Assignment03_main

November 29, 2021

1 Assigment 3

This notebook covers the exercises for the next two weeks.

1.1 Exercise 5: Block Matching and Harris Corner Detection

1.2 Ex. 5.1 Dense Optical Flow by Block Matching

- implement the block matching method as shown in the lecture
- take two frames from the datasets "lane_detection" or "copter_flight" with variable distances in time (1, 2, x) and compute the vector flow field
- display a subset of flow vectors on the gray-value version of the first image, by drawing a respective line. adjust the grid density such that not too many vectors overlap (**RESULT**)

```
[]: from google.colab import drive drive.mount('/content/drive')
```

Mounted at /content/drive

```
[ ]: path_to_project = "/content/drive/MyDrive/University/computer vision/"
```

```
[]: %matplotlib inline
import matplotlib.pyplot as plt
from skimage import io, data, feature, color, draw
import numpy as np

# Chose other images if you like
lane1 = io.imread(path_to_project + 'images/lane_detection/f00000.png')
lane2 = io.imread(path_to_project + 'images/lane_detection/f00001.png')

# Footage from our Neurocopter project:
# http://berlinbiorobotics.blog/projects/neurocopter/
copter1 = io.imread(path_to_project + 'images/copter_flight/frame050.jpg')
copter2 = io.imread(path_to_project + 'images/copter_flight/frame052.jpg')

fig = plt.figure(figsize=(15, 10))
ax11 = plt.subplot(2, 2, 1)
ax12 = plt.subplot(2, 2, 2)
ax21 = plt.subplot(2, 2, 3)
```

```
ax22 = plt.subplot(2, 2, 4)
ax11.imshow(lane1)
ax12.imshow(lane2)
ax21.imshow(copter1)
ax22.imshow(copter2)
```

[]: <matplotlib.image.AxesImage at 0x7fae08dd75d0>

500



500

400

```
[]: import cv2

def find_vector_flow(image_1, image_2, coords_of_block = (350, 300, 450, 200)):
    """
    image_1: initial image from timestamp 1
    image_2: image from timestamp x

    coords_of_block: tuple with 4 integers indicating bbox (x1, y1, x2, y2)
    """
    x1, y1, x2, y2 = coords_of_block
    width = x2-x1
    height = y1-y2

image_1 = np.copy(image_1)
```

```
image_2 = np.copy(image_2)
   # extract initial block
   initial_block = image_1[y2 : y1, x1 : x2]
   # ranges of x and y-shifts
   delta_x_range = np.arange(-width // 2, width // 2, 1)
   delta_y_range = np.arange(-height // 2, height // 2, 1)
   shifts_arr = np.zeros((len(delta_y_range), len(delta_x_range)))
   # value added to the shift ( to do prober indexing in shifts array )
   x_incrementor = np.abs(delta_x_range[0])
   y_incrementor = np.abs(delta_y_range[0])
   for x_shift in delta_x_range:
       for idx, y_shift in enumerate(delta_y_range):
           x1_new, y1_new = x1 + x_shift, y1 + y_shift
           x2_new, y2_new = x2 + x_shift, y2 + y_shift
           new_block = image_2[y2_new : y1_new, x1_new : x2_new]
           sim = np.linalg.norm(initial_block * new_block) # np.linalg.
\rightarrow norm(initial block - new block)
           shifts_arr[y_shift + y_incrementor, x_shift + x_incrementor] = sim
   min_coords = np.where(shifts_arr == np.min(shifts_arr))
   x1_new, y1_new = x1 + min_coords[1] - x_incrementor, y1 + min_coords[0] -__
\rightarrowy_incrementor
   x2 new, y2 new = x2 + min coords[1] - x incrementor, y2 + min coords[0] -
→y_incrementor
   cv2.rectangle(image_1, (x2, y2), (x1, y1), (170,110,30),3)
   cv2.rectangle(image_2, (x2, y2), (x1, y1), (170,110,30),3)
   cv2.rectangle(image_2, (x2_new, y2_new), (x1_new, y1_new), (0,255,0),3)
   cv2.putText(image_2, "New block", (x2_new - 50, y2 - 10),cv2.
→FONT_HERSHEY_SIMPLEX, 1, (0,255,0), 2, cv2.LINE_AA)
   fig, ax = plt.subplots(1,3, figsize = (26,26))
   ax[0].imshow(image_1)
   ax[1].imshow(image_2)
```

