Sector Consumo $\Rightarrow$ $C_t = K_{ct}^{\theta} (A_{ct} \cdot n_{ct})^{1-\theta}$ → PRODUCCIÓN S. CONSUMO

Sector Inversión $\Rightarrow$ $X_t = K_{xt}^{\theta} (A_{xt} \cdot n_{xt})^{1-\theta}$ → PRODUCCIÓN S. INVERSIÓN

Son cobb-douglas normales con 2 factores $\begin{cases} \text{capital } K \\ \text{Trabajo } n \end{cases}$

- El progreso tecnológico multiplica al trabajo -

Con $A_{it+1} = (1 + \gamma_i) A_{it}$ con $\gamma_i > 0$, $i \in \{a, x\}$

Progreso tecnológico dinámico que aumenta de forma constante

Por tanto:

$K_t = K_{ct} + K_{xt}$ → Capital Total = Suma de los Capitales

$1 = n_{ct} + n_{xt}$ → Trabajo total = Suma de los trabajos

↳ ver que no existe ocio en el modelo.

$P_t$: Precio del Consumo en términos de la inversión.

- Precio de $C_t$ en términos de $X_t$ -

2 Partes $\begin{cases} \text{Parte de la producción} \rightarrow \text{Importante} \\ \text{Parte de los hogares} \rightarrow \begin{cases} \text{Ramsey} \\ \text{Solow} \end{cases} \end{cases}$

PARTE DE LA PRODUCCIÓN (2 empresas, 1 produce C y otra X)

Empresa produce consumo

$$\Pi_c = \underbrace{P_t}_{\text{Precio C}} \cdot \underbrace{K_{ct}^{\theta} (A_{ct} \cdot n_{ct})^{1-\theta}}_{\text{Producción C}} - \underbrace{R_t}_{\text{Precio K}} \cdot \underbrace{K_{ct}}_{\text{Cantidad K}} - \underbrace{W_t}_{\text{Precio n}} \cdot \underbrace{n_{ct}}_{\text{cantidad n}}$$

Empresa produce inversión

$$\Pi_x = K_{xt}^{\theta} (A_{xt} \cdot n_{xt})^{1-\theta} - R_t K_{xt} - W_t \cdot n_{xt}$$

↳ No hay $P_t$ porqué si recordamos $P_t$ era el precio de $C_t$ en términos de $X_t$, digamos que $X_t$, ya está en términos de $X_t$.

CONDICIONES DE PRIMER ORDEN

Para $\pi c = P_t K_{ct}^{\theta} (A_{ct} \cdot n_{ct})^{1-\theta} - R_t K_{ct} - W_t \cdot n_{ct}$

$$\frac{\partial \pi c}{\partial K_{ct}} = \theta P_t K_{ct}^{\theta-1} (A_{ct} \cdot n_{ct})^{1-\theta} - R_t = 0 \quad (1)$$

$$\frac{\partial \pi c}{\partial n_{ct}} = (1-\theta) P_t K_{ct}^{\theta} (A_{ct} \cdot n_{ct})^{-\theta} \cdot A_{ct} - W_t = 0$$

$$W_t = (1-\theta) P_t K_{ct}^{\theta} (A_{ct} \cdot n_{ct})^{-\theta} \cdot A_{ct}$$

$$W_t = (1-\theta) P_t \cdot \frac{K_{ct}^{\theta}}{(A_{ct} \cdot n_{ct})^{\theta}} \cdot A_{ct}$$

$$W_t = (1-\theta) P_t \cdot \frac{K_{ct}^{\theta}}{A_{ct}^{\theta} \cdot n_{ct}^{\theta}} \cdot A_{ct}$$

$$W_t = (1-\theta) P_t \cdot \frac{K_{ct}^{\theta}}{n_{ct}^{\theta}} \cdot A_{ct} \cdot A_{ct}^{-\theta}$$

