Design of cooperative dynamic regulators for synchronization of multi-agents systems

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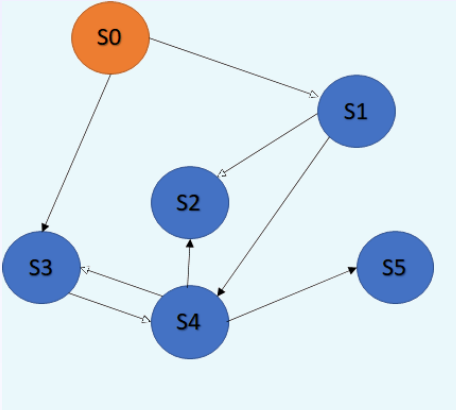
General formulation

- Here we will focus on CPS described as multi-agents systems modeled by means of directed graphs (digraphs)



- Each node S_i of the graph is a dynamical LTI system
- Edges are used to model communication between agents

General formulation

We will consider multi-agents control problems (generically called here *synchronization problems*) where the CPS is made up of

- 1 *leader node* S_0
- N *follower nodes* S_i $(i = 1, 2, \ldots, N)$
- The agents cooperate in order to perform a task possibly dictated (to some extent) by the leader node
- In order to perform the assigned task the follower nodes exploit information shared on the communication network represented by the digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$

General formulation: cooperative tracking problems

- In this part we will mainly focus on *cooperative tracking problem* where the task to be performed is dictated by the leader node S_0 which generates the desired target trajectory (acting as a command generator exosystem)

- We will use the term *cooperative regulator problem* when the consensus is sought among agents in the absence of a leader node

- Assumptions: the N follower agent are identical

Reference book

B5 *Cooperative Control of Multi-Agent Systems*, F. Lewis et al., Springer London , 2013

Agents mathematical models

- The dynamics of the **leader node** S_0 is described by

$$\dot{x}_0 = Ax_0, \;\; y_0 = Cx_0 \tag{1}$$

  where $x_0 \in \mathbb{R}^n$, $y_0 \in \mathbb{R}^p$

- The dynamics of the N identical **follower nodes** S_i is described by

$$\dot{x}_i = Ax_i + Bu_i, \;\; y_i = Cx_i \tag{2}$$

  where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $y_0 \in \mathbb{R}^p$ and $i \in \mathcal{N} = \{1, 2, \ldots, N\}$
- The triple (A, B, C) is *stabilizable* and *detectable*

Communication network modeling

- The **follower nodes** S_i share information through a communication network represented as a digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with N nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and a set of edges (arcs) $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$
- The **adjacency matrix** associated to $\mathcal{G}$ is $\mathcal{A} = [a_{ij}] \in \mathbb{R}^N$
- $a_{ij} > 0$ **is the weight for edge** (v_j, v_i) implying that node i can get information from node j (node j is a neighbor of node i)
- The neighbor set of node i is denoted as $\mathcal{N}_i = \{j | a_{ij} > 0\}$
- We assume there is no self-loop ($a_{ii} = 0, \forall i$)
- To describe connection between the leader node and the graph $\mathcal{G}$ of follower nodes we define the **augmented graph** $\overline{\mathcal{G}} = \{\overline{\mathcal{V}}, \overline{\mathcal{E}}\}$ where $\overline{\mathcal{V}} = \{v_0, v_1, \ldots, v_N\}$, $\overline{\mathcal{E}} \subset \overline{\mathcal{V}} \times \overline{\mathcal{V}}$

Cooperative state variables feedback (SBVF) control

Fundamental assumptions

- The **leader node** S_0 can only be observed by a small subset of the follower nodes
- **If node i receives information form the leader**, node i is said to be **pinned to the leader**. Therefore an edge (v_0, v_i) **exists** in the augmented graph $\overline{\mathcal{G}}$ with **weight g_i (pinning gain)**
- **The control law of each agent can only use the local neighborhood information** of that agent, according to the graph topology (fully distributed control scheme)
- There exists at least **one directed path from the leader node to every follower node**.
- **The state variables of the agents are not directly measurable ($C \neq I$ in equation (2))** $\Longrightarrow$ we need to design a **Dynamic Regulator (Observer + State-feedback)**

Cooperative observer design

Cooperative observer design for agent (node) i

Objective of cooperative observer design

We want to design a device (cooperative observer/cooperative asymptotic estimator) able to provide an estimate $\hat{x}_i$ of the state vector x_i such that

$$\lim_{t \to \infty} \hat{x}_i(t) = x_i, \forall i = 1, \ldots, N \tag{3}$$

Local output estimation error at node i

We define the *local output estimation error as*

$$\tilde{y}_i = y_i - \hat{y}_i \tag{4}$$

where $\hat{y}_i = C\hat{x}_i$ is the *local output estimate*

Cooperative observer design

Cooperative observer design for agent (node) i

Neighborhood output estimation error of node i

$$\xi_i = \sum_{j=1}^{N} a_{ij}(\tilde{y}_j - \tilde{y}_i) + g_i(\tilde{y}_0 - \tilde{y}_i) \tag{5}$$

Cooperative observer for agent (node) i

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i - cF\xi_i \tag{6}$$

- coupling gain: $c > 0$
- observer gain: $F \in \mathbb{R}^{n \times p}$

Cooperative observer design

Cooperative observer design for agent (node) i

Remarks

- The state estimation algorithm considered here is *completely distributed* in the sense that each observer only requires its local output estimation error + the output estimation errors of the neighborhood agents

