On the workspace of suspended cable-driven parallel robots

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***Abstract*— Workspace calculation for cable-driven parallel robot (CDPR) differs from the one for robots with rigid legs as the main limiting factor is the satisfaction of the mechanical equilibrium, being given a model for the cable behavior. We are considering here 6-dof CDPR with $n \geq 6$ cables and are investigating the calculation of horizontal cross-sections of the workspace under the assumption that the orientation of the platform is constant. We show that for non deformable or elastic mass-less cables it is possible to calculate the border of the workspace under the constraints $\tau_{min} \leq \tau \leq \tau_{max}$ where τ is the cable tension and τ_{min}, τ_{max} are constant thresholds. For sagging cables the calculation is much more complex: we present however a preliminary, computer intensive algorithm that allow to obtain an approximation of the cross-section as a set of boxes. We also emphasize that for CDPR the definition of the workspace as the set of poses that can be reached has to be considered stricto sensu and cannot be used for other purpose: for example the inclusion of a trajectory in the workspace is not a proof of the feasibility of this trajectory.**

I. INTRODUCTION

Workspace is an important performance criteria for robots and therefore play a major role for their design. Workspace calculation is usually more complex for parallel robots compared to serial manipulator as there is a coupling between the translational and orientational capabilities. However workspace calculation for parallel robots with rigid legs (PRRL) is a well-addressed subject [1], [2]. The main constraints that limit the workspace of PRRL are the limited stroke of the actuators, the limited rotations of the passive joints, the singularities and leg interference. In this paper we will consider a special type of parallel robot, cable-driven parallel robots (CDPR), where the rigid legs are substituted with coilable cables. We will assume 6-dof CDPR that are in a *suspended* configuration where no cable has the ability to pull the platform downward.

For CDPRs the passive joints that are classically used for PRRL, such as S and U joints for a Gough platform, are substituted by cable deformation at the anchor point, hence drastically reducing the rotation limitation constraints. In the same manner possible changes of leg lengths are much larger than for PRRL and the limitation on these changes play usually a limiting role only on the vertical excursion of the platform motion. On the other hand a major limiting role for the platform motion is the unilateral action of the cables that can only pull but cannot push. Hence for a given wrench $\mathcal{F}$ acting on the platform a pose will be reachable if and only if the tension in the cables are compatible with the nature of the cables. This compatibility will depend on the cable model and in this paper we will consider 3 cable models:

- *straight line cable*: the cable profile is the segment joining the cable anchor points and there is no elasticity in the cable
- *straight line linear elastic cable*: the cable profile is the segment joining the cable anchor points and the length of the cable changes with the tension in the cable
- *sagging cable*: the cable profile is influenced by the cable own mass and its elasticity

As for PRRL there is a coupling between the possible translation and orientation of the platform of a CDPR. Hence to simplify the workspace calculation we will consider a specific workspace, the *constant orientation workspace*, i.e. the 3D set of poses that can be reached with a given orientation of the platform. Furthermore we will even restrict this workspace by imposing that the altitude of the platform will be constant, i.e. we will calculate horizontal cross-sections of the constant orientation workspace and impose that the 6-dof of the platform may be controlled.

Singularities of PRRL are obtained when the direct kinematic (DK) equations have an infinite number of solutions or, equivalently, when the determinant of the matrix of the linear system of static equations is equal to 0. When considering 2D cross-section of the workspace singularity loci curves can be obtained analytically [3]. For CDPR with straight line cable model the DK equations are similar to the one of PRRL, leading to the same loss of control consequence. Therefore singularities must also be taken into account for workspace calculation. As for leg interference methods [4] used for PRRL may be used for CDPR with straight line cable models but more complex algorithms must be used for sagging cable [5], [6]: in this paper we will however not consider leg interference.