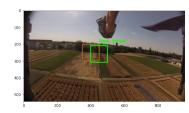
```
x,y = np.meshgrid(np.linspace(x1,x2,3),np.linspace(y2,y1,3))
u = min_coords[1] - x_incrementor
v = min_coords[0] - y_incrementor

ax[2].imshow(color.rgb2gray(image_1), cmap = 'gray')
ax[2].quiver(x,y,u,v, alpha = 0.9)

plt.show()
return min_coords
```

[]: find_vector_flow(copter1, copter2)







[]: (array([58]), array([99]))

1.3 Ex. 5.2 Harris Corner Detection

- implement the Harris Corner Detector as discussed in the lecture
- compute corners in the first image and track them with Lucas-Kanade (use e.g. the function "calcOpticalFlowPyrLK" in OpenCV)
- \bullet mark the positions of your Harris corners and draw the flow vectors found by Lucas-Kanade on the gray-value versions of the first image (**RESULT**)

```
[]: # from previous assignment

def conv2d(image, kernel, pad=0.5):
    """
    If kernel has an even number the "hangover pixel"
    will be to the top/left.
    """
    result = np.zeros(image.shape)
```

```
# Convert kernel into kernel with even sides
    kernel = np.array(kernel)
    if kernel.shape[0] % 2 == 0:
        new_kernel = np.zeros((kernel.shape[0] + 1, kernel.shape[1]))
        new_kernel[:kernel.shape[0]] = kernel
        kernel = new kernel
    if kernel.shape[1] % 2 == 0:
        new kernel = np.zeros((kernel.shape[0], kernel.shape[1] + 1))
        new_kernel[:,:kernel.shape[1]] = kernel
        kernel = new kernel
    # pad the image
    pad_y, pad_x = kernel.shape[0] // 2, kernel.shape[1] // 2
    padded = np.empty((image.shape[0] + pad_y * 2, image.shape[1] + pad_x * 2))
    if not pad_y and not pad_x:
        raise Exception("Bad Kernel")
    elif not pad_y:
        padded[:, pad_x:-pad_x] = image
        padded[:,:pad_x], padded[:,-pad_x:] = pad, pad
    elif not pad x:
        padded[pad_y:-pad_y,:] = image
        padded[:pad_y], padded[-pad_y:] = pad, pad
    else:
        padded[pad_y:-pad_y, pad_x:-pad_x] = image
        padded[:pad_y], padded[-pad_y:] = pad, pad
        padded[:,:pad_x], padded[:,-pad_x:] = pad, pad
    # Shift image for every kernel value
    for kernel_y in range(kernel.shape[0]):
        for kernel_x in range(kernel.shape[1]):
            displacement_y = kernel_y + image.shape[0]
            displacement_x = kernel_x + image.shape[1]
            shifted_image = np.empty_like(image)
            shifted_image = padded[kernel_y:displacement_y, kernel_x:
→displacement_x]
            result += shifted_image * kernel[kernel_y, kernel_x]
    kernel_sum = np.sum(kernel)
    return result / kernel_sum if kernel_sum else result
def dnorm(x, mu, sd):
    return 1 / (np.sqrt(2 * np.pi) * sd) * np.e ** (-np.power((x - mu) / sd, 2)_
 \rightarrow/ 2)
```

```
def gaussian_kernel(size, sigma=1, verbose=False):
    kernel_1D = np.linspace(-(size // 2), size // 2, size)
    for i in range(size):
        kernel_1D[i] = dnorm(kernel_1D[i], 0, sigma)
    kernel_2D = np.outer(kernel_1D.T, kernel_1D.T)

kernel_2D *= 1.0 / kernel_2D.max()

if verbose:
    plt.imshow(kernel_2D, interpolation='none',cmap='gray')
    plt.title("Image")
    plt.show()

return kernel_2D
```

```
[]: def get_harris_responce(img, k = 0.05):
        if len(img.shape) > 2:
             img_gray = color.rgb2gray(img)
        else:
            img_gray = img
         # apply sobel kernel to find gradients
        sobel_kernel = np.array([[-1,0,1],[-2,0,2],[-1,0,1]])
        # compute x and y derivatives of image
        gradient_along_x = conv2d(img_gray, sobel_kernel)
        gradient_along_y = conv2d(img_gray,sobel_kernel.T)
        gradient_magnitude = np.sqrt(np.square(gradient_along_x) + np.
     # compute products of derivatives at every pixel
        gradient_along_x2 = gradient_along_x * gradient_along_x
        gradient_along_y2 = gradient_along_y * gradient_along_y
        gradient_along_xy = gradient_along_x * gradient_along_y
         # compute the sums of the products of derivatives at each pixel (apply_
     \rightarrow qaussian filter)
        kernel_gaus = gaussian_kernel(3,1)
        gradient_along_x2 = conv2d(gradient_along_x2, kernel_gaus)
        gradient_along_y2 = conv2d(gradient_along_y2, kernel_gaus)
        gradient_along_xy = conv2d(gradient_along_xy, kernel_gaus)
        # compute the response of the detector at each pixel
```

```
det = gradient_along_x2 * gradient_along_y2 - gradient_along_xy *□

→gradient_along_xy

trace = gradient_along_x2 + gradient_along_y2

r = det - k * (trace ** 2)

return r
```

