Structural Transformation

Advanced Macroeconomics

Novemberer 2022

References

- "Growth and Structural Transformation", Berthold Herrendorf, Richard Rogerson and Akos Valentinyi, *Handbook of Economic Growth*, volume 2b, Elsevier.
- "Introduction to Modern Economic Growth", Daron Acemoglu, Princeton Univ. Press.
  - Chapter 20.

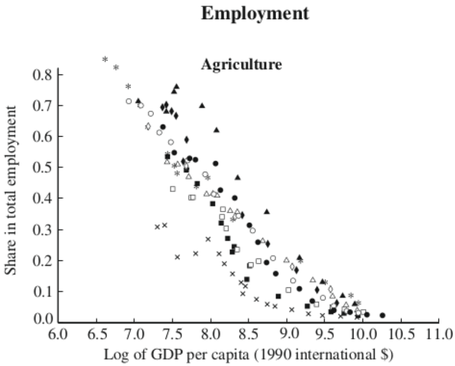
Structural Transformation

- **DEF**: Reallocation of economic activity across three broad sectors (agriculture, manufactures and services) that accompanies the process of economic growth.
- The process of Economic Growth entails the increase in income (GDP per capita).
- However, the process of Economic Growth is also accompanied by deeper changes in the sectoral structure of production.
- There is a process of industrialization by which resources are employed away from agriculture toward nonagriculture activities.
- The models that we have studied so far (Solow/Ramsey) can explain (at least partially) the increases in income per capita, but are not able to explain this other process of structural change.

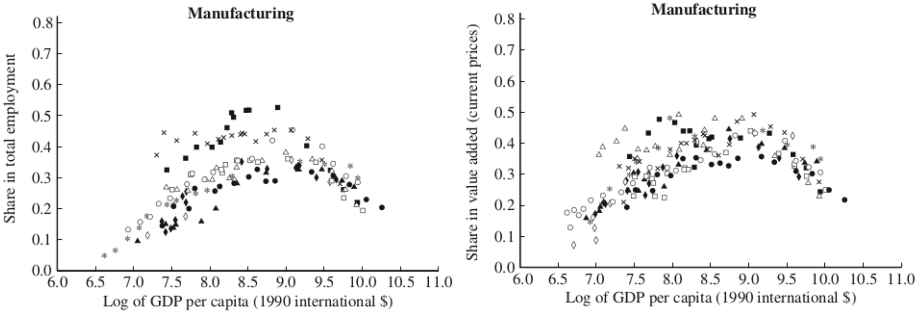
How to measure Structural Transformation

- There are three main measures:
  - Employment shares
  - Value Added shares
  - Final Consumption expenditure shares
- Problem: Employment and VA shares are related to production while Consumption Exp. shares are related to consumption...
- ... and they are not necessarily the same.
- **Example**: Cotton shirt
  - From the point of view of consumption is a good and therefore the entire expenditure will be measured as consumption expenditure of the manufacturing sector
  - But in terms of VA, the same purchase will be broken down into three components: One from the Agricultural sector, one from Manufacturing, and one from Services.

