№18 Ласка Иван

382003_2.

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma'(z) = \frac{-(-e^{-z})}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2}$$

$$\sigma(1-\sigma) = \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}} = \frac{e^{-z}}{(1+e^{-z})^2}$$

№19

1) $$g_k(S_1, \dots, S_k) = \frac{e^{S_k}}{\sum_{i=1}^{k} e^{S_i}}; \quad R^{(i)} = -\sum_{k=1}^{k} I(y^{(i)} = k)$$

$$-\ln g_k(S_1, \dots, S_k)$$

2) $$\frac{\partial g_k}{\partial S_l} = \begin{cases} e^{S_k} \cdot \left(-\frac{e^{S_l}}{\left(\sum_i e^{S_i}\right)^2}\right) = \frac{-e^{S_l}}{\sum_i e^{S_i}} = -g_l \cdot g_k, & k \neq l \\ \frac{e^{S_k}\left(\sum_i e^{S_i}\right) - e^{S_k} \cdot e^{S_l}}{\left(\sum_i e^{S_i}\right)^2} = \frac{e^{S_k}}{\sum_i e^{S_i}} - \frac{e^{S_k}}{\sum_i e^{S_i}} \cdot \frac{e^{S_l}}{\sum_i e^{S_l}} = g_k(1-g_l), & k = l \end{cases}$$

$$I(k=l) = \begin{cases} 1, & k=l \\ 0, & k \neq l \end{cases} \Rightarrow \frac{\partial g_k}{\partial s_l} = g_k \cdot (I(k=l) - g_l)$$

2) $$\frac{\partial R^{(i)}}{\partial g_k} = -I(y^{(i)}=k)(\ln g_k(s_1, \dots s_k)) =$$

$$= -\frac{I(y^{(i)}=k)}{g_k(s_1, \dots s_k)}$$

3) $$\frac{\partial R^{(i)}}{\partial s_l} = \sum_k \frac{\partial R^{(i)}}{\partial g_k} \cdot \frac{\partial g_k}{\partial s_l} = -\sum_k \frac{I(y^{(i)}=k)}{g_k(s_1, \dots s_k)}$$

$$\cdot g_k (I(k=l) - g_l) = -(I(y^{(i)}=l) - g_l) =$$

$$= g_l - I(y^{(i)}=l)$$