatic joints in the articulated traveling plate. Indeed, used at high speed, commercial prismatic joints have a short service life, due to high acceleration and pressure exerted on balls. Thus, I4 is well suited for high force/ moderate acceleration application (machining for example).

For high speed robots, the use of revolute joints in the articulated traveling plate seems to be more adapted. That's why, Par4 has been developed with the constraint of using only revolute joints on its articulated traveling plate.

B. From H4 to Par4

H4 uses revolute joints, but a weak point of this robot is the arrangement of its actuators. This particularity is due to singularity configurations.

Singularities occur for particular poses of the mechanism and lead to a bad behavior of the end-effector. In [13], a classification of singularities is proposed. They can be parted into three categories: under-mobilities [14], over-mobilities [14], and internal singularities [15]. These notions can be partly explained using linear kinematic equation [16]:

$$\mathbf{J}_x \dot{\mathbf{x}} = \mathbf{J}_q \dot{\mathbf{q}} \quad (1)$$

where $\dot{\mathbf{x}}$ is the vector of operational velocities and $\dot{\mathbf{q}}$ is the vector of actuated joints velocities.

In order to explain these categories of singularities, a simple two-*dof* parallel mechanism is proposed:

- On one hand, under-mobilities occur when \mathbf{J}_q is singular. In that case, a velocity $\dot{\mathbf{q}}$ can be applied without producing any motion on the traveling plate (Fig. 3a),
- On the other hand, over-mobilities occur when \mathbf{J}_x is singular. In that case, it is possible to have $\dot{\mathbf{x}} \neq 0$ without moving the actuators (Fig. 3b).

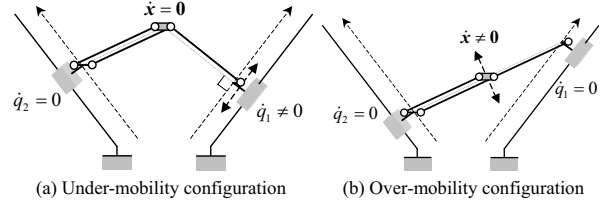


Fig. 3 Presentation of singularities

At last, “internal” singularities can occur for some mechanisms. They cannot be enlightened thanks to \mathbf{J}_x or \mathbf{J}_q , and a more complete study has to be done. On Fig. 4, an internal singularity occurs when the parallelogram becomes flat. In that case, orientation of traveling plate cannot be guaranteed (this orientation does not belong to operational velocities).

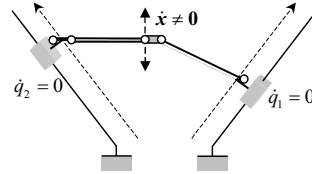


Fig. 4 Presentation of internal singularities

Placing actuators of H4 with a homogeneous repartition, *i.e.* placed at 90° one relatively to each other, leads to internal singularities. Thus, a particular arrangement has to be adopted. This particularity involves a non-homogeneous behavior of H4 in the workspace and a bad stiffness, as demonstrated in [17].

As said before, the study of classical matrices \mathbf{J}_x and \mathbf{J}_q presented in equation (1) is not enough to enlighten internal singularities.

That's why, a **complete** kinematic analysis has been done on Par4 in order to show that this architecture overcomes

drawbacks of H4 and I4 while keeping all the advantages for high speed. The method of the analysis has been introduced in [11] and [18] for I4 case. The analysis assumes that the mechanism is constituted of 2 sub-unit (actuators and traveling plate) linked by 8 rods having spherical joints. Each rod adds between those two sub-sets a pure geometrical length constraint.