Numerous works have addressed the problem of workspace calculation of CDPR [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18] but all of them assume a straight line cable model. For elastic or sagging cables only a few discretisation-based methods, that are extremely computer intensive, have been proposed[19],[20]. For straight line cable models our approach is closer to [21], [17] which calculates the workspace border equations for planar robots but we will consider here spatial robot and will consider more closely the impact of redundancy.

We must however point out a severe limitation of workspace calculation for CDPR: its definition has to be considered *stricto sensu* i.e. a pose inside the calculated workspace is reachable. For PRRL if a trajectory is included in the calculated workspace then it can usually be performed without drastic changes in the actuation forces and in the platform pose provided that non-singular change in the

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assembly modes are taken into account [22], [23]. This is not the case for CDPR that may exhibit large change in cable tensions (even simply due to the effect of discrete-time control [24] and/or small errors in the stiffness parameters of elastic cables [25]) and even platform pose. Hence for CDPR the inclusion of a trajectory in the workspace is not a proof that it can be realized.

In his paper we will consider full 6 dof CDPR with $n \geq 6$ cables. Cables i outputs from its coiling system at point A_i while the other extremity of the cable is attached on the platform at point B_i. The set of parameters that describe the pose is denoted by $\mathbf{X}$. Part of $\mathbf{X}$ describe the location of the platform center of mass C in a reference frame $O, \mathbf{x}, \mathbf{y}, \mathbf{z}$ where $\mathbf{z}$ is the local vertical. Another part of $\mathbf{X}$ allows one to describe the constant platform orientation. As we are interested only in horizontal cross-sections of the workspace only the x, y components of $\mathbf{X}$ will be of interest. As static equations play a fundamental role we have to consider which wrench is applied on the platform: in this paper we will assume that the platform is submitted only to gravity.

For the numerical example we will consider a CDPR with 8 cables. The coordinates of the A, B points are provided in tables I,II and are derived from the robot presented in [26].

	A_1	A_2	A_3	A_4
x	-7.175120	-7.315910	-7.302850	-7.160980
y	-5.243980	-5.102960	5.235980	5.372810
z	5.462460	5.472220	5.476150	5.485390
	A_5	A_6	A_7	A_8
x	7.182060	7.323310	7.301560	7.161290
y	5.347600	5.205840	-5.132550	-5.269460
z	5.488300	5.499030	5.489000	5.497070

TABLE I

COORDINATES OF THE A POINTS (IN METER)

	B_1	B_2	B_3	B_4
x	0.503210	-0.509740	-0.503210	0.496070
y	-0.492830	0.350900	-0.269900	0.355620
z	0.000000	0.997530	0.000000	0.999540
	B_5	B_6	B_7	B_8
x	-0.503210	0.499640	0.502090	-0.504540
y	0.492830	-0.340280	0.274900	-0.346290
z	0.000000	0.999180	-0.000620	0.997520

TABLE II

COORDINATES OF THE B POINTS IN THE MOBILE FRAME (IN METER)

The external wrench applied on the platform is supposed to be a downward vertical force of 1 Newton.

II. WORKSPACE FOR STRAIGHT LINE CABLE

A. Preamble

We consider a CDPR with $n \geq 6$ cables whose lengths will be denoted ρ while the tensions in the cables will be τ. For this cable model there is a unique solution of the inverse kinematic equations i.e for each cable i there is an unique ρ_i that will correspond to a given pose $\mathbf{X}$. Note that ρ_i is simply obtained as $\sqrt{||\mathbf{A_iB_i}||^2}$. On the opposite as soon as 6 cables are under tension, then the platform pose is fixed, being one of the up to 40 solutions of the DK. As we cannot assume an infinite accuracy of the cable lengths controller a direct consequence is that at any time there will be at most 6 cables under tension while the $n-6$ other cables are slack. Note that this fact is independent from the cable controlled variable whether length or tension. Indeed for non-deformable cable and suspended CDPR (i.e. with no cable that can exert an opposing force), tension control amounts to changing the cable length which will lead either to the cable being slack or to a change in the pose. A *cable configuration* is a set of 6 cables under tension among the n cables but not all cable sextuplets represent a cable configuration as for some of them the solution in τ of the static equations will lead to cables(s) that have to push and therefore are not valid. For such cable model the static equations are written as

