Annual Reviews in Control 55 (2023) 142–164

Contents lists available at ScienceDirect

Annual Reviews in Control

journal homepage: www.elsevier.com/locate/arcontrol

Review article

A review of current state-of-the-art control methods for lower-limb powered prostheses

Rachel Gehlhar [a,*], Maegan Tucker [a], Aaron J. Young [b,c], Aaron D. Ames [a,d]

[a] Department of Mechanical and Civil Engineering, California Institute of Technology, 1200 E. California Blvd., Pasadena, 91125, CA, USA
[b] Woodruff School of Mechanical Engineering, Georgia Institute of Technology, North Avenue, Atlanta, 30332, GA, USA
[c] Institute for Robotics and Intelligent Machines, Georgia Institute of Technology, North Avenue, Atlanta, 30332, GA, USA
[d] Department of Computing and Mathematical Sciences, California Institute of Technology, 1200 E. California Blvd., Pasadena, 91125, CA, USA

ARTICLE INFO

Keywords:
Prostheses
Control
Robotics
Lower-extremity
User-customization

ABSTRACT

Lower-limb prostheses aim to restore ambulatory function for individuals with lower-limb amputations. While the design of lower-limb prostheses is important, this paper focuses on the complementary challenge—the control of lower-limb prostheses. Specifically, we focus on powered prostheses, a subset of lower-limb prostheses, which utilize actuators to inject mechanical power into the walking gait of a human user.

In this paper, we present a review of existing control strategies for lower-limb powered prostheses, including the control objectives, sensing capabilities, and control methodologies. We separate the various control methods into three main tiers of prosthesis control: High-level control for task and gait phase estimation, mid-level control for desired torque computation (both with and without the use of reference trajectories), and low-level control for enforcing the computed torque commands on the prosthesis. In particular, we focus on the high- and mid-level control approaches in this review. Additionally, we outline existing methods for customizing the prosthetic behavior for individual human users. Finally, we conclude with a discussion on future research directions for powered lower-limb prostheses based on the potential of current control methods and open problems in the field.

1. Introduction

The number of individuals living with major lower limb loss in the United States was estimated to be over 600,000 in 2005 and is expected to double by 2050 due to an increasing rate of diabetes (Ziegler-Graham, MacKenzie, Ephraim, Travison, & Brookmeyer, 2008). Currently, the most commonly prescribed device for lower-limb amputation is a passive lower-limb prosthesis. However, several advantages have been demonstrated with powered prostheses compared to passive prostheses, albeit at the cost of increased complexity. Throughout this survey we will outline various control strategies leveraged for powered prostheses, discuss the associated benefits and limitations of these approaches, and highlight open problems in the field as well as potential future research directions.

1.1. Advantages of powered prostheses

While passive prostheses are able to provide some energy absorption, they are unable to provide energy generation as human muscles do (DeVita, Helseth, & Hortobagyi, 2007; Winter, 1983); interestingly, biological ankles provide up to 60% of the energy generated by a limb in a gait cycle (Teixeira-Salmela, Nadeau, Milot, Gravel, & Requião, 2008; Winter, 1983). Hence, powered prostheses pose an advantage through their contribution of net positive work in addition to their energy absorption. This is especially important for alleviating compensatory behaviors of prosthesis users. For example, during stair climbing, transtibial (amputation above the ankle) and transfemoral (amputation above the knee) amputees often develop asymmetries (Hobara et al., 2011; Schmalz, Blumentritt, & Marx, 2007; Torburn, Schweiger, Perry, & Powers, 1994; Vack, Nielsen, & Shurp, 1999) due to the inability to produce the net positive work required (in either the stance knee or stance ankle) to raise the user's center of mass (Riener, Rabuffetti, & Frigo, 2002). Powered prostheses have demonstrated reduction in these compensatory behaviors (Armannsdottir, Tranberg, Halldorsdottir, & Briem, 2018) including the reduction of loading impulse thereby reducing the risk of knee osteoarthritis (Morgenroth et al., 2011). Similarly, powered prostheses have been demonstrated to increase self-selected walking speed (Herr & Grabowski, 2011; Martinez-Villalpando, Mooney, Elliott, & Herr, 2011) compared to passive prosthesis users

* Corresponding author.
E-mail address: rgehlhar@caltech.edu (R. Gehlhar).

https://doi.org/10.1016/j.arcontrol.2023.03.003
Received 31 December 2022; Accepted 3 March 2023
Available online 3 April 2023

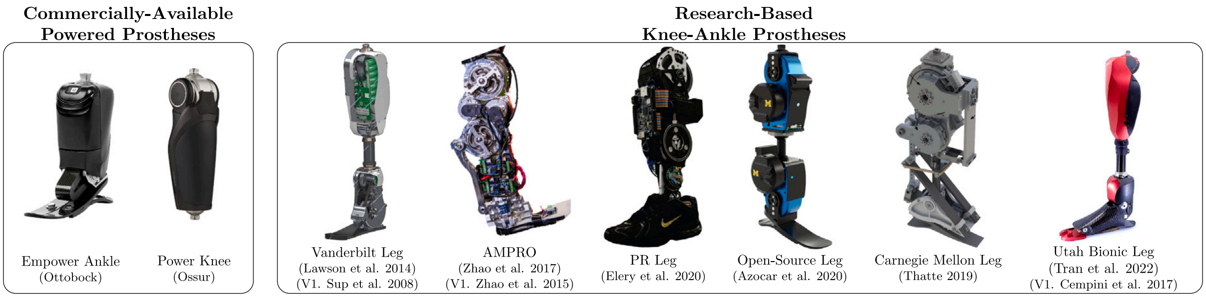

Fig. 1. Existing powered prostheses. While several dual-actuated (knee–ankle) prostheses have been developed and demonstrated in research settings, there are currently only two commercially-available powered prostheses which are both single-actuated. These commercially-available devices are shown on the left, with some of the dual-actuated devices used in research settings shown on the right. Note that the images are taken from the publications listed below each image, with the references for the first versions of the devices indicated by "V1".

who experience shorter step length (Gailey, 2008), slower walking cadence (Waters, Perry, Antonelli, & Hislop, 1976), and a decrease in comfortable walking speed (Waters & Mulroy, 1999) compared to able-bodied walking.

Another known limitation of passive prostheses is that their use results in increased metabolic energy expenditure (Esposito, Rodriguez, Ràbago, & Wilken, 2014; Molen, 2004; Waters & Mulroy, 1999). Comparatively, powered prostheses are hypothesized to enable more energy efficient behavior relative to passive prostheses; this metabolic reduction has been demonstrated in preliminary studies (Au, Weber, & Herr, 2009; Herr & Grabowski, 2011; Ledoux & Goldfarb, 2017; Martinez-Villalpando et al., 2011). This potential improvement is particularly important for individuals with above-the-knee amputation since higher levels of amputation have been shown to result in less efficient gaits and higher O_2 costs (Nowroozi, Salvanelli, & Gerber, 1983; Waters et al., 1976).

1.2. Current limitations of powered prostheses

Despite the clear benefits associated with powered lower-limb prostheses, there are also several drawbacks that limit their ability to be commercially viable. First, powered lower-limb prostheses tend to have increased weight compared to microprocessor or passive prostheses due to the addition of sensing, actuation, and batteries. This additional weight increases the load on an amputee's intact limb, inducing further stresses at the socket interface. Additionally, this higher weight has been shown to increase human joint torque and metabolic expenditure; for example, a study conducted on healthy walking found that net metabolic rate rose with both increased loading and more distal placements of the mass (Browning, Modica, Kram, & Goswami, 2007).

This increase in mechanical and electrical complexity also drives up the cost of powered prostheses, decreasing the likelihood of insurance covering these devices and inhibiting their potential viability. Furthermore, the addition of sensors and actuators introduces more opportunities for failures, including both electronic and mechanical failures. Aside from the physical components, powered prostheses rely on a control system to drive the actuation, which introduces unique challenges. Predominately, these controllers are specifically designed for certain conditions, including the locomotion mode (i.e. level-ground walking or stair climbing Ledoux & Goldfarb, 2017) and characteristics of the prosthesis user (Huang, Si, Brandt, & Li, 2021; Quintero Reznick et al., 2018). Thus, it can be challenging to develop prosthesis controllers that perform well across a variety of conditions. To date, the benefits of powered prostheses only emerge once the prosthesis control is customized to each subject and each locomotion mode. Similarly, since most prosthesis controllers only consider level-ground walking, it is challenging for powered prostheses to handle varying terrain. Moreover, for dual-actuated prostheses, the challenges associated with the control system are amplified since the controller needs to coordinate the motion of two joints simultaneously and the number of control parameters that require tuning at least doubles. This motivates developing systematic control methods that extend to high-dimensional parameter spaces and faster methods of user-customization.