This complete study can be done writing equiprojectivity of speeds for the 8 rods joining the actuators to the traveling plate. It leads to the following equation:

$$\mathbf{J}_p \dot{\mathbf{x}}_I = \mathbf{J}_{act} \dot{\mathbf{q}} \quad (2)$$

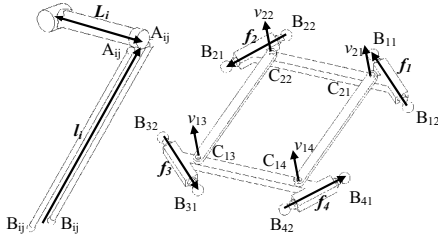


Fig. 5 Par4 parameters for complete singularity analysis

As shown in Fig. 5, the following parameters are introduced:

- i : number of kinematic chain. $i = 1, 4$
- j : number of rod in each kinematic chain. $j = 1, 2$
- k : number of half traveling plate. $k = 1, 2$
- A_{ij} : center of spherical joints on actuated side of forearms
- B_{ij} : center of spherical joints on traveling plate side of forearms
- A_i : geometrical point situated at the middle of A_{i1} and A_{i2}
- B_i : geometrical point situated at the middle of B_{i1} and B_{i2}
- C_{ki} : center of revolute joints of traveling plate
- D : controlled point (located on one of the parts of traveling plate)
- \mathbf{l}_i : vector between B_i and A_i
- \mathbf{f}_i : vector between B_{i1} and B_{i2}
- \mathbf{v}_{ki} : unitary vector of collinear revolute joint axis ki
- \mathbf{d}_i : vector linking C_{ki} and B_i
- \mathbf{c}_k : vector linking C_{ki} and D
- $\mathbf{e}_i = \mathbf{c}_k + \mathbf{d}_i$
- $\dot{\mathbf{e}}_{ki}$: velocity of part k respectively to parallelogram rod (in revolute joint # ki oriented by \mathbf{v}_{ki})
- $\omega_x, \omega_y, \omega_z$: internal angular velocities
- $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$: reference frame axes, with \mathbf{e}_z describing the vertical axis

Assuming that the PI joint in the traveling plate induces a constant parallelism of rods, it produces a coupling. Thus, velocities of revolute joints of traveling plate are equal and the following simplifications can be made:

$$\dot{\mathbf{e}}_{14} = \dot{\mathbf{e}}_{13} = \dot{\mathbf{e}} \quad (3)$$

$$\dot{\mathbf{e}}_{21} = \dot{\mathbf{e}}_{22} = \dot{\mathbf{e}} \quad (4)$$

This coupling is the key difference with the complete kinematic modeling of H4.

Additionally, the following assumptions are made:

$$\mathbf{v}_{14} = \mathbf{v}_{13} = \mathbf{e}_z \quad (5)$$

$$\mathbf{v}_{21} = \mathbf{v}_{22} = \mathbf{e}_z \quad (6)$$

Note that $\dot{\mathbf{x}}$ presented in (1) is the vector composed of operational velocities whereas $\dot{\mathbf{x}}_I$ in (2) is the vector composed of velocities of the complete articulated traveling plate, including internal velocities. Thus, $\dot{\mathbf{x}}_I$ is the following vector:

$$\dot{\mathbf{x}}_I = [\dot{x} \ \dot{y} \ \dot{z} \ \omega_x \ \omega_y \ \omega_z \ \dot{\mathbf{e}}]^T \quad (7)$$

where $\dot{x}, \dot{y}, \dot{z}, \omega_x, \omega_y, \omega_z$ are operational velocities.

It is now possible to write the $[8 \times 7]$ matrix \mathbf{J}_p as described in (8).