$$\boxed{W_t = (1-\theta) P_t \cdot \left(\frac{K_{ct}}{n_{ct}}\right)^{\theta} \cdot A_{ct}^{1-\theta}}$$

$$(1) \quad \theta P_t K_{ct}^{\theta-1} (A_{ct} \cdot n_{ct})^{1-\theta} = R_t$$

$$R_t = \theta P_t \cdot \frac{K_{ct}^{\theta-1}}{(A_{ct} \cdot n_{ct})^{\theta-1}} = \theta P_t \left(\frac{K_{ct}}{A_{ct} \cdot n_{ct}}\right)^{\theta-1}$$

$$\boxed{R_t = \theta P_t \cdot \left(\frac{K_{ct}}{n_{ct}}\right)^{\theta-1} \cdot A_{ct}^{1-\theta}}$$

y exactamente igual para $\Pi_x = K_{xt}^{\theta}(A_{xt}\cdot n_{xt})^{1-\theta} - R_t K_{xt} - W_t \cdot n_{xt}$

$$R_t = \theta\left(\frac{K_{xt}}{n_{xt}}\right)^{\theta-1}\cdot A_{xt}^{1-\theta}$$

$$w_t = (1-\theta)\cdot\left(\frac{K_{xt}}{n_{xt}}\right)^{\theta}\cdot A_{xt}^{1-\theta}$$

Puede verse que es exactamente igual pero sin "$P_t$".

e igualando

$$\begin{cases} R_t = \theta P_t \cdot \left(\frac{K_{ct}}{n_{ct}}\right)^{\theta-1}\cdot A_{ct}^{1-\theta} = \theta\left(\frac{K_{xt}}{n_{xt}}\right)^{\theta-1}\cdot A_{xt}^{1-\theta} \\ W_t = (1-\theta)P_t\cdot\left(\frac{K_{ct}}{n_{ct}}\right)^{\theta}\cdot A_{ct}^{1-\theta} = (1-\theta)\cdot\left(\frac{K_{xt}}{n_{xt}}\right)^{\theta}\cdot A_{xt}^{1-\theta} \end{cases}$$

Dividiendo

$$\frac{\theta P_t \cancel{\left(\frac{K_{ct}}{n_{ct}}\right)^{\theta-1}} A_{ct}^{1-\theta}}{\theta \cancel{\left(\frac{K_{xt}}{n_{ct}}\right)^{1-\theta}} A_{xt}^{1-\theta}} = 0$$

asumiendo que el ratio capital/trabajo son iguales

$$\boxed{P_t = \left(\frac{A_{xt}}{A_{ct}}\right)^{1-\theta}}$$

## VALOR TOTAL DEL CONSUMO EN EL MERCADO

$$P_t \cdot C_t = P_t \cdot K_{ct}^{\theta} (A_{ct} \cdot n_{ct})^{1-\theta}$$

$$\Rightarrow P_t \cdot C_t = P_t \cdot K_{ct}^{\theta} \cdot A_{ct}^{1-\theta} \cdot \frac{n_{ct}}{n_{ct}^{\theta}}$$

$$\Rightarrow P_t \cdot C_t = P_t \left(\frac{K_{ct}}{n_{ct}}\right)^{\theta} \cdot A_{ct}^{1-\theta} \cdot n_{ct}$$

$$\Rightarrow P_t \cdot C_t = \left(\frac{A_{xt}}{\cancel{A_{ct}}}\right)^{1-\theta} \cdot \left(\underbrace{\frac{K_{ct}}{n_{ct}}}\right)^{\theta} \cdot \cancel{A_{ct}^{1-\theta}} \cdot n_{ct}$$

$$\Rightarrow P_t \cdot C_t = \underbrace{K_t^{\theta}} \cdot A_{xt}^{1-\theta} \cdot n_{ct}$$

