CS 325 Spring 2019, Homework 6

0.1 Shortest Paths using Linear Programming

Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

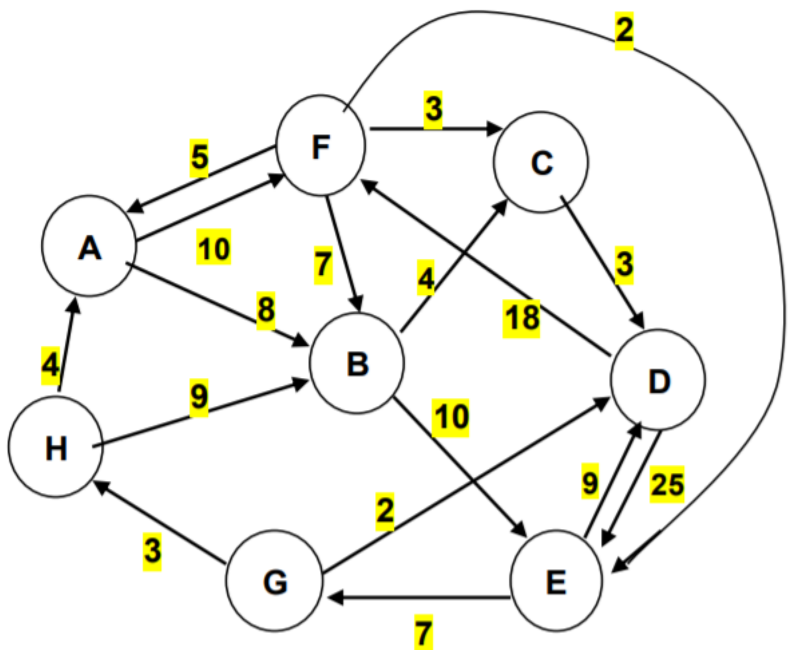
- We can compute the shortest path from s to t in a weighted directed graph by solving

```
max dt
subject to
    ds = 0
    dv - du <= w(u, v) for all (u, v) in E
```

- We can compute the single-source by changing the objective function to

$$\max \sum_{v \in V} dv$$

Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.



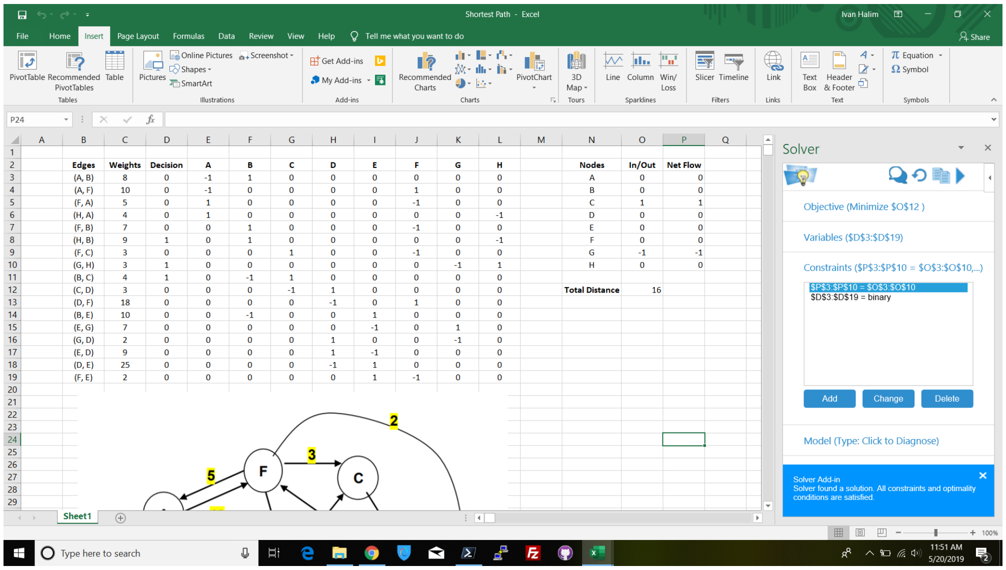
Graph Shortest Path.

