№1.

$$I_n(\alpha) = \int_0^1 \frac{x^n}{x+\alpha}\,dx = \int_0^1 \frac{x^{n-1}\cdot x}{x+\alpha}\,dx =$$

$$= \int_0^1 \frac{x^{n-1}(x-\alpha+\alpha)}{x+\alpha}\,dx = \int_0^1 x^{n-1}dx - \int_0^1 \frac{\alpha x^{n-1}}{x+\alpha} = \frac{1}{n}x^n\Big|_0^1 -$$

$$-\alpha\int_0^1 \frac{x^{n-1}}{x+\alpha}\,dx$$

$$\Downarrow$$

$$\boxed{I_n(\alpha) = \frac{1}{n} - \alpha I_{n-1}}$$

прямая

$$I_0(\alpha) = \int_0^1 \frac{x^0}{x+\alpha}\,dx = \int_0^1 \frac{dx}{x+\alpha} = \ln|x+\alpha|\Big|_0^1 =$$

$$= \ln|1+\alpha| - \ln|\alpha| = \ln\left|\frac{1+\alpha}{\alpha}\right|$$

Обратная рекурсия:

$$I_n(\alpha) = \frac{1}{n} - \alpha I_{n-1}$$

$$I_{n-1} = \frac{1}{n\alpha} - \frac{I_n(\alpha)}{\alpha}$$

$$\Downarrow$$

$$\boxed{I_n = \frac{1}{\alpha(n+1)} - \frac{I_{n+1}(\alpha)}{\alpha}}$$

обратная

№3.

$a_0 = 1$, $a_1 = -3$

$a_n = 6a_{n-2} - a_{n-1}$

Примем $a_n = \lambda^n$, тогда:

$$\lambda^2 / 1 = 6 \cdot \frac{1}{\lambda^2} - \lambda / \frac{1}{\lambda}$$

$\Downarrow$

$$\lambda^2 + \lambda - 6 = 0$$

$\Downarrow$

$$\begin{cases} \lambda = 2 \\ \lambda = -3 \end{cases}$$

$$a_n = \alpha 2^n + \beta (-3)^n$$

Найдем $\alpha, \beta = const$: $\begin{cases} \alpha + \beta = a_0 \\ 2\alpha - 3\beta = a_1 \end{cases} \Rightarrow \begin{cases} \alpha + \beta = 1 \\ 2\alpha - 3\beta = -3 \end{cases} \Rightarrow \begin{cases} \alpha = 0 \\ \beta = 1 \end{cases}$

$$a_{2020} = (-3)^{2020} = \boxed{3^{2020}}$$

№4.

$$A = \begin{pmatrix} 1 & 10 \\ \delta & 1 \end{pmatrix}, \quad \varepsilon(\delta) = \max \lambda(A), \quad \delta > 0$$

$k(\delta) = ?$ $(\delta = 10$ и $\delta = 0{,}1)$

Решение:

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 10 \\ \delta & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 10\delta = 0$$

$$\Downarrow$$

$$1 - 2\lambda + \lambda^2 - 10\delta = 0$$

$$\lambda^2 - 2\lambda + (1 - 10\delta) = 0$$

$$D = 4 - 4(1 - 10\delta)$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{40\delta}}{2} = 1 \pm \sqrt{10\delta}$$

$$\lambda_{max} = 1 + \sqrt{10\delta}$$

$$k(\delta) = \frac{d\varepsilon(\delta)}{d\delta} = \sqrt{10} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\delta}} = \frac{\sqrt{10}}{2} \cdot \delta^{-\frac{1}{2}}$$

$$k(\delta = 10) = \frac{\sqrt{10}}{2} \cdot \frac{1}{\sqrt{10}} = \boxed{\frac{1}{2}}$$

$$k(\delta = 0{,}1) = \frac{\sqrt{10}}{2} \cdot \frac{1}{\sqrt{\frac{1}{10}}} = \frac{10}{2} = \boxed{5}$$