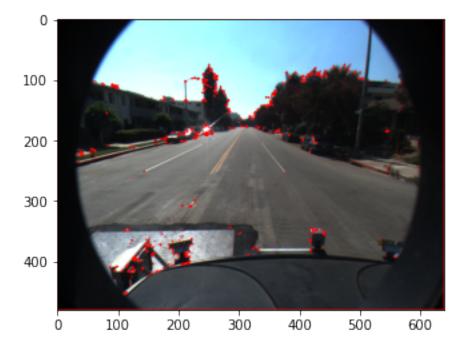
```
def get_corners_coords(img, threshold_percentile = 95):
    harris_response = get_harris_responce(img)
    image_with_corners = np.copy(img)
    corners_coords = np.zeros_like(harris_response)

    threshold = np.percentile(harris_response, threshold_percentile)

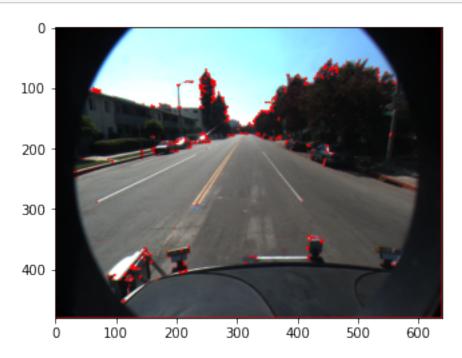
# iterate through values of harris response at each pixel
for y in range(harris_response.shape[0]):
    for x in range(harris_response.shape[1]):
        if harris_response[y, x] > threshold:
            corners_coords[y, x] += 1
            image_with_corners[y, x] = [255,0,0]

plt.imshow(image_with_corners)
return corners_coords
```

[]: corners_coords_lane1 = get_corners_coords(img = lane1, threshold_percentile = →98)



```
[]: corners_coords_lane2 = get_corners_coords(img = lane2, threshold_percentile = →98)
```



