DC Motors

1.0 Basic Operation of a DC Motor

A DC motor consists of a stationary magnetic field (stator) and a rotating magnetic field (rotor or armature). A magnetic field can be generated using a permanent magnet or an electromagnet. An electromagnet consists of an insulated wire wound around a iron core. When current flows through the windings a magnetic field is generated. The polarity of the magnetic field (orientation of the N and S poles) is determined by the direction of the current flow, as shown in the Figure 1.0-1 below.

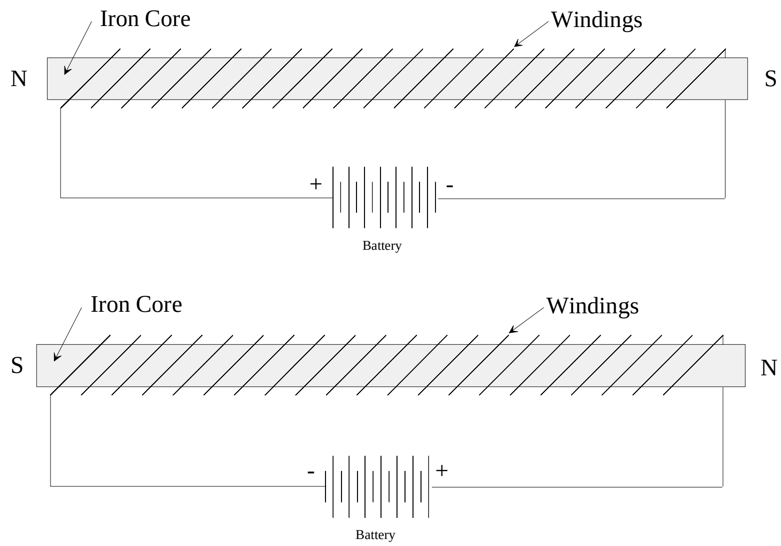


Figure 1.0-1: Electromagnet illustrating the change in polarity of the magnetic field with the change in the direction of current flow

The most common type of DC motor uses a permanent magnet for the fixed magnetic field and an electromagnet for the rotating magnetic field, as shown in Figure 1.0-2. The electromagnet is free to pivot on the axel and the permanent magnet is fixed in place. For the battery polarity shown the electromagnet would rotate to the orientation shown, where the N and S poles of the permanent and electromagnet would align. If the polarity of the battery were reversed the polarity of the electromagnet would reverse and the electromagnet would rotate 180 degrees again aligning the N and S poles of the permanent and electromagnet. At this point this is not a very useful motor, it is capable of rotating only 180 degrees! However what if just as the electromagnet rotates towards the pole alignment point the polarity of the battery is switched. Momentum would carry the rotating electromagnet past the pole alignment point and the reversed polarity of the electromagnet would rotate it another 180 degrees. The key to achieving continuous rotation is to switch the polarity of the electromagnet as it approaches the pole alignment point. This switching is accomplished by using a commutator as shown in Figure 1.0-3. Note the commutator rotates with the electromagnet and connects to the windings.

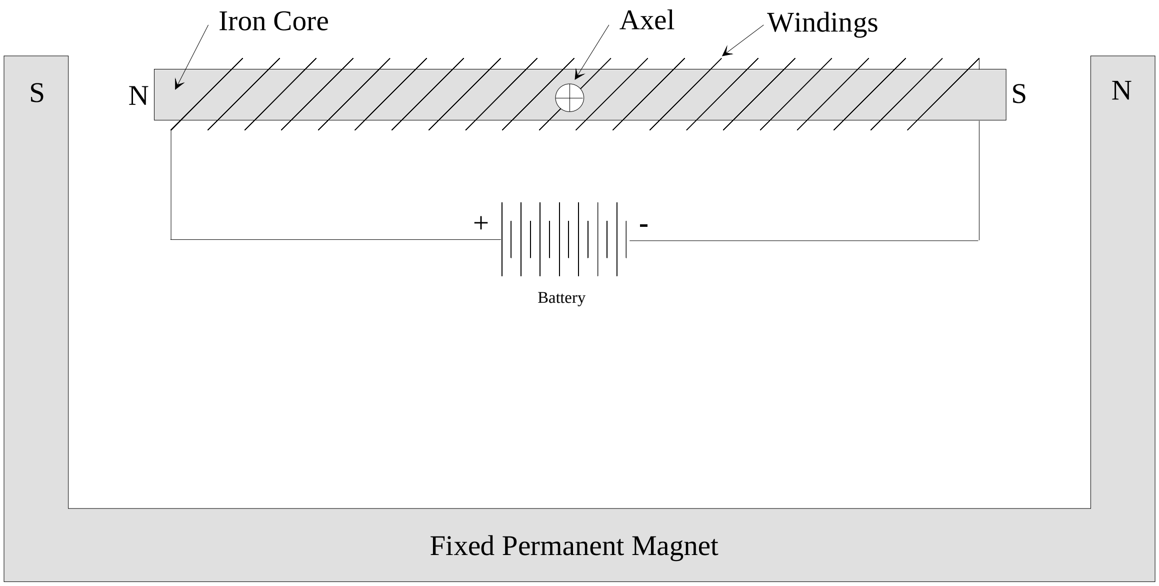


Figure 1.0-2: Simplified DC Motor

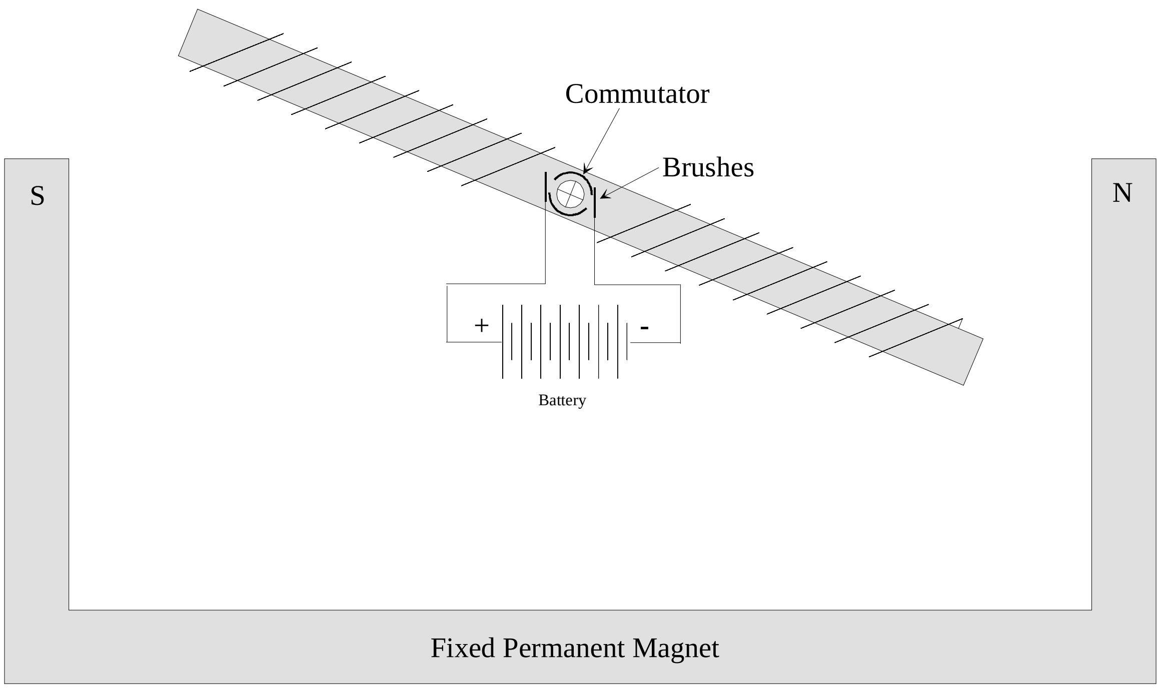


Figure 1.0-3: Simplified DC Motor, with Commutator and Brushes

The combination of the brushes and the commutator switch the polarity of the electromagnet at just the proper time to create a continuous torque and rotational motion of the motor.