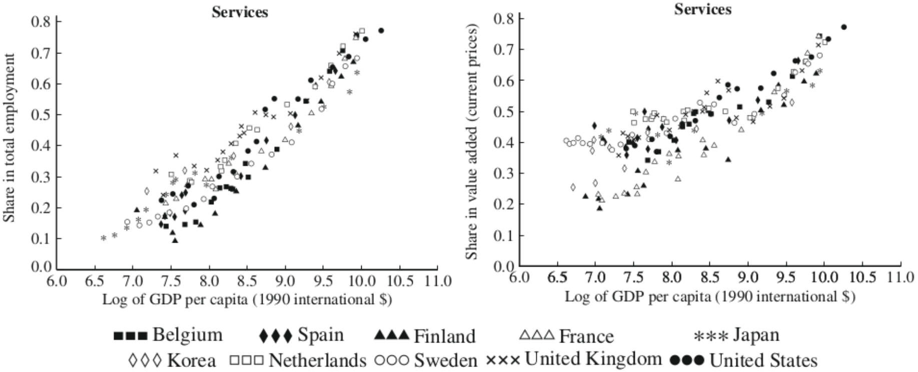
Empirical Evidence: Agriculture



Empirical Evidence: Manufacturing



Empirical Evidence: Services



Empirical Evidence: Summary

- An increase in Income (GDP) implies:
  - **Agriculture**: Decrease in employment and VA shares.
  - **Services**: Increase in employment and VA shares.
  - **Manufacturing**: Hump-shape.
- Also for **Agriculture**: For low levels of development the VA share $<$ Employment share. This means that poor countries have most of the their workers in the less productive sector (Agric).
- **Services** are bounded away from zero: VA share (20%), Employment share (10%).
- **Services**: Acceleration in VA share around *log* GDP $= 9$.

A Two-Sector Model: Production I

- Consumption sector: $C_t = k_{ct}^{\theta}(A_{ct}n_{ct})^{1-\theta}$
- Investment sector: $X_t = k_{xt}^{\theta}(A_{xt}n_{xt})^{1-\theta}$
- where $A_{it+1} = (1+\gamma_i)A_{it}$, $\gamma_i > 0$, $i \in \{a, x\}$
- Feasibility constraints:

$$\begin{aligned} K_t &= k_{ct} + k_{xt} \\ 1 &= n_{ct} + n_{xt} \end{aligned}$$

- P_t: Price of C_t in terms of X_t

A Two-Sector Model: Production II

- Profit functions:

$$\begin{aligned}\pi_c &= P_t k_{ct}^{\theta}(A_{ct}n_{ct})^{1-\theta} - R_t k_{ct} - W_t n_{ct} \\ \pi_x &= k_{xt}^{\theta}(A_{ct}n_{xt})^{1-\theta} - R_t k_{xt} - W_t n_{xt}\end{aligned}$$

- F.O.C.:

$$\begin{aligned}R_t = &\quad \theta P_t \left(\frac{k_{ct}}{A_{ct}n_{ct}}\right)^{\theta-1} &= \theta \left(\frac{k_{xt}}{A_{xt}n_{xt}}\right)^{\theta-1} \\ W_t = &\quad (1-\theta)P_t \left(\frac{k_{ct}}{n_{ct}}\right)^{\theta} A_{ct}^{1-\theta} &= (1-\theta)\left(\frac{k_{xt}}{n_{xt}}\right)^{\theta} A_{xt}^{1-\theta}\end{aligned}$$

- Divinding the foc

$$\frac{k_{it}}{n_{it}} = \frac{\theta}{1-\theta}\frac{W_t}{R_t}$$

A Two-Sector Model: Production III

- and therefore

$$\frac{k_{ct}}{n_{ct}} = \frac{k_{xt}}{n_{xt}} = K_t$$

- Then, from the previous equation and the foc

$$P_t = \left(\frac{A_{xt}}{A_{ct}}\right)^{1-\theta}$$

- The market value of total consumption is

$$P_t C_t = \left(\frac{k_{ct}}{n_{ct}}\right)^{\theta} P_t A_{ct}^{1-\theta} n_{ct} = K_t^{\theta} A_{xt}^{1-\theta} n_{ct}$$

- and aggregate production is defined as

$$Y_t = X_t + P_t C_t = \left(\frac{k_{xt}}{n_{xt}}\right)^{\theta} A_{xt}^{1-\theta} n_{xt} + K_t^{\theta} A_{xt}^{1-\theta} n_{ct} = K_t^{\theta} A_{xt}^{1-\theta}$$

A Two-Sector Model: Production IV

- and

$$
\begin{aligned}
R_t &= \theta K_t^{\theta-1} A_{xt}^{1-\theta} \\
W_t &= (1-\theta) K_t^{\theta} A_{xt}^{1-\theta}
\end{aligned}
$$

