

Acknowledgements

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Appendix. The calculation of tention exerted by the brush of linear polyelectrolyte chains at the grafting surface

Within the strong stretching approximation there is a universal relation between polymer density profile, $c_p(z)$, local stretching function, $E(z, z')$, of the chain with the end position at z' and normalized to unity distribution of the end segments, $g(z')$, as

$$c_p(z) = \frac{1}{s} \int_z^H \frac{g(z') dz'}{E(z, z')} \quad (31)$$

The stretching function $E(z, z')$ is given by

$$E(z, z') = a \sqrt{1 - \frac{\cos^2(\pi z'/2L)}{\cos^2(\pi z/2L)}}$$

and thus eq 31 can be used for obtaining the end segments distribution $g(z)$. By introducing new variables

$$\xi = \cos^2\left(\frac{\pi x'}{2L}\right) - \cos^2\left(\frac{\pi H}{2L}\right)$$

$$\eta = \cos^2\left(\frac{\pi x}{2L}\right) - \cos^2\left(\frac{\pi H}{2L}\right)$$

eq 31 can be reduced to a standard Abel's integral equation

$$\phi(\eta) = \int_0^\eta \frac{f(\xi) d\xi}{\sqrt{\eta - \xi}}$$

where

$$\phi(\eta) = \frac{\pi s}{2La^2} \frac{c_p(x(\eta))}{\cos\left(\frac{\pi x(\eta)}{2L}\right)}$$

and

$$f(\xi) = \frac{g(x'(\xi))}{\sin \frac{\pi x'(\xi)}{L}}$$

with the soution

$$f(\xi) = \frac{1}{\pi} \left(\frac{\phi(0)}{\sqrt{\xi}} + \int_0^\xi \frac{(d\phi(\eta)/d\eta)d\eta}{\sqrt{\xi - \eta}} \right)$$

Hence the (normalized to unity) distribution of the end segments is given by

$$g(z) = \frac{sa}{2L} \sin\left(\frac{\pi z}{L}\right) \left(\frac{c_p(H)}{\cos(\frac{\pi H}{2L}) \sqrt{\cos^2(\frac{\pi z}{2L}) - \cos^2(\frac{\pi H}{2L})}} - \int_z^H dz' \frac{\sec(\frac{\pi z'}{2L}) \frac{dc_p(z')}{dz'} + \frac{\pi}{2L} c_p(z') \tan(\frac{\pi z'}{2L}) \sec(\frac{\pi z'}{2L})}{\sqrt{\cos^2(\frac{\pi z}{2L}) - \cos^2(\frac{\pi z'}{2L})}} \right) \quad (32)$$

Tension exerted at the grafting surface, $z = 0$, by the chain with the end point localised at z'

$$\frac{f(z = 0, z')a}{k_B T} = \frac{3}{2} \ln \frac{1 + E(z = 0, z')/a}{1 - E(z = 0, z')/a} \quad (33)$$

is expressed though the local stretching function

$$E(z = 0, z')/a = \left(\sqrt{1 - \frac{\cos^2 \pi z'/2L}{\cos^2 \pi z/2L}} \right)_{z=0} = \sin \frac{\pi z'}{2L} \quad (34)$$

By averaging tension given by eqs 33, 34 with respect to the position z' of the free chain end with the distribution function $g(z')$, given by eq 32, we obtain an expression for the average tension exerted by the brush per unit area of the grafting surface

$$\begin{aligned} \frac{T(0)a}{k_B T} &= \frac{3}{2s} \int_0^H g(z') \ln \left(\frac{1 + \sin \frac{\pi z'}{2L}}{1 - \sin \frac{\pi z'}{2L}} \right) dz' = \\ &= \frac{3a}{4L} \left[\int_0^H \sin \frac{\pi z'}{L} \ln \left(\frac{1 + \sin \frac{\pi z'}{2L}}{1 - \sin \frac{\pi z'}{2L}} \right) \frac{c_p(H)}{\cos(\frac{\pi H}{2L}) \sqrt{\cos^2(\frac{\pi z'}{2L}) - \cos^2 \frac{\pi H}{2L}}} dz' \right. \\ &\quad \left. - \int_0^H dz' \sin \frac{\pi z'}{L} \ln \left(\frac{1 + \sin \frac{\pi z'}{2L}}{1 - \sin \frac{\pi z'}{2L}} \right) \int_{z'}^H dz \frac{(\frac{dc_p(z)}{dz}) \sec(\frac{\pi z}{2L}) + (\frac{\pi}{2L}) c_p(z) \tan(\frac{\pi z}{2L}) \sec(\frac{\pi z}{2L})}{\cos^2(\frac{\pi z'}{2L}) - \cos^2(\frac{\pi z}{2L})} \right] \quad (35) \end{aligned}$$

After changing the order of integration in the second integral in eq 35 and using the relation

$$\int_0^b \ln \frac{1+y}{1-y} \frac{y dy}{\sqrt{b^2 - y^2}} = \pi(1 - \sqrt{1 - b^2})$$

we obtain a general expression for $T(0)$

$$\frac{T(0)a}{k_B T} = 3a \left[c_p(H) \left(\sec \frac{\pi H}{2L} - 1 \right) - \int_0^H dz \left(1 - \cos \frac{\pi z}{2L} \right) \left(\frac{dc_p(z)}{dz} \sec \left(\frac{\pi z}{2L} \right) + \frac{\pi}{2L} c_p(z) \tan \left(\frac{\pi z}{2L} \right) \sec \left(\frac{\pi z}{2L} \right) \right) \right] \quad (36)$$

applicable at any shape of the polymer density profile $c_p(z)$, that is, at any type of interactions in the brush.

Consider now the brush of linear polyelectrolyte chains of length $L = na$ grafted with the density a^2/s to planar surface.

The electrostatic potential in the linear polyelectrolyte brush with finite extensibility is specified as

$$\psi_{in}(x) = \frac{U(x)}{\alpha k_B T} = \frac{3}{\alpha} \ln \frac{\cos(\pi x/2L)}{\cos(\pi H/2L)} \quad (37)$$

By applying similar arguments as above, we find the polymer concentration profile

$$\alpha c_p(z) = \frac{3\pi}{16l_B \alpha L^2} \sec^2 \left(\frac{\pi z}{2L} \right) + \frac{1}{2\pi l_B \tilde{\Lambda}^2} \left[\frac{\cos(\pi z/2L)}{\cos(\pi H/2L)} \right]^{\frac{3}{\alpha}} \quad (38)$$

where

$$\tilde{\Lambda} = \frac{4}{3\pi} \frac{\alpha L}{\tan(\pi H/2L)} = \frac{H_0^2}{H} \frac{(\pi H/2L)}{\tan(\pi H/2L)} \quad (39)$$

Then eq 38 can be presented in the form

$$c_p(z) = \frac{1}{2\pi l_B \alpha H_0^2} \left(\sec^2 \frac{\pi z}{2L} + \frac{3}{2\alpha} \frac{\pi H}{2L} \left[\frac{\cos(\pi z/2L)}{\cos(\pi H/2L)} \right]^{\frac{3}{\alpha}} \right) \quad (40)$$

and we find finite expression for the tension exerted to the unit area of the surface as

$$\frac{T(0)a}{k_B T} = \frac{3a}{4\pi l_B \alpha H_0^2} \left[\tan^2 \left(\frac{\pi H}{2L} \right) + \frac{\pi H}{2L} \tan \left(\frac{\pi H}{2L} \right) \left(\sec^{3/\alpha} \left(\frac{\pi H}{2L} \right) - 1 \right) \right] \quad (41)$$