1. **Find the distance of the shortest path from G to C**

   - **Variables**: x_e, for each edge e.
   - **Objective function**: Minimize $\sum_{e \in E} wt(e) x_e$.
   - **Constraints**:
     - $0 \leq x_e \leq 1$, for every edge e.
     - $x_e \in \mathbb{Z}$, for every edge e.
   - For every node u, \$ $\sum$ \$

Find the distances of the shortest paths from G to all other vertices

- **G to A**: 7
- **G to B**: 12
- **G to D**: 2
- **G to E**: 19
- **G to F**: 17
- **G to H**: 3



Shortest Path Solution.

0.2 Product Mix

Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, `profit per tie = selling price - labor cost - material cost`. Labor cost is 0.75 per tie for all four types of ties. The material requirements and costs are given below.

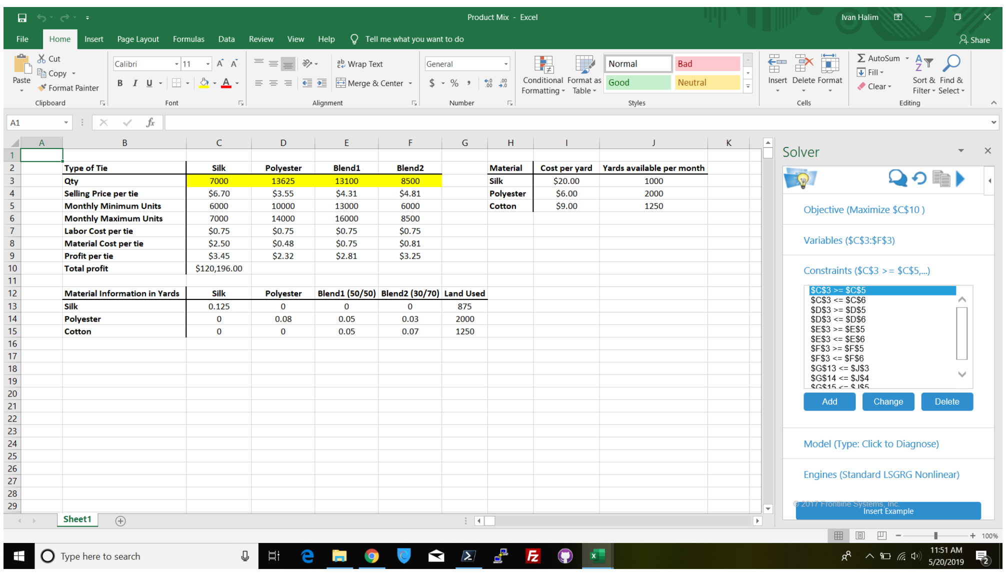
Material	Cost per yard	Yards available per month
Silk	20	1000
Polyester	6	2000
Cotton	9	1250

Product Information	Silk = s	Poly = p	Blend1 = b	Blend2 = c
Selling Price per tie	6.70	3.55	4.31	4.81
Monthly Minimum units	6000	10000	13000	6000
Monthly Maximum units	7000	14000	16000	8500

Material Information in Yards	Silk	Poly	Blend1 (50/50)	Blend2 (30/70)
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal number of ties of each type to maximize profit?

- **Variables**: x_p, for each product p.
- **Objective function**: Maximize $\sum_{p \in P} profit(p) \cdot x_p$ where,
  - $profit(p) = price(p) - laborcost - \sum_{m \in M} yard_m(p) \cdot cost_m$
- **Constraints**:
  - $\min(x_p) \leq x_p \leq \max(x_p)$, for each product p.
  - $x_p \in \mathbb{Z}$, for each product p.
  - $\sum_{p \in P} yard_m(p) \leq max(yard_m)$, for each material m.