Household's Problem I

- Household preferences

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log C_t$$

subject to

$$P_t C_t + K_{t+1} = (1 - \delta + R_t) K_t + W_t, \ \forall \ t = 0 ... \infty$$

- Which comes from the corresponding
  - Budget Constraint (BC) $W_t + R_t K_t = P_t C_t + X_t$.
  - Law of motion (LM), $K_{t+1} = X_t + (1 - \delta) K_t$.
- Building the Lagrangean and computing the foc yields the Euler equation

$$\frac{P_{t+1} C_{t+1}}{\beta P_t C_t} = 1 + R_{t+1} - \delta$$

Generalized Balanced Growth Path: GBGP

- A GBGP only requires that the real interest rate is constant (Kaldor Stylized Facts)
- It can be shown that in the GBGP K_t and Y_t grow at constant rates.
- From the value of $R_t = \theta K_t^{\theta-1} A_{xt}^{1-\theta}$ and the fact that R_t is constant

$$\frac{K_{t+1}}{K_t} = \frac{A_{xt+1}}{A_{xt}}$$

- For Y_t note that $Y_t = K_t^{\theta} A_{xt}^{1-\theta}$ and therefore

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{A_{xt+1}}{A_{xt}}\right)^{1-\theta} \left(\frac{K_{t+1}}{K_t}\right)^{\theta} = (1+\gamma_x)$$

- This also implies that $\frac{K_t}{Y_t}$ and $\frac{R_t K_t}{Y_t}$ are constant along the GBGP.

An ST utility function

- In order to explain ST we have to divide the aggregate consumption sector into three sectors $i \in \{a, m, s\}$
- We say that there is ST if either employment shares (n_{it}) or sectoral value added shares ($\frac{P_{it}C_{it}}{Y_t}$) are not constant for all three consumption sectors.
- Thus, we define total consumption (C_t above) as a (CES) composite of the three types of consumption (agriculture, manufactures and services)

$$C_t = \left[\omega_a^{\frac{1}{\epsilon}}(c_{at} - \bar{c}_a)^{\frac{\epsilon-1}{\epsilon}} + \omega_m^{\frac{1}{\epsilon}}(c_{mt})^{\frac{\epsilon-1}{\epsilon}} + \omega_s^{\frac{1}{\epsilon}}(c_{st} + \bar{c}_s)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$$

- If $\epsilon = 1$ this boils down to the standard Cobb-Douglas aggregator

$$C_t = (c_{at} - \bar{c}_a)^{\omega_a}(c_{mt})^{\omega_m}(c_{st} + \bar{c}_s)^{\omega_s}$$

- $\bar{c}_a$ is the subsistence level of consumption of the agricultural good. If c_{at} is below that level utility is negative
- $\bar{c}_s$ allows to have zero consumption levels for services

Sectoral Production Functions I

- Consumption sector:

$$c_{it} = k_{it}^{\theta}(A_{it}n_{it})^{1-\theta};\ i \in \{a, m, s\}$$

- Investment sector:

$$X_t = k_{xt}^{\theta}(A_{xt}n_{xt})^{1-\theta}$$

- Feasibility constraints:

$$\begin{aligned} K_t &= k_{at} + k_{mt} + k_{st} + k_{xt} \\ 1 &= n_{at} + n_{mt} + n_{st} + n_{xt} \end{aligned}$$

- p_{it}: Price of sector i relative to X.