$$\mathcal{F} = \mathbf{J^{-T}}\tau \tag{1}$$

where $\mathcal{F}$ is the wrench applied on the platform (here the gravity) and $\mathbf{J^{-T}}$ the transpose of the inverse kinematic jacobian. The i-th column J_i of this matrix is written as:

$$J_i = (\frac{\mathbf{A_iB_i}}{\rho_i} \quad \frac{\mathbf{CB_i} \times \mathbf{A_iB_i}}{\rho_i})$$

With this notation a cable i is under tension if $\tau_i < 0$ (note that we may fall back to the more classical condition $\tau_i > 0$ by reverting $\mathcal{F}$.. We introduce another matrix $\mathbf{J_m^{-T}}$ which is derived from $\mathbf{J^{-T}}$ by removing all the ρ that appear as denominator in the elements of $\mathbf{J^{-T}}$. To each cable configuration is associated a specific workspace and the robot workspace is the sum of these workspaces, that is sometimes called the wrench-closure workspace.

B. Computing the border of the workspace defined by $\tau_i = 0$

In this section we are considering a specific cable configuration and the workspace constraint $\tau_i \leq 0$ for all cables of the configuration. Necessary and sufficient conditions for a pose $\mathbf{X_0}$ to be part of the border are:

1) there is j such that $\tau_j = 0$
2) there are two vectors $\mathbf{N_1}, \mathbf{N_2}$ such that for an arbitrary small ϵ we have $\tau_j > 0$ for the pose $\mathbf{X_0} + \epsilon\mathbf{N_1}$ and $\tau_j < 0$ for the pose $\mathbf{X_0} + \epsilon\mathbf{N_2}$

to which must be added the possibility that the robot is singular at the pose.

If we apply Cramer's rule on the linear system (1) each τ_k will be obtained as

$$\tau_k = \rho_k P_k / D \tag{2}$$

where D is the determinant of $\mathbf{J_m^{-T}}$ and P_k the minor associated to column k. Using symbolic software tools the analytical expressions of P_k, D can easily be obtained. Both P_k, D are polynomial in x, y of total degree 3 and maximal degree 3. Consequently the border of the workspace will be composed of the curves $P_k = 0$ for all 6 cables and $D = 0$ or part of these curves. We will denote by $\mathcal{C}$ the set of the 6 curves $P_k = 0$ and $D = 0$. An ordered set of points on the curve (that may have several connected components) may be obtained with symbolic software tools: we will denote by S_k the set of points for the curve C_k of $\mathcal{C}$.