Porqué sabíamos que $\frac{K_{ct}}{n_{ct}} = \frac{K_{xt}}{n_{xt}} = K_t$

## PRODUCCIÓN TOTAL: $Y_t = X_t + P_t \cdot C_t$

• $Y_t = \underbrace{X_t} + K_t^{\theta} \cdot A_{xt}^{1-\theta} \cdot n_{ct}$

$$K_{xt}^{\theta} \cdot (A_{xt} \cdot n_{xt})^{1-\theta} = K_{xt}^{\theta} \cdot A_{xt}^{1-\theta} \cdot \frac{n_{xt}}{n_{xt}^{\theta}}$$

$$\to \left(\frac{K_{xt}}{n_{xt}}\right)^{\theta} \cdot A_{xt}^{1-\theta} \cdot n_{xt}$$

• $Y_t = \left(\underbrace{\frac{K_{xt}}{n_{xt}}}\right)^{\theta} \cdot A_{xt}^{1-\theta} \cdot n_{xt} + K_t^{\theta} \cdot A_{xt}^{1-\theta} \cdot n_{ct}$

y operando $K_t = \frac{K_{xt}}{n_{xt}} = \frac{K_{ct}}{n_{xt}}$

$K_t^{\theta} A_{xt}^{1-\theta} \cdot n_{xt} + K_t^{\theta} \cdot A_{xt}^{1-\theta} \cdot n_{ct}$ → sacando factor común a $K_t^{\theta} \cdot A_{xt}^{1-\theta}$

$$K_t^{\theta} \cdot Ax_t^{1-\theta} \underbrace{(n_{xt} + n_{ct})} = K_t^{\theta} \cdot Ax_t^{1-\theta}$$

sabemos que
$n_{ct} + n_{xt} = 1$

## PROBLEMA DE LOS HOGARES

- Maximización intertemporal define un cantidad total de consumo para todos los periodos que ahora los hogares han de decidir como repartir en cada periodo concreto entre agricultura, manufacturas y servicios. - Maximización sujeto a restricción -

$$\max_{\{C_{at}, C_{mt}, C_{st}\}_{t=0}^{\infty}} \omega_a \log(C_{at} - \bar{C}_a) + \omega_m \log(C_{mt}) + \omega_s \log(C_{st} + \bar{C}_s)$$

sujeto a $P_{at} \cdot C_{at} + P_{mt} \cdot C_{mt} + P_{st} \cdot C_{st} = TE$

TOTAL EXPENDITURE (dado por el problema intertemporal)

CONSTRUIR LANGRAGEANO

$$\mathcal{L}(\dots) = \omega_a \log(C_{at} - \bar{C}_a) + \omega_m \log(C_{mt}) + \omega_s \log(C_{st} + \bar{C}_s) - \lambda \left[P_{at} \cdot C_{at} + P_{mt} \cdot C_{mt} + P_{st} \cdot C_{st} - TE\right]$$

$$(1) \quad \frac{\partial \mathcal{L}}{\partial C_{at}} = \frac{\omega_a}{C_{at} - \bar{C}_a} - \lambda P_{at} = 0$$

$$(2) \quad \frac{\partial \mathcal{L}}{C_{mt}} = \frac{\omega_m}{C_{mt}} - \lambda P_{mt} = 0$$

$$(3) \quad \frac{\partial \mathcal{L}}{C_{st}} = \frac{\omega_s}{C_{st} + \bar{C}_s} - \lambda P_{st} = 0$$

$$(4) \quad \frac{\partial \mathcal{L}}{\partial \lambda} = P_{at} \cdot C_{at} + P_{mt} \cdot C_{mt} + P_{st} \cdot C_{st} - TE = 0$$

Despejando $\lambda$

- $(1) \Rightarrow \lambda = \dfrac{\omega_a}{P_{at}(C_{at} - \bar{C}_a)}$
- $(2) \Rightarrow \lambda = \dfrac{\omega_m}{P_{mt} \cdot C_{mt}}$
- $(3) \Rightarrow \lambda = \dfrac{\omega_s}{P_{st}(C_{st} + \bar{C}_s)}$

e igualando las $\lambda$.