Most DC motors have several sets of windings to smooth out the motor torque and motion.

So in summary the key components of a DC motor are a rotating electromagnet, a stationary magnet and a method of switching the polarity of the electromagnet at the proper time.

It is interesting to note that a DC motor results if electrical power is provided and a DC generator results if the electromagnet is rotated by an external mechanical power source.

2.0 Types of DC Motors

2.1 Permanent Magnet DC Motor

The fixed magnetic field is generated using a permanent magnet. The permanent magnet DC motor is the most common and the most cost effective motor up to approximately three horsepower.

2.2 Shunt Wound DC Motor

The fixed magnetic field is generated using an electromagnet rather that a permanent magnet. The shunt wound DC motor is more cost effective in motors above approximately three horsepower. Electromagnets are more cost effective in the higher power ranges.

2.3 Brushless DC Motor

A brushless DC motor eliminates the commutator and brushes and does the necessary switching of the polarity of the rotating electromagnet electronically. This eliminates brush wear and the maintenance associated with replacing the brushes. However brushless motors tend to be more expensive than brush type motors.

2.4 Stepper Motor

A stepper motor is controlled by sending the motor a pulse stream. The motor increments a small value for each pulse and then holds the motor in that position. A stepper motor exhibits high torque at low speeds and can simplify the motor control.

3.0 System Equations for a DC Motor

The figure below shows a schematic drawing of a DC (direct current) motor. The main components are the rotor to which the armature windings are attached and the stator which generates a magnetic field. The magnetic field can be generated by field windings or as in the case shown below a permanent magnet. Additional components include the armature brushes that pass the armature current and the commutator which switches the direction or polarity of the armature current and armature magnetic field at the appropriate time.

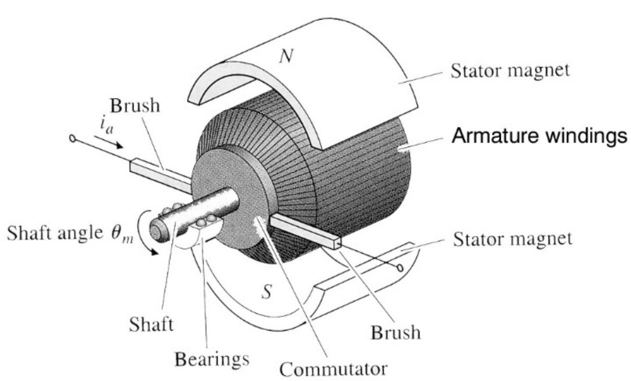


Figure 3.0-1: Schematic Drawing of a DC Motor
(from Feedback Control of Dynamic Systems, Franklin and Powell)

The input to the motor is an input voltage and the output is the motor torque. The motor can then drive an external load as shown in the Figure below.

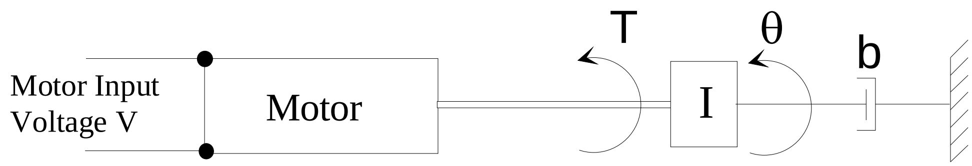


Figure 3.0-2: DC Motor with a Load

This type of DC motor consists of an armature and field circuit as shown below. The armature and field circuit can be independently excited (V and V_f) to control the position and speed of the motor. In most cases one voltage is held constant and the other is varied to control the motor.

The field circuit generates the magnetic field that produces a motor torque in the presence of an armature current (i).

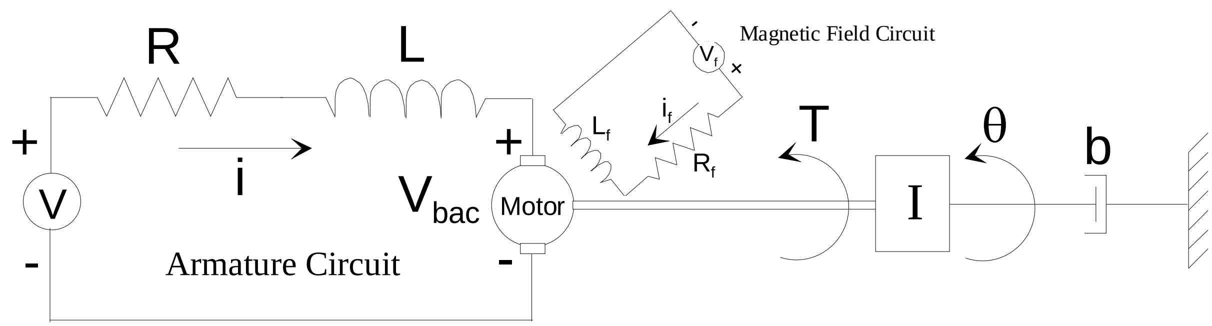


Figure 3.0-3: Electrical Schematic of a DC Motor with a Load

3.1 Armature Controlled (Permanent Magnet) DC Motor

If the magnetic field voltage is held constant the motor is controlled by varying the armature voltage (V). In many cases the field circuit is a permanent magnet that generates the required magnetic field. In this case the magnetic field is constant which provides a linear relationship between the armature current and the motor torque. The armature controlled permanent magnet DC motor is the most common type of DC motor.