$$\mathbf{J}_p \dot{\mathbf{x}}_I = \begin{bmatrix} \mathbf{l}_1^T & [\mathbf{e}_1 \times \mathbf{l}_1]^T & (\mathbf{e}_z \times \mathbf{d}_1) \cdot \mathbf{l}_1 \\ \mathbf{l}_2^T & [\mathbf{e}_2 \times \mathbf{l}_2]^T & (\mathbf{e}_z \times \mathbf{d}_2) \cdot \mathbf{l}_2 \\ \mathbf{l}_3^T & [\mathbf{e}_3 \times \mathbf{l}_3]^T & (\mathbf{e}_z \times \mathbf{d}_3) \cdot \mathbf{l}_3 \\ \mathbf{l}_4^T & [\mathbf{e}_4 \times \mathbf{l}_4]^T & (\mathbf{e}_z \times \mathbf{d}_4) \cdot \mathbf{l}_4 \\ \mathbf{0} & [\mathbf{f}_1 \times \mathbf{l}_1]^T & (\mathbf{e}_z \times \mathbf{f}_1) \cdot \mathbf{l}_1 \\ \mathbf{0} & [\mathbf{f}_2 \times \mathbf{l}_2]^T & (\mathbf{e}_z \times \mathbf{f}_2) \cdot \mathbf{l}_2 \\ \mathbf{0} & [\mathbf{f}_3 \times \mathbf{l}_3]^T & (\mathbf{e}_z \times \mathbf{f}_3) \cdot \mathbf{l}_3 \\ \mathbf{0} & [\mathbf{f}_4 \times \mathbf{l}_4]^T & (\mathbf{e}_z \times \mathbf{f}_4) \cdot \mathbf{l}_4 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \dot{\mathbf{e}} \end{bmatrix} \quad (8)$$

where \times and \cdot are respectively cross-product and dot-product.

Matrix \mathbf{J}_p expressed in (8) can be manipulated in order to enlighten blocks. It is then possible to rewrite expression (2). It leads to the following equation:

$$\begin{bmatrix} \mathbf{l}_1^T & (\mathbf{c}_1 \times \mathbf{l}_1) \cdot \mathbf{e}_z & [\mathbf{e}_1 \times \mathbf{l}_1]^T \\ \mathbf{l}_2^T & (\mathbf{c}_1 \times \mathbf{l}_2) \cdot \mathbf{e}_z & [\mathbf{e}_2 \times \mathbf{l}_2]^T \\ \mathbf{l}_3^T & (\mathbf{c}_2 \times \mathbf{l}_3) \cdot \mathbf{e}_z & [\mathbf{e}_3 \times \mathbf{l}_3]^T \\ \mathbf{l}_4^T & (\mathbf{c}_2 \times \mathbf{l}_4) \cdot \mathbf{e}_z & [\mathbf{e}_4 \times \mathbf{l}_4]^T \\ \mathbf{0} & 0 & [\mathbf{f}_1 \times \mathbf{l}_1]^T \\ \mathbf{0} & 0 & [\mathbf{f}_2 \times \mathbf{l}_2]^T \\ \mathbf{0} & 0 & [\mathbf{f}_3 \times \mathbf{l}_3]^T \\ \mathbf{0} & 0 & [\mathbf{f}_4 \times \mathbf{l}_4]^T \end{bmatrix} \begin{bmatrix} \dot{x} \\ \omega_z \\ \omega_x \\ \omega_y \\ \omega_z + \dot{\mathbf{e}}_1 \end{bmatrix} = \mathbf{J}_q \dot{\mathbf{q}} \quad (9)$$

We recognize equation (1) with additional terms:

$$\begin{bmatrix} \mathbf{J}_x & \mathbf{J}_x^{int} \\ \mathbf{0} & \mathbf{J}_{int} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{v}_{int} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_q \dot{\mathbf{q}} \\ \mathbf{0} \end{bmatrix} \quad (10)$$

With this modeling, \mathbf{J}_{int} has the following expression:

$$\mathbf{J}_{int} = \begin{bmatrix} [\mathbf{f}_1 \times \mathbf{l}_1]^T \\ [\mathbf{f}_2 \times \mathbf{l}_2]^T \\ [\mathbf{f}_3 \times \mathbf{l}_3]^T \\ [\mathbf{f}_4 \times \mathbf{l}_4]^T \end{bmatrix} \quad (11)$$

and,

$$\mathbf{v}_{int} = [\omega_x \ \omega_y \ (\omega_z + \dot{\varepsilon})]^T \quad (12)$$

