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clear all
close all
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EXERCISE 1

In this exercise we want to show that the notions of consistency and convergence are not the same. For that we will show that there is a consistent method that is not convergent.

Given the differential equation of the form

$$\begin{cases} y'(t) = f(t, y(t)) \\ y(0) = y_0 \end{cases}$$

take the multistep method given by the difference equation

$$\begin{cases} w_{n+1} = -\frac{3}{2}w_n + 3w_{n-1} - \frac{1}{2}w_{n-2} + 3hf(t_n, w_n) \\ w_0 = y_0 \end{cases},$$

where $t_n = hn$. That is, the method has a fixed step size h.

A) Show that the method is consistent following the steps below

i) Write the taylor polynomials of order one of $y(t_{n+1})$, $y(t_{n-1})$ and $y(t_{n-2})$ centered at $y(t_n)$ including the error term.

ii) Find an expression for the error in the approximation

$$y(t_{n+1}) + \frac{3}{2}y(t_n) - 3y(t_{n-1}) + \frac{1}{2}y(t_{n-2}) \approx 3hy'(t_n).$$

iii) Compute the local truncation error of the method using the result from the previous step.

iv) Show that the local truncation error tends to zero when h tends to zero.

NOTE: Point A) does not requiere any coding. Your answer should be added here in the final pdf.

B) Apply the method to solve the differential equation

$$\begin{cases} y'(t) = -y(t) \\ y(0) = 1 \end{cases}$$

with step sizes $h = 0.2, 0.19, 0.18, \ldots, 0.05$ in the interval $[0, 2]$.

i) For each value of h plot the approximated solution and the real solution. Each graph should have a title that tells the value of h and a legend to distinguish the real from the approximated solution.

ii) For each value of h compute the error of the approximation at $t = 2$. Note: make sure that the method is actually reaching the final time $t = 2$.

```
%Your code and outputs for B)i) and  ii) goes here. The only outputs should
%be the graphs from i)
```

iii) Plot the error at time $t = 2$ versus the step size h. Explain using this graph why the method is not convergent.

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%Your code and outputs for B)iii) goes here.
```

EXERCISE 2

Apply RK4 to solve the differential equation

$$\begin{cases} y'(t) = -y(t) \\ y(0) = 1 \end{cases}$$

with step sizes $h = 0.2, 0.19, 0.18, \ldots, 0.05$ in the interval $[0, 2]$.

A) For each value of h plot the approximated solution and the real solution. Each graph should have a title that tells the value of h and a legend to distinguish the real from the approximated solution.

B) For each value of h compute the error of the approximation at $t = 2$. Note: make sure that the method is actually reaching the final time $t = 2$.

```
%Your code and outputs for B)i) and  ii) goes here. The only outputs should
%be the graphs from i)
```

C) Plot the error at time $t = 2$ versus the step size h. Explain using this graph why the method is convergent.

```
%Your code and outputs for B)iii) goes here.
```

EXERCISE 3

A) Apply RK45 to solve the differential equation

$$\begin{cases} y'(t) = -y(t) \\ y(0) = 1 \end{cases}$$

within the interval $[0, 2]$ with an error smaller than a prescribed tolerance TOL. Note: TOL should be a variable whose value should be chosen by the user. Before submitting be sure that it is working by checking the error yourself for some different values of TOL. To submit set up TOL = 10^{-5} but take into account that I will test for other values as well. The only output at this point should be the approximated value of y at $t = 2$.

B) Modify RK45 to output the time discretization $T = [t_1, \ldots, t_n]$$ used and the approximated values $Y = [y_1, \ldots, y_n]$ of y at those points. Do not print T or Y on the screen. Plot Y vs T with markers that show the discretization.

```
%Your code for A) and B) goes here.
```

ADDITIONAL INSTRUCTIONS FOR ALL EXERCISES:

- All graphs should have titles and legends
- After you complete the code in this template, export it as a .docx. Add the solutions to the exercises without code in the .docx document. You can change the sizes, alignments and formats of the pictures and written parts in the .docx document but **do not eliminate or change the order of any output**. After you do that save as a pdf and upload to Gradescope.
- All the code should be properly commented. If you declare a function the meaning of its inputs and outputs should be specified together with their format so the user knows how to properly apply the function.
- Eliminate **by code** all output that is not required.