1.3.1 Lukas-Kanade corner tracking

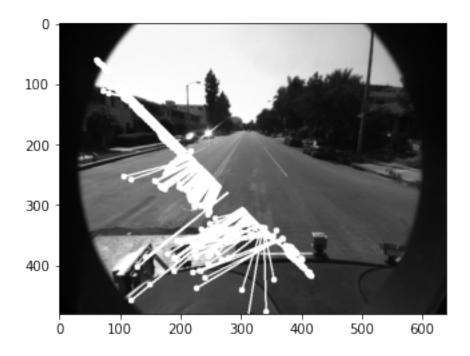
```
[]: ?cv2.calcOpticalFlowPyrLK # np.where(corners_coords_lane2 !=0 )
```

```
# nextPts, status, err = cv2.calcOpticalFlowPyrLK(prevImg = lane1, nextImg = lane2, prevPts = corners_coords_lane1, nextPts = None )
prevImg = np.copy(cv2.cvtColor(lane1, cv2.COLOR_BGR2GRAY))
corners_coords_lane1_reshaped = np.copy(np.stack(np.where(corners_coords_lane1 ! = 0)).reshape(-1,2)).astype('float32')
corners_coords_lane2_reshaped = np.copy(np.stack(np.where(corners_coords_lane2 ! = 0)).reshape(-1,2)).astype('float32')

nextPts, status, err = cv2.calcOpticalFlowPyrLK(prevImg = prevImg, nextImg = locv2.cvtColor(lane2, cv2.COLOR_BGR2GRAY), prevPts = locv2.cvtColor(lane2, cv2.COLOR_BGR2GRAY), prevPts = locv2.corners_coords_lane1_reshaped, nextPts = None )
```

[]: plt.imshow(prevImg, cmap = 'gray')

[]: <matplotlib.image.AxesImage at 0x7f5ed4639250>



1.4 Exercise 6: Viola & Jones

Read the paper by Viola and Jones. You can find it in "mycampus/resources/papers".

1.5 Ex. 6.1 Rectangular Features

Implement the construction of all 2-rect features (horizontal and vertical) within a given window (like the 24x24 px window from the paper). How many exist? (**RESULT**) Display three of these overlaid onto an image of a face. (**RESULT**)

```
[]: !mkdir data
!wget http://www.ai.mit.edu/courses/6.899/lectures/faces.tar.gz
!tar -xzvf "/content/faces.tar.gz" -C "/content/data/"
!tar -xzvf "/content/data/face.train.tar.gz" -C "/content/data/"
```

```
[]: import os
     from skimage import io
     positive data = '/content/data/train/face'
     negative_data = '/content/data/train/non-face'
     data = []
     labels = []
     for file_name in os.listdir(positive_data):
       x = io.imread(os.path.join(positive_data,file_name),as_gray = True)
       y = 1
       data.append(x)
      labels.append(y)
     for file_name in os.listdir(negative_data):
       x = io.imread(os.path.join(negative data,file name),as gray = True)
       y = -1
       data.append(x)
       labels.append(y)
     face1 = data[0]
```

```
[]: np.unique(labels, return_counts = True)
```

```
[]: (array([-1, 1]), array([4548, 2429]))
```

```
[]: import numpy as np

class RectangleRegion:
    def __init__(self, x, y, width, height):
        self.x = x
        self.y = y
```

```
self.width = width
        self.height = height
    def compute_feature(self, ii):
        Computes the value of the Rectangle Region given the integral image
        Args:
            integral image : numpy array, shape (m, n)
            x: x coordinate of the upper left corner of the rectangle
            y: y coordinate of the upper left corner of the rectangle
            width: width of the rectangle
            height: height of the rectangle
        11 11 11
        return ii[self.y+self.height][self.x+self.width] + ii[self.y][self.x] -

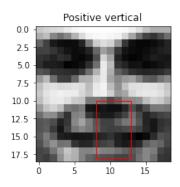
→(ii[self.y+self.height][self.x]+ii[self.y][self.x+self.width])
def build_features(image_shape):
        Builds the possible features given an image shape
            image shape: a tuple of form (height, width)
          Returns:
            an array of tuples. Each tuple's first element is an array of the⊔
\negrectangle regions which positively contribute to the feature. The second
 \hookrightarrowelement is an array of rectangle regions negatively contributing to the \sqcup
 \hookrightarrow feature
        height, width = image_shape
        features = []
        for w in range(1, width+1):
            for h in range(1, height+1):
                i = 0
                while i + w < width:
                     j = 0
                     while j + h < height:
                         #2 rectangle features
                         immediate = RectangleRegion(i, j, w, h)
                         right = RectangleRegion(i+w, j, w, h)
                         if i + 2 * w < width: #Horizontally Adjacent
                             features.append(([right], [immediate]))
                         bottom = RectangleRegion(i, j+h, w, h)
                         if j + 2 * h < height: #Vertically Adjacent</pre>
                             features.append(([immediate], [bottom]))
                         j += 1
                     i += 1
```