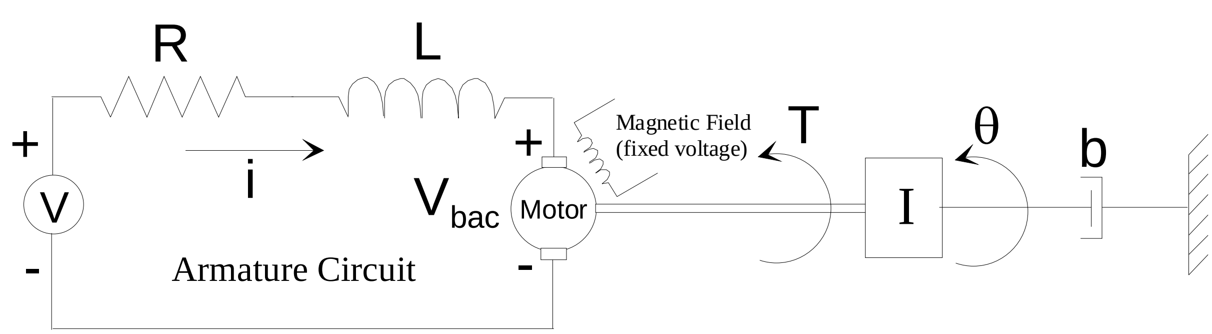


Figure 3.1-1: Electrical Schematic of a Armature Controlled (Permanent Magnet) DC Motor_with a Load

V	Motor Input Voltage (Armature Voltage)
R	Armature Resistance
L	Armature Inductance
i	Armature Current
V_{bac}	Motor Back EMF Voltage ($V_{bac} = K_{bac}\,\dot{\theta}$)
T	Motor Output Torque $(T = K_t\, i)$
K_{bac}	Motor Back EMF Constant
K_t	Motor Torque Constant
I	Motor and Load Inertia
b	Motor and Load Damping
θ	Motor Shaft Angle

The relationship between the armature current and motor torque T is:

$$T = K_t\, i \tag{3.1-1}$$

The back EMF motor voltage (V_{bac}) is related to the motor speed:

$$V_{bac} = K_{bac}\,\dot{\theta} \tag{3.1-2}$$

Defining $\omega = \dot{\theta}$ the equation above becomes:

$$V_{bac} = K_{bac}\,\omega \tag{3.1-2a}$$

K_t and K_{bac} are motor constants provided by the motor manufacturer.

The relationship between the input armature voltage (V) and the motor velocity (ω) will now be derived. Kirchhoff's voltage and current laws as well as Newton's second Law will be used in the development.

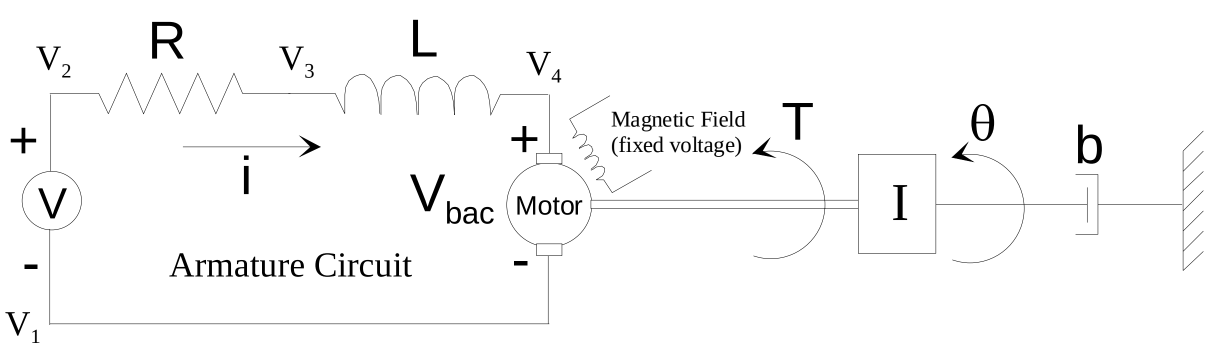


Figure 3.1-2: Electrical Schematic of a Armature Controlled (Permanent Magnet) DC Motor_with a Load Showing the Intermediate Voltages

Applying Kirchhoff's voltage law to the motor circuit above.

$$V_{12} + V_{23} + V_{34} + V_{41} = 0 \tag{3.1-3}$$

Where

$V_{12} = -V$ and $V_{41} = V_{bac} = K_{bac}\,\omega$

Equation 3.1-3 becomes:

$$V_{23} + V_{34} + K_{bac}\,\omega = V \tag{3.1-3a}$$

The elemental relations for the resistor and inductor are:

$$V_{23} = R\,i \qquad V_{34} = L\frac{di}{dt}$$

Equation 3.1-3a now becomes:

$$L\frac{di}{dt} + R\,i + K_{bac}\,\omega = V \tag{3.1-3b}$$

The motor output torque (T) is proportional to the motor current:

$$T = K_t\,i \tag{3.1-4}$$

The free body diagram for the mechanical components of the motor is shown below.

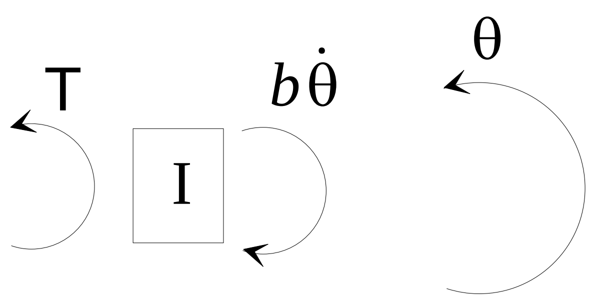


Figure 3.1-3: Mechanical Components – Free Body Diagram

Summing moments yields:

$$\sum M = T - b\dot{\theta} = I\,\ddot{\theta}$$

$$I\,\ddot{\theta} + b\dot{\theta} = T \tag{3.1-5}$$

Equation 3.1-5 can be written in terms of the angular velocity ($\omega = \dot{\theta}$).

$$I\,\dot{\omega} + b\,\omega = T \tag{3.1-5a}$$

We now need to combine equations 3.1-3b, 3.1-4, and 3.1-5a to get the desired relationship between the input voltage to the (V) and the output motor velocity (ω).

$$L\frac{di}{dt}+R\,i+K_{bac}\,\omega=V \qquad (3.1\text{-}3b)$$
$$T=K_t\,i \qquad (3.1\text{-}4)$$
$$I\,\dot{\omega}+b\,\omega=T \qquad (3.1\text{-}5a)$$

This can be accomplished by solving equation 3.1-3b for i and substituting this result and equation 3.1-4 into equation 3.1-5a. Using the s operator ($s=\frac{d}{dt}$) equation 3.1-3b can be rewritten as:

$$L\,s\,i+R\,i+K_{bac}\,\omega=V \qquad (3.1\text{-}3c)$$
$$(L\,s+R)\,i+K_{bac}\,\omega=V \qquad (3.1\text{-}3d)$$

Solving for i:

$$i=\frac{V-K_{bac}\,\omega}{(L\,s+R)} \qquad (3.1\text{-}3e)$$

Substituting equation 3.1-3e and 3.1-4 into 3.1-5a yields:

$$I\,\dot{\omega}+b\,\omega=K_t\,i \qquad (3.1\text{-}5b)$$

$$I\,\dot{\omega}+b\,\omega=K_t\,\frac{V-K_{bac}\,\omega}{(L\,s+R)} \qquad (3.1\text{-}5c)$$

Converting back to differential form:

$$(L\,s+R)(I\,\dot{\omega}+b\,\omega)=K_t\,(V-K_{bac}\,\omega) \qquad (3.1\text{-}5d)$$

$$(I\,L)\,\ddot{\omega}+(b\,L+I\,R)\,\dot{\omega}+(R\,b+K_t\,K_{bac})\,\omega=K_t\,V \qquad (3.1\text{-}5e)$$

Equation 3.1-5e is the differential equation relating the output motor velocity (ω) and the input motor voltage (V).

