Master of science degree in Computer Engineering
Academic Year 2024-2025, Second Semester

01UDSOV - Modeling and control of cyberphysical systems

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Part II, Lab activity n. 1

Main learning objectives

Upon successful completion of this homework, students will

1. Be able to compute Least Squares parameter estimation for Discrete-time LTI systems.
2. Be able to analyze properties and limitations of the considered estimator.

Problem

In this problem we assume that the plant is a continuous time LTI dynamical system assumed to be exactly described by the following transfer function:

$$G_p(s) = \frac{100}{s^2 + 1.2s + 1} \tag{1}$$

The plant transfer function is unknown to the user, and it has only to be used to generate the plant input-output data by means of a simulation, as here explained:

- Assuming that the input-output data have been collected with a sample time of Ts = 1s, compute a discrete-time model for the plant as follows (see help of the Matlab command `c2d`):

  $$G_d(z) = \text{c2d}(G_p,1,\text{'zoh'}) \tag{2}$$

  The discrete-time model will be given by $G_d(z) = N_d(z)/D_d(z)$.

- Use the obtained discrete-time model to generate input-output data by applying to the system a random sequence of H samples and amplitude 1 as input (see commands `rand` and command `lsim`). Collect the input sequence in the array u and the output sequence in the array w.

- Using the array u and w, build matrix A and array b required for the computation of the least squares estimate of the discrete-time model parameters, according to the theory and examples about least square estimation presented in the classroom.

- Compare the estimated parameters with the *true* ones obtained in (2).

- Repeat the exercise with different values of H.

- Repeat the exercise (for different values of H) by adding a random *equation error* e while simulating the data, i.e. in such a way that the collected measurement y are given by:

  $$D_d(q^{-1})y_t = N_d(q^{-1})u_t + e_t \tag{3}$$

  where e_t is a normally (Gaussian) distributed signal of standard deviation 5 (use the command `randn`).

- Repeat the exercise (for different values of H) by adding a random *output measurement error* η as follows:

  $$w(t) = G_d(q^{-1})u(t), \; y(t) = w(t) + \eta(t) \tag{4}$$

  where η is a normally (Gaussian) distributed signal of standard deviation 5.