$Ч\_1\_23\_08$

$$y'' - 2y' + y = \frac{e^t}{t+1}, y(0) = 0, y'(0) = 0$$

рассмотрим уравнение

$$y_1'' - 2y_1' + y_1 = 1, y_1(0) = 0, y_1' = 0$$

$$y_1(t) = Y(p), y_1'(t) = pY(p), y_1''(t) = p^2Y(p)$$

$$p^2Y(p) - 2pY(p) + Y(p) = \frac{1}{p} \Rightarrow Y(p) = \frac{1}{(p-1)^2} - \frac{1}{p-1} + \frac{1}{p} \Rightarrow$$

$$\Rightarrow y_1 = te^t - e^t + 1 \Rightarrow y_1' = t \cdot e^t$$

Решение исходного уравнения найдем с помощью формулы Дюамеля

$$y(t) = \int_0^t f(\tau) \cdot y_1'(t-\tau) d\tau = \int_0^t \frac{e^\tau}{1+\tau}(t-\tau)e^{t-\tau} d\tau =$$

$$= e^t \int_0^t \frac{e^\tau}{1+\tau}(t-\tau)e^{-\tau} d\tau = e^t \int_0^t \frac{t-\tau}{1+\tau} d\tau = e^t \int_0^t \left(\frac{t}{1+\tau} - \frac{\tau}{1+\tau} \right) d\tau =$$

$$= e^t \left(t\int_0^t \frac{1}{1+\tau} d\tau - \int_0^t \frac{\tau}{1+\tau} d\tau \right) = e^t \left(t\ln(1+\tau)\Big|_0^t - \int_0^t \frac{1+\tau-1}{1+\tau} d\tau \right) =$$

$$= e^t \left(t\ln(1+\tau)\Big|_0^t - \int_0^t \left(1 - \frac{1}{1+\tau}\right) d\tau \right) = e^t \left(t\ln(1+\tau)\Big|_0^t - \left(\tau - \ln(1+\tau)\right)\Big|_0^t \right) =$$

$$= e^t \Big(\big(t\ln(1+t) - t\ln(1+0)\big) - \Big(\big(t - \ln(1+t)\big) - \big(0 - \ln(1+0)\big)\Big) \Big) =$$

$$= e^t \big((t+1)\ln(1+t) - t \big)$$

$Ч\_1\_24\_08$

$y'' + 2y' = 2 + e^t,\ y(0) = 1,\ y'(0) = 2$

Пусть $y(t) = Y(p) \Rightarrow y'(t) = pY(p) - y(0) = pY(p) - 1$

$y''(t) = p^2Y(p) - p \cdot y(0) - y'(0) = p^2Y(p) - p - 2$

Если оригинал функции равен $2 + e^t$ *, то изображение функции равно* $\dfrac{2}{p} + \dfrac{1}{p-1}$

$$p^2Y(p) - p - 2 + 2\left(pY(p) - 1\right) = \frac{2}{p} + \frac{1}{p-1}$$

$$\left(p^2 + 2p\right)Y(p) = \frac{2(p-1) + p}{p(p-1)} + 4 + p$$

$$Y(p) = \frac{2(p-1) + p + (4+p)\cdot p \cdot (p-1)}{p(p-1)\left(p^2 + 2p\right)} = \frac{p^3 + 3p^2 - p - 2}{p^2(p-1)(p+2)} \overset{(1)}{=} \frac{1}{p^2} + \frac{1}{p} - \frac{1}{3}\cdot\frac{1}{p+2} + \frac{1}{3}\cdot\frac{1}{p-1}$$

$$Y(p) = \frac{1}{p^2} + \frac{1}{p} - \frac{1}{3}\cdot\frac{1}{p+2} + \frac{1}{3}\cdot\frac{1}{p-1} \Rightarrow y(t) = t + 1 - \frac{1}{3}e^{-2t} + \frac{1}{3}e^t$$