1.3. Commercial availability of powered prostheses

Ultimately, the benefits of powered prostheses observed in research settings do not yet outweigh the limitations, which likely contributes to why there are only two powered lower-limb prostheses currently available on the commercial market, as illustrated in Fig. 1. The first device, "Empower", is a powered ankle prosthesis that was released in 2010 from Ottobock (formerly known as the "Biom" Au & Herr, 2008). The second device, "Power Knee", is a powered knee prosthesis from Ossur, Reykjavik, and Iceland (2018) that was first released in 2006. While the addition of these devices to the commercial market emphasizes the potential impact of powered prostheses, it is important to note that there are currently no powered dual-actuated lower-limb prostheses commercially available. This is due in part to the increased challenge of translating dual-actuated devices to clinical settings due to the increased complexity of the control system and required user customization (Nowroozi et al., 1983; Waters et al., 1976). This poses a disadvantage to transfemoral amputees considering that dual-actuated knee–ankle prostheses offer a unique solution towards controlling the coordination of multiple joints which plays an important role in ankle push-off (Winter, 1983).

However, as also illustrated in Fig. 1, there exist many dual-actuated powered prostheses utilized in research settings. To date, the purpose of these devices is predominantly to develop and evaluate novel control strategies (Azocar et al., 2020; Elery, Rezazadeh, Nesler, & Gregg, 2020; Lawson et al., 2014; Sup, Bohara, & Goldfarb, 2008; Thatte, 2019; Zhao, Ambrose and Ames, 2017). Recently, there has also been a push towards developing open-source prostheses and test platforms since the difference between various devices hinders comparisons between control strategies (Azocar et al., 2020). Lastly, efforts have been made towards decreasing the weight associated with powered prostheses; of particular note is the Utah Knee (Tran, Gabert, Cempini, & Lenzi, 2019) and powered polycentric ankle prosthesis (Cempini, Hargrove, & Lenzi, 2017).

One of the main factors limiting powered prostheses from being clinically practical is the time-consuming nature of tuning control parameters for individual prosthesis users (Quintero, Reznick et al., 2018; Simon et al., 2014). This tuning process becomes increasingly complex when considering multiple locomotion modes as well as multiple joints. To examine the limitations and promising recent advancements of current control methods, we survey the control strategies for lower-limb powered prostheses, followed by a discussion on existing techniques for user-specific customization.

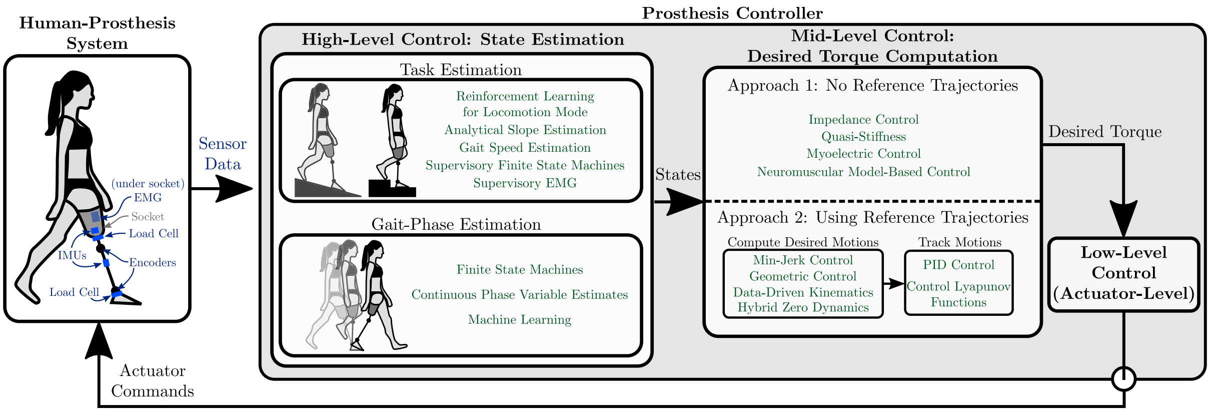

Fig. 2. Prosthesis control architecture. While there exist various prosthesis control strategies, most control architectures utilize the illustrated hierarchy. Overall, the controller receives sensor data from the human-prosthesis system and returns (typically joint-level) torques. Furthermore, all controllers generally contain the following shared components: high-level task and gait phase estimation; mid-level desired torque computation either with or without a reference trajectory, and a low-level controller that commands the desired torque from the actuators. The various control methods that will be discussed in the survey are illustrated in green. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

1.4. Overview of the paper

While there exist several reviews on the design and control of lower-limb powered prostheses (Price, Beckerle, & Sup, 2019; Tucker et al., 2015; Voloshina & Collins, 2020; Windrich, Grimmer, Christ, Rinderknecht, & Beckerle, 2016), many of these reviews focus on the mechanical design and provide a high-level treatment of the available control methodologies. In contrast, this survey focuses solely on the existing control strategies. This survey allows us to explore the heuristic and formal nature of current methods to examine their customizability and their potential to generalize across users. Ultimately, a method that can both generalize across users and have clinically friendly parameters for user-customization could bring these devices into every day use.

To provide context for the control methods presented in this paper, we will begin in Section 2 by discussing specific challenges amputees face with passive prostheses that then motivate various control objectives. Section 3 will then introduce the various sensing methods available as inputs to prosthesis control methods. We categorized the control method components into four categories, as shown in Fig. 2. The first component, presented in Section 4, is a high-level controller that estimates the desired task for the prosthesis, such as walking speed, walking incline, and locomotive modes. A high-level controller that generally follows task estimation is gait phase estimation, discussed in Section 5. The phase of the gait cycle is generally determined in a discrete way through a finite state machine or in a continuous way through a phase variable. Following these high-level controllers, we discuss mid-level controllers, which we consider to fit under one of two approaches. The first approach, discussed in Section 6, focuses on computing torque using task and phase estimates, kinematics, EMG signals, or a combination of these. The second approach, in Section 7, consists of two steps. The first step, in Section 7.1, determines a desired reference trajectory for the prosthesis, which is a desired motion of a joint defined with respect to time or gait phase. The second step, in Section 7.2, determines a torque to track the reference trajectory. Finally, in the prosthesis control-architecture, a low-level controller commands the desired torques from the actuators. These low-level controllers will not be discussed in this survey. All of the control methods we present require some level of tuning. Various tuning approaches used for user-customization will be outlined in Section 8. Section 9 will then discuss open questions regarding prosthesis control methods and important considerations in working towards better prosthesis performance, followed by a brief conclusion in Section 10.

2. Control objectives for prostheses

A major consideration when developing controllers for powered prostheses is how to assess the performance for any given controller. We consider three categories that encompass the main control metrics used: *naturalness* of the gait, *efficiency* of the gait for the human and prosthetic device, and the prosthesis' *responsiveness* to the human's behavior and the environment.

2.1. Naturalness

One of the most common control objectives is the ability of a prosthesis to mimic healthy human walking patterns. This goal is motivated by the gait asymmetry prosthesis users exhibit kinematically (Boonstra, Schrama, Fidler, & Eisma, 1995; Hurley, McKenney, Robinson, Zadravec, & Pierrynowski, 1990), kinetically (Nolan & Lees, 2000; Nolan et al., 2003), and temporally (Isakov, Burger, Krajnik, Gregorič, & Marinček, 1997; Jaegers, Arendzen, & de Jongh, 1995). Part of this asymmetry arises from the limitations of passive devices as well as from amputees favoring their intact leg. Favoring one side leads to loading asymmetry (Engsberg, Lee, Tedford, & Harder, 1993; Suzuki, 1974), which increases with walking speed (Nolan et al., 2003). This tendency also leads to changes in biomechanical motion, degenerative changes to their limbs, and pain in their joints and back (Gailey, 2008).