Global cooperative observer dynamics

$$\dot{\hat{x}} = A_o \hat{x} + (I_N \otimes B)u + c((L + G) \otimes F)y \tag{7}$$

- $A_o = (I_N \otimes A) - c((L + G) \otimes FC)$
- $\hat{x} = col(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N) \in \mathbb{R}^{nN}$
- $y = col(y_1, y_2, \ldots, y_N) \in \mathbb{R}^{pN}$
- $u = col(u_1, u_2, \ldots, u_N) \in \mathbb{R}^{mN}$

Cooperative observer design

Analysis of the global estimation error dynamics

Global state estimation error

$$\tilde{x}(t) = x(t) - \hat{x} \tag{8}$$

Global state estimation error dynamics

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = A_o \tilde{x} \tag{9}$$

$$A_o = (I_N \otimes A) - c((L + G) \otimes FC) \tag{10}$$

Cooperative observer design

Objective and problem solution

Objective of the cooperative observer design problem

The cooperative observer design problem is solved if

$$\lim_{t \to \infty} \tilde{x}(t) = 0 \tag{11}$$

Cooperative observer design problem solution

The global state estimation error converges to 0 if and only if matrix

$$A_o = (I_N \otimes A) - c((L + G) \otimes F \tag{12}$$

is *Hurwitz* (i.e., it has all the eigenvalues with strictly negative real part)

Cooperative observer design

Global observer eigenvalues

- The following Lemma provides useful insight about the eigenvalues of global observer

Lemma 2 (Global observer eigenvalues)

$$eig(A_0) = \bigcup_{i=1}^{N} eig(A - c\lambda_i FC) \tag{13}$$

where $\lambda_i,\ i = 1, \ldots, N$ are the eigenvalues of the matrix $L + G$

- Lemma 2 can be easily proved by analyzing the structure of matrix A_o in (12)

Design of observer parameter F and c

Theorem 2 (Global observer design)

Consider the cooperative observer in equation (6). Design the observer gain F as

$$F = PC'R^{-1} \tag{14}$$

where P is the unique positive definite solution of the algebraic Riccati equation (ARE)

$$AP + PA' + Q - PC'R^{-1}CP = 0 \tag{15}$$

Then

$$\lim_{t\to\infty} \tilde{x}(t) = 0 \tag{16}$$

if

$$c \geq \frac{1}{2\min_{i\in\mathcal{N}} Re(\lambda_i)} \tag{17}$$

Cooperative dynamic regulator design

Connecting the SVFB protocol to the Cooperative observer

SVFB distributed protocol

$$u_i = cK\hat{\varepsilon}_i \tag{18}$$

$$\hat{\varepsilon}_i = \sum_{j=1}^{N} a_{ij}(\hat{x}_j - \hat{x}_i) + g_i(\hat{x}_0 - \hat{x}_i) \tag{19}$$

Cooperative (distributed) observer

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i - cF\xi_i \tag{20}$$

$$\xi_i = \sum_{j=1}^{N} a_{ij}(\tilde{y}_j - \tilde{y}_i) + g_i(\tilde{y}_0 - \tilde{y}_i) \tag{21}$$

Cooperative dynamic regulator design

Global dynamic control protocol equations

Local dynamic controller of agent (node) i

$$u_i = cK\hat{\varepsilon}_i \tag{22}$$

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i - cF\xi_i \tag{23}$$

Local closed-loop dynamics for agent (node) i

$$\dot{x}_i = Ax_i + cBK\left(\sum_{j=1}^{N} a_{ij}(\hat{x}_j - \hat{x}_i) + g_i(\hat{x}_0 - \hat{x}_i)\right) \tag{24}$$

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i - cF\left(\sum_{j=1}^{N} a_{ij}(\tilde{y}_j - \tilde{y}_i) + g_i(\tilde{y}_0 - \tilde{y}_i)\right) \tag{25}$$

Cooperative dynamic regulator design

Global closed-loop dynamics

Theorem 3

The global closed-loop dynamics obtained by applying the cooperative dynamics regulator is described by the following equation:

$$\begin{bmatrix} \dot{\delta} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ 0 & A_o \end{bmatrix} \begin{bmatrix} \delta \\ \tilde{x} \end{bmatrix} \tag{26}$$

Corollary (Separation principles)

Theorem 3 tell us that:

$$\dot{\delta} = A_c \delta \tag{27}$$

global disagreement error dynamics only depends on c and K no matter if the state variables are directly available or not

$$\dot{\tilde{x}} = A_o \tilde{x} \tag{28}$$

global estimation error dynamics only depends on c and F no matter how you design the SVFB protocol

Cooperative SVFB protocol + local observes

Agent controller with local observer

Local dynamic controller of agent (node) i

$$u_i = cK\hat{\varepsilon}_i \tag{29}$$

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i - cF\tilde{y}_i \tag{30}$$

A possible alternative approach is to design

Theorem 4

The global closed-loop dynamics obtained by applying the cooperative SVFB protocol with local observers is described by the following equation:

$$\begin{bmatrix} \dot{\delta} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ 0 & I \otimes (A + cFC) \end{bmatrix} \begin{bmatrix} \delta \\ \tilde{x} \end{bmatrix} \tag{31}$$

Local observer design

Local observer dynamics

- The following Lemma provides useful insight about the eigenvalues of global observer

Lemma 3 (Local observer dynamics)

$$eig(I \otimes (A + cFC)) = \bigcup_{i=1}^{N} eig(A + cFC) \tag{32}$$

- Lemma 3 shows that in order to drive the state estimation error $\tilde{x}$ to 0 asymptotically we have just to design F such that $(A + cFC)$ is Hurwitz.