Distributed optimal SVFB cooperative control of multi-agents systems

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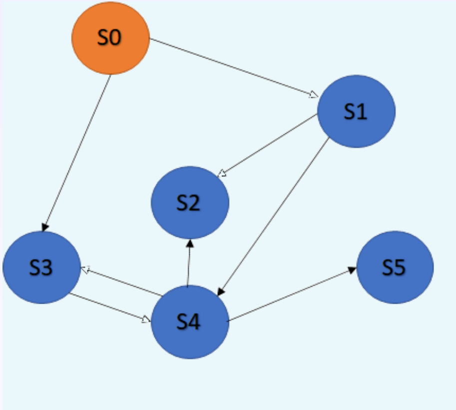
General formulation

- Here we will focus on CPS described as multi-agents systems modeled by means of directed graphs (digraphs)



- Each node S_i of the graph is a dynamical LTI system
- Edges are used to model communication between agents

General formulation

We will consider multi-agents control problems (generically called here *synchronization problems*) where the CPS is made up of

- 1 *leader node* S_0
- N *follower nodes* S_i $(i = 1, 2, \ldots, N)$
- The agents cooperate in order to perform a task possibly dictated (to some extent) by the leader node
- In order to perform the assigned task the follower nodes exploit information shared on the communication network represented by the digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$
- Examples: autonomous vehicles platooning, unmanned air vehicles (UAVs) formation control, control of autonomous mobile robots team, ...

General formulation: cooperative tracking problems

- In this part we will mainly focus on *cooperative tracking problem* where the task to be performed is dictated by the leader node S_0 which generates the desired target trajectory (acting as a command generator exosystem)
- We will use the term *cooperative regulator problem* when the consensus is sought among agents in the absence of a leader node
- Assumption: the N follower agent are identical

Reference book

B5 *Cooperative Control of Multi-Agent Systems*, F. Lewis et al., Springer London , 2013

Agents mathematical models

- The dynamics of the **leader node** S_0 is described by

$$\dot{x}_0 = Ax_0, \; y_0 = Cx_0 \tag{1}$$

where $x_0 \in \mathbb{R}^n$, $y_0 \in \mathbb{R}^p$

- The dynamics of the N identical **follower nodes** S_i is described by

$$\dot{x}_i = Ax_i + Bu_i, \; y_i = Cx_i \tag{2}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $y_0 \in \mathbb{R}^p$ and $i \in \mathcal{N} = \{1, 2, \ldots, N\}$

- The triple (A, B, C) is *stabilizable* and *detectable*

Communication network modeling

- The **follower nodes** S_i share information through a communication network represented as a digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with N nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and a set of edges (arcs) $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$
- The **adjacency matrix** associated to $\mathcal{G}$ is $\mathcal{A} = [a_{ij}] \in \mathbb{R}^N$
- $a_{ij} > 0$ **is the weight for edge** (v_j, v_i) implying that node i can get information from node j (node j is a neighbor of node i)
- The neighbor set of node i is denoted as $\mathcal{N}_i = \{j | a_{ij} > 0\}$
- We assume there is no self-loop ($a_{ii} = 0, \forall i$)
- To describe connection between the leader node and the graph $\mathcal{G}$ of follower nodes we define the **augmented graph** $\overline{\mathcal{G}} = \{\overline{\mathcal{V}}, \overline{\mathcal{E}}\}$ where $\overline{\mathcal{V}} = \{v_0, v_1, \ldots, v_N\}$, $\overline{\mathcal{E}} \subset \overline{\mathcal{V}} \times \overline{\mathcal{V}}$

Cooperative state variables feedback (SBVF) control

Fundamental assumptions

- All the **state variables** of the agents are assumed to be measurable ($C = I$ in equation (2))
- The **leader node** S_0 can only be observed by a small subset of the follower nodes
- **If node i observes the leader**, node i is said to be **pinned to the leader**. Therefore an edge (v_0, v_i) **exists** in the augmented graph $\overline{\mathcal{G}}$ with **weight** g_i **(pinning gain)**
- **The control law of each agent can only use the local neighborhood information** of that agent, according to the graph topology (fully distributed control scheme)
- There exists at least **one directed path from the leader node to every follower node**.