This loading asymmetry and the amount of body weight supported by the prosthesis can be assessed by the ground reaction forces (Hunt, Hood, & Lenzi, 2021; Jayaraman et al., 2018; Simon et al., 2016). Additionally, comparing the center of pressure (CoP) trajectories of able-bodied walking to that of a prosthesis foot reveals human-likeness, and to that of an amputee's sound foot indicates the presence of compensatory strategies (Gregg, Lenzi, Hargrove and Sensinger, 2014; Schmid, Beltrami, Zambarbieri, & Verni, 2005). Furthermore, prosthesis control aims to improve the kinetic naturalness at the joint level, often with the intent of improving the loading asymmetry. These metrics include comparing actuator torques, power, and work (Eilenberg, Geyer, & Herr, 2010; Gregg, Lenzi et al., 2014; Ledoux & Goldfarb, 2017; Thatte & Geyer, 2015; Zhao, Horn, Reher, Paredes and Ames, 2016), as well as the relationship between torques and the joint angles (the quasi-stiffness) (Latash & Zatsiorsky, 1993; Lenzi, Hargrove, & Sensinger, 2014; Rouse, Gregg, Hargrove, & Sensinger, 2013) to that of able-bodied walking.

There are certain kinematic features that have been found to play an important role in human gait, such as knee flexion in early stance to cushion the transition to weight-bearing mode (Blumentritt, Scherer, Wellershaws, & Michael, 1997) and ankle plantar flexion in late stance to propel the body forward (Inman, 1966; Winter, 1983). Motivated by the way these kinematic behaviors lead to desirable full body dynamic effects, prosthesis performance is commonly assessed by comparing the joint kinematic profiles to able-bodied kinematic data (Eilenberg et al., 2010; Gehlhar & Ames, 2023; Gregg, Lenzi et al., 2014; Ledoux & Goldfarb, 2017; Sup et al., 2008; Thatte, Shah and Geyer, 2019). Gait symmetry is also assessed by comparing the kinematic trajectories of the prosthesis to the sound side (Hood, Gabert, & Lenzi, 2022).

Other metrics for assessing the naturalness of prosthesis walking include spatio-temporal parameters, such as step length, time spent on each leg, duration of gait phases between each limb, and maximum joint angles in specific phases of walking (Lenzi et al., 2014). At the muscular level, measuring muscle activation through electromyography (EMG) assesses the muscle activation symmetry between limbs during prosthesis walking (Huang, Wensman, & Ferris, 2014; Hunt et al., 2021; Jayaraman et al., 2018). Lastly, kinematic properties of the walking are assessed using postural observations including the evolution of the center of mass during walking (Brandt & Huang, 2019).

Lastly, prosthetic control is often aimed at replicating biomechanical relationships observed in human walking, such as the relationships between ankle work and walking speed (Lenzi et al., 2014), between ankle work and positive slope angle (Eilenberg et al., 2010), and between maximum flexion angle and speed (Lenzi et al., 2014). These relationships provide insight into how well the control method captures a human's underlying objectives to exhibit similar responses to given conditions.

2.2. *Efficiency*

Metabolic cost of transport is commonly used to evaluate a human's locomotive efficiency. This metric is determined by both the user's metabolic rate and their walking speed (Brockway, 1987), with the control objective being to decrease metabolic rate and increase walking speed. This section will discuss how performance assessed with this metric degrades with an amputation, which then motivates using powered prostheses to restore higher levels of energy efficiency.

It is well understood that passive prostheses increase hip power and metabolic expenditure (Winter, 1991). Specifically, transfemoral amputees exert three times more hip power and torque on their amputated side in walking than able-bodied individuals (Winter, 1991). Furthermore, transtibial and transfemoral amputees respectively expend around 20% and 30% more energy while walking (Molen, 2004; Waters & Mulroy, 1999). To mitigate this decline in efficiency, prosthesis control aims to reduce metabolic cost of transport (Jayaraman et al., 2018; Price et al., 2019).

However, it is unclear whether powered prostheses are yet capable of restoring metabolic expenditure to a level comparable to walking with intact limbs. While there exists previous work that has demonstrated metabolic reduction using a powered ankle prosthesis (Au et al., 2009; Herr & Grabowski, 2011; Ledoux & Goldfarb, 2017; Martinez-Villalpando et al., 2011), it should be noted that these results have not been repeatable across powered prosthetic devices (Welker, Voloshina, Chiu, & Collins, 2021). Specifically, other results indicate that powered ankle prostheses fail to reduce metabolic cost across a wide variety of ankle push-off strategies (Quesada, Caputo, & Collins, 2016). Note that these existing results are largely limited to powered ankle prostheses. Thus, the potential for dual-actuated prostheses to reduce metabolic expenditure for transfemoral amputees is still an open question.

Moreover, it is unclear how to develop control methods that directly improve metabolic efficiency. One existing approach is to systematically select mid-level control parameters to minimize cost of transport of a modeled amputee-prosthesis system (Li, Tucker, Gehlhar, Yue, & Ames, 2022; Thatte & Geyer, 2015; Zhao, Hereid, Ambrose and Ames, 2016). In addition to reducing the energy cost to the human, these offline optimization approaches and other online optimizations (Zhao, Horn, Reher, Paredes and Ames, 2017) can reduce the mechanical work of the device which reduces actuator demands.

As mentioned earlier, the last component of efficiency is related to the self-selected walking speed of the prosthesis user. Typically humans' *self-selected walking speed* is between 1.3 and 1.5 m/s, slower than their maximum speed of around 2 m/s (Bohannon, 1997), but close to their optimal speed, of 1.2 m/s, with respect to energy expenditure (Ralston, 2004). Amputees walking with passive prostheses experience a significant decrease in self-selected walking speed (Jaegers et al., 1995) — an 11% speed decrease for transtibial amputees, and 35% for transfemoral (Waters & Mulroy, 1999). Motivated by this decline in performance, prosthesis control methods are assessed by a user's self-selected walking speed and aimed at increasing it (Fritz & Lusardi, 2009; Herr & Grabowski, 2011; Jayaraman et al., 2018; Martinez-Villalpando et al., 2011).

2.3. *Responsiveness*

The final control objective for powered prostheses is the ability to respond appropriately to both the user and the environment. This requires a prosthetic device to adapt its behavior to satisfy user intent. Responsiveness also refers to a prosthesis' ability to respond to disturbances in a way that keeps the user stable and safe.

First, the ability of a prosthesis to respond to a human user is often characterized as *volitional ability*. This objective is especially considered in direct EMG control and is quantified by the precision of motion under the user's volition. Specifically, the following metrics have been examined to assess volitional control: excess distance traveled when moving to a target angle, time the subject held the prosthesis in target window, and total wasted motion of the prosthesis in accomplishing various tasks (Clites et al., 2018). Volitional ability is also assessed in control methods that use mechanical sensors, by evaluating the prosthesis' ability to respond to changes in speed and phase (Gregg, Lenzi et al., 2014; Lenzi et al., 2014; Rezazadeh et al., 2019).

Second, powered prostheses must also be responsive to changes in the environment. This includes both transitioning between locomotion modes as well as maintaining stability. In terms of transitions, it is expected that for daily locomotion, prostheses must be able to traverse a variety of terrain types: upward and downward slopes, ascending and descending stairs, and uneven terrain. Generally, two approaches are taken towards realizing these various types of locomotion: develop task estimators that discretely classify the terrain type or locomotive mode (Young & Hargrove, 2015), or focus on developing a unified control method that adapts to the user's behavior (Best, Embry, Rouse, & Gregg, 2021; Hood et al., 2022; Hunt et al., 2021; Mendez, Hood, Gunnel, & Lenzi, 2020; Thatte & Geyer, 2015). A prosthesis' responsiveness to the environment can be evaluated using classification accuracy and response time.

To put the importance of such responsiveness into context, even for small mode transition errors (1%), a user who ambulates 5000 steps a day would experience 50 transition errors daily (Young & Hargrove, 2015). To decrease the number of potential falls from misclassifications, powered prostheses must have both highly accurate methods of task estimation as well as controllers that can seamlessly recover from misclassification errors. More generally, this notion of recovery refers to a prosthesis' ability to respond to unexpected disturbances (i.e., maintaining balance in the event of a trip).

