Dynamic Modelling and Mass Matrix Evaluation of the DELTA Parallel Robot for Axes Decoupling Control

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Abstract

The recent development of the direct drive DELTA, a 3 to 4 degree-of-freedom parallel robot dedicated to pick and place operations, opened a number of new perspectives in the domain of fast and accurate handling of light objects. The elimination of reduction gearing in these designs although improving acceleration capabilities has the disadvantage of increasing mechanical couplings between robot axes and reflects the full varying inertias directly onto each motor axis. To resolve this problem, the computed-torque control method has been applied to many serial robots over the past few years. Its application to parallel robots has however been restrained by the difficulty in establishing a simple dynamic model that can easily be calculated in real time. In this paper, an efficient method based on the direct application of the virtual work principle is proposed. Contrarily to the other methods developed up to now, it has the advantage of providing explicitly the mass matrix of the robot, which can be used in the control algorithm for axes decoupling. The use of the dynamic model in a feedforward control strategy leads to an improvement of trajectory tracking by a factor 6.

1. Introduction

In recent years an increase of interest in parallel robots has been observed. These structures actually possess a number of advantages when compared to serial arms. The most important one is certainly the possibility to keep the motors fixed on the base thus allowing a large reduction of the active mobile mass of the robot structure. Besides keeping the motors on the base of the robot is a requirement when a direct drive is used. Parallel robots are thus well-suited to direct drive actuation. Another advantage of parallel robots is their higher rigidity. These features allow more precise and much faster manipulations. They however mostly suffer from a limited workspace that can be seen as the intersection of the individual workspace of each serial arm constituting the robot. This limitation has partially been resolved by the discovery of the DELTA robot, a new kind of parallel robot dedicated to the handling of light objects [2]. To fully utilise the DELTA robot potential, a direct drive version has been developed [4]. The advantages of direct drive robots are well known [1]: i.e. simple mechanical structure, elimination of backlash, reduction of friction and noise, higher rigidity for some of them. This concept however requires a greater effort and sophistication at the controller level because of the mechanical coupling and inertia variations effects which are directly reflected on each motor axis. Many control schemes have been proposed in the literature in order to solve this problem. Model-based methods have given improved results for serial direct drive robots [7]. Implementations of the algorithm on planar parallel (five bar linkage) robots have also been reported [8] but are very scarce. The greatest difficulty in this approach lies in developing a numerically simple dynamic model. If this has been mastered in the case of applications for serial robot structures, the same cannot be said for parallel structures. The few attempts to provide a systematic method for the dynamic modelling of parallel robots led to complicated formulations that are not readily applicable for the control of the robot.

In this paper, an efficient dynamic model of the DELTA robot is proposed. It is based on the virtual work principle and has the advantage of giving explicitly the mass matrix of the robot. This latter is also evaluated based on kinetic energy

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considerations. The model has been used in a feedforward control scheme which allows the reduction of tracking errors by a factor 6 compared to a classical PD regulator. A proposition for the axes decoupling control of the DELTA robot is also presented.

2. Geometric parameters of the DELTA

As illustrated in fig. 1, the DELTA robot is made of 3 parallel kinematic chains linked at the travelling plate (3). Each chain is moved, driven by a motor (4) fixed to the robot base. Motions of the travelling plate are achieved by the combination of movements of the arms (1) which is transmitted to the plate by the system of parallel rods (2) through a pair of ball-and-socket passive joints. These parallel rods, also called forearms, assure that the travelling plate always remains parallel to the robot base.

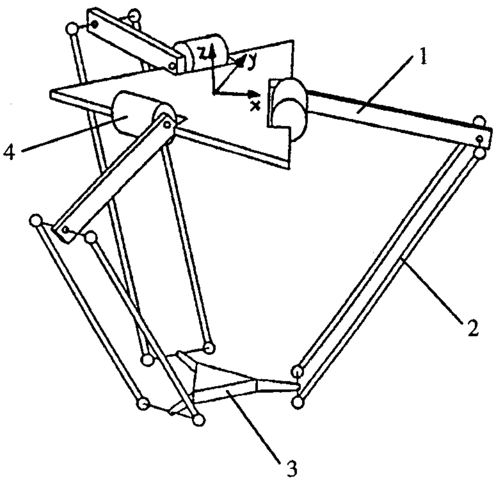

Fig. 1 : The DELTA robot

The absolute reference frame $\{R\}$ is chosen as shown in figure 1, i.e. at the centre of the triangle drawn by the axes of the 3 motors, z pointing upward, and x being perpendicular to the axis of motor 1.