Expressing equation 3.1-5e in transfer function form:

$$\frac{\omega}{V}=\frac{K_t}{(I\,L)\,s^2+(b\,L+I\,R)\,s+(R\,b+K_t\,K_{bac})} \qquad (3.1\text{-}5f)$$

3.2 Field Controlled DC Motor

When the armature voltage or current is held constant the motor can be controlled by varying the voltage in the field circuit. This type of motor is called a field controlled DC motor.

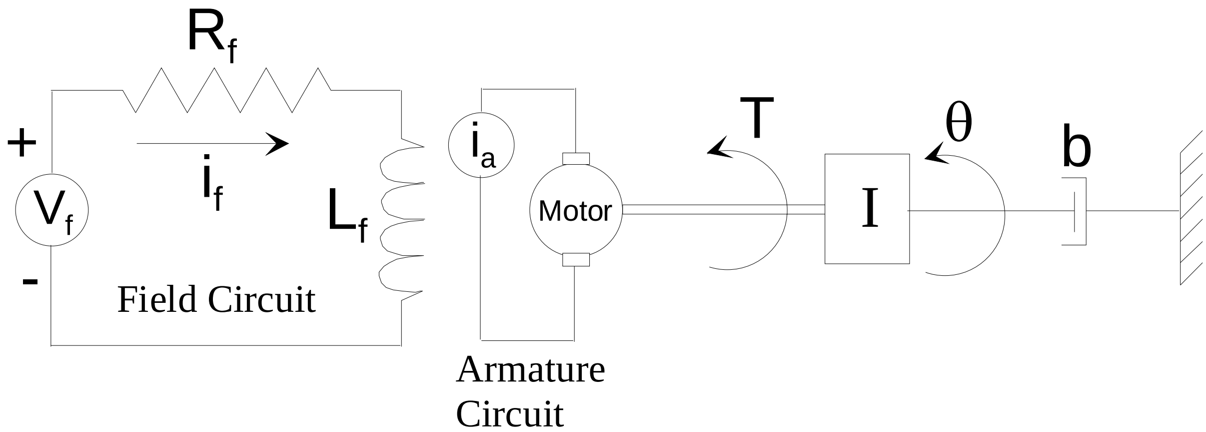


Figure 3.2-1: Electrical Schematic of a Field Controlled DC Motor with a Load

V_f	Motor Input Voltage (Field Voltage)
R_f	Field Resistance
L_f	Field Inductance
i_f	Field Current
i_a	Armature Current (usually constant)
T	Motor Output Torque
I	Motor and Load Inertia
b	Motor and Load Damping
θ	Motor Shaft Angle
K_t	Motor Torque Constant

For field controlled DC motors the torque is proportional to the field current.

$$T = K_t\, i_f \tag{3.2-1}$$

The differential equation relating the motor current (i_f) to the input field voltage (V_f) is:

$$\left(L_f\, s + R_f\right) i_f = V_f \tag{3.2-2}$$

Note that for a field controlled DC motor there is no back EMF voltage.

The equation for the motor and load mechanical elements remains the same:

$$I\,\dot{\omega} + b\,\omega = T \tag{3.1-5a}$$

Following the same procedure the differential equation relating the output motor speed (ω) to the input field voltage (V_f) is:

$$\left(I\,L_f\right)\ddot{\omega} + \left(b\,L_f + I\,R_f\right)\dot{\omega} + \left(R_f\, b\right)\omega = K_t V_f \tag{3.2-3}$$

Expressing equation 3.2-3 in transfer function form:

$$\frac{\omega}{V_f} = \frac{K_t}{\left(I\,L_f\right)s^2 + \left(b\,L_f + I\,R_f\right)s + \left(R_f\,b\right)} \tag{3.2-3a}$$

4.0 Block Diagrams

It is enlightening to express the differential equations above in block diagram form. The block diagram form graphically shows the interconnections between the various elements.

4.1 Armature Controlled DC Motor – Block Diagram

The three differential equations representing the motor system are shown below.

$$(L\,s+R)i+K_{bac}\,\omega=V \qquad (3.1\text{-}3d)$$
$$T=K_t\,i \qquad (3.1\text{-}4)$$
$$I\,\dot{\omega}+b\,\omega=T \qquad (3.1\text{-}5a)$$

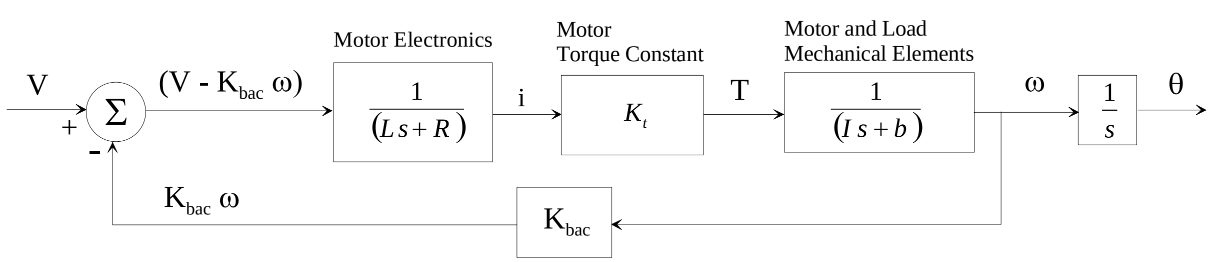
Equation 3.1-5a can be expressed in s operator form as:

$$(I\,s+b)\omega=T \qquad (4.1\text{-}1)$$

Further each of these equations can be expressed in transfer function form.

$$\frac{i}{(V-K_{bac}\,\omega)}=\frac{1}{(L\,s+R)} \quad \text{or} \quad \frac{(V-K_{bac}\,\omega)}{i}=(L\,s+R) \qquad (4.1\text{-}2)$$
$$\frac{i}{T}=\frac{1}{K_t} \quad \text{or} \quad \frac{T}{i}=K_t \qquad (4.1\text{-}3)$$
$$\frac{\omega}{T}=\frac{1}{(I\,s+b)} \quad \text{or} \quad \frac{T}{\omega}=(I\,s+b) \qquad (4.1\text{-}4)$$

Now these transfer functions can be used to construct the block diagram.