Lastly, locomotive stability is important, reflected by the fact that amputees have a greater risk of falling than healthy individuals (Miller, Speechley and Deathe, 2001). Passive prosthesis users tend to have less robust gaits (Kulkarni, Wright, Toole, Morris, & Hirons, 1996), falling more often than non-amputees (Bukowski, 2006). Kulkarni et al. (1996) polled amputees and found 45% had fallen while wearing their prosthesis in the previous year. However, there is currently no universal metric for stability/balance apart from recording falls in a long term study (Miller, Deathe, Speechley and Koval, 2001; Wong, Wilska, &

Stern, 2012). Thus, it is difficult to compare the effect of various controllers on locomotive stability. Though, it should be noted that test environments are being developed to introduce walking disturbances in a controlled setting to assess a human and prosthesis' stability in the presence of perturbations. These efforts along with other research focused around fault detection and fall prevention will be discussed in Section 9.1.2.

3. Sensing for prosthesis control

One factor that influences a controller's capabilities to achieve the aforementioned objectives is the available sensing. We present four categories of sensing used in powered prosthesis control: kinematic sensors such as inertial measurement units (IMUs) and encoders to measure joint and limb motion; force sensors to provide force feedback at the ground and the user's socket; EMG sensors to detect human muscle activity; and computer vision and range sensors to infer information about the user's environments.

3.1. Kinematics — IMU, encoders

Almost all microprocessor controlled devices have an encoder for joint angle measurements and many devices also include an IMU for additional state feedback (Azocar et al., 2020; Elery et al., 2020; Lawson et al., 2014; Thatte, 2019; Tran et al., 2019; Zhao, Ambrose et al., 2017). Encoders measure prosthesis joint angles and velocities, and an IMU is typically used to measure the global orientation and velocity of the prosthesis-side shank or the human's residual limb. These measurements are used in many forms of prosthesis control, including impedance control laws, reference trajectory tracking controllers, and gait phase estimators.

3.2. Forces sensors

Many devices include a force sensor, typically a load cell distal to the ankle joint (Elery et al., 2020; Gregg, Lenzi et al., 2014), in the shank (Azocar et al., 2020; Lawson et al., 2014), or proximal to the knee joint (Sup et al., 2008; Tran et al., 2019) to detect ground contact, ground reaction forces, or CoP. Most commonly, these force sensor measurements are used to determine transitions between discrete gait phases in a finite state machine (FSM), especially between stance and swing phase. The work of Gregg, Lenzi et al. (2014) used the CoP to encode and modulate virtual constraints. Recently, a load cell was incorporated at the socket interface of a transfemoral prosthesis and a pressure sensor into the shoe of the prosthesis to provide real-time force feedback to complete the modeled prosthesis dynamics for model-based control (Gehlhar, Yang, & Ames, 2022).

3.3. EMG sensors

EMG sensors measure electrical signals generated by muscles during contraction. Broadly speaking, there are two kinds of EMG sensors: surface-mount and implantable EMG sensor interfaces (Fleming et al., 2021). To our knowledge, there is only one study that utilized surgically implanted wireless intramuscular EMG sensors in lower-limb amputees for prosthesis control (Sigurðardóttir, 2015); these surgically implanted EMG sensors are more commonly used in upper-limb prostheses (Dewald et al., 2019; Ortiz-Catalan, Brånemark, Håkansson, & Delbeke, 2012; Pasquina et al., 2015; Vaskov et al., 2022). The majority of existing studies on myoelectric control of lower-limb prostheses instead use bipolar surface EMG electrodes (Clites et al., 2018; Fleming et al., 2021; Hunt et al., 2021; Wang, Kannape, & Herr, 2013).

A major limitation of surface EMG signals is that the recorded signals vary over time due to changes in skin impedance, day-to-day variations in electrode placement, and relative motion between the electrodes and underlying muscles during movements (Dewald et al., 2019). Moreover, electrode placement within the prosthesis socket can cause physical disturbances that induce noise in the EMG recordings, can compromise the socket suspension, and ultimately degrade user comfort (Hong & Mun, 2005; Sensinger, Lock, & Kuiken, 2009; Zhang & Huang, 2015). Solutions for improving EMG signal reliability include further investigating implantable sensors (Dewald et al., 2019; Ortiz-Catalan et al., 2012; Pasquina et al., 2015), novel flexible electrode design (Myers, Huang, & Zhu, 2015; Yeon et al., 2021), custom prosthesis sockets with integrated electrodes (Wang et al., 2013), and developing EMG decoding algorithms and control paradigms (Fleming et al., 2021). Notably, novel flexible electrodes are particularly promising for improving the reliability and comfort of EMG sensors since they (1) are more compatible with a subject's prescribed prosthetic socket and liner compared to other within-socket surface EMG sensors, (2) will likely be less expensive than fabricating a custom socket and liner for each individual subject, and (3) have been shown to be more comfortable than commercial surface-mount electrodes (Yeon et al., 2021).

Lastly, another challenge with using EMG signals in prosthesis control is that individuals with limb loss often exhibit variations in residual muscle activation and coordination (Huang & Ferris, 2012; Wentink, Prinsen, Rietman, & Veltink, 2013). Thus, before EMG control can be effective for subjects who exhibit abnormal muscle patterns, further research is needed towards understanding how limb amputation influences EMG sensing.

3.4. Computer vision and range sensors

To translate prosthetic walking to environments other than level-ground walking, additional sensors are required to estimate elements of the environment. Most commonly these sensors include cameras and range sensors.

Notably, cameras used for computer vision have demonstrated highly accurate predictions of complex terrain environments. For example, large datasets of wearable camera images of walking environments have been used to train convolutional neural networks for real-world stair environments (Kurbis, Laschowski, & Mihailidis, 2022). Additionally, visual features have been extracted from similar large datasets and images have been classified using a Bag of Words method for terrain identification (Diaz, da Silva, Zhong, Huang, & Lobaton, 2018). Furthermore, specialized cameras, such as depth cameras, have been used to directly estimate various properties of the environment such as stair height and stair depth (Krausz, Lenzi, & Hargrove, 2015).

Similarly, range sensors provide depth information about the terrain. One example of range sensors is laser distance meters, which have been used with decision trees to classify terrain type as either ascending/descending ramps, ascending/descending stairs, or level ground (Liu, Wang, & Huang, 2015). Another example is LIDAR, which has been used (in combination with an IMU and joint encoders) to estimate the position of the prosthesis leg with respect to the ground for real-time reactive control for trip avoidance (Thatte, Srinivasan and Geyer, 2019). Notably, this was the first work to incorporate visual feedback into real-time planning of prosthesis control.

4. High-level task estimators

As shown in Fig. 2, after receiving input signals from the aforementioned sensors, the first component of the prosthesis controller is determining the desired tasks (i.e., locomotion mode and walking speed). This component can also be otherwise interpreted as deciphering user intent. In this section we will discuss the approaches to high-level control for estimating both locomotion mode and gait speed.

4.1. Estimating locomotion mode

The goal of locomotion mode estimation is to quickly and accurately predict the intended locomotion modes of a prosthesis user based on real-time input signals (including kinematic, dynamic, and neuromuscular signals collected from both the device and user). Commonly estimated locomotion modes include level ground walking, ramp ascent and descent, and stair ascent and descent (Young & Hargrove, 2015; Young, Simon, Fey, & Hargrove, 2014). In general, there are two approaches for this: analytical algorithms and machine learning classifiers. Additionally, some machine learning classifiers include the use of EMG sensing, which has been shown to improve classification accuracy but at the cost of relying on user-dependent classifiers.

First, we will discuss analytical algorithms, which use predefined event triggers with sensor information to switch between locomotion modes. FSMs can be used as a supervisory controller to switch between locomotion modes based on foot contacts, direction of ankle motion, shank position, and time (Culver, Bartlett, Shultz, & Goldfarb, 2018). Additionally, slope incline can be estimated by estimating the angle of the foot using forward kinematics and global orientation of the thigh (as measured by an IMU placed on the residual thigh) (Best et al., 2021). Lastly, ground slope can also be estimated using foot load sensors (Sup, Varol, & Goldfarb, 2011; Svensson & Holmberg, 2005; Ulf Holmberg, 2006).