Cooperative state variables feedback (SBVF) control

Local controller at each node i

Neighborhood tracking error of node i

$$\varepsilon_i = \sum_{j=1}^{N} a_{ij}(x_j - x_i) + g_i(x_0 - x_i) \tag{3}$$

SVFB control protocol for each node i

$$u_i = cK\varepsilon_i \tag{4}$$

- coupling gain: $c > 0$
- feedback gain matrix: $K \in \mathbb{R}^{m \times n}$

Cooperative state variables feedback (SBVF) control

Closed-loop control system

Closed-loop system of each node i

$$\dot{x}_i = Ax_i + cBK\left(\sum_{j=1}^{N} a_{ij}(x_j - x_i) + g_i(x_0 - x_i)\right) \tag{5}$$

Global closed-loop system dynamics

$$\dot{x} = (I_N \otimes A - c(L+G) \otimes BK)x + (c(L+G) \otimes BK)\underline{x}_0 \tag{6}$$

- $x = col(x_1, x_2, \ldots, x_N) \in \mathbb{R}^{nN}$, $\underline{x}_0 = col(x_0, x_0, \ldots, x_0) \in \mathbb{R}^{nN}$
- Laplacian matrix of $\mathcal{G}$, $L = [l_{ij}] = D - \mathcal{A}$, $D = diag(d_1, d_2, \ldots, d_N)$, d_i in-degree of node i
- Pinning matrix $G = diag(g_1, g_2, \ldots, g_N)$, $\otimes$ is the Kronecker product

Cooperative state variables feedback (SBVF) control

Disagreement error δ

Local disagreement error at each node i

$$\delta_i(t) = x_i(t) - x_0(t) \tag{7}$$

Global disagreement error and global disagreement error dynamics

$$\delta(t) = x(t) - \underline{x}_0(t) = col(\delta_1, \delta_2, \ldots, \delta_N) \tag{8}$$

$$\dot{\delta} = \dot{x} - \dot{\underline{x}}_0 = A_c \delta \tag{9}$$

$$A_c = I_N \otimes A - c(L + G) \otimes BK \tag{10}$$

Cooperative state variables feedback (SBVF) control

Objective and solution

Objective of the cooperative tracking problem

The cooperative tracking problem is solved if

$$\lim_{t\to\infty} \delta(t) = 0 \tag{11}$$

Cooperative tracking problem solution

The global disagreement error converges to 0 if and only if matrix

$$A_c = I_N \otimes A - c(L+G) \otimes BK \tag{12}$$

is *Hurwitz* (i.e., it has all the eigenvalues with strictly negative real part)

Cooperative state variables feedback (SBVF) control

Closed-loop CPS eigenvalues

- The following Lemma provides useful insight about the eigenvalues of the closed-loop multi-agents system

Lemma 1 (closed-loop eigenvalues)

$$eig(A_c) = \bigcup_{i=1}^{N} eig(A - c\lambda_i BK) \tag{13}$$

where $\lambda_i,\ i = 1, \ldots, N$ are the eigenvalues of the matrix $L + G$

- Lemma 1 can be easily proved by analyzing the structure of matrix A_c in (12)

Cooperative state variables feedback (SBVF) control

Controller design (I)

The following crucial considerations are directly obtained from Lemma 1:

- The closed-loop dynamics of the whole multi-agents system depends both on the local controller parameters K and c and on the eigenvalues of matrix $L+G$ accounting for the communication network effect

- Stability of the single agent dynamics does not imply stability of the multi-agents systems

$$(A-BK) \text{ Hurwitz} \not\Rightarrow A_c \text{ Hurwitz} \tag{14}$$

- The coupling parameter c is introduced to cope with the effect of λ_i on the global multi-agent system dynamics

Controller design (II)

Theorem 1 (Cooperative controller design)

Consider the local distributed control protocols given in equation (4). Design the SVFB control gain K as

$$K = R^{-1}B'P \tag{15}$$

where P is the unique positive definite solution of the algebraic Riccati equation (ARE)

$$A'P + PA + Q - PBR^{-1}B'P = 0 \tag{16}$$

Then

$$\lim_{t \to \infty} \delta(t) = 0 \tag{17}$$

if

$$c \geq \frac{1}{2 \min_{i \in \mathcal{N}} Re(\lambda_i)} \tag{18}$$