We can notice that the $[4 \times 3]$ matrix (11) shows that the mechanism is over-constrained (over-determined system). In addition, developing equation (10) leads to:

$$\begin{cases} \mathbf{J}_x \dot{\mathbf{x}} + \mathbf{J}_x^{int} \mathbf{v}_{int} = \mathbf{J}_q \dot{\mathbf{q}} \\ \mathbf{J}_{int} \mathbf{v}_{int} = \mathbf{0} \end{cases} \quad (13)$$

The second equation of system (13) shows that additional terms \mathbf{v}_{int} will be null if \mathbf{J}_{int} is a full-rank matrix. It means that mechanism will not have “internal singularities”:

$$\text{rank}(\mathbf{J}_{int}) = 3 \quad (14)$$

Additionally, the usual kinematic relation (1) will be derived.

Considering that the four pairs of rods have a symmetric contribution on the mechanism, (14) leads to calculate one determinant D_{ijk} (among the four possible ones), and to verify that:

$$\exists (i, j, k) \in \{(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)\}, D_{ijk} \neq 0 \quad (15)$$

where,

$$D_{ijk} = ((\mathbf{f}_i \times \mathbf{l}_i) \times (\mathbf{f}_j \times \mathbf{l}_j)) \cdot (\mathbf{f}_k \times \mathbf{l}_k) \quad (16)$$

This working condition remains always true while placing the motors at 90° one relatively to each other and scanning the whole workspace. This fact is a key issue for robot performance.

To summarize, Par4 has been developed with the aim of reaching high speed and acceleration. Thus, the adopted configuration is a good compromise between H4 and I4 robots: passive joints used in the articulated traveling plate are only revolute joints and motors are placed in homogenous arrangement.

IV. OPTIMIZATION OF PICK-AND-PLACE ROBOTS

Having the general configuration of the mechanism, it is necessary to optimize its geometrical parameters. The aim of this optimization is to satisfy some industrial constraints, while guaranteeing our final goal: to reach high speed and acceleration. Thus, an optimization method dedicated to pick-and-place robots has been developed.

A. Principle of optimization method

The aim of this method is to determine the geometrical parameters in order to: *i)* obtain a robot with the smallest footprint, *ii)* with the largest workspace, *iii)* while guaranteeing the possibility of performing a classical pick-and-place cycle (named Adept Motion: see § III.B) in a given time, *iv)* and having a homogenous behavior in the workspace.

We decided to minimize the cost function Ψ :

$$\Psi = \frac{L+l}{D} \quad (17)$$

where L is length of arms, l is length of forearms, and D is the diameter of workspace. Lengths are chosen equal for each kinematic chain.

This cost function was chosen in order to satisfy conditions *i)* and *ii)* at the same time.

The cost function Ψ is minimized under the following constraints:

- Verifying that maximum joint velocities of actuators are always lower than a given value while performing the Adept Motion (see §III.B) for a given time,
- Verifying that conditioning number of the weighted jacobian matrix is always lower than a given value.

This first constraint characterizes condition *iii)* of optimization goals and the second one characterizes condition *iv)*. The optimization process can be summarized as shown on Fig. 6.

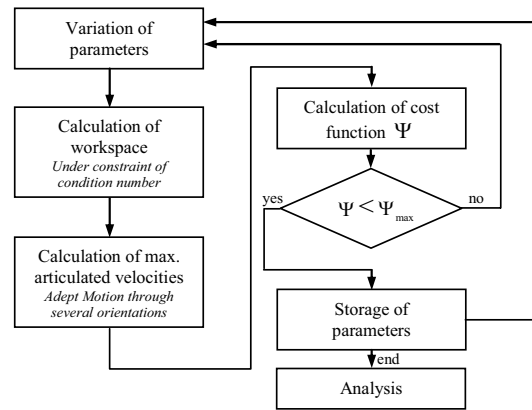


Fig. 6 Optimization process

For each set of parameters, the cost function is calculated using workspace and adept motion computations. If this cost is lower than a given value, the parameters are stored. This computation is done with other set of parameters until the end of the computation.