Due of the triple-symmetry of the robot, each arm can be treated separately. Its geometric parameters are defined in figure 2. The index i ($i=1,2,3$) is used to identify the arm number. Each arm is separated by an angle of 120°. For each arm, a corresponding frame is chosen located at the same place as $\{R\}$ but rotated by an angle $\theta_i = 0°$, 120°, 240°, for arm 1, 2 and 3 respectively. The transformation matrix between frame $\{R_i\}$ and $\{R\}$ is given by

$$ {}^{R}_{i}R = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

As the travelling plate can only be translated, a frame attached to it will always keep the same orientation as $\{R\}$. This fact allows us to consider the distance from the reference frame $\{R\}$ to the motor as being $R = R_A - R_B$ and thus $P=B_1=B_2=B_3$, i.e. the travelling plate is reduced to a single point. This definition will simplify the derivation of the model without affecting the results.

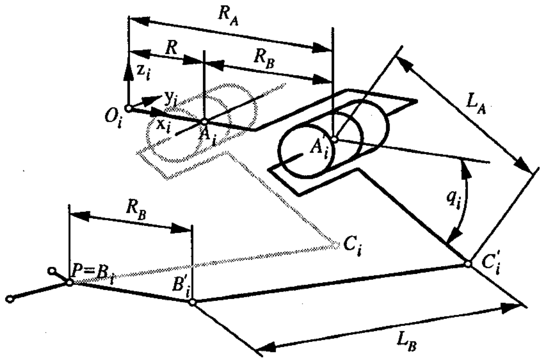

Fig. 2: The geometric parameters of a revolute actuated DELTA

With these definitions, the direct and inverse geometric model can be established as proposed in [3] or [15]. It should however be noted that, in both formulations, the inverse geometric model suffers from mathematical singularities of the type "0/0" that lead to disturbances in the control of the robot at low velocities. To eliminate these singularities, a new model is proposed in [4].

3. Jacobian matrix of the DELTA

The Jacobian matrix J of a robot expresses the relation between operational space velocities and joint velocities as follows :

$$\dot{X} = J\dot{q}. \qquad (2)$$

For serial robots, a systematic approach can be used in order to find this matrix. Unfortunately, this is more difficult for parallel robots [10]. Very often, a loop closure constraint equation is used and differentiated to obtain the velocity relationships [17].

In the case of the delta robot, the Jacobian matrix was first established by Codourey [4]. This was based on a numerical computation of the partial derivatives of the direct geometric model with respect to the joint variables, i.e. :

$$\dot{X}_n = \begin{bmatrix} \frac{\partial f_x}{\partial q_1} & \frac{\partial f_x}{\partial q_2} & \frac{\partial f_x}{\partial q_3} \\ \frac{\partial f_y}{\partial q_1} & \frac{\partial f_y}{\partial q_2} & \frac{\partial f_y}{\partial q_3} \\ \frac{\partial f_z}{\partial q_1} & \frac{\partial f_z}{\partial q_2} & \frac{\partial f_z}{\partial q_3} \end{bmatrix} \dot{q} \qquad (3)$$

where $X_n = f(q)$ represents the direct geometric model of the robot. f_x, f_y, f_z are the functions related to the x, y and z components of X_n respectively. The partial derivative is calculated as follows for the first term above :

$$\frac{\partial f_x}{\partial q_1} = \frac{f_x(q_1 + \Delta, q_2, q_3) - f_x(q_1, q_2, q_3)}{\Delta}, \qquad (4)$$

and so on for the other terms. Δ has to be chosen small enough to reduce as much as possible the errors in the computation of the Jacobian, but large enough in order to avoid numerical noise. This formulation requires four evaluations of the geometric model for the computation of the Jacobian matrix, these are $f(q1,q2,q3)$, $f(q1+\Delta,q2,q3)$, $f(q1,q2+\Delta,q3)$ and $f(q1,q2,q3+\Delta)$.