For determining the border we will first compute the intersection points of all the pairs of curves in $\mathcal{C}$. This is simply done by computing the resultant of the pair of polynomials C_k, C_l in one variables (say x) leading to a polynomial in y that is solved. Being given a root for y the roots of C_l, C_k in x are compared and the common root is kept, leading to all points that belongs both to C_k and C_l. For a given curve in $\mathcal{C}$ the set of intersection points with all the other curves will be called the *critical points* of the curve. The critical points of each curve C_k will be added to the set S_k. If $P^m_{S_k}$ denote the m-th point of the set S_k we build an augmented set of ordered critical points $\mathcal{T}$ on the curve by adding to the the critical points the first and last points of the curve. We consider now the arcs $\mathcal{A}_k$ of the curve C_k that are constituted of the points between two successive critical points T_l, T_{l+1}. The q arc $\mathcal{A}^q_k$ of $\mathcal{A}_k$ may or may not be part of the border. If any point M of $\mathcal{A}^q_k$ belongs to the border, then the whole arc is part of the border. Such a point satisfy by definition the first necessary and sufficient conditions to be on the border i.e. $P_k = 0$ or $D = 0$. We will then compute the normal vector $\mathbf{N}$ to the curve at M as $(\partial C_k/\partial x, \partial C_k/\partial y)$ computed at M and we will set $\mathbf{N}_1 = \mathbf{N}/||\mathbf{N}||$ and $\mathbf{N}_2 = -\mathbf{N}/||\mathbf{N}||$. To check the second part of the conditions we have to determine a positive ϵ small enough such that the points $M_1 = M + \epsilon\mathbf{N}$ and $M_2 = M - \epsilon\mathbf{N}$ do not belong to another component of the workspace. For that purpose we note that the coordinates of the points M_1, M_2 are only function of ϵ. We then consider each curve C_l with $l \neq k$ and examine if M_1, M_2 may belong to C_l, which leads to two polynomials in ϵ that are solved, retaining only the positive solutions. If there is such positive solution we consider the lowest one $\epsilon_m$and set ϵ to $\epsilon_m/2$. If no positive solution is found then we set ϵ to any arbitrary positive value. Such a value of ϵ guarantee that we have not selected points inside another workspace component. We then compute the cable tension at point M_1, M_2 and if one component is positive for one point and negative for the other one, then the arc is part of the border. This treatment involves only checking if two points belong or not to the workspace. An alternate method will be to check the value of the derivatives of the tension when moving along $\mathbf{N_1}, \mathbf{N_2}$ but will more computer intensive. Note a special treatment of the curve $D = 0$: if the tension are all positive at point M_1 and M_2, then the curve is an *internal border* that splits the workspace in components. Going from one component to another component involves crossing a singularity.Note that the workspace procedure may seem to be tedious because we may have used directly available tracing routine based on implicit definition of the workspace but in practice the approach is almost as fast, is guaranteed to be exact and furthermore provide explicit equations for the border curves which will allow to compute exactly the workspace area.

As an example we set $z = 2$ and the platform orientation matrix equal to the identity. Figure1 shows the $x - y$ cross-section of the workspace for the cable configurations [1, 2, 3, 4, 5, 6] and [1, 2, 3, 4, 5, 7].

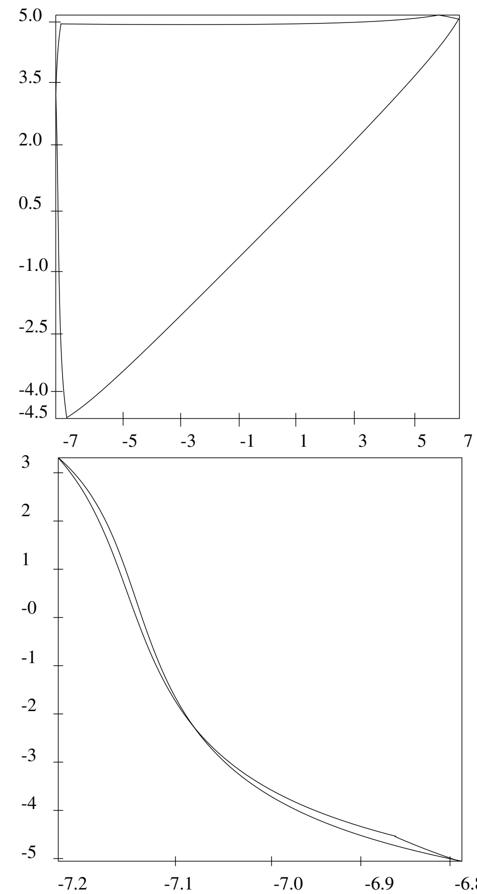

Fig. 1. Workspace for $z = 2, \mathbf{R} = \mathbf{I_3}$ for the cable configurations [1, 2, 3, 4, 5, 6] and [1, 2, 3, 4, 5, 7].