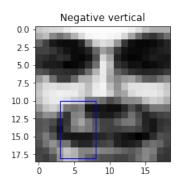
```
return np.array(features)

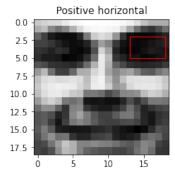
features_face = build_features(data[0].shape)
```

```
[]: from matplotlib import pyplot as plt
     import matplotlib.patches as patches
     def drawROI(image, rec, out = plt, color='red'):
         out.add_patch(
             patches.Rectangle(
                 (rec.x, rec.y),
                 rec.width,
                 rec.height,
                 fill=False,
                 edgecolor=color
             )
         )
         out.imshow(image, cmap='gray')
     fig, (ax0, ax1, ax2) = plt.subplots(ncols=3, figsize=(12, 3))
     drawROI(face1, features_face[17000][0][0], ax0)
     ax0.set_title("Positive vertical")
     drawROI(face1, features_face[17000][1][0], ax1, 'blue')
     ax1.set_title("Negative vertical")
     drawROI(face1, features face[16000][0][0], ax2)
     ax2.set_title("Positive horizontal")
```

[]: Text(0.5, 1.0, 'Positive horizontal')







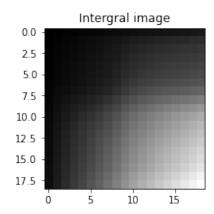
[]:

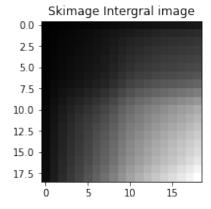
1.6 Ex. 6.2 Integral Image

Implement a function that computes the integral image of a given input image. Display the integral image for an input of you choice (**RESULT**). Compare it to the output of skimage's integral_image() function. (**RESULT**)

```
[]: import skimage.transform
     def integral_image(image):
         Computes the integral image representation of a picture. The integral image_
      \hookrightarrow is defined as following:
         1. s(x, y) = s(x, y-1) + i(x, y), s(x, -1) = 0
         2. ii(x, y) = ii(x-1, y) + s(x, y), ii(-1, y) = 0
         Where s(x, y) is a cumulative row-sum, ii(x, y) is the integral image, and
      \rightarrow i(x, y) is the original image.
         The integral image is the sum of all pixels above and left of the current \sqcup
      \hookrightarrow pixel
           Args:
              image: an numpy array with shape (m, n)
         ii = np.zeros(image.shape)
         s = np.zeros(image.shape)
         for y in range(len(image)):
             for x in range(len(image[y])):
                  s[y][x] = s[y-1][x] + image[y][x] if y-1 >= 0 else image[y][x]
                  ii[y][x] = ii[y][x-1]+s[y][x] if x-1 >= 0 else s[y][x]
         return ii
     fig, (ax0, ax1) = plt.subplots(ncols=2, figsize=(12, 3))
     ax0.set_title("Intergral image")
     ax0.imshow(integral_image(face1) , cmap='gray')
     ax1.set_title("Skimage Intergral image")
     ax1.imshow(skimage.transform.integral.integral_image(face1) , cmap='gray')
```

[]: <matplotlib.image.AxesImage at 0x7fadfc1f2310>





1.7 Ex. 6.3 AdaBoosting

Implement the AdaBoost algorithm as outlined in the paper. Train a 20-feature classifier using these datasets:

faces and non-faces: www.ai.mit.edu/courses/6.899/lectures/faces.tar.gz

Test your classifier with different the sholds and visualize the ROC curve (RESULTS)

```
[]: |#test = load_test()
     !tar -xzvf "/content/data/face.test.tar.gz" -C "/content/data/"
     positive_data = '/content/data/test/face'
     negative_data = '/content/data/test/non-face'
     data_test = []
     labels_test = []
     for file in os.listdir(positive_data):
       x = io.imread(os.path.join(positive_data,file),as_gray = True)
      y = 1
      data_test.append(x)
      labels_test.append(y)
     for file in os.listdir(negative_data):
       x = io.imread(os.path.join(negative_data,file),as_gray = True)
      v = -1
      data_test.append(x)
      labels_test.append(y)
     print(np.unique(labels_test, return_counts = True))
```

```
[]: # extract 2-rect features

# integral_image(face1)

def extract_n_features(img, n_features = 20, rect=np.ones((4,2))):
    img = integral_image(img)

    y = 2
    x = 2

    features = []

    for i in range(int(n_features//2)):
```

```
region_pos = img[y: y + rect.shape[0], x: x + rect.shape[1]]
             pos = region_pos[-1][1] - region_pos[-1][0] - region_pos[0][1] + 
      →region_pos[0][0]
             region_neg = img[y: y + rect.shape[0], x + rect.shape[1]: x + rect.
      \rightarrowshape[1] * 2]
             neg = region_neg[-1][1] - region_neg[-1][0] - region_neg[0][1] + 
      →region_neg[0][0]
             y += 1
             x += 1
             features.append(pos - neg)
         y = 2
         x = 2
         rect = rect.T
         for i in range(int(n_features // 2)):
             region_pos = img[y: y + rect.shape[0], x: x + rect.shape[1]]
             pos = region_pos[-1][1] - region_pos[-1][0] - region_pos[0][1] + 
      →region_pos[0][0]
             region_neg = img[y: y + rect.shape[0], x + rect.shape[1]: x + rect.
      \rightarrowshape[1] * 2]
             neg = region_neg[-1][1] - region_neg[-1][0] - region_neg[0][1] + 
      →region_neg[0][0]
             v += 1
             x += 1
             features.append(pos - neg)
         return features
[]: from tqdm import tqdm
     n_features = 20
     train_features_array = np.empty((len(data), n_features))
     for idx, img in tqdm(enumerate(data)):
         features = extract_n_features(img)
         train_features_array[idx,:] = features
[]: test_features_array = np.empty((len(data_test), n_features))
     for idx, img in tqdm(enumerate(data_test)):
         features = extract_n_features(img)
         test_features_array[idx, :] = features
[]: # helping functions
     from sklearn.metrics import recall_score, f1_score, confusion_matrix,_
      →classification_report
```