As mentioned before, another way to compute the Jacobian of parallel robots is to consider a set of constraint equations linking the operational space variables to the joint space variables. This method was first applied to the DELTA robot by Guglielmetti [6]. We describe in the following, a simplified version of this latter formulation.

The three constraint equations in the case of the DELTA robot can be chosen as:

$$\|C_i B_i\|^2 - L_B{}^2 = 0 \qquad i = 1,2,3. \qquad (5)$$

i.e. signifying that the length of the forearms must be constant. Let s_i be the vector C_iB_i. The previous equation can then be written as :

$$s_i^T \cdot s_i - L_B{}^2 = 0 \qquad i = 1,2,3. \qquad (6)$$

with

$$s_i = O_i B_i - (O_i A_i + A_i C_i)$$

$$= \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} - {}^R_i R \left(\begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} L_A \cos q_i \\ 0 \\ -L_A \sin q_i \end{bmatrix} \right)$$

$$i = 1,2,3. \qquad (7)$$

The time derivative of equation (6) then leads to:

$$s_i^T \dot{s}_i + \dot{s}_i^T s_i = 0 \qquad i = 1,2,3. \qquad (8)$$

Due to the commutativity property of the product, this can be rewritten as

$$s_i^T \dot{s}_i = 0 \qquad i = 1,2,3. \qquad (9)$$

where the time derivative of s_i is given by :

$$\dot{s}_i = \begin{bmatrix} \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{bmatrix} + {}^R_i R \begin{bmatrix} L_A \sin q_i \\ 0 \\ L_A \cos q_i \end{bmatrix} \dot{q}_i = \dot{X}_n + b_i \dot{q}_i$$

$$i = 1,2,3. \qquad (10)$$

where

$$b_i = {}^R_i R \begin{bmatrix} L_A \sin q_i \\ 0 \\ L_A \cos q_i \end{bmatrix} \qquad i = 1,2,3. \qquad (11)$$

Rearranging equation (10), with the definitions of equations (7) and (11), into a vector form, the following is obtained :

$$\begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix} \dot{X}_n + \begin{bmatrix} s_1^T b_1 & 0 & 0 \\ 0 & s_2^T b_2 & 0 \\ 0 & 0 & s_3^T b_3 \end{bmatrix} \dot{q} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (12)$$

where $\dot{q} = [\dot{q}_1 \ \dot{q}_2 \ \dot{q}_3]^T$ is the joint space velocity vector. From this last equation the Jacobian matrix of the robot is obtained,

$$\dot{X}_n = J\dot{q}$$

with

$$J = -\begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix}^{-1} \begin{bmatrix} s_1^T b_1 & 0 & 0 \\ 0 & s_2^T b_2 & 0 \\ 0 & 0 & s_3^T b_3 \end{bmatrix} \qquad (13)$$

It is worth noting here that the Jacobian matrix J is not only a function of q as is usually in the case for serial robots, but also a function of the end effector position X_n, evaluated using the direct geometric model of the robot.

3.1. Acceleration

After time derivation of equation (12) above and some transformations, we find:

$$\ddot{X}_n = -\begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix}^{-1} \left(\begin{bmatrix} \dot{s}_1^T \\ \dot{s}_2^T \\ \dot{s}_3^T \end{bmatrix} J + K \right) \dot{q} + J\ddot{q} \quad (14)$$

with:

$$K = \begin{bmatrix} \dot{s}_1^T b_1 + s_1^T \dot{b}_1 & 0 & 0 \\ 0 & \dot{s}_2^T b_2 + s_2^T \dot{b}_2 & 0 \\ 0 & 0 & \dot{s}_3^T b_3 + s_3^T \dot{b}_3 \end{bmatrix}$$

where $\dot{s}_i$ has been calculated above, and $\dot{b}_i$ is given by:

$$\dot{b}_i = {}_i^R R \begin{bmatrix} L_A \cos q_i \\ 0 \\ -L_A \sin q_i \end{bmatrix} \dot{q}_i \quad i = 1,2,3. \qquad (15)$$

In equation (14), the time derivative of the Jacobian $\dot{J}$ can be identified as the term multiplying $\dot{q}$.