C. Computing the border of the workspace defined by $\tau_{min} \leq |\tau_i| \leq \tau_{max}$

Now let us assume that beside the constraint $\tau_i \leq 0$ we want to impose an upper bound for the cable tension i.e. we want to compute the set of poses such that $\tau_i \leq 0$ and $\tau_{min} \leq |\tau_i| \leq \tau_{max}$ where τ_{min}, τ_{max} are arbitrary positive thresholds. Such a workspace is usually called the *wrench-feasible* workspace. With this additional constraint part of the border of the workspace may be constituted of poses at which we have $\tau_i \leq 0$ and $|\tau_i| = \tau_{max}$ or $|\tau_i| = \tau_{min}$. Note however that the later equations are not algebraic as the expression of τ_i (2) is not algebraic because of the square root of ρ_i. But this problem may be dealt with by considering the curve $U_k = \rho_k^2 P_k^2 - \tau_{max}^2 D^2 = 0$ and $V_k = \rho_k^2 P_k^2 - \tau_{min}^2 D^2 = 0$. Such curves regroup the loci of the pose where $\tau_k = \pm\tau_{max}(\tau_{min})$ but this will not create an issue.

For the management of these constraints we will add the 12 curves U_k, V_k to the previously determined set of curves $\mathcal{C}$ and we proceed with the same method than described in the previous section, except that a given arc between two successive critical points belongs to the border by checking if $\tau(M_i) < -\tau_{max}, \tau(M_j) > -\tau_{max}$ or $\tau(M_i) < -\tau_{min}, \tau(M_j) > -\tau_{min}$ where i, j are either 1 or 2.

As an example we consider the cross-section obtained for $z = 2$ and the platform orientation matrix equal to the identity. Figure 2 shows the $x - y$ cross-section of the workspace for the cable configurations [1, 2, 3, 4, 5, 6] and [1, 2, 3, 4, 5, 7] with the constraint $0.01 \leq |\tau_i| \leq 0.5$.

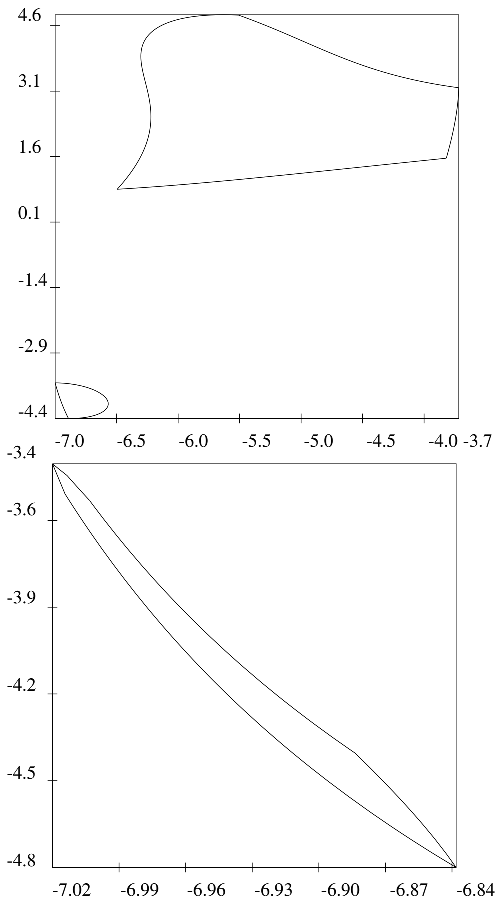

Fig. 2. Workspace for $z = 2, \mathbf{R} = \mathbf{I_3}$ for the cable configurations [1, 2, 3, 4, 5, 6] and [1, 2, 3, 4, 5, 7] with the constraint $0.01 \leq |\tau_i| \leq 0.5$.