The second method of estimating locomotion mode is to train machine learning classifiers using precollected training data. Such classifiers enable smooth and automatic transitions between locomotion modes, reducing the cognitive burden placed on prosthesis users, but require the collection of training data. Some common methods of classification for lower-limb prosthesis mode include support vector machines (Ceseracciu et al., 2010), artificial neural networks, linear discriminant analysis, maximum likelihood (Huang et al., 2011; Myung, 2003), and Bayesian networks (Hargrove, Englehart, & Hudgins, 2007; Nefian, Liang, Pi, Liu, & Murphy, 2002)).

As mentioned, some machine learning classifiers involve EMG sensors (Hargrove et al., 2013; Huang, Kuiken, Lipschutz, et al., 2008; Hussain et al., 2020; Jin, Yang, Zhang, Wang, & Zhang, 2006; Peeraer, Aeyels, & Van der Perre, 1990). The use of EMG for estimating locomotion mode is often termed *supervisory EMG*. One important component to supervisory EMG control is extracting features from the EMG signals and classifying patterns of these features for various locomotion modes. However, since EMG signals are non-stationary over walking gait cycles, it can be difficult to extract key features for walking; comparatively, extracting key features is easier for upper-limb prosthesis control. One common approach of supervisory EMG control for lower-limb prostheses is to divide the gait cycle into discrete phases, and extract key features for each phase separately (Huang et al., 2008).

Generally, adding EMG signals to classifiers improves classification accuracy (Hargrove et al., 2015; Huang et al., 2011; Zhang & Huang, 2013) compared to only using mechanical sensors. However, the performance of such classifiers relies on the accuracy of EMG signals from the prosthesis user. While many studies have demonstrated supervisory EMG using EMG signals from residual limb muscles (Huang et al., 2008), studies have found that including additional sensors to intact muscles improves accuracy but also consequently increases complexity for daily use and sensor setup (Fleming et al., 2021). For example, Huang et al. (2008, 2011) used a combination of 10–16 EMG signals across both the residual and intact limbs. It is interesting to note that the reliability of EMG signals on the residual limb depends on the amputation level of the subject. One solution to improve the accuracy of EMG signals is to leverage TMR surgery to record signals directly from reinnervated residual muscles (Hargrove et al., 2013, 2015).

One open question when training machine learning classifiers is the amount of data that should be provided during the model training (or specifically, the amount of "time history"). An existing study that compared strategies with and without time history found that including time history improved locomotion mode intent recognition accuracy (Young et al., 2014). Another open question regarding classifiers for locomotion modes is if the classifiers should be user-independent or user-dependent. The benefit of developing user-independent classifiers is that they generalize to new users without requiring new training data to be collected per subject. However, such classifiers have been found to be less accurate compared to user-dependent classifiers. For example, Young and Hargrove (2015) found misclassification rates for user-independent classifiers to be significantly higher than those for the user-dependent system. It was hypothesized that this is in part due to the variance in subjects' walking patterns. Similarly, Bhakta, Camargo, Donovan, Herrin and Young (2020) compared subject-independent and subject-dependent intent recognition across three machine learning algorithms (linear discriminant analysis (LDA), neural networks (NN), and a gradient tree boosting method called XGBoost) and also found that subject-independent classifiers result in significantly higher misclassification errors.

A major limitation of classifiers for locomotion modes is the reliance on rich sensor information. As argued in Camargo, Flanagan, Csomay-Shanklin, Kanwar and Young (2021), although additional sensors and features tend to increase the performance of machine learning models, their inclusion also greatly increases the model complexity. Future research includes studying the minimum complexity of sensor information required for accurate classifiers. For example, Camargo, Flanagan et al. (2021) performed an analysis of sensor importance for a knee–ankle prosthesis and concluded that mechanical sensors (IMUs and goniometers) were generally more important than EMG sensors. Another major limitation is that even for a user-independent classifier, a prosthesis with different sensors would require a new set of training data. Thus, future research directions include strategies for reducing the initial user-independent misclassification rate.

4.2. Estimating gait speed

The second component of high-level task estimation for lower-limb prosthesis control is estimating gait speed. As with estimating locomotion modes, the two common approaches for estimating gait speed are (1) analytical algorithms which directly compute walking speed from sensor measurements and assumptions about the underlying kinematic gait model, and (2) regression modeling and machine learning algorithms (Bhakta, Camargo, Compton, Herrin, & Young, 2021).

The most common method is analytical algorithms that directly utilize IMU sensors due to their simplicity. However, these sensors suffer from long-term drift which tends to degrade the gait speed estimation accuracy over time (Sabatini, Martelloni, Scapellato, & Cavallo, 2005). Thus, algorithms that incorporate kinematic models are advantageous since they incorporate a variety of sensor information. However, kinematic models have been shown to be less accurate without subject-dependent calibration (Aminian, Najafi, Büla, Leyvraz, & Robert, 2002; Hu, Sun, & Cheng, 2013; Li, Young, Naing, & Donelan, 2010; Mariani et al., 2010). One example of this method is Lenzi et al. (2014) which uses prosthesis shank, knee, and angle angles in a three-link planar leg model to compute the forward hip velocity in the sagittal plane. In stance, gyroscope data is used, and in swing, accelerometer outputs are integrated. Another example of an analytical algorithm is that of Best et al. (2021), which estimates gait speed based the displacement of the foot (approximated by the prosthesis-side leg geometry) and the time between steps.

One disadvantage associated with analytical algorithms is that they typically only update their prediction of gait speed once per gait cycle or step. This motivates the second method, machine learning classifiers, which provide a continuous estimation for gait speed. This method leverages similar classifiers as discussed for estimating locomotion mode (Bhakta et al., 2021; He & Zhang, 2011; Vathsangam, Emken,

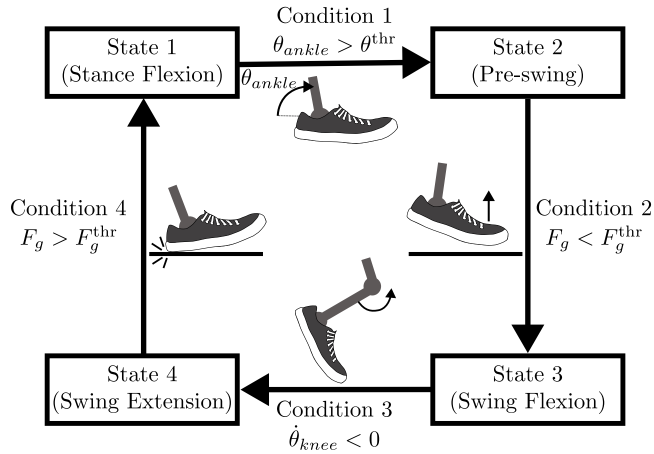

Fig. 3. Finite state machine. An example of an FSM used to determine gait phase, based on Sup et al. (2008). Here the gait cycle is divided into four phases, or states, and conditions based on the joint angle, velocity, and ground reaction forces dictate state transitions. In state 1, when the ankle angle θ_{ankle} reaches the threshold θ^{thr}, state 2 begins. Once the foot lifts off the ground, determined when the ground reaction force F_g is less than a threshold F_g^{thr}, the controller switches to state 3. This state ends when the knee velocity, $\dot{\theta}_{knee}$ becomes negative. Finally, the controller starts back at state 1 when the foot strikes the ground in state 4, causing F_g to exceed its threshold.

Spruijt-Metz, & Sukhatme, 2010; Zihajehzadeh & Park, 2016). Specifically, the work of Bhakta et al. (2021) evaluates three algorithms for determining gait speed: linear regression (LR), extreme gradient boosting (XGBoost), and neural networks (NN) for both subject-independent and subject-dependent datasets. The results found that the machine learning algorithms performed competitively or better than analytical algorithms for both subject-independent and subject-dependent models.

Another advantage of leveraging machine learning classifiers is that they can be trained to detect multiple important control variables simultaneously. For example, Camargo, Flanagan et al. (2021) introduced a combined locomotion mode classifier and environmental parameter estimator to provide accurate information on a user's current ambulation state. Specifically, the classifier in Camargo, Flanagan et al. (2021) identified mode classification, walking speed estimation, ground slope estimation, and stair height estimation.