4. Dynamic model

The development of robot dynamic models has been a subject of intense research interest over the past decades. The principal difficulty lies in the finding of a solution which is sufficiently representative of the real system and that can easily be calculated in real-time for implementation into the control algorithm. For parallel structures, the problem is even more complex than for serial robots, mainly because of the analytical difficulty presented by the joint variables interdependencies. Research into methods generally applicable to modelling such robots has been carried out but their results do not readily lend themselves to real-time processing. A way that has often been explored is to cut out the closed chain mechanism at passive joints and consider firstly the dynamics of the tree-structure robot thus created. The closure condition is then relaxed either by the use of Lagrange multipliers [19] or the application of the d'Alembert virtual work principle by the mean of some Jacobian matrix [8]. For special parallel mechanisms, the direct application of the Newton-Euler method has also been studied [13][14]. Its application to the DELTA robot can be found in [4] or [5]. Methods based on the virtual work principle [18], on Lagrange's formalism [9][12] or on Hamilton's principle [11] have also been investigated. Most of the authors agree that a complete model, taking into account the masses and inertias of all the links leads to very complicated solutions. Thus, for control purposes, some simplifying hypotheses have to be used. The dynamic model of the DELTA robot, developed next uses such simplifications.

4.1. Simplifying hypothesis

For the DELTA robot, the complexity of the model arises mainly due to the movement of the forearms. We can simplify this problem if we choose to neglect their rotational inertias. This assumption is not very restricting due to use of carbon fibres in their construction. Thus, the force between the travelling plate and the arm is in a direction given by the orientation of the forearm. The model developed for the DELTA robot is based on the Newton-Euler method with the following simplifying hypotheses :

- the rotational inertias of forearms are neglected
- for analytical purposes the masses of forearms are optimally separated into two portions and placed at their two extremities, i.e. 2/3 at its upper extremity (elbow) and 1/3 at its lower extremity (travelling plate). This is easily justifiable by the fact that the inertia of a rigid rod of length L and mass m about one of its extremities is given by $I = 1/3\, mL^2$. This is equivalent to placing 1/3 of the mass at the mobile extremity.
- friction effects and elasticity are neglected.

4.2. Dynamic parameters

In the following, several dynamic parameters are used to establish the dynamic model of the robot. They are listed here for convenience.

At the travelling plate level, only the total mass is used, i.e.:

$$m_{nt} = m_n + m_{payload} + 3(1-r)m_{ab}$$

that is the sum of the mass of the travelling plate m_n, the payload and the 3 reported masses of each of the 3 forearms. $r \in [0,1]$ is the ratio of the mass of the forearms that is located at their upper articulation. As explained above, for an optimal distribution of this mass, r is chosen to be equal to 2/3.

At the arm level, the position of the centre of mass of the arm is calculated as:

$$r_{Gb} = L_A \frac{\frac{1}{2}m_{br} + m_c + rm_{ab}}{m_b} \quad (16)$$

with $m_b = m_{br} + m_c + rm_{ab}$,

where m_{br} is the mass of the arm,

m_c the mass of the elbow,

m_{ab} the mass of the forearm, and

$r = 2/3$, the portion of m_{ab} that is placed at the elbow.

The inertia of the arm I_{bi} is the sum of the inertia of the motor I_m and the arm I_{br} and is given by :

$$I_{bi} = I_m + I_{br} \quad (17)$$

where

$$I_{br} = L_A{}^2\left(\frac{m_{br}}{3} + m_c + rm_{ab}\right) \quad (18)$$

4.3. Dynamic model based on the virtual work principle

The use of the Jacobian matrix J of the robot developed in the last section leads to a very simple simplified model for the DELTA robot. With the above mentioned simplifying hypothesis, the robot can be reduced to 4 bodies only: the travelling plate and the 3 upper arms.