Before presenting results obtained with this method applied to Par4, the following paragraph will present Adept Motion and the way to obtain it.

B. Adept Motion

This motion is well-known for commercial pick-and-place robots. It permits to evaluate performances of a robot for a pick-and-place task. It is defined by a length and an altitude. The indicator is the time made by the robot to complete a “round trip”. Its shape is given at Fig. 7.

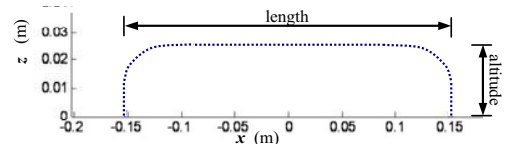


Fig. 7 Adept Motion shape

As seen previously, we used the simulation of Adept Motion in order to determine maximum joint velocities.

Applying a time law, it is possible to determine absolute position and velocity function of time, $x(t)$ and $\dot{x}(t)$. By consequence $q(t)$ and $\dot{q}(t)$ are obtained using models of robot.

Three options have been studied in order to realize Adept Motion: Bezier curves, clotoide, and half ellipses.

1) Bezier curves

Bezier curves are defined using controlled points. As proposed in [19], a time law can be applied on it using a 4th degree polynomial interpolation [20]. However, this interpolation may cause velocity discontinuities.

2) Clotoide

The second option studied is clotoide equations in order to avoid discontinuity while connecting two lines [21]. Absolute coordinates are expressed using Fresnel integrals that do not have analytical solution and have to be resolved by Taylor's series.

3) Half-ellipse

A half-ellipse can be defined using three points: start, end, and transition points. Knowing these points, half-perimeter of ellipse can be approximated [23] by:

$$d \approx \pi \sqrt{1/2(A^2 + B^2)} \quad (18)$$

where A is the length of big half-axis of ellipse and B is length of little half-axis.

As proposed in [22], equation of ellipse can be easily approximated using projection of curvilinear abscissa of a circle on the ellipse. Applying a time law $s(t)$, absolute positions can be expressed:

$$X(t) = A \cdot \cos(\pi \cdot s(t)/d) \cdot e_x + B \cdot \cos(\pi \cdot s(t)/d) \cdot e_z + C \quad (19)$$

where C is the center of ellipse.

This simple solution has been kept in the optimization algorithm described at § III.A.

C. Optimization results

The described method has been applied to Par4 robot. Optimized parameters (see Fig. 8) were length of arms L , length of sub-arms l , position of actuators R (corresponding to a radius) and the altitude of workspace center Z_0 . In order to be in accordance with industrial requirements, diameter of workspace D has been fixed to 1 meter.

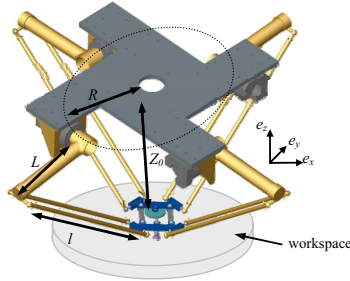


Fig. 8 Optimized parameters

Characteristics of Adept Motion are: length of 305 mm, altitude of 25 mm, and round trip time of 0.28 s.

For each L , a set of “optimum parameters” has been obtained (see Fig. 9).