Where,

$$\frac{\omega}{V}=\frac{K_t}{(I\,L)\,s^2+(b\,L+I\,R)\,s+(R\,b+K_t\,K_{bac})} \qquad (4.1\text{-}5)$$

$$\frac{\theta}{V}=\frac{K_t}{s\left[(I\,L)\,s^2+(b\,L+I\,R)\,s+(R\,b+K_t\,K_{bac})\right]} \qquad (4.1\text{-}6)$$

4.2 Field Controlled DC Motor – Block Diagram

It is enlightening to express the equations above in block diagram form. The block diagram form graphically shows the interconnections between the various elements. The three differential equations representing the motor system are shown below.

$$\left(L_f\, s + R_f\right) i_f = V_f \qquad (3.2\text{-}2)$$

$$T = K_t\, i_f \qquad (3.2\text{-}1)$$

$$I\,\dot{\omega} + b\,\omega = T \qquad (3.1\text{-}5a)$$

Equation 3.1-5a can be expressed in s operator form as:

$$\left(I\, s + b\right)\omega = T \qquad (4.2\text{-}1)$$

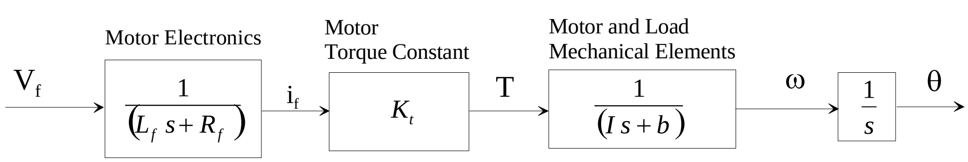
Further each of these equations can be expressed in transfer function form.

$$\frac{i_f}{V_f} = \frac{1}{\left(L_f\, s + R_f\right)} \quad \text{or} \quad \frac{V_f}{i_f} = \left(L_f\, s + R_f\right) \qquad (4.2\text{-}2)$$

$$\frac{i_f}{T} = \frac{1}{K_t} \quad \text{or} \quad \frac{T}{i_f} = K_t \qquad (4.2\text{-}3)$$

$$\frac{\omega}{T} = \frac{1}{\left(I\, s + b\right)} \quad \text{or} \quad \frac{T}{\omega} = \left(I\, s + b\right) \qquad (4.2\text{-}4)$$

Now these transfer functions can be used to construct the block diagram.



Where,

$$\frac{\omega}{V_f} = \frac{K_t}{\left(I\, L_f\right)s^2 + \left(b\,L_f + I\,R_f\right)s + \left(R_f\, b\right)} \qquad (4.2\text{-}5)$$

$$\frac{\theta}{V_f} = \frac{K_t}{s\left[\left(I\, L_f\right)s^2 + \left(b\,L_f + I\,R_f\right)s + \left(R_f\, b\right)\right]} \qquad (4.2\text{-}6)$$

By comparing the block diagram for the field controlled DC Motor with the Block diagram for the Armature Controlled DC Motor it is evident that the main difference is the lack of the back EMF voltage in the field controlled motor.

5.0 Approximations for the Motor Time Constants [Palm]

Manufacture's catalogs often give approximate values for the motor time constants. The time constants are given as the mechanical time constant $\left(\tau_{mech}\right)$ and the electrical time constant $\left(\tau_{elec}\right)$ as shown below.

5.1 Armature Controlled DC Motor – Approximations for the Motor Time Constants

The approximate time constants for an armature controlled DC motor are shown below.

$$\tau_{mech} = \frac{R\,I}{K_{bac}\,K_t} \tag{5.1-1}$$

$$\tau_{elec} = \frac{L}{R} \tag{5.1-2}$$

It should be noted that these formulas are approximations and will only give accurate results in certain circumstances. The development below will illustrate the circumstances in which these equations will give accurate results.

Starting with the differential equation for the motor system:

$$\left(I\,L\right)\ddot{\omega} + \left(b\,L + I\,R\right)\dot{\omega} + \left(R\,b + K_t\,K_{bac}\right)\omega = K_t\,V \tag{3.1-5e}$$

The characteristic equation becomes:

$$\left(I\,L\right)s^2 + \left(b\,L + I\,R\right)s + \left(R\,b + K_t\,K_{bac}\right) = 0 \tag{5.1-3}$$

Using the quadratic formula the roots of the characteristic equation become:

$$s = \frac{-\left(b\,L + I\,R\right) \pm \sqrt{\left(b\,L + I\,R\right)^2 - 4\left(I\,L\right)\left(R\,b + K_t\,K_{bac}\right)}}{2\left(I\,L\right)} \tag{5.1-4}$$

This can be rewritten as:

$$s = \frac{\left(b\,L + I\,R\right)}{2\left(I\,L\right)}\left[-1 \pm \sqrt{1 - \frac{4\left(I\,L\right)\left(R\,b + K_t\,K_{bac}\right)}{\left(b\,L + I\,R\right)^2}}\;\right] \tag{5.1-5}$$

If $\left[\dfrac{4\left(I\,L\right)\left(R\,b + K_t\,K_{bac}\right)}{\left(b\,L + I\,R\right)^2}\right] << 1$ (5.1-6) then using the approximation

$$\sqrt{1-x} \approx \left(1-\frac{x}{2}\right) \qquad (5.1\text{-}7)$$

$$s = \frac{(bL+IR)}{2(IL)}\left[-1 \pm \left[1-\frac{2(IL)(Rb+K_t K_{bac})}{(bL+IR)^2}\right]\right] \qquad (5.1\text{-}8)$$

The two roots then become:

$$s_1 = \frac{(bL+IR)}{2(IL)}\left[-\frac{2(IL)(Rb+K_t K_{bac})}{(bL+IR)^2}\right] = \left[-\frac{(Rb+K_t K_{bac})}{(bL+IR)}\right] \qquad (5.1\text{-}9)$$

$$s_2 = \frac{(bL+IR)}{2(IL)}\left[-2+\frac{2(IL)(Rb+K_t K_{bac})}{(bL+IR)^2}\right] \qquad (5.1\text{-}10)$$

But based on the approximation $\left[\frac{4(IL)(Rb+K_t K_{bac})}{(bL+IR)^2}\right] << 1$ the second term in the bracketed expression can be neglected yielding the following second root.

$$s_2 \approx -\frac{(bL+IR)}{(IL)} \qquad (5.1\text{-}11)$$

The time constants now become:

$$\tau_{mech} = -\frac{1}{s_1} = \frac{(bL+IR)}{(Rb+K_t K_{bac})} \qquad (5.1\text{-}12)$$

$$\tau_{elec} = -\frac{1}{s_2} = \frac{(IL)}{(bL+IR)} \qquad (5.1\text{-}13)$$

If the damping b is small the expressions above which are base on the approximation $\left[\frac{4(IL)(Rb+K_t K_{bac})}{(bL+IR)^2}\right] << 1$ reduce to the simpler expressions above (equations 5.1-1 and 5.1-2).