III. Workspace for straight line elastic cables

In this case a cable is a force generator and the elasticity law is used to determine what should be the cable lengths at rest so that when the platform reaches its desired pose the overall cable tensions will equilibrate the external wrench applied on the platform. Hence even if $n > 6$ all cables may be under tension. If $n = 6$ the cable tensions are uniquely determined and the workspace border will be obtained in the same manner than for non elastic cable. Thus we consider only the case where $n > 6$. Equations (1) is a linear system of 6 equations in n unknowns and we may choose any set $\mathcal{H} = \{\tau_m, \ldots \tau_u\}$ of $n-6$ tensions and solve the equations to obtain 6 cable tensions as functions of these $n - 6$ tensions. If τ_H^j denotes the j-th element in the set $\mathcal{H}$ we will obtain the value of a τ_i that does not belong to $\mathcal{H}$ as

$$\tau_i = \rho_i(\sum_{k=1}^{k=n-6} a_k \tau_H^k + a_{n-5})/D \tag{3}$$

where the a_k are coefficients that depends algebraically upon the pose of the platform. If τ_i is positive for a given choice of the τ_H^j and if we do not impose limits on the τ_H^j, then we may get $\tau_i < 0$ if there is at least one of the a_k that is positive by just decreasing the corresponding τ_H^j. Hence a necessary condition for a pose to be on the border is that all $a_k = 0$ and $a_{n-5} = 0$ while the condition $D = 0$ has still to be considered. For a given $\mathcal{H}$ we will get $6 \times (n-5)$ constraint equations ($a_k = 0$, $a_{n-5} = 0$) plus one ($D = 0$). As we have to consider all possible $\mathcal{H}$ the total number of curves is $C_6^n(1 + 6 \times (n-5))$ which leads to 532 curves for $n = 8$. Note however that some of these curves will be identical as equation (3)may be used to obtain one of the τ_H^k as function of τ_i and of the other τ_H^k with the same constraint curves. For $n = 8$ there will be only 84 different curves to consider. In spite of this relatively high number of curves testing their intersection is very fast as most of curve pairs do not intersect. Typically the highest number of critical points we have found in our test was less than 20.

The critical points will then be obtained in the same manner than in the previous section. It remains then to produce a test to determine if the arc of a curve between two successive critical points is part of the border. As in the previous section we will select arbitrarily a point on the arc and calculate the coordinates of the point M_1, M_2 that are on both side of the curve. For each of them we will then simply use the simplex feasibility test to determine if there is a solution of the equations (1) under the constraints $\tau_i < 0$. If the result is true for one of the M_1, M_2 and wrong for the other one, then the arc is part of the border.

As an example figure 3 presents the robot workspace for $z = 2, \mathbf{R} = I_3$ and for $z = 2$ and a rotation of the platform of $\pi/3$ radian around the x axis.

Note that this algorithm may be extended to manage constraint of the type $\tau_{min} \leq \tau \leq \tau_{max}$. For that purpose we will use equation (3) that we will square to get rid of the non algebraic ρ_i to get

$$\tau_i^2 D^2 - \rho_i^2(\sum_{k=1}^{k=n-6} a_k \tau_H^k + a_{n-5})^2 = 0 \tag{4}$$

A border curve (i.e. a curve whose part is an element of the border) will be obtained when the values of the τ_i, τ_H^k are either equal to τ_{min} or to τ_{max}, leading to an algebraic curve. Hence as we are considering all possible combination for these $n-5$ τ we will get 2^{n-5} constraint equation. Hence for $n = 8$ we will have to consider 672 (8×84) curves.

The most computer intensive part of the workspace algorithm is getting a significative number of points on the curves, with about 15 seconds for each curve. The remaining of the computation is neglectible compared to this part.

