

# Part II: Control of Cyber-physical systems

Diego Regruto and Sophie M. Fosson



**Politecnico  
di Torino**

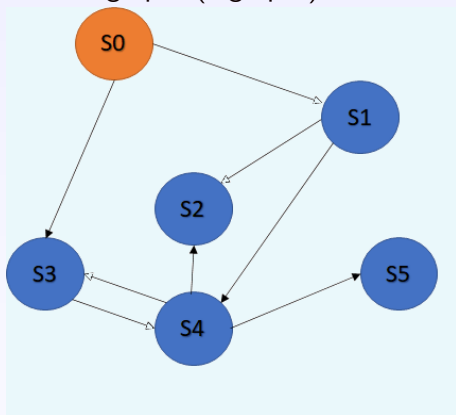
Department of Control and  
Computer Engineering



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# General formulation

- In this second part we will consider CPS described as multi-agents systems modeled by means of directed graphs (digraphs)



- Each node  $S_i$  of the graph is a dynamical LTI system
- Edges are used to model communication between agents

We will consider multi-agents control problems (generically called here *synchronization problems*) where the CPS is made up of

- 1 leader node  $S_0$
- $N$  follower nodes  $S_i$  ( $i = 1, 2, \dots, N$ )
- The agents cooperate in order to perform a task possibly dictated (to some extent) by the leader node
- In order to perform the assigned task the follower nodes exploit information shared on the communication network represented by the digraph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$
- Examples: autonomous vehicles platooning, unmanned air vehicles (UAVs) formation control, control of autonomous mobile robots team, ...

# General formulation: cooperative tracking problems

- In this part we will mainly focus on *cooperative tracking problem* where the task to be performed is dictated by the leader node  $S_0$  which generates the desired target trajectory (acting as a command generator exosystem)
- We will use the term *cooperative regulator problem* when the consensus is sought among agents in the absence of a leader node
- Assumption: the  $N$  follower agent are identical
- Possible peculiar problems of CPSs:
  - ▶ Each agents may be subjected to actuator/sensor faults/attacks
  - ▶ Information flow through the network may be affected by time-delay

# Course material: Part II

- **Lecture Slides/Notes**
- **Scientific papers** (suggested during lessons)

Further readings: one more book....

**B5** *Cooperative Control of Multi-Agent Systems*, F. Lewis et al., Springer London , 2013



# Agents mathematical models

- The dynamics of the **leader node**  $S_0$  is described by

$$\dot{x}_0 = Ax_0, \quad y_0 = Cx_0 \quad (1)$$

where  $x_0 \in \mathbb{R}^n$ ,  $y_0 \in \mathbb{R}^p$

- The dynamics of the  $N$  identical **follower nodes**  $S_i$  is described by

$$\dot{x}_i = Ax_i + Bu_i, \quad y_i = Cx_i \quad (2)$$

where  $x_i \in \mathbb{R}^n$ ,  $u_i \in \mathbb{R}^m$ ,  $y_0 \in \mathbb{R}^p$  and  $i \in \mathcal{N} = \{1, 2, \dots, N\}$

- The triple  $(A, B, C)$  is *stabilizable* and *detectable*

# Communication network modeling

- The **follower nodes**  $S_i$  share information through a communication network represented as a digraph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$  with  $N$  nodes  $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$  and a set of edges (arcs)  $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$
- The **leader node**  $S_0$  send information to some of the follower nodes
- The **leader node**  $S_0$  is not affected by any of the follower node
- To describe connection between the leader node and the graph  $\mathcal{G}$  of follower nodes we define the **augmented graph**  $\overline{\mathcal{G}} = \{\overline{\mathcal{V}}, \overline{\mathcal{E}}\}$  where  $\overline{\mathcal{V}} = \{v_0, v_1, \dots, v_N\}$ ,  $\overline{\mathcal{E}} \subset \overline{\mathcal{V}} \times \overline{\mathcal{V}}$

# Part II outline

- Experimental modeling of the agents
  - ▶ Set-membership identification
- Modeling of the communication network
  - ▶ review of basic graph-theory
  - ▶ basic consensus problem with continuous-time agents
- Cooperative control of multi-agents systems
  - ▶ Structural properties of LTI systems (review and extension)
  - ▶ Optimal linear quadratic (LQ) control of continuous-time systems
  - ▶ Distributed optimal control of multi-agent systems
- Additional topics possibly considered (depending on available time)
  - ▶ Formation control
  - ▶ Platooning
  - ▶ Cooperative control in presence of actuator/sensors attacks
  - ▶ Cooperative control in presence of communication network delay