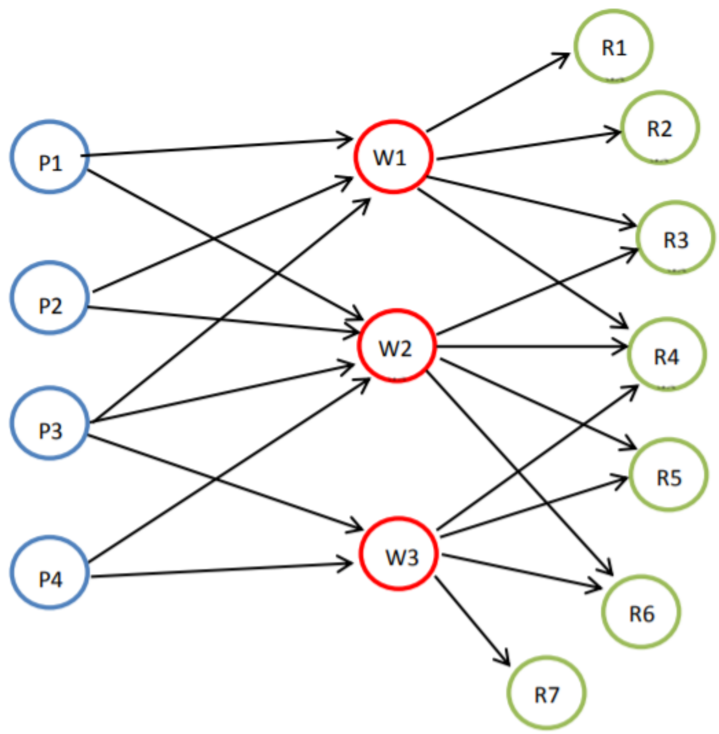


Product Mix.

0.3 Transshipment Model

This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (p_i) must be shipped to a Warehouse (w_j) before being shipped to the Retailer (r_k). Each Plant will have an associated supply (s_i) and each Retailer will have a demand (d_k). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i, j) from plant (p_i) to warehouse (w_j) have costs associated denoted $cp(i, j)$. The edges (j, k) from a warehouse (w_j) to a retailer (r_k) have costs associated denoted $cw(j, k)$.

The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



Transshipment Model.

Below are the costs of shipping from a plant to a warehouse and then a warehouse to a retailer. If it is impossible to ship between the two locations, an X is placed in the table.

cost	W1	W2	W3
P1	10	15	X
P2	11	8	X
P3	13	8	9
P4	X	14	8

cost	R1	R2	R3	R4	R5	R6	R7
W1	5	6	7	10	X	X	X

cost	R1	R2	R3	R4	R5	R6	R7
W2	X	X	12	8	10	14	X
W3	X	X	X	14	12	12	6

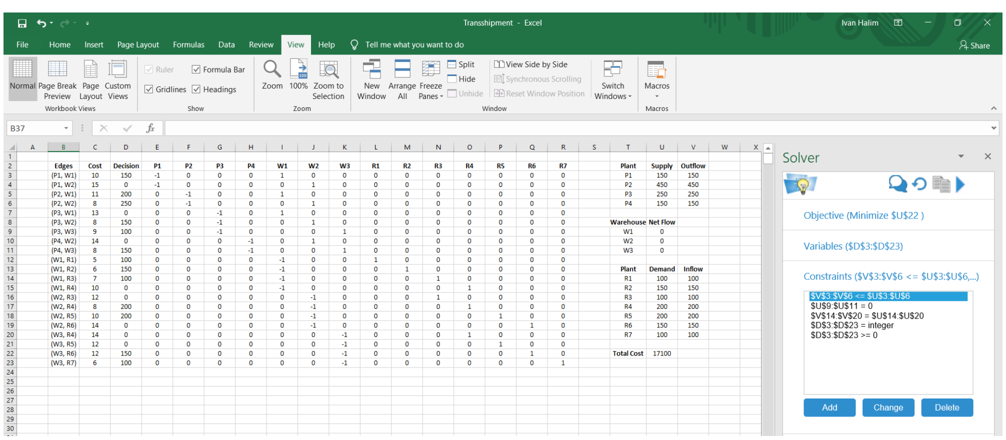
The tables below give the capacity of each plant (supply) and the demand for each retailer (per week).

Plants	P1	P2	P3	P4
Supply	150	450	250	150

Retailers	R1	R2	R3	R4	R5	R6	R7
Demand	100	150	100	200	200	150	100

Part A: Determine the number of refrigerators to be shipped from the plants to the warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost?

- **Variables**: x_e, for each edge e.
- **Objective function**: Minimize $\sum_{e \in E} wt(e) x_e$.
- **Constraints**:
  - $x_e \geq 0$, for every edge e.
  - $x_e \in \mathbb{Z}$, for every edge e.
  - For every plant p, $\sum_{e \in out(p)} x_e \leq supply(p)$.
  - For every warehouse w, $\sum_{e \in in(w)} x_e = \sum_{e \in out(w)} x_e$.
  - For every retailer r, $\sum_{e \in in(r)} x_e = demand(r)$.