Sectoral Production Functions II

- proceeding as in the 2-sector model we obtain

$$\frac{k_{it}}{n_{it}} = K_t \;\; i \in \{a, m, s\}$$

- Then, from the previous equation and the foc

$$p_{it} = \left(\frac{A_{xt}}{A_{it}}\right)^{1-\theta}$$

- and

$$Y_t = p_{at}c_{at} + p_{mt}c_{mt} + p_{st}c_{st} + X_t = K_t^{\theta}A_{xt}^{1-\theta}$$

- and also

$$\begin{aligned} R_t = & \quad \theta K_t^{\theta-1}A_{xt}^{1-\theta} \\ W_t = & \quad (1-\theta)K_t^{\theta}A_{xt}^{1-\theta} \end{aligned}$$

Case 1: Income Effects (Kongsamut et al. (2001)) I

- **Engel's Law**: "as household's income increases, the fraction that it spends on food declines and increases on services"
- We assume $\bar{c}_a$, $\bar{c}_s > 0$, $\gamma_i = \gamma_j \; i \neq j$ and $\epsilon = 1$ (Cobb-Douglas).
- In order to solve the optimal problem we divide it into an aggregate problem (two-sector model above) and an intraperiod problem in which the individual chooses the optimal amounts of (c_{at}, c_{mt}, c_{st}) given $P_t C_t$ obtained from the aggregate problem.
- **Household's static problem**: given an expenditure level $(P_t C_t)$ the household has to decide the consumption levels for (c_{at}, c_{mt}, c_{st})

$$\max_{\{c_{at}, c_{mt}, c_{st}\}_{t=0}^{\infty}} \omega_a \log(c_{at} - \bar{c}_a) + \omega_m \log(c_{mt}) + \omega_s \log(c_{st} + \bar{c}_s)$$

Case 1: Income Effects (Kongsamut et al. (2001)) II

subject to

$$p_{at}c_{at} + p_{mt}c_{mt} + p_{st}c_{st} = TE$$
$$\forall\ t = 0...\infty$$

- where TE is the total expenditure (given by the intertemporal problem)
- Building the Lagrangean, computing the foc, and dividing by (c_{at})

$$\left.\begin{aligned} &(c_{at})\ \frac{\omega_a}{c_{at}-\bar{c}_a} = \lambda_t p_{at} \\ &(c_{mt})\ \frac{\omega_m}{c_{mt}} = \lambda_t p_{mt} \\ &(c_{st})\ \frac{\omega_s}{c_{st}+\bar{c}_s} = \lambda_t p_{st} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} p_{mt}c_{mt} &= \frac{\omega_m}{\omega_a} p_{at}(c_{at}-\bar{c}_a) \\ p_{st}(c_{st}-\bar{c}_s) &= \frac{\omega_s}{\omega_a} p_{at}(c_{at}-\bar{c}_a) \end{aligned}\right.$$

Case 1: Income Effects (Kongsamut et al. (2001)) III

- We can define an aggregate price index as

$$P_t = p_{at}^{\omega_a} p_{mt}^{\omega_m} p_{st}^{\omega_s} \text{ where } \sum_i \omega_i = 1$$

- It can then be proved that

$$p_{at}(c_{at} - \bar{c}_a) + p_{mt} c_{mt} + p_{st}(c_{st} + \bar{c}_s) = P_t C_t$$

- Then substituting in this equation the ratios obtained with the foc

$$p_{at}(c_{at} - \bar{c}_a)[\omega_a + \omega_m + \omega_s] = \omega_a P_t C_t$$

- which yields the following system of demand equations

$$c_{at} = \frac{\omega_a P_t C_t}{p_{at}} + \bar{c}_a; \quad c_{mt} = \frac{\omega_m P_t C_t}{p_{mt}}; \quad c_{st} = \frac{\omega_s P_t C_t}{p_{st}} - \bar{c}_s$$

Case 1: Income Effects (Kongsamut et al. (2001)) IV

- Moreover, since $\gamma_i = \gamma_j \Rightarrow p_{it} = \left(\frac{A_{xt}}{A_{it}}\right)^{1-\theta}$ all grow at same rate and therefore relative prices are constant

$$\frac{p_{it}}{P_t} = \frac{p_{io}}{P_0} \text{ for } i \in \{a, m, s\}$$

- Hence, $\begin{bmatrix} c_{at} \\ c_{mt} \\ c_{st} \end{bmatrix}$ grows at a $\begin{bmatrix} \text{slower} \\ \text{equal} \\ \text{higher} \end{bmatrix}$ rate than C_t.
- and given rel. prices are constant, $\frac{p_{it} c_{it}}{P_t C_t}$ is $\begin{bmatrix} \text{decreasing} \\ \text{constant} \\ \text{increasing} \end{bmatrix}$ for $\begin{bmatrix} \text{agriculture} \\ \text{manufacturing} \\ \text{services} \end{bmatrix}$

