Distributed formation control for LTI multi-agents systems

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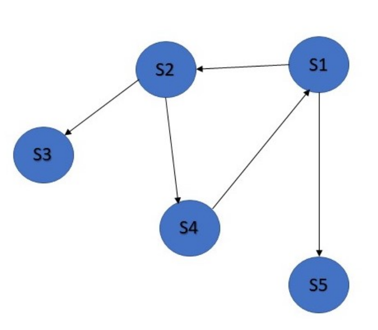
General formulation

- Here we will focus on CPS described as multi-agents systems modeled by means of <u>**directed**</u> graphs



- Each node S_i of the graph is a dynamical LTI system
- We assume that no agent is explicitly playing the role of a leader (we will partly remove this assumption later on in the examples)

General formulation

We will consider a multi-agents formation control problems where the CPS is made up of

- N agents S_i $(i = 1, 2, \ldots, N)$
- The agents cooperate to reach a suitably specified formation (details in the next slides)
- In order to perform the assigned formation task the follower agents (nodes) exploit information shared on the communication network represented by the digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$
- The N agents are assumed to be identical.

General formulation: agents model

- The dynamics of the N identical agents S_i is described by

$$\dot{x}_i = Ax_i + Bu_i, \;\; y_i = Cx_i \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $y_i \in \mathbb{R}^p$ and $i \in \mathcal{N} = \{1, 2, \ldots, N\}$

- The triple (A, B, C) is *stabilizable* and *detectable*

- Assumption 1: the N agents are identical

- Assumption 2: $C = I$ (i.e., all the state variables are directly measurable)

Communication network modeling

- The **agents** S_i share information through a communication network represented as a digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with N nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and a set of edges (arcs) $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$
- The **adjacency matrix** associated to $\mathcal{G}$ is $\mathcal{A} = [a_{ij}] \in \mathbb{R}^N$
- $a_{ij} > 0$ **is the weight for edge** (v_j, v_i) implying that node i can get information from node j (node j is a neighbor of node i)
- The neighbor set of node i is denoted as $\mathcal{N}_i = \{j | a_{ij} > 0\}$
- We assume there is no self-loop $(a_{ii} = 0, \forall i)$
- The Laplacian matrix of $\mathcal{G}$ is defined as $L = [l_{ij}] = D - \mathcal{A}$, $D = diag(d_1, d_2, \ldots, d_N)$, d_i in-degree of node i

General formulation: formation specification

- The desired *formation* is defined in terms of values assumed by the state variables of agents S_i relative to the values of the state variables of the other agents in the CPS

- Explicitly, we will use the matrix

$$H = [h_1' \ h_2' \ \dots \ h_N']' \in \mathbb{R}^{nN} \tag{2}$$

where $(h_j - h_i)$, for $j = 1, \dots, N$ with $i \neq j$, define the desired behaviour of S_i with respect to the other agents S_j.

- Assumption 3: time-invariant formation (i.e., h_i does not depend on time $\longrightarrow$ formation with static shape)
- For example, H can be used to describe the vertex of a generic polygon formation

Distributed state-feedback formation control

Local controller at each node i

Neighborhood formation error of node i

$$\varepsilon_i = \sum_{j=1}^{N} a_{ij}[(x_j - x_i) - (h_j - h_i)] \tag{3}$$

State-feedback formation protocol for each node i

$$u_i = cK\varepsilon_i \tag{4}$$

- coupling gain: $c > 0$
- feedback gain matrix: $K \in \mathbb{R}^{m \times n}$

Distributed state-feedback formation control

Closed-loop control system

Local formation error δ_i

Let us define:

$$\delta_i = (x_i - x_1) - (h_i - h_1), \; i = 2, 3, \ldots, N \tag{5}$$

$$\delta = col(\delta_2, \delta_3, \ldots, \delta_N), \tag{6}$$

$$\delta_H = [(h2 - h1), (h3 - h1), \ldots, (h_N - h_1)]' \tag{7}$$

Distributed state-feedback formation control

Global formation error dynamics

Global formation error dynamics

$$\dot{\delta}(t) = [I_{N-1} \otimes A - (L_{22} + \mathbf{1}_{N-1}\alpha') \otimes cBK]\delta(t) + (I_{N-1} \otimes A)\delta_H \quad (8)$$

where

$$L_{22} = \begin{pmatrix} d_2 & -a_{23} & \dots & -a_{2N} \\ -a_{32} & d_3 & \dots & -a_{3N} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ -a_{N2} & -a_{N2} & \dots & d_N \end{pmatrix} \quad (9)$$

$$\alpha = (a_{12}, a_{13}, \dots, a_{1N})' \quad (10)$$

$$\mathbf{1}_{N-1} = (1, 1, \dots, 1) \quad (11)$$

Distributed state-feedback formation control

Objective and solution

Objective of the distributed formation control

The distributed formation control problem is solved if

$$\lim_{t \to \infty} \delta(t) = 0 \tag{12}$$

Distributed formation control problem solution

The global formation error converges to 0 if the following two conditions are satisfied:

- Formation stability:

$$A_c = [I_{N-1} \otimes A - (L_{22} + \mathbf{1}_{N-1}\alpha') \otimes cBK] \ \text{ is } \mathit{Hurwitz} \tag{13}$$

- Formation feasibility:

$$(I_{N-1} \otimes A)\delta_H = 0 \tag{14}$$

Distributed state-feedback formation control

Closed-loop eigenvalues

- The following Lemma provides useful insight about the eigenvalues of the closed-loop multi-agents system

Lemma 3 (closed-loop eigenvalues)

$$eig(A_c) = \bigcup_{i=2}^{N} eig(A - c\lambda_i BK) \tag{15}$$

where $\lambda_i, \; i = 1, \ldots, N-1$ are the eigenvalues of the matrix $L22$ (which, in turn, are the nonzero eigenvalues of L)

Distributed state-feedback formation control

Controller and formation design (I)

Theorem 4 (Cooperative controller and formation design)

Consider the local distributed control protocols given in equation (4).
The formation control problem is solvable if and only if:

- The graph $\mathcal{G}$ describing the network topology has a spanning tree
- The gains c and K are such that A_c is Hurwitz (see, e.g., design approach proposed for the *synchronization* problem)
- The formation H is selected in such a way that condition (14) is satisfied

References

The content of this presentation is based on the following journal papers where you can find additional theoretical details (and in particular full details about the proofs sketched in the classroom lectures):

P1 Cuiqin MA, Jifeng ZHANG – *On Formability of Linear Continuous-Time Multi-Agent Systems*, Journal of Systems Science and Complexity (2012), Vol. 25, pp. 13–29

P2 G. LAFFERRIERE, A.WILLIAMS, J. CAUGHMAN, J.J.P. VEERMAN – *Decentralized control of vehicle formations*, Systems & Control Letters (2005), Vol.54, pp. 899 – 910