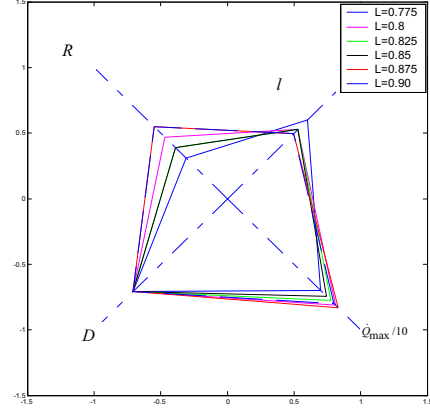


Fig. 9 Sets of optimal parameters

Optimal parameters chosen are a compromise among the sets of parameters described on Fig. 9 :

$$L = 0.825 \text{ m}, l = 0.375 \text{ m}, R = 0.275 \text{ m}, Z_0 = -0.58 \text{ m}$$

It leads to the maximum articulated velocity:

$$\dot{Q}_{\max} = 21.9 \text{ rad.s}^{-1}$$

This compromise has been chosen in order to obtain a correct maximum actuated joint velocity while geometrical constraints L , l and R have reasonable values.

IV. EXPERIMENTAL RESULTS

Using optimal parameters presented in § III, and architecture developed with the aim of reaching very high speed in § II, a prototype (see Fig. 2) has been built.

All tests have been performed using Adept Motion (as described at III.B.3) with a length of 305 mm and an altitude of 25 mm. The control loop is P/PI applied on actuated variables.

The first experimentations have been done with the following characteristics: maximal velocity (v) = 2.5 m.s⁻¹, maximal acceleration (a) = 100 m.s⁻². Obtained cycle time (round trip) is: 0.45 s. Records of this experimentation are presented at Fig. 10.

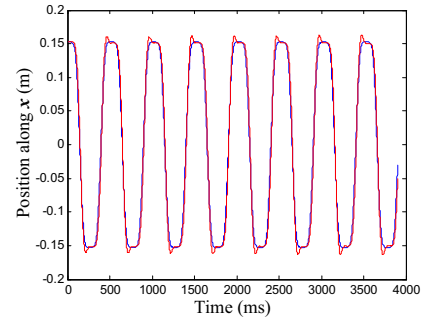


Fig. 10 Evolution of position along x ($v=2.5 \text{ m.s}^{-1}$, $a=100 \text{ m.s}^{-2}$)

The second experimentations have been made applying high speed and accelerations: $v = 3.8 \text{ m.s}^{-1}$, $a = 130 \text{ m.s}^{-2}$. The obtained cycle time is: 0.28 s. (see Fig. 11).

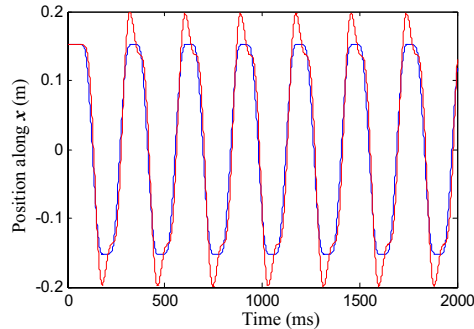


Fig. 11 Evolution of position along x ($v = 3.8 \text{ m.s}^{-1}$, $a = 130 \text{ m.s}^{-2}$)

These experimentations show that we are able to have very good performances and to complete short cycle time. However, an overshoot is obtained as shown on Fig. 11. This restriction is partially due to torque limitation of actuators used on prototype.

IV. CONCLUSION

This paper introduces a new four-*dof* parallel manipulator dedicated to pick-and-place and developed to perform high speed and acceleration. It shows that this robot is an improvement of H4, I4 and Delta robots and its architecture has been developed to overcome drawbacks of these existing robots. The key point of this architecture is its symmetrical arrangement of actuators (involving homogenous behavior) and the use of articulated traveling plate made of revolute joints. This key point has been proved making a complete analysis of all singularities. In addition, the Adept Motion, a reference motion used by industrial people to evaluate pick-and-place robots, has been used to run an optimization. The concrete result obtained is a prototype able to reach an acceleration of 13 G and a cycle time equal to 0.28s.

The future work we will make on this robot will be based on dynamic modeling and dynamic control in order to improve the behavior of the robot while reaching high acceleration.

A demonstration video of Par4 prototype can be consulted at:

<http://www.lirmm.fr/~nabat/par4.wmv>

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