Case 2: Rel. Price Effects (Ngai and Pissarides (2007)) I

- Now we consider the case in which ST is generated purely from changes in relative prices.
- Accordingly, there are no income effects and $\bar{c}_a = \bar{c}_s = 0$ and the rates of technological progress are different $\gamma_i \neq$ for all i. Finally we drop the Cobb-Douglas assumption and $\epsilon \neq 1$
- This implies that P_t will not grow at a constant rate but it can still be proved the existence of a GBGP and that along the GBGP $P_t C_t$ grows at the constant rate γ_x.

Case 2: Rel. Price Effects (Ngai and Pissarides (2007)) II

- From the foc and $p_{it} = \left(\frac{A_{xt}}{A_{it}}\right)^{1-\theta}$

$$\frac{c_{at}}{c_{mt}} = \frac{\omega_a}{\omega_m}\left(\frac{A_{at}}{A_{mt}}\right)^{\epsilon(1-\theta)}$$

$$\frac{c_{st}}{c_{mt}} = \frac{\omega_s}{\omega_m}\left(\frac{A_{st}}{A_{mt}}\right)^{\epsilon(1-\theta)}$$

- Also since capital-labor ratios are equal across sectors $c_{it} = K_t^{\theta} A_{it}^{1-\theta} n_{it}$ and therefore,

$$\frac{n_{at}}{n_{mt}} = \frac{\omega_a}{\omega_m}\left(\frac{A_{mt}}{A_{at}}\right)^{(1-\epsilon)(1-\theta)}$$

$$\frac{n_{st}}{n_{mt}} = \frac{\omega_s}{\omega_m}\left(\frac{A_{mt}}{A_{st}}\right)^{(1-\epsilon)(1-\theta)}$$

Case 2: Rel. Price Effects (Ngai and Pissarides (2007)) III

- If $\epsilon = 1$ then n_{it} are constant for all i, as well as $\frac{p_{it}c_{it}}{P_t C_t}$ and $\frac{p_{it}c_{it}}{Y_t}$.
- If $\epsilon \neq 1$ the model can generate ST as long as the γ_i's differ among the three consumption sectors.
- Let us now assume $\gamma_a > \gamma_m > \gamma_s$ and $\epsilon < 1$,
  - n_{it}, $\frac{p_{it}c_{it}}{P_t C_t}$ and $\frac{p_{it}c_{it}}{Y_t}$ are $\begin{bmatrix} \text{decreasing} \\ \text{increasing} \end{bmatrix}$ for $\begin{bmatrix} \text{agriculture} \\ \text{services} \end{bmatrix}$
  - for manufacturing these magnitudes are either decreasing or hump-shaped (see Ngai and Piss.)
  - In this case the three goods are complements ($\epsilon < 1$) and the agriculture goods is becoming cheaper ($\gamma_a >$), thus the individual devotes more resources to the more expensive goods (m and s).
  - If $\epsilon > 1$ (substitutes) the individual would devote more resources to the cheapest good (agric) and less to the other two.

Qualitative Assessment

Income Effects (Kongsamut et al.)

- The model accounts for the increase in services and decrease in agriculture along the CBGP, but does not generate a hump shape for the manufacturing sector.
- Implies that in sufficiently poor economies the household will consume zero services and $n_s = 0$ which is counterfactual.

Relative Price Effects (Ngai and Piss.)

- Accounts for the increase in services and decrease in agriculture along the CBGP, and can produce a hump-shaped dynamics for manufacturing for some parameter values.