Two kinds of forces act on the travelling plate; the gravity force G_n and the inertial force F_n. They are respectively given by:

$$G_n = m_{nt}[0 \quad 0 \quad -g]^T, \quad (19)$$

and $F_n = m_{nt}\ddot{X}_n$. (20)

The contribution of these two forces to each motor can then be calculated by multiplying it with the transpose of the Jacobian matrix.

$$\Gamma_n = J^T F_n = J^T m_{nt} \ddot{X}_n \quad (21)$$

$$\Gamma_{Gn} = J^T G_n = J^T m_{nt}[0 \quad 0 \quad -g]^T \quad (22)$$

According to the virtual work principle, the contribution of all non-inertial forces must equal the contribution of all inertial forces. This applied at the joint level leads to:

$$\Gamma + J^T G_n + \Gamma_{Gb} = I_b\ddot{q} + J^T F_n, \quad (23)$$

or

$$\Gamma = I_b\ddot{q} + J^T m_{nt}\ddot{X}_n - J^T G_n - \Gamma_{Gb} \quad (24)$$

where

Γ is the vector of torques that have to be applied to the motors,

Γ_{Gb} is the torque produced by the gravitational force of the arms and is given by:

$$\Gamma_{Gb} = m_b r_{Gb} g[\cos q_1 \quad \cos q_2 \quad \cos q_3]^T \quad (24a)$$

I_b is the inertia matrix of the arms in joint space, and is given by:

$$I_b = \begin{bmatrix} I_{b1} & 0 & 0 \\ 0 & I_{b2} & 0 \\ 0 & 0 & I_{b3} \end{bmatrix} \quad (24b)$$

In equation (24), $\ddot{X}_n$ and $\ddot{q}$ are not independent, but actually linked by the following equation where J and $\dot{J}$ are given by equations (13) and (14) respectively:

$$\ddot{X}_n = J\ddot{q} + \dot{J}\dot{q} \quad (25)$$

Substituting this into equation (24) gives:

$$\Gamma = I_b\ddot{q} + J^T m_{nt}\left(J\ddot{q} + \dot{J}\dot{q}\right) - J^T G_n - \Gamma_{Gb} \quad (26)$$

or,

$$\Gamma = \left(I_b + m_{nt}J^T J\right)\ddot{q} + J^T m_{nt}\dot{J}\dot{q} - J^T G_n - \Gamma_{Gb} \quad (27)$$

where we identify

- the mass matrix A of the robot

$$A = I_b + m_{nt}J^T J \quad (27a)$$

- the Coriolis and centrifugal contributions

$$\Gamma_{c\&c} = m_{nt}J^T\dot{J}\dot{q} \quad (27b)$$

- and the gravity part

$$\Gamma_{gravity} = -J^T m_{nt}[0 \quad 0 \quad -g]^T - \Gamma_{Gb} \quad (27c)$$

In equation (27b), $\dot{J}\dot{q}$ can be identified to the first term of equation (14) and is thus easily computed. An alternative for obtaining the required torque Γ is to consider equation (24) directly, where $\ddot{X}_n$ is calculated by numerically differentiating the direct coordinate transformation twice with respect to time, i.e. :

$$\ddot{X}_n = \frac{d^2 f(q)}{dt^2} \quad (28)$$

In a computational sense, this is more efficient than calculating $\dot{J}\dot{q}$, but can lead to numerical noise, especially when the accelerations of the robot are small. Since the DELTA robot is usually driven with very high accelerations, this is not too troublesome, but is a point to be retained.