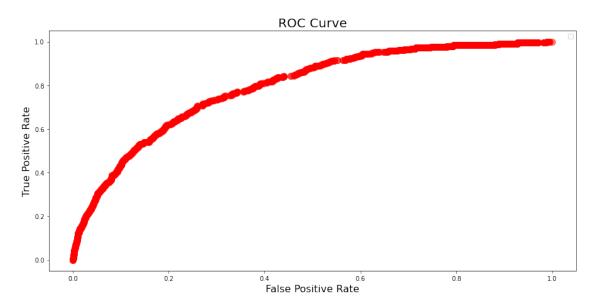
```
[]: class Classifier:
         def accuracy(self, labels, predictions):
             return np.mean(labels == predictions)
         def confusion_matrix(self, labels, predictions):
             size = len(set(labels))
             matrix = np.zeros((size, size))
             for correct, predicted in zip(labels.astype(int), predictions.
     →astype(int)):
                 matrix[correct][predicted] += 1
             return matrix
     class DecisionStump(Classifier):
         def __init__(self, feature, label, alternative_label, threshold=0):
             self.feature = feature
             self.label = label
             self.alternative_label = alternative_label
             self.threshold = threshold
         def predict(self, X):
             return np.where(X[:, self.feature] >= self.threshold, self.label, self.
     →alternative_label)
     def create_classifier_pool(X):
         classifier_pool = []
         for feature in range(X.shape[1]):
             for threshold in sorted(set(X[:, feature])):
                 classifier_pool += [
                     DecisionStump(feature, 1, -1, threshold),
                     DecisionStump(feature, -1, 1, threshold)
                 ]
         return classifier_pool
     class AdaBoost(Classifier):
         def __init__(self, classifier_pool, num_classifiers):
             self.classifier_pool = list(classifier_pool)
             self.num_classifiers = num_classifiers
             self.classifiers = []
             self.weights = []
```

```
def fit(self, X, y):
             scouting_matrix = np.array([clf.predict(X) != y for clf in self.
      →classifier_pool])
             w = np.ones(len(y)) # Initially, all weights are the same
             for _ in range(self.num_classifiers):
                 # Step 1
                 errors = scouting_matrix@w
                 best_remaining = errors.argmin()
                 # Step 2
                 We = errors[best_remaining]
                 W = w.sum()
                 em = (W - We) / W
                 self.classifiers += [self.classifier_pool[best_remaining]]
                 self.weights += [0.5 * -np.log((1 - em) / em)] # alphas
                 # Step 3
                 w = w * np.exp(np.where(scouting_matrix[best_remaining], 1, -1) *_{\sqcup}
      \rightarrowself.weights[-1])
                 scouting_matrix = np.delete(scouting_matrix, best_remaining, axis=0)
                 del self.classifier_pool[best_remaining]
         def predict(self, X, return_weighted_preds = False):
             preds = np.array([cl.predict(X) for cl in self.classifiers])
             weighted preds = np.dot(self.weights, preds)
             if return_weighted_preds:
                 return weighted_preds
             return np.where(weighted_preds >= 0, 1, -1)
[]: classifier_pool = create_classifier_pool(train_features_array)
     print ("Created %d weak learners" % len(classifier_pool))
    Created 19266 weak learners
[]: model = AdaBoost(classifier_pool, num_classifiers=200)
     model.fit(train_features_array, labels)
[]:
[]: predictions = model.predict(train_features_array)
     print("\nTrain: ", classification_report(labels, predictions))
     predictions = model.predict(test features array)
     print("\nTest: ", classification_report(labels_test, predictions))
```

```
Train:
                          precision
                                       recall f1-score
                                                           support
              -1
                       0.95
                                 0.95
                                            0.95
                                                      4548
                       0.91
               1
                                 0.91
                                            0.91
                                                      2429
                                            0.94
                                                      6977
        accuracy
                                            0.93
                                                      6977
       macro avg
                       0.93
                                 0.93
    weighted avg
                       0.94
                                 0.94
                                            0.94
                                                      6977
    Test:
                         precision
                                      recall f1-score
                                                          support
                       0.98
                                 0.97
                                                     23573
              -1
                                            0.98
                       0.12
                                 0.22
                                            0.15
                                                       472
               1
        accuracy
                                            0.95
                                                     24045
                                            0.56
                                                     24045
       macro avg
                       0.55
                                 0.59
    weighted avg
                       0.97
                                 0.95
                                            0.96
                                                     24045
    1.7.1 Plot ROC
[]:
[]: def sigmoid(x):
         return 1 / (1 + np.exp(-x))
     sigmoid_vec = np.vectorize(sigmoid)
     weighted_preds = model.predict(test_features_array, return_weighted_preds =__
     →True)
     preds_probabilities = sigmoid_vec(weighted_preds)
[]: from sklearn.metrics import roc_curve
     fpr, tpr, thresholds = roc_curve(labels_test, preds_probabilities)
     plt.figure(figsize=(15, 7))
     plt.scatter(fpr, tpr, s=100, alpha=0.5, color="red")
     plt.title("ROC Curve", fontsize=20)
     plt.xlabel("False Positive Rate", fontsize=16)
     plt.ylabel("True Positive Rate", fontsize=16)
     plt.legend()
```

No handles with labels found to put in legend.

[]: <matplotlib.legend.Legend at 0x7fadf1998d90>



1.7.2 Congratz, you made it through Assignment 3! You can now try to solve this optional exercise.

This exercise is not graded, but might be a good preparation for the exam.

Please go to the following link: https://forms.gle/f7mx9SM2J2vUHwMo9.

In the form, you will find the optional task (and the corresponding consent form in case you agree with us processing your data). The task is formulated in German, and we would prefer German as the language for your answer. However, if you don't feel comfortable with that, please feel free to solve it in English.