(1) $\dfrac{w_a}{P_{at}(C_{at}-\bar{C}_a)} = \dfrac{w_m}{P_{mt}\cdot C_{mt}}$

$$\rightarrow \underbrace{P_{mt}\cdot C_{mt}}_{(1)} = \frac{w_m P_{at}\cdot(C_{at}-\bar{C}_a)}{w_a} = \frac{w_m}{w_a} P_{at}(C_{at}-\bar{C}_a)$$

(2) $\dfrac{w_a}{P_{at}\cdot(C_{at}-\bar{C}_a)} = \dfrac{w_s}{P_{st}(C_{st}+C_s)} \rightarrow \underbrace{P_{st}(C_{st}+C_s)}_{(2)} = \dfrac{w_s P_{at}(C_{at}-\bar{C}_a)}{w_a}$

$$= \frac{w_s}{w_a}\cdot P_{at}(C_{at}-\bar{C}_a)$$

**DEFINIMOS UN ÍNDICE DE PRECIOS AGREGADO**

$$P_t = P_{at}^{w_a}\cdot P_{mt}^{w_m}\cdot P_{st}^{w_s} \quad \text{dónde } \sum w_i = 1$$

Puede demostrarse que $P_{at}(C_{at}-\bar{C}_a) + \overbrace{P_{mt}\cdot C_{mt}}^{(1)} + \overbrace{P_{st}(C_{st}+C_s)}^{(2)} = P_t\cdot C_t$

y sustituyendo, m.c. denominador

$$P_t\cdot C_t = \frac{w_a P_{at}(C_{at}-\bar{C}_a)}{w_a} + \frac{w_m P_{at}(C_{at}-\bar{C}_a)}{w_a} + \frac{w_s P_{at}(C_{at}-\bar{C}_a)}{w_a}$$

$$w_a P_t C_t = w_a P_{at}(C_{at}-\bar{C}_a) + w_m P_{at}(C_{at}-\bar{C}_a) + w_s P_{at}(C_{at}-\bar{C}_a)$$

$$w_a P_t\cdot C_t = P_{at}(C_{at}-\bar{C}_a)\,[w_a + w_m + w_s]$$

Para $C_{at}$

$$\boxed{C_{at} = \frac{W_a P_t \cdot C_t}{P_{at}}}$$

Para $C_{mt}$ (sustituyendo)

$$\underbrace{P_{mt} \cdot C_{mt}}_{1} = \frac{W_m P_{at}\left(\frac{W_a P_t \cdot C_t}{P_{at}} \cancel{+\bar{C}_a} \cancel{-\bar{C}_a}\right)}{W_a/1} = \frac{W_m \cancel{P_{at}} \cdot \cancel{W_a} \cdot P_t \cdot C_t}{\cancel{P_{at}} \cdot \cancel{W_a}}$$

$$\rightarrow P_{mt} \cdot C_{mt} = W_m \cdot P_t \cdot C_t \rightarrow \boxed{C_{mt} = \frac{W_m P_t C_t}{P_{mt}}}$$

Para $C_{st}$

$$\underbrace{P_{st}(C_{st} + C_s)}_{2} = \frac{W_s P_{at} \frac{W_a P_t \cdot C_t}{P_{at}} \cancel{+\bar{C}_a} \cancel{-\bar{C}_a}}{W_a/1} =$$

$$= \frac{W_s \cancel{P_{at}} \cancel{W_a} P_t \cdot C_t}{\cancel{P_{at}} \cancel{W_a}} \rightarrow P_{st}(C_{st} + C_s) = W_s \cdot P_t \cdot C_t$$

$$\rightarrow C_{st} + C_s = \frac{W_s P_t \cdot C_t}{P_{st}} \rightarrow \boxed{C_{st} = \frac{W_s P_t \cdot C_t}{P_{st}} - \bar{C}_s}$$