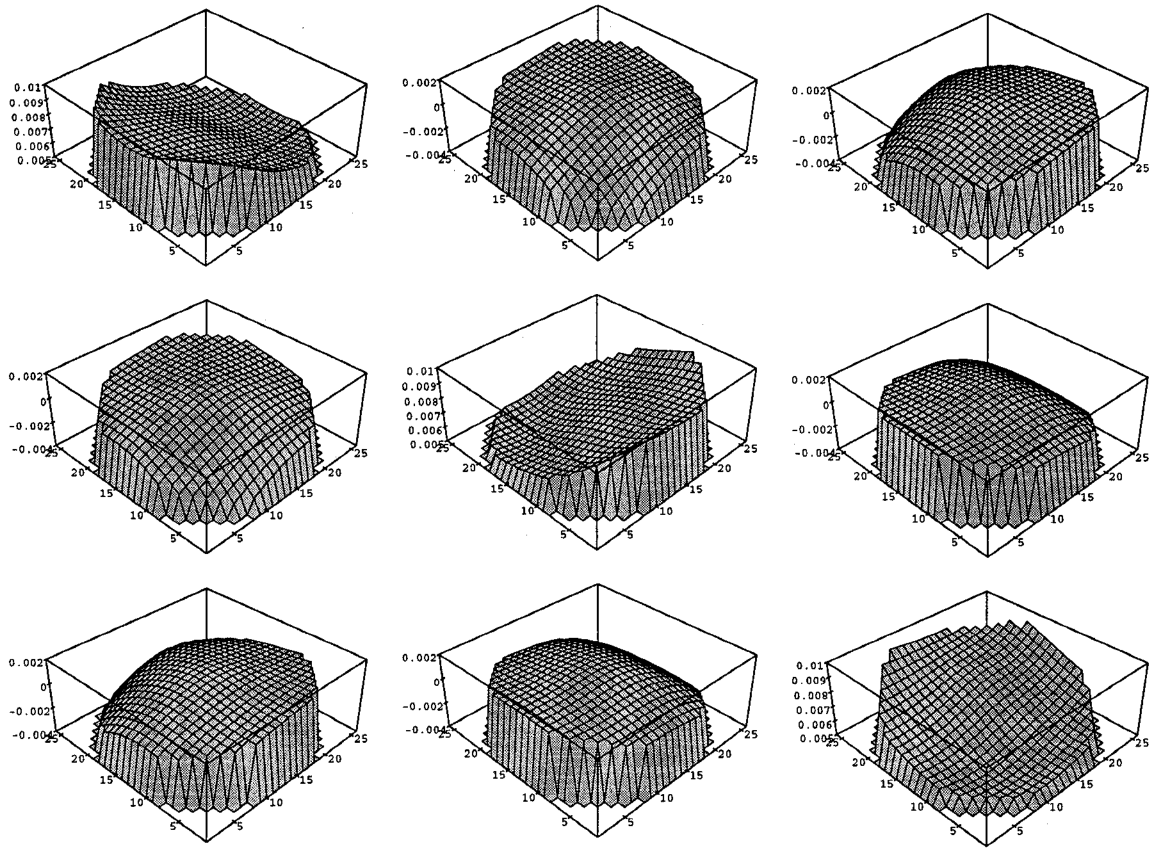

Fig. 4: Mass matrix of the DELTA robot for an horizontal cut into its workspace at a height z = -230 mm. The upward axis represents the inertia seen from the motor (unit: kgm²). The scale on the two other axes represents the sample number in the xy plane of the robot, 0 corresponding to -250 mm and 25 to +250 mm.

4.4. Mass matrix of the DELTA robot

The determination of the mass matrix of a robot is very important when one would like to decouple the individual axes in the overall robot control. The mass matrix of any robot can be derived without having to establish its dynamic model first. Kinetic energy considerations can be used as the basis for the establishment of this matrix. For the DELTA robot, the same simplifications as for the model developed above are used, i.e. only the inertia of the 3 arms and the mass of the travelling plate contribute to the kinetic energy of the robot. The total kinetic energy of the robot is thus :

$$T = T_{arms} + T_n \tag{29}$$

where

$$T_{arms} = \frac{1}{2}\dot{q}^T I_b \dot{q} \tag{30}$$

$$T_n = \frac{1}{2}\dot{X}_n^T M_n \dot{X}_n \tag{31}$$

and

$$I_b = \begin{bmatrix} I_{b1} & 0 & 0 \\ 0 & I_{b2} & 0 \\ 0 & 0 & I_{b3} \end{bmatrix},$$

$$M_n = \begin{bmatrix} m_{nt} & 0 & 0 \\ 0 & m_{nt} & 0 \\ 0 & 0 & m_{nt} \end{bmatrix}.$$

where $I_{b1} = I_{b2} = I_{b3} = I_{bi}$ and m_{nt} were defined previously.