5.2 Field Controlled DC Motor – Approximations for the Motor Time Constants

Starting with the differential equation for the motor system:

$$(IL_f)\ddot{\omega} + (bL_f + IR_f)\dot{\omega} + (R_f b)\omega = K_t V_f \qquad (3.2\text{-}3)$$

The characteristic equation becomes:

$$\left(I\,L_f\right)s^2+\left(b\,L_f+I\,R_f\right)s+\left(R_f\,b\right)=0 \tag{5.2-1}$$

Using the quadratic formula the roots of the characteristic equation become:

$$s=\frac{-\left(b\,L_f+I\,R_f\right)\pm\sqrt{\left(b\,L_f+I\,R_f\right)^2-4\left(I\,L_f\,R_f\,b\right)}}{2\left(I\,L_f\right)} \tag{5.2-2}$$

This can be rewritten as:

$$s=\frac{-\left(b\,L_f+I\,R_f\right)\pm\sqrt{\left(b\,L_f-I\,R_f\right)^2}}{2\left(I\,L_f\right)}=\frac{-\left(b\,L_f+I\,R_f\right)\pm\left(b\,L_f-I\,R_f\right)}{2\left(I\,L_f\right)} \tag{5.2-3}$$

The two roots then become:

$$s_1=\frac{-\left(b\,L_f+I\,R_f\right)-\left(b\,L_f-I\,R_f\right)}{2\left(I\,L_f\right)}=\frac{-2\,b\,L_f}{2\left(I\,L_f\right)}=\frac{-b}{I} \tag{5.2-4}$$

$$s_2=\frac{-\left(b\,L_f+I\,R_f\right)+\left(b\,L_f-I\,R_f\right)}{2\left(I\,L_f\right)}=\frac{-2\,I\,R_f}{2\left(I\,L_f\right)}=\frac{-R_f}{L_f} \tag{5.2-5}$$

The time constants now become:

$$\tau_{mech}=-\frac{1}{s_1}=\frac{I}{b} \tag{5.2-6}$$

$$\tau_{elec}=-\frac{1}{s_2}=\frac{R_f}{L_f} \tag{5.2-7}$$

Note that in this case no approximation was necessary in arriving at these roots and time constants. In the case of a field controlled DC motor the response will always be overdamped (two real roots).

6.0 Example - Armature Controlled DC Motor

The following information was obtained from the motor manufacture's catalog.

R	=	1.1 Ω	(armature resistance)
L	=	0.002 H	(armature inductance)
b_m	=	1.3e-5 N-m-s/rad	(motor damping)
I_m	=	3.8e-5 N-m-s^2/rad	(motor inertia)
K_t	=	.060 N-m/amp	(motor torque constant)
K_{bac}	=	.050 V/(rad/sec)	(motor back EMF constant)

In this example the dynamics of the motor alone will be examined. Therefore the load inertia and damping will be zero and the total inertia and damping becomes:

$I = I_m = 3.8e\text{-}5$ N-m-s^2/rad
$b = b_m = 1.3e\text{-}5$ N-m-s/rad

The differential equation for the Armature Controlled DC motor is:

$$(I\,L)\,\ddot{\omega} + (b\,L + I\,R)\,\dot{\omega} + (R\,b + K_t\,K_{bac})\,\omega = K_t V \qquad (3.1\text{-}5e)$$

Dividing through by K_t.

$$\frac{(I\,L)}{K_t}\,\ddot{\omega} + \frac{(b\,L + I\,R)}{K_t}\,\dot{\omega} + \frac{(R\,b + K_t\,K_{bac})}{K_t}\,\omega = V \qquad (6.0\text{-}1)$$

Substituting the parameter values:

$$\frac{(3.8e-5)(.002)}{.060}\,\ddot{\omega} + \frac{((1.3e-5)(.002)+(3.8e-5)(1.1))}{.060}\,\dot{\omega} + \frac{((1.1)(1.3e-5)+(.06)(.05))}{.060}\,\omega = V$$

$$(1.267e-6)\,\ddot{\omega} + (6.971e-4)\dot{\omega} + (5.023e-2)\,\omega = V \qquad (6.0\text{-}2)$$

The roots of the characteristic equation are:

$s_1 = -\,85.2713$
$s_2 = -\,464.9261$

or in terms of the mechanical and electrical time constants:

$$\tau_{mech} = -\frac{1}{s_1} = .01172 \text{ sec}$$

$$\tau_{elec} = -\frac{1}{s_2} = .002151 \text{ sec}$$

Comparing these values to the calculated approximate values for the mechanical and electrical time constants:

$$\tau_{mech} == \frac{(b\,L + I\,R)}{(R\,b + K_t\,K_{bac})} = \frac{((1.3e-5)(.002) + (3.8e-5)(1.1))}{((1.1)(1.3e-5) + (.06)(.05))} = .01388 \text{ sec}$$

$$\tau_{elec} = -\frac{1}{s_2} = \frac{(I\,L)}{(b\,L + I\,R)} = \frac{(3.8e-5)(.002)}{((1.3e-5)(.002) + (3.8e-5)(1.1))} = .001817 \text{ sec}$$

Note the actual time constant values are close but not exactly equal to the approximate calculated values.

The figure below shows the unit (1 Volt) step response of the motor system.

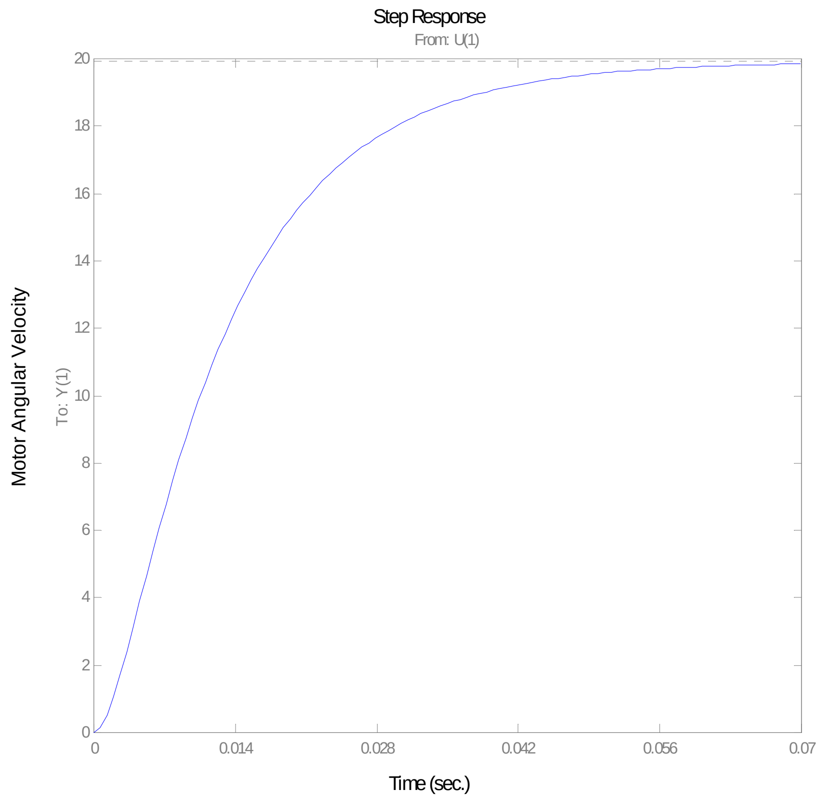


Figure 6.0-1: Unit Step Response

7.0 References

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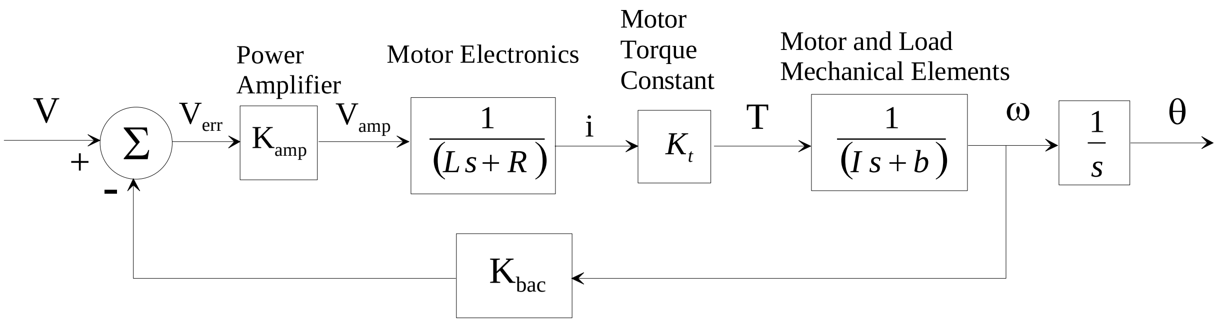
Appendix A: Armature Controlled DC Motor with Position Feedback