(1)

$$\frac{p^3 + 3p^2 - p - 2}{p^2(p-1)(p+2)} = \frac{a}{p} + \frac{b}{p^2} + \frac{d}{p-1} + \frac{g}{p+2} =$$

$$= \frac{a \cdot p \cdot (p+2)(p-1) + b\cdot(p-1)(p+2) + d \cdot p^2(p+2) + g \cdot p^2 \cdot (p-1)}{p^2(p-1)(p+2)}$$

$$a \cdot p \cdot (p+2)(p-1) + b\cdot(p-1)(p+2) + d \cdot p^2(p+2) + g \cdot p^2 \cdot (p-1) = p^3 + 3p^2 - p - 2 \Rightarrow$$

$$\Rightarrow \begin{cases} p = 0 : -2b = -2 \\ p = -2 : -12g = 4 \\ p = 1 : 3d = 1 \\ p = -1 : 2a - 2b + d - 2g = 1 \end{cases} \Rightarrow \begin{cases} b = 1 \\ g = -1/3 \\ d = 1/3 \\ 2a - 2 + 1/3 + 2/3 = 1 \end{cases} \Rightarrow \begin{cases} b = 1 \\ g = -1/3 \\ d = 1/3 \\ a = 1 \end{cases}$$

$$\frac{p^3 + 3p^2 - p - 2}{p^2(p-1)(p+2)} = \frac{1}{p} + \frac{1}{p^2} + \frac{1/3}{p-1} - \frac{1/3}{p+2} = \frac{1}{p^2} + \frac{1}{p} - \frac{1}{3}\cdot\frac{1}{p+2} + \frac{1}{3}\cdot\frac{1}{p-1}$$

$Ч\_1\_26\_08$

$$\begin{cases} \dot{x} = -3x - 4y + 1 \\ \dot{y} = 2x + 3y \end{cases}$$

$x(0) = 0, y(0) = 2$

$x(t) = X(p) \Rightarrow \dot{x}(t) = pX(p) - x_0 = pX(p)$

$y(t) = Y(p) \Rightarrow \dot{y}(t) = pY(p) - y_0 = pY(p) - 2$

$$\begin{cases} pX = -3X - 4Y + 1/p \\ pY - 2 = 2X + 3Y \end{cases}$$

$$\begin{cases} (p+3)X + 4Y = 1/p \\ -2X + (p-3)Y = 2 \end{cases}$$

Решим систему методом Крамера

$$\Delta = \begin{vmatrix} p+3 & 4 \\ -2 & p-3 \end{vmatrix} = (p+3)(p-3) - 4\cdot(-2) = p^2 - 1$$

$$\Delta_X = \begin{vmatrix} 1/p & 4 \\ 2 & p-3 \end{vmatrix} = \frac{1}{p}\cdot(p-3) - 2\cdot 4 = -7 - \frac{3}{p}$$

$$\Delta_Y = \begin{vmatrix} p+3 & 1/p \\ -2 & 2 \end{vmatrix} = (p+3)\cdot 2 - \frac{1}{p}\cdot(-2) = 6 + \frac{2}{p} + 2p$$

$$\begin{cases} X = \Delta_X / \Delta = \dfrac{-7 - \dfrac{3}{p}}{p^2 - 1} = \dfrac{-7p - 3}{p(p-1)(p+1)} = \dfrac{-5}{p-1} + \dfrac{3}{p} + \dfrac{2}{p+1} \\ Y = \Delta_Y / \Delta = \dfrac{6 + \dfrac{2}{p} + 2p}{p^2 - 1} = \dfrac{2\left(1 + 3p + p^2\right)}{p(p-1)(p+1)} = \dfrac{5}{p-1} - \dfrac{2}{p} - \dfrac{1}{p+1} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x(t) = -5e^t + 3 + 2e^{-t} \\ y(t) = 5e^t - 2 - e^{-t} \end{cases}$$