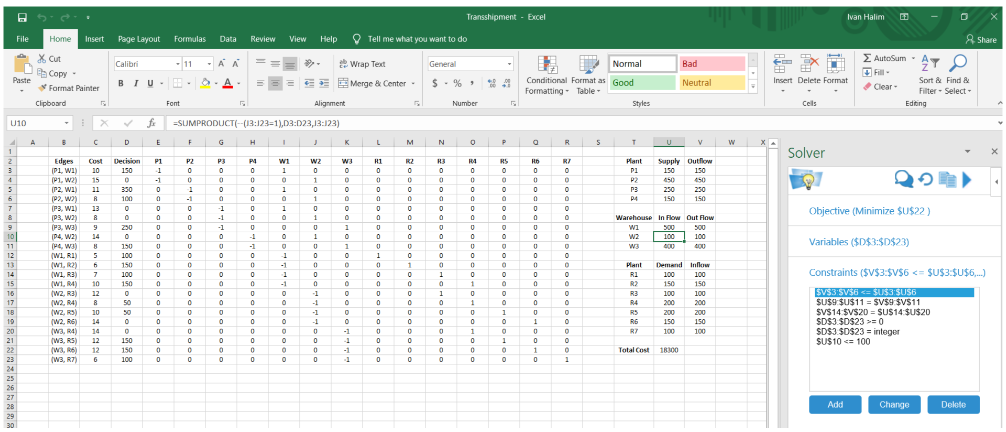


Transshipment Solution 1.

Part B: Due to old infrastructure, Warehouse 2 is going to close, eliminating all of the associated routes. What is the optimal solution for this modified model? Is it feasible to ship all the refrigerators to either Warehouse 1 or 3 and then to the retailers without using Warehouse 2? Why or why not?

It is not feasible. Without Warehouse 2, Plant 1 cannot send a supply to Retailer 5 and 6. Plant 2 cannot send a supply to Retailer 5 and 6. Plant 4 cannot send a supply to Retailer 3. As a result, Retailer 3, 5 and 6 are inadequately supplied.

Part C: Instead of closing Warehouse 2, management has decided to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so, what is the optimal solution when warehouse 2 is limited to 100 refrigerators?



Edges	Cost	Decision	P1	P2	P3	P4	W1	W2	W3	R1	R2	R3	R4	R5	R6	R7
(P1, W1)	10	150	-1	0	0	0	1	0	0	0	0	0	0	0	0	0
(P1, W2)	15	0	-1	0	0	0	0	1	0	0	0	0	0	0	0	0
(P2, W1)	11	350	0	-1	0	0	1	0	0	0	0	0	0	0	0	0
(P2, W2)	8	100	0	-1	0	0	0	1	0	0	0	0	0	0	0	0
(P3, W1)	13	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0
(P3, W2)	8	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0
(P3, W3)	9	250	0	0	-1	0	0	0	1	0	0	0	0	0	0	0
(P4, W2)	14	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0
(P4, W3)	8	150	0	0	0	-1	0	0	1	0	0	0	0	0	0	0
(W1, R1)	5	100	0	0	0	0	-1	0	0	1	0	0	0	0	0	0
(W1, R2)	6	150	0	0	0	0	-1	0	0	0	1	0	0	0	0	0
(W1, R3)	7	100	0	0	0	0	-1	0	0	0	0	1	0	0	0	0
(W1, R4)	10	150	0	0	0	0	-1	0	0	0	0	0	1	0	0	0
(W2, R3)	12	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0
(W2, R4)	8	50	0	0	0	0	0	-1	0	0	0	0	1	0	0	0
(W2, R5)	10	50	0	0	0	0	0	-1	0	0	0	0	0	1	0	0
(W2, R6)	14	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0
(W3, R4)	14	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0
(W3, R5)	12	150	0	0	0	0	0	0	-1	0	0	0	0	1	0	0
(W3, R6)	12	150	0	0	0	0	0	0	-1	0	0	0	0	0	1	0
(W3, R7)	6	100	0	0	0	0	0	0	-1	0	0	0	0	0	0	1

Plant	Supply	Outflow
P1	150	150
P2	450	450
P3	250	250
P4	150	150

Warehouse	In Flow	Out Flow
W1	500	500
W2	100	100
W3	400	400

Plant	Demand	Inflow
R1	100	100
R2	150	150
R3	100	100
R4	200	200
R5	200	200
R6	150	150
R7	100	100
Total Cost	18300	