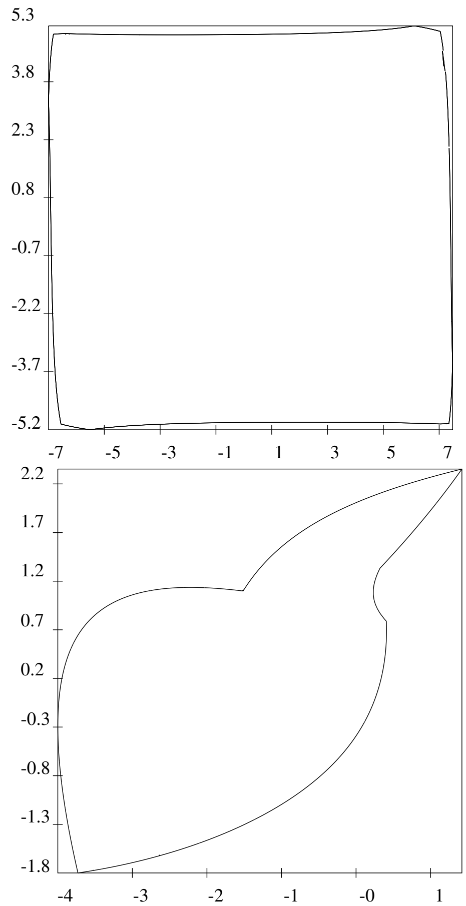

Fig. 3. Workspace of a robot with elastic cables for $z = 2, \mathbf{R} = \mathbf{I_3}$ and $z = 2$ and a rotation of the platform of $\pi/3$

IV. Workspace for sagging cables

A sagging cable model is usually established in the cable vertical plane by assigning the origin of the plane frame to A. The coordinates of B in this frame are (x_b, z_b) while F_x, F_z are the components of the force exerted by the platform on the cable (F_z is the vertical component and F_x is assumed to be strictly positive). With these notations the sagging model provide non algebraic relationships between x_b, z_b, F_x, F_z and the length at rest of the cable l_0 [27], [28], [29]. Sagging has a drastic influence on the kinematics problem of CDPR. For example the IK for the two previous cable models were relatively simple: basically it consists in determining if the linear system (1) may have only negative solutions, a problem that is easily addressed with classical linear algebra methods. This is no more the case for sagging cables as the IK equations are no more linear, may have multiple solutions and are complex to solve [30]. The complexity and computation time of solving the IK prohibits using a discretisation method for estimating even cross-sections of the workspace. Currently there is no known method to obtain an analytical form of the border curves.

We will now suggest a method that allows one to compute an approximation of the cross-section of the workspace. We will first consider the case where $n = 6$ for which the IK is a square system of 18 equations. Let us assume that we are able to find a pose $\mathbf{X_s} = (x_s, y_s)$ such that the IK has at least one solution $\mathbf{P_s}$. An important theorem for determining an IK solution is the Kantorovitch theorem. Let $\mathbf{X_0}$ be a possible solution of an equation system $\mathbf{F}$ with n equations and define a ball U centered at $\mathbf{X_0}$ with radius B_0. We denote $||\mathbf{G}||$ the norm of a vector or a matrix $\mathbf{G}$ of dimension p or $p \times p$. This norm may be arbitrary but we will use here $\mathrm{Max}(|G_i|) \forall i \in [1, p]$ for vectors and $\mathrm{Max}(\sum_{j=1}^{j=p} |G_{ij}|) \forall i \in [1, p]$. Let's assume that the following conditions hold:

1) the Jacobian matrix of the system has an inverse Γ_0 at $\mathbf{X_0}$ such that $||\Gamma_0|| \leq A_0$
2) $||\Gamma_0 \mathbf{F}(\mathbf{X_0})|| \leq 2B_0$
3) $\forall \mathbf{X} \in U \; \sum_{k=1}^{n} |\frac{\partial^2 F_i(\mathbf{X})}{\partial x_j \partial x_k}| \leq C$ for $i, j = 1, \ldots, n$

Then if $2nA_0B_0C \leq 1$ there is an unique solution of $\mathbf{F}$ in U and the Newton method, used with $\mathbf{X_0}$ as initial estimate of the solution, will converge toward this solution.

In an interval analysis-based solving algorithm $\mathbf{X_0}$ is taken as the centers of the boxes that are the result of the bisection process and therefore is a scalar vector.