Replacing the expression of the Jacobian into equation (31), the kinetic energy of the robot can be written as:

$$T = \frac{1}{2}\dot{q}^T I_b \dot{q} + \frac{1}{2}\dot{q}^T J^T M_n J \dot{q} \qquad (32)$$

or

$$T = \frac{1}{2}\dot{q}^T \left(I_b + J^T M_n J\right)\dot{q} \qquad (33)$$

where the kinetic energy matrix (or mass matrix) A of the DELTA robot can be identified as:

$$A = I_b + J^T M_n J \qquad (34)$$

This verifies the result obtained in the previous section. The shape of this mass matrix is shown in figure 4. The 9 graphs in the figure each represent a term of the 3x3 mass matrix for a horizontal cut into the workspace of the robot at the height z = -230 mm. We firstly recognise the 3-symmetry of the robot by analysing the diagonal terms of the matrix. The ratio between the maximum and minimum inertias is about 2. The coupling between the axes is very important and has the same order of magnitude as the proper inertia of each individual axis. This shows the importance of incorporating a decoupling strategy in the control of the robot. Equation (34) is a very simple form that can readily be used for that purpose, as it is discussed in the next section. The Jacobian can be evaluated either by numerical differentiation of the geometric model or by using equation (13).

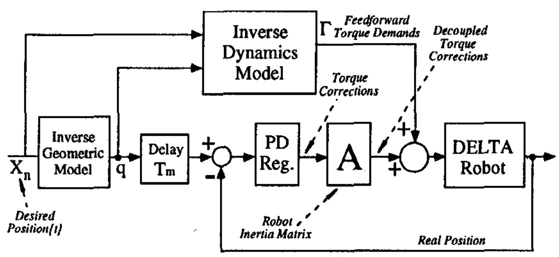

Fig. 5: Feedforward control of the direct-drive DELTA robot with a mass matrix based regulator output command decoupling.

5. Experimental Results

The scheme used for the control of the direct drive DELTA robot is shown in figure 5. It is composed of a feedforward block in which the inverse dynamic model of the robot is calculated based on equation (24). This block has as inputs both the joint space (q) and operational space (X_n) position and acceleration signals. The accelerations are obtained by the double numerical differentiation of the desired motor angles and travelling plate position. In this controller configuration, the trajectory signal input to the robot regulator is phase-shifted with respect to the feedforward torque generation block. This trick allows us to take account of an interval of T_m = 2 milliseconds which corresponds to the rise time of the current in the motor. The controller is a standard Proportional-Derivative (PD) regulator. The inertia matrix, calculated with the formulation of equation (34), is placed in series with the regulator. This allows the elimination of the interaction or coupling between the three robot axes and their linearization. This control strategy has been implemented successfully on the DELTA robot [4][5][16]. A great improvement in trajectory following for high speed pick and place movements was observed. As shown in figure 6, the error reduction is as high as 600% when the dynamic model is used.

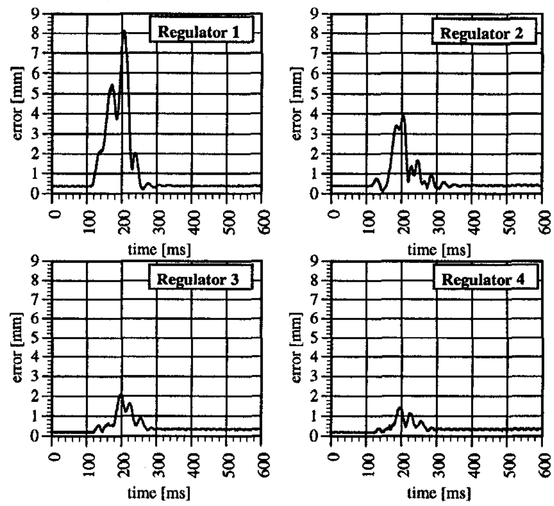

Fig. 6: Tracking errors for different level of sophistication in the controller: 1) PD, 2) PD+Constant Inertia Feedforward, 3) PD+Dynamic Model Feedforward, 4) same as 3 with "phase-shift"

6. Conclusion

In this article, the decoupled control of the DELTA parallel robot has been presented. To this end, a simplified dynamic model of the robot was firstly developed, before the establishment of its mass matrix. The proposed method uses the robot Jacobian matrix to project the forces acting at the operational point onto the joint space according to the virtual work principle. This has the advantage of giving explicit solutions for the robot mass matrix, the Coriolis and centrifugal forces, and the gravity contribution. An evaluation of the mass matrix based on kinetic energy considerations is also presented. This mass matrix is used to decouple the PD torque control components of the robot's command, as in the computed torque method. This control scheme has successfully been implemented on the DELTA robot, allowing fast pick and place movements at a speed of up to 3 Hz. The DELTA robot is thus today one of the fastest robots in the world.

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