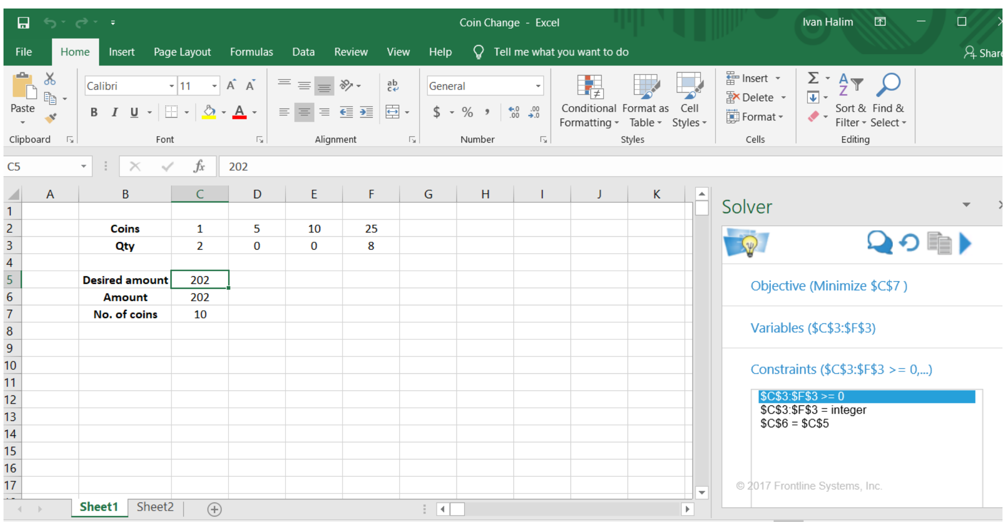
Transshipment Solution 2.

0.4 Making Change

Given coins of values $1 = v_1 < v_2 < ... < v_n$, we wish to make change for an amount A using as few coins as possible. Assume that v_i's and A are integers. Since $v_1 = 1$ there will always be a solution. Solve the coin change using integer programming. For each of the following denomination sets and amounts, formulate the problem as an integer program with an objective function and constraints, determine the optimal solution. What is the minimum number of coins used in each case and how many of each coin is used? Include a copy of your code.

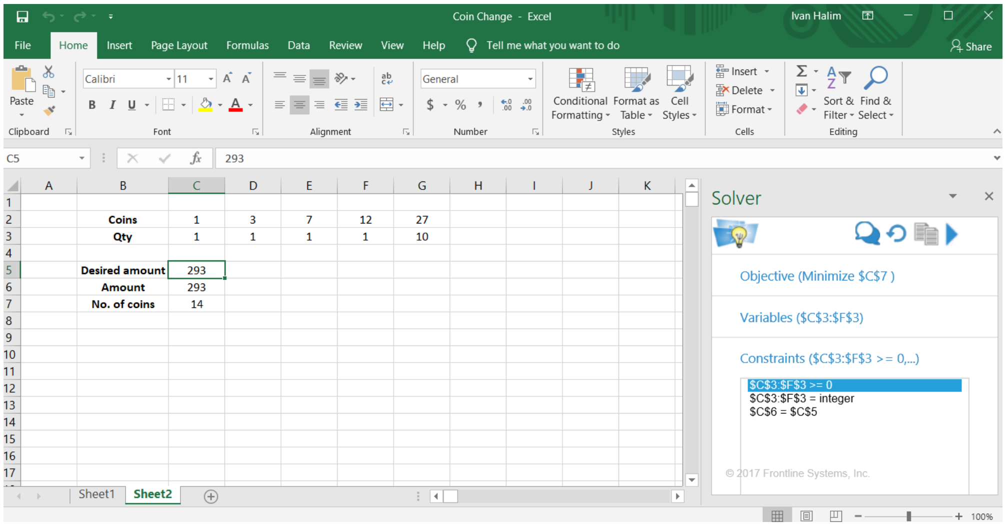
1. **V = [1, 5, 10, 25] and A = 202**

    - **Variables**: x_c, for each coin c.
    - **Objective function**: Minimize $\sum_{c \in C} x_c$.
    - **Constraints**:
        - $x_c \geq 0$, for each coin c.
        - $x_c \in \mathbb{Z}$.
        - $\sum_{c \in C} V(c) \cdot x_c = A$.



Coin Change Solution 1.

2. **V = [1, 3, 7, 12, 27] and A = 293**



Coin Change Solution 2.