Shown below is the block diagram for a armature controlled DC motor which includes the power amplifier.



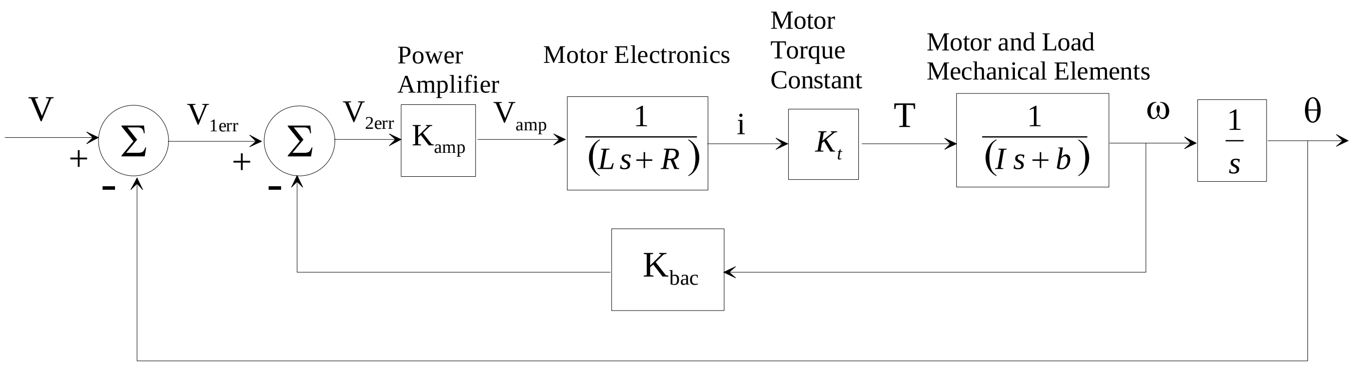
Where,

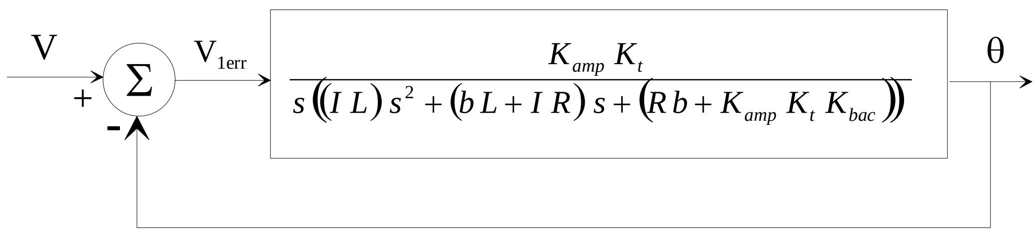
V_{amp}	Motor Input Voltage (Armature Voltage)
R	Motor Armature Resistance
L	Motor Armature Inductance
i	Motor Armature Current
V_{bac}	Motor Back EMF Voltage ($V_{bac} = K_{bac}\,\dot{\theta}$)
T	Motor Output Torque $(T = K_t\, i)$
K_{bac}	Motor Back EMF Constant
K_t	Motor Torque Constant
K_{amp}	Power Amplifier Gain
V_{amp}	Output Voltage of the Power Amplifier
I	Motor and Load Inertia
b	Motor and Load Damping
ω	Motor Shaft Velocity
θ	Motor Shaft Angle

$$\frac{\omega}{V} = \frac{K_{amp}\,K_t}{(I\,L)\,s^2 + (b\,L + I\,R)\,s + (R\,b + K_{amp}\,K_t\,K_{bac})} \qquad \text{(A-1)}$$

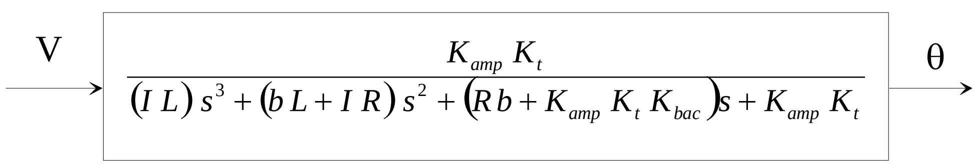
$$\frac{\theta}{V} = \frac{K_{amp}\,K_t}{s\left((I\,L)\,s^2 + (b\,L + I\,R)\,s + (R\,b + K_{amp}\,K_t\,K_{bac})\right)} \quad \text{(Open Loop)} \qquad \text{(A-2)}$$

Add Position Feedback:



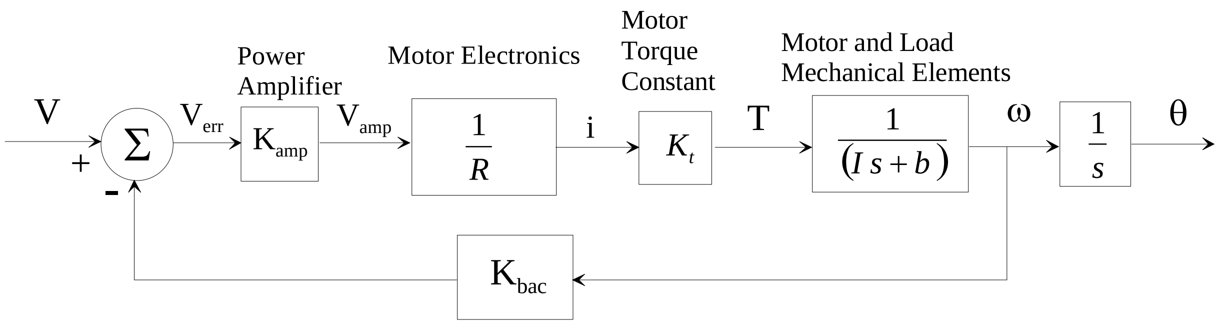


$$\frac{\theta}{V}=\frac{K_{amp}\,K_t}{\left(I\,L\right)s^3+\left(b\,L+I\,R\right)s^2+\left(R\,b+K_{amp}\,K_t\,K_{bac}\right)s+K_{amp}\,K_t} \quad \text{(Closed Loop)} \qquad \text{(A-3)}$$



