· SOLUTION OF A GENERIC POLYNOMIAL OPT. PROB. VIA "SPARSE POP"

· $\min_{x \in \mathbb{R}^n} f_0(x)$

s.t.

$f_k(x) \geq 0 \quad (k = 1, 2, \ldots, \ell)$

$f_k(x) = 0 \quad (k = \ell+1, \ldots, m)$

$lbd_i \leq x_i \leq ubd_i \quad (i = 1, \ldots, n)$

where f_0, f_N are multivariate polynomial function of the optimization variable $x = [x_1, \ldots, x_n]$

. Example:

$$\min_{x \in \mathbb{R}^3} \Big(\underbrace{-2x_1 + 3x_2 - 2x_3}_{f_0(x)} \Big)$$

s.t.

{1} $6x_1^2 + 3x_2^2 - 2x_2x_3 + 3x_3^2 - 17x_1 + 8x_2 - 14x_3 \geq -19$

{2} $x_1 + 2x_2 + x_3 \leq 5$

{3} $5x_1 + 2x_3 = 7$

$0 \leq x_1 \leq 2$

$0 \leq x_2 \leq 1$

$\Downarrow$

· Sparse data structure:

· STEP 1: define the objective function

objPoly.typeCone = 1 (always 1)

objPoly.dimVar = 3 (#opt. variables in the problem)

objPoly.degree = 1 (degree of objective function $f_0(x)$)

objPoly.noTerms = 3 (number of monomial in the obs. function)

$\left\{\begin{array}{l}\text{objPoly.supports = support}\\ \text{objPoly.coef = coef}\end{array}\right.$

data structures used to describe the obs. function.

· support: matrix with · number of rows = noTerms

· number of columns = dimVar

each entry of support is a real number representing the degree of the optimization variable involves in the considered term

$$f_0(x) = -2x_1 + 3x_2 - 2x_3 \implies \text{support} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

· coef: is a column vector with coefficient of the different terms in the objective function

$$\Longrightarrow \quad coef = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$$

To do, to compute
$\forall$ row of the constraints

· STEP 2: we can proceed similarly to build the constraints

ineqPolySys{1}.typeCone = 1 $\left(\begin{matrix} 1 \to \geq \\ -1 \to = \end{matrix}\right)$
ineqPolySys.dimVar = 3 (optimiz. variable in the entire problem)
ineqPolySys{1}.degree = 2
ineqPolySys{1}.noTerm = 8
ineqPolySys{1}.supports = support1
ineqPolySys{1}.coef = coeff1

cs{1}

{1} $19 - 17x_1 + 8x_2 - 14x_3 + 6x_1^2 + 3x_2^2 - 2x_2x_3 + 3x_3^2 \geq 0$

$$support1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \qquad coef1 = \begin{bmatrix} 19 \\ -17 \\ 8 \\ -14 \\ 6 \\ 3 \\ -2 \\ 3 \end{bmatrix}$$

in order

ho riordinato i termini:
noti / x_k^1 / x_k^2

· The same for the other two rows {2}{3} of constraints

· Regarding the bounds on the variable:

it's the same as say unbounded

(upper bound) ubd = [2 1 $1e^{10}$]

(lower bound) lbd = [0 0 $-1e^{10}$]

· Set param.POPsolver = 'active-set' % refinement method