We will now explain how this theorem may be used to determine a ball that is included in the robot workspace. Consider a pose $\mathbf{X_s^0}$ that is very close to $\mathbf{X_s}$ and a box $\mathcal{B}_x$ centered at $\mathbf{X_s^0}$ with radius ϵ_1 and a box $\mathbf{P_S^0}$ centered at $\mathbf{P_s}$ with radius ϵ_2. In the Kantorovitch theorem $\mathbf{X_0}$ is taken as the ball $\mathcal{B}_x$ and therefore is an interval vector. Consequently the jacobian matrix of the system $\mathbf{J_0}$ is an interval matrix but nevertheless an inverse interval matrix may be calculated (this matrix includes all possible inverse matrices of any scalar matrix instance included in $\mathbf{J_0}$). The norm of this inverse is an interval and A_0 will be set to the upper bound of this interval. In the same way $\Gamma_0 \mathbf{F}(\mathbf{X_0})$ is an interval vector and B_0 will be set to the upper bound of its norm. If the condition $2nA_0B_0C \leq 1$ holds, then the IK has a single solution for any pose in $\mathcal{B}_x$. We have thus designed an *expansion algorithm* that determine if for any $\mathbf{X}$ in $\mathcal{B}_x$ there will be an unique solution of the IK problem. If this algorithms fails, then we decrease the value of ϵ_1, ϵ_2. Note that this method also manage the singularity problem as we are using the inverse of the jacobian matrix of the system: if ϵ_1, ϵ_2 have to be set to a value lower than a threshold we may suspect that there is a singularity nearby (and other methods may be used to locate exactly this singularity) or that $\mathbf{X_s}$ is very close to the border.

We then consider the four poses located at the midpoint of the edges of $\mathcal{B}_x$ (called the *root box*) and calculate their IK solution in order to obtain 4 new starting poses on which the expansion algorithm is applied. This *propagation process* allows one to calculate an approximation of the workspace as a set of boxes.

This algorithm may be extended to consider robots with $n > 6$ cables. First remember that the *width* of an interval is defined as the difference between its upper and lower bound while the width of an interval vector is the largest width of its elements. If $n > 6$, then the IK system is no more square and has more unknowns than equations. However interval analysis may still be used to locate possible IK solutions for a given pose. Indeed a box for the unknowns will be assumed to contain a solution if its width is lower than a small threshold α while the interval evaluations of the IK equations all include 0: this approach will be called the *root detection algorithm*. If no such box is found then we may guarantee that the IK has no solution, while if there is an IK solution such box should exist, although not all of them may include a solution. If such a box is found we set $n-6$ unknowns to the mid-point of their respective intervals so that the IK is now a square system. We then solve the equations with interval analysis which is appropriate as we are looking for a solution within a small search space. If no solution is found we decrease α until either not a single box is found or a solution $\mathbf{P_s}$ is obtained.

We then proceed to the expansion algorithm by setting the $n-6$ unknowns to their values in $\mathbf{P_s}$. The propagation process may however leads to very small box because the values of the $n-6$ unknowns chosen for the root box are not the most appropriate. If this is the case we will run the root detection algorithm to obtain better values.

These algorithms have not been fully implemented but the root detection and the expansion algorithms have been fully implemented with the sagging model proposed in [29]. For the proposed robot it appears that the width of a box $\mathcal{B}_x$ is of the order 2.10^{-7} meters for the middle of the workspace and may decrease to 2.10^{-10} when getting closer to the border. Hence although the expansion algorithm is very fast a full coverage of the workspace will be computer intensive.

V. CONCLUSIONS

Workspace calculation for CDPR is quite different than for PRRL as the constraints limiting the workspace are basically related to the cable tensions. However, as soon as the cable model allows to preserve the linear relationship between the cables tensions and the external wrench, determining the workspace border is relatively easy to manage. For sagging cables the situation is much more complex as there is no clear analytical way to determine the border equation. We have proposed for the first time an algorithm that allow to cover the workspace with boxes but it is quite computer intensive. Clearly a better understanding of the conditions that are required to have a pose on the border of the workspace has to be investigated..

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