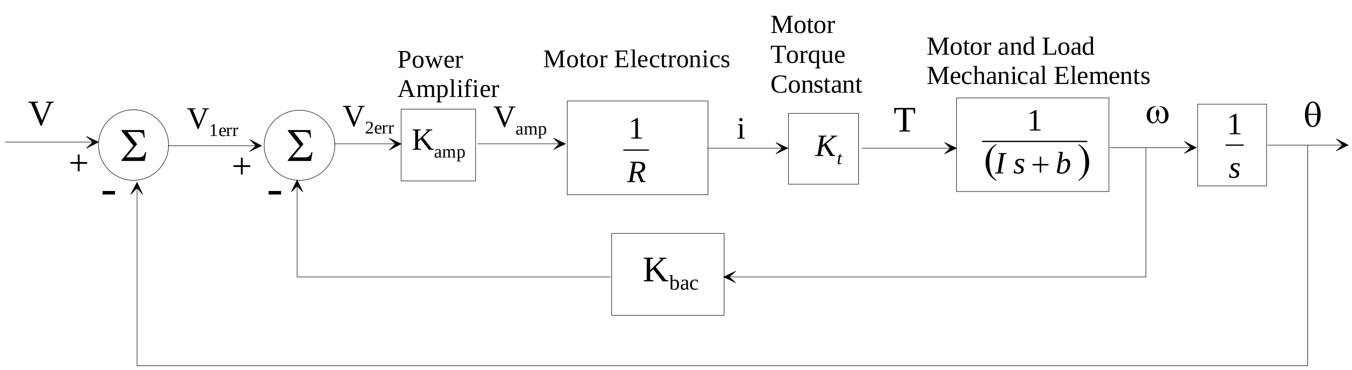
Appendix B: Simplification of Armature Controlled DC Motor

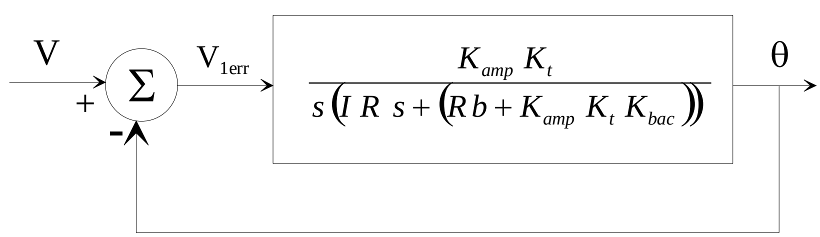
Sometimes the motor inductance L is small enough such that it can be neglected. In this is the case the simplified transfer functions become.



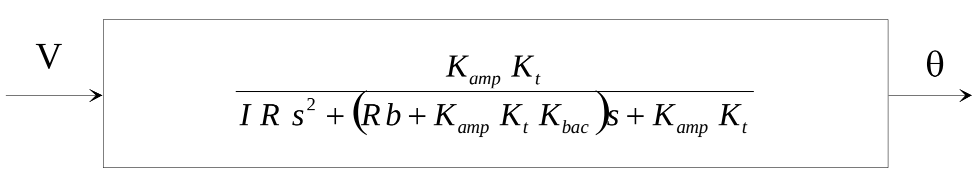
$$\frac{\omega}{V} = \frac{K_{amp}\,K_t}{I\,R\;s + \left(R\,b + K_{amp}\,K_t\,K_{bac}\right)} \qquad \text{(B-1)}$$

$$\frac{\theta}{V} = \frac{K_{amp}\,K_t}{s\left(I\,R\;s + \left(R\,b + K_{amp}\,K_t\,K_{bac}\right)\right)} \qquad \text{(B-2)}$$



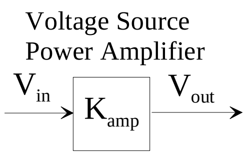


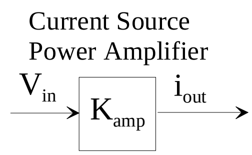
$$\frac{\theta}{V} = \frac{K_{amp}\,K_t}{I\,R\;s^2 + \left(R\,b + K_{amp}\,K_t\,K_{bac}\right)s + K_{amp}\,K_t} \qquad \text{(B-3)}$$



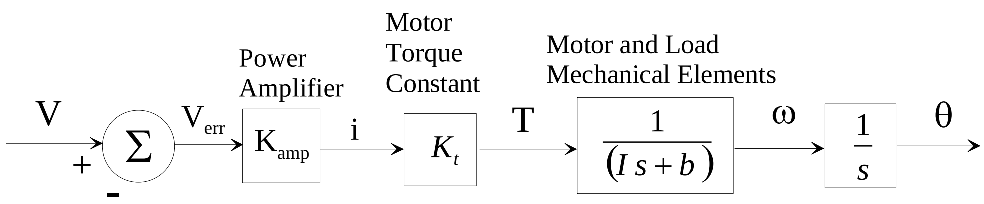
Appendix C: Armature Controlled DC Motor with a Current Supply Power Amplifier

The power amplifier used above is a voltage source type amplifier. That is the input to the amplifier is a voltage V_{in} and the output of the amplifier is $V_{out} = K_{amp} * V_{in}$ and whatever necessary current to maintain that voltage. A current source amplifier provides an output current instead of a voltage ($i_{out} = K_{amp} * V_{in}$) and whatever voltage that is necessary to maintain that current.



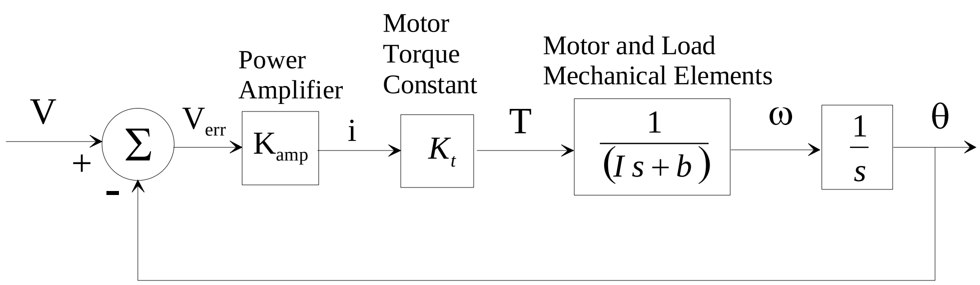


A current source power amplifier is often used in position control applications because the motor resistance, inductance and back emf voltage are eliminated from the transfer function resulting in a much simpler transfer function for the system.



$$\frac{\omega}{V} = \frac{K_{amp} K_t}{(I s + b)} \quad \text{(C-1)}$$

$$\frac{\theta}{V} = \frac{K_{amp} K_t}{s(I s + b)} \quad \text{(C-2)}$$



$$\frac{\theta}{V} = \frac{K_{amp} K_t}{I s^2 + b s + K_{amp} K_t} \quad \text{(C-3)}$$

$$V \longrightarrow \boxed{\frac{K_{amp} K_t}{I s^2 + b s + K_{amp} K_t}} \longrightarrow \theta$$