While these ML methods demonstrate promising success towards accurately estimating gait speed, they suffer from the same limitations as the ML classifiers discussed in Section 4.1; the classifiers do not easily generalize across users without either manual tuning or collecting additional training data.

5. High-level gait phase estimators

In addition to estimating locomotive tasks, the other component of high-level control is estimating gait phase. It is widely agreed-upon that quickly and accurately estimating gait phase during prosthetic locomotion is an important component of prosthesis control (Vu et al., 2020). Typically, a full gait cycle is defined as the periodic cycle starting from the impact of one foot on the ground to the following occurrence of the same impact event for the same foot. Note that gait phase can be estimated in either a discrete or continuous manner.

5.1. Finite state machines for gait phase

In biomechanics, the human gait cycle is divided into different phases (Winter, 2009). Lower-limb prosthesis controllers are commonly modeled in a similar fashion using FSMs, as illustrated in Fig. 3, whereby each state represents a phase of the gait cycle (Au, Berniker, & Herr, 2008; Eilenberg et al., 2010; Lawson, Varol, Huff, Erdemir, & Goldfarb, 2013; Sup et al., 2008). State transitions are then determined using a fixed set of rules or *transition conditions* including pre-specified thresholds of ground reaction forces and joint configurations. Within each state, specific control laws are implemented. Most commonly, impedance control laws are used as discussed in Section 6. Due to their simplicity and flexibility, FSMs are widely used in powered prosthesis control methods and are also used in commercial microprocessor-controlled knee prostheses (passive) (Fleming et al., 2021; Fluit, Prinsen, Wang, & Kooij, 2019).

Separate gait phase FSMs can be constructed for different locomotive modes (Simon et al., 2014), and a supervisory FSM, as described in 4.1, can switch between these modes. For example, Culver et al. (2018) constructed gait phase FSMs for stair ascent, stair descent, standing, and level-ground walking modes and a supervisory FSM dictated which gait phase FSM to use. An alternative approach to realize multiple locomotive modes with an FSM involves using multiple transition conditions for a single gait phase FSM state, such that the controller can execute different sequences of states to realize modes. For example, Culver, Vailati, and Goldfarb (2022) used a 6-state FSM, with multiple transition conditions between states to realize 8 different activities of daily living (ADLs) without explicitly identifying the ADL.

FSMs are simple to develop, flexible in their application, and the transition rules are intuitive to tune given the threshold parameters are typically based on ground reaction forces and joint angles. However, realizing multiple locomotive modes often requires more FSMs or more complex FSMs. This increases the number of parameters to tune and some of these parameter need to retuned for each user. Even after tuning, FSMs have robustness issues in performance (Thatte & Geyer, 2022). For example, in Thatte, Shah et al. (2019), when the user encountered unexpected ground height disturbances, the user's resultant abnormal kinematics caused the FSM to skip a state. This led to large, sudden torque changes, which sometimes resulted in a fall.

5.2. Continuous phase variables

To avoid the tuning and torque discontinuities that can accompany discretizing a gait cycle, phase variables have been used to develop unified and continuous control methods (Gregg, Lenzi et al., 2014; Holgate, Sugar, & Bohler, 2009; Sinnet, Zhao, & Ames, 2011). A phase variable approach provides a continuous mechanical representation of the gait cycle. Bipedal robotic control methods use state-based phase variables to parameterize desired kinematic trajectories for a robot in a time-invariant way, such that the motion being enforced on the robot is dictated by the state of the robot (Westervelt, Grizzle, Chevallereau, Choi, & Morris, 2018; Westervelt, Grizzle, & Koditschek, 2003).

Any measurable quantity of a walker's motion that is monotonic during a gait cycle can be used for the phase variable. One example, as shown in Fig. 4, is the linearized horizontal forward progression of the stance hip, which continuously increases during the stance phase of a gait cycle (Jiang et al., 2012). This quantity demonstrates how a phase variable can be a function of multiple joint coordinates. By defining $\boldsymbol{\theta} \in \mathbb{R}^n$ to be the vector of the n prosthesis configuration coordinates measured by kinematic sensors, a phase variable $\phi_\theta : \mathbb{R}^n \to \mathbb{R}$ is generally defined by,

$$\phi_\theta(\boldsymbol{\theta}) = \frac{p(\boldsymbol{\theta}) - p_0}{p_f - p_0}. \quad (1)$$

Here, the state-based monotonic function $p : \mathbb{R}^n \to \mathbb{R}$ is parameterized by its initial and final values ($p_0, p_f \in \mathbb{R}$), such that $\phi_\theta(\boldsymbol{\theta})$ goes from 0 to 1 over the gait cycle.

One consideration in selecting a phase variable for a prosthesis is finding a quantity that the human has some volitional control over, such that the speed of gait progression will be continuously modulated by the human body's progression. These options can vary with the level of amputation to ensure the selected variable is both affected by the human's motion and measurable by the prosthetic device. In the

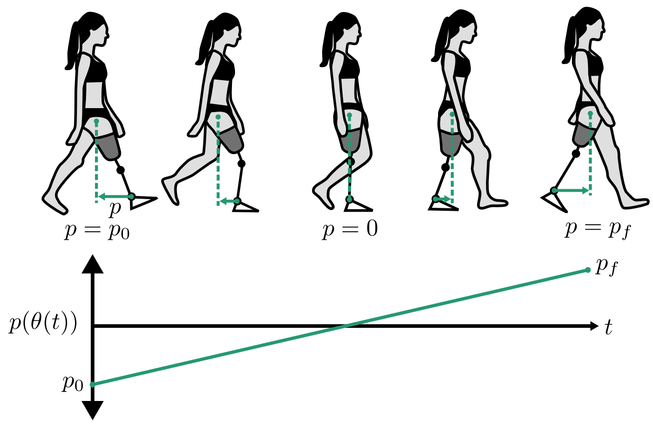

Fig. 4. Progression of monotonic phase variable. The horizontal forward progression of the stance hip relative to the ankle is an example of a physical quantity that is monotonic in a gait cycle, as explained in Jiang, Partrick, Zhao, and Ames (2012), allowing it to be used as a phase variable in a prosthesis stance controller. The initial and final positions of the hip provide the parameters p_0 and p_f, respectively, to parameterize the phase variable such that it goes from 0 to 1 during stance phase.

first instance of phase-based prosthesis control, Holgate et al. (2009) used the global tibia angle as the phase variable for a powered ankle prosthesis. This quantity is piece-wise monotonic in a gait cycle and can be measured by a sensor on an ankle prosthesis. However, when the foot of a knee–ankle prosthesis is flat on the ground, the prosthesis fully controls the tibia angle, removing the human's volitional control. Alternative approaches towards estimating gait phase when the prosthesis is in stance include using the linearized forward hip progression (Sinnet et al., 2011; Zhao, Horn et al., 2017), the CoP trajectory (Gregg, Lenzi et al., 2014), and multiple joint angle and velocity measurements fused in an extended Kalman filter (Thatte, Shah et al., 2019).

While these methods achieved human-prosthesis walking on a knee–ankle prosthesis with phase-based control in stance, the selected phase variables cannot be used to realize phase-based control in swing since they require sensing on the human stance leg. Moreover, while it is possible to place IMUs on the intact limb in research settings (Zhao, Horn et al., 2016), wearing these IMUs in daily life would likely be inconvenient. To avoid cumbersome sensors on the human, Villarreal and Gregg (2016) and Villarreal, Poonawala, and Gregg (2017) examined potential phase variable candidates and determined that the polar coordinate of the hip's phase portrait provides a robust parameterization of the human gait cycle for both stance and swing phase. This phase variable proved beneficial in rhythmic tasks (i.e. walking) (Bartlett, King, Goldfarb, & Lawson, 2022; Quintero, Villarreal, Lambert, Kapp and Gregg, 2018), but its requirement of a well-defined thigh orbit inhibits its application to non-rhythmic tasks (i.e. kicking a ball). With the global residual thigh angle shown to be piece-wise monotonic (Villarreal & Gregg, 2014), both rhythmic and non-rhythmic tasks were realized (Best et al., 2021; Quintero, Villarreal et al., 2018). Best et al. (2021) showed how this phase variable could realize walking at different inclines by online automatic updating of phase parameters based on the user's behavior.

Phase-based control approaches pose an advantage by providing some volitional control to the user through the use of a mechanical sensors while avoiding the challenges posed by EMG sensors, as discussed in 3.3. Phase-based strategies also naturally adapt to changes in walking speed. Demonstrated benefits of phase-based control for multi-joint prostheses include coordinating ankle and knee motion, realizing walking patterns that resembled healthy human patterns, and enabling amputees to walk at different speeds and inclines with a single controller (Best et al., 2021; Gregg, Lenzi et al., 2014; Quintero, Villarreal et al., 2018; Rezazadeh et al., 2019). Ideally, the phase variable will be invariant across subjects (Villarreal et al., 2017), such that the control strategy will generalize across subjects. Compared to impedance and neuromuscular methods that require tuning of a large number of parameters, these approaches can reduce the number of tuning parameters and depend on clinically intuitive parameters (Bartlett et al., 2022). The strategy of Gregg, Lenzi et al. (2014), for example, only required five parameters to be tuned.

One outstanding challenge with phase-based methods for prostheses is the uncertainty regarding whether the prosthesis should be fully dictated by a state-based phase variable or if a time-dependent term should be included. For example, when using the global thigh angle as a phase variable, Rezazadeh et al. (2019) observed a slow ankle push-off and Best et al. (2021) observed a pause in the kinematic reference trajectories. This issue was mitigated by introducing a time-based component in the controller (Best, Welker, Rouse, & Gregg, 2023). The approaches in Zhao, Horn et al. (2016, 2017) required a velocity modulating output in one phase which also introduced a time-based component into the phase-based controller. However, introducing a time-based component removes some volitonal control from the human. In summary, it remains unclear which choice of phase variable is best in terms of the following considerations: if the phase variable can be detected in both stance and swing; if the phase variable provides human volitional control; and if the phase variable progresses forward at a natural rate in the closed-loop human-prosthesis system.

5.3. Machine learning for gait phase

Another method of estimating gait phase is to leverage machine learning models. As with machine learning methods for estimating gait speed, these algorithms can provide either discrete estimates of gait cycle events or continuous estimates throughout the gait cycle. Some examples of machine learning algorithms for estimating gait phase include: neural network classifiers to estimate gait phase discretized to 1% intervals using offline IMU data (Vu et al., 2018); using offline data with hidden Markov models (HMMs) to detect 4–6 gait cycle events (Bae & Tomizuka, 2011; Evans & Arvind, 2014; Mannini, Genovese, & Sabatini, 2013; Sánchez Manchola, Bernal, Munera, & Cifuentes, 2019; Taborri, Scalona, Palermo, Rossi, & Cappa, 2015); neural network models to filter raw measurements and provide an HMM with classifications (Zhao et al., 2019); neural networks to classify stance and swing phases based on EMG data (Nazmi, Rahman, Yamamoto, & Ahmad, 2019); and estimation of foot strike and toe off events using deep learning with neural networks (Gadaleta et al., 2019; Lempereur et al., 2020).

It is known that the choice of sensor used with the machine learning algorithm heavily influences the classification accuracy. A review of machine learning methods for gait phase detection found that after comparing several available wearable sensors for gait phase detection algorithms, foot switches and foot pressure insoles yielded the highest accuracy (Taborri, Palermo, Rossi, & Cappa, 2016). However, due to the sensitivity of these sensors to wear and placement, they are not considered suitable for daily activity applications. In comparison, IMUs are more favorable since they provide rich information about the walking cycle and are low-cost, low-energy, and durable. However, since they can be sensitive to movement artifacts, these sensors still require pre-processing. This pre-processing step can be accomplished within the learning framework (Vu et al., 2018).

6. Mid-level control via torque computation without reference trajectories

After estimating the behavior and gait phase, the mid-level control then computes a desired torque for the prosthesis. We separate this mid-level task into two categories: controllers that compute desired torque using reference trajectories and those that do not. In this section, we will survey various mid-level control strategies that do not utilize a reference trajectory.

6.1. Impedance control

Impedance control was one of the first mid-level control techniques implemented on powered prostheses (Sup et al., 2008), and continues to be one of the most popular methods because of its simplicity and ability to replicate natural walking. The motivation behind why impedance control produces natural walking is that human joints have been shown to behave like variable impedance controllers (Hogan, 1984a). However, this technique requires careful tuning of various parameters per subject. Specifically, an impedance control law for the torque τ of a single joint is as follows:

$$\tau(\theta, \dot{\theta}) = -k(\theta - \theta_e) - b\dot{\theta}, \tag{2}$$

where θ denotes joint position, $\dot{\theta}$ denotes joint velocity, k is the positive stiffness coefficient, θ_e is the equilibrium point (reference set-point), and b is the positive damping coefficient. Simply put, this control law relates position and velocity to torque, thus regulating joint torque by the state of the system and imposing a parallel spring-damper behavior on the joint (Hogan, 1984b).

Impedance control strategies are typically paired with an FSM, as described in 5.1, where impedance control laws with constant parameters are defined for each discrete phase within the FSM and thus enforce passive dynamics within a state. These parameters differ between states of an FSM, which in part replicates the variable impedance observed in human joints and allows for the injection or removal of net energy. This creates a piecewise control law for the gait cycle. Recent works have defined impedance control laws with parameters that vary within a state (Best et al., 2023; Fey, Simon, Young, & Hargrove, 2014; Hong, Paredes, Chao, Patrick, & Hur, 2019). This approach does not enforce passive dynamics and may be able to better capture the variable impedance behavior of human limbs (Lee & Hogan, 2014; Rouse, Hargrove, Perreault and Kuiken, 2014; Shorter & Rouse, 2018).

Impedance control is also capable of accomplishing behaviors other than walking through the adjustment of coefficients. For example, Lawson et al. (2013) realized the first instance of powered transfemoral prosthesis stair ascent and descent. Notably, this control strategy achieved knee and ankle kinematic profiles that matched healthy human profiles more closely than that of a passive knee–ankle prosthesis in stair ascent, and matched ankle profiles more closely in stair descent. Other achievements of impedance control within an FSM include the first powered knee–ankle prosthesis running (Shultz, Lawson, & Goldfarb, 2015), upslope walking (Sup et al., 2011), and variable cadence bilateral amputee walking (Lawson, Ruhe, Shultz, & Goldfarb, 2015).

As mentioned, the primary disadvantage of impedance control is the need to tune the coefficients within each FSM state for each prosthesis user and locomotion mode. Thus, there exist several methods in the literature for how to select these parameters. These user-customization methods are discussed further in Section 8.

6.1.1. Quasi-stiffness control

A subset of impedance control strategies is quasi-stiffness control. This approach determines the impedance parameters used to define the state-torque relationship through matching the relationship between a human's joint torque τ_h and angle θ_h. The slope of this relationship $\frac{d\tau_h}{d\theta_h}$ is called "quasi-stiffness" (Latash & Zatsiorsky, 1993; Rouse et al., 2013). The desired prosthesis torque τ is then determined by,

$$\tau(\theta) = \frac{d\tau_h}{d\theta_h}(\theta)(\theta - \theta_e), \tag{3}$$

where the quasi-stiffness, $\frac{d\tau_h}{d\theta_h} : \mathbb{R} \to \mathbb{R}$, is a function of the joint angle θ. While quasi-stiffness and stiffness are equivalent in passive prostheses that only contain a spring, these concepts are distinct in powered prosthesis control (Rouse et al., 2013). It is interesting to note that the torque–angle relationship in human ankles is approximately linear for most of stance at normal walking speeds, and this relationship changes as walking speed changes (Hansen, Childress, Miff, Gard and Mesplay, 2004).

The torque–angle relationships prescribed to prosthesis joints are often modified for changing conditions to emulate these biological trends. For example, Lenzi et al. (2014) created a look up table of quasi-stiffness profiles, obtained from able-bodied walking experiments, for different walking speeds. By inputting current joint angle and walking speed, the table output a desired joint torque, normalized by the body mass of the user, for both a knee and an ankle joint for each phase of walking. For stair-climbing, Hood et al. (2022) modified prosthesis torque–angle relationships based on the estimated stair height; this resulted in increased torque with taller stair heights, imitating this biomechanical relationship observed in humans (Riener et al., 2002).

Quasi-stiffness methods have been shown to improve temporal symmetry between the prosthesis and intact leg and achieve physiological gait energetics over a wide range of walking speeds without subject-specific or speed-specific tuning (Lenzi et al., 2014). Enforcing quasi-stiffness profiles allows the prosthesis to modulate its mechanical work independent of the actual joint velocity.

6.2. Torque-based controllers

Instead of aiming to emulate the human kinetics through an impedance model, some approaches directly command torque based on a predetermined desired torque profile. There are two main methods for obtaining reference torque profiles: using human joint torque data or handcrafting a profile. These profiles are generally parameterized by a progression variable $\phi_{\{\theta,t\}}$, which can be state-based, i.e. $\phi_\theta = \phi_\theta(\theta)$, or time-based, i.e. $\phi_t = t$. Simply, this means the commanded torque τ is equal to the reference torque profile $\tau_r : \mathbb{R} \to \mathbb{R}$ evaluated at either ϕ_θ or ϕ_t:

$$\tau = \tau_r(\phi_{\{\theta,t\}}). \tag{4}$$

Sometimes, additional terms are added to this expression, such as a PD controller on a kinematic reference trajectory, as done in Thatte, Shah et al. (2019).

The first approach can be used to generate desired torque profiles in a task-dependent way, such as the control surfaces developed with respect to phase velocity and gait phase in Thatte, Shah et al. (2019). The level of assistance can also be altered by scaling the desired peak torque magnitude, as done in Lawson, Ledoux, and Goldfarb (2017). This approach allows desired torque to be determined directly from human data instead of depending on a large set of tuned parameters, and could be conducted in a subject- and task-specific way with appropriate datasets. However, replaying human torque profiles on a prosthesis may yield different performance results on various devices given their differing physical parameters, such as mass and inertia, which would influence the device dynamics.

The second approach typically handcrafts desired torque profiles based on a prespecified function form and tunable shape parameters. While this method of control is not as common in the field of lower-limb prostheses, it is widely prevalent in the field of lower-limb exoskeletons (Baud, Manzoori, Ijspeert, & Bouri, 2021). One example on lower-limb prostheses is that of Welker et al. (2021) wherein a time-based torque profile was constructed for an ankle prosthesis emulator using four parameters: peak time, rise time, fall time, and peak magnitude. The benefit of such a parameterized profile is that the behavior of the device can be adjusted via the control parameters for individual prosthesis users using a human-in-the-loop optimization problem. However, the results of Welker et al. (2021) found that despite the user-specific customization (which has been shown to reduce metabolic cost for lower-limb exoskeleton users Zhang et al., 2017), the controller failed to significantly reduce metabolic expenditure.

6.3. Neuromuscular model-based control

Thus far in this section, the control approaches focused on replicating recorded torque profiles through an impedance or gait phase parameterized model. These methods depend on human data for the specific scenario at hand, meaning they require different sets of data to achieve various locomotive tasks. Neuromuscular model-based control methods instead aim to model the human's underlying neuromuscular system that produces these torques in various scenarios. If a human's high-level objectives are encoded in their neuromuscular system, controlling a prosthesis in a similar manner may provide a single, unified control method that leads to more natural and robust walking in a variety of environments.

As described in Thatte and Geyer (2015), joint torque produced by muscle forces (modeled using a Hill-type muscle tendon unit (MTU) van Leeuwen, 1992) can be calculated using the formula:

$$\tau(\theta) = r^m(\theta)F^m(S^m(t), l^m_{\text{mtu}}, l^m_{\text{ce}}), \tag{5}$$

where $r^m : \mathbb{R} \to \mathbb{R}$ is the moment arm of muscle m around the joint depending on the joint angle $\theta \in \mathbb{R}$. Additionally, $F^m : \mathbb{R}\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}$ is the force produced by the MTU that depends on the time-varying muscle stimulation $S^m : \mathbb{R}^+ \to \mathbb{R}$, MTU overall length l^m_{mtu}, and MTU contractile element length l^m_{ce}. These MTU lengths l^m_{mtu} and l^m_{ce} are computed using several muscle-specific reference parameters such as the ones explicitly listed in Geyer and Herr (2010). The stimulation value is computed using various muscle-specific parameters:

$$S^m(t) = S^m_0 + \sum_n G^m_n P^m_n(t - \Delta t^m_n), \tag{6}$$

where S^m_0 is a constant pre-stimulation parameter, G^m_n is the gain on the time-delayed length or force signal $P^m_n : \mathbb{R} \to \mathbb{R}$ from muscle n acting on muscle m, and Δt^m_n is the time delay constant for muscle n acting on muscle m.

These neuromuscular model parameters can be determined through optimization; for example, Eilenberg et al. (2010) optimized the parameters of one MTU to fit the ankle torque–angle profile to that measured from a weight- and height-matched able-bodied subject. For multi-joint control, it is also possible to optimize parameters of multiple MTUs simultaneously; for example, Thatte and Geyer (2015) optimized the parameters of 7 MTUs to minimize the cost of transport and maximize the distance traveled for a human-prosthesis model.

With the resultant parameter sets, these control strategies have led to adaptive behavior on prostheses when confronted with unexpected slope changes and trips. These results suggest this neuromuscular model-based approach captures some balance recovery techniques that humans exhibit, making it more robust to variations in terrain and disturbances without explicit detection or change in control policy. However, one potential drawback of using neuromuscular models is the large number of parameters that require tuning. Moreover, these parameters are typically challenging to tune because they do not directly relate to position, speed, or some other physical quantity.

6.4. Myoelectric control

Another neuromuscular-inspired approach is *myoelectric* control which, instead of trying to emulate the neuromuscular function of biological limbs through a model, leverages neuromuscular control signals recorded directly from muscles. These signals are typically measured by EMG sensors located inside the socket on the residual limb as mentioned in Section 3. The advantage of utilizing these signals in prosthesis control is that they provide real-time insight into the desired motion of the human user. For example, Wang et al. (2013) introduced a volitional EMG controller to modulate powered ankle plantar flexion by proportionally increasing the plantar flexion ankle torque based on EMG activity of the calf. The specific control law used in this work was of the form:

$$\tau(\theta) = K_{\text{emg}}s_{\text{emg}} + K_0(\theta - \theta_0), \tag{7}$$

where K_{emg} is a gain term that is adjusted based on the walking velocity and s_{emg} is the muscle activation captured by the EMG signal. Note that K_0 is an initial stiffness term based on an equilibrium joint angle θ_0 and actual joint angle θ that is added to ensure a biomimetic plantarflexion toe-off angle. Enabling volitional ankle control through this type of proportional EMG control approach has been shown to lead to more natural gait compared to walking with a passive prosthesis (Huang et al., 2014).

Aside from enabling volitional control during walking, prosthesis controllers that utilize EMG signals are advantageous in that they enable non-cyclic tasks and adaptation to different environments. For example, the proportional EMG knee controller in Hunt et al. (2021) allowed users to adapt the prosthesis movements to other motions including sit-to-stand, squatting, lunging, and smooth motion transitions without explicitly classifying these movements. Lastly, it is interesting to note that compared to passive prostheses, using an active prosthesis with myoelectric control also improved the users' weight bearing symmetry and decreased their associated muscle effort (Hunt et al., 2021).

In summary, the advantage of myoelectric control is that by interfacing directly with the prosthesis user, EMG sensing enables versatile prosthesis control that is able to adapt to various situations (Fleming et al., 2021). Despite this clear advantage, there are several limitations inherent to EMG sensing, as discussed in Section 3. Also, in the case of transfemoral amputation, the muscles used for controlling the ankle joint are no longer available for EMG sensing.

6.4.1. Neural engineering of human physiology

A possible solution to handle the lack of ankle musculature in transfemoral amputation is to engineer human physiology to recover amputated neural pathways using techniques such as targeted motor reinnervation (TMR) surgery. This surgery transfers residual nerves to alternative muscle sites during scheduled amputation procedures. To date, this technique has been predominantly applied towards upper-limb control of prosthetic arms (Hijjawi et al., 2006; Kuiken, Dumanian, Lipschutz, Miller, & Stubblefield, 2004; Kuiken et al., 2009) but there are few studies that demonstrate the potential of TMR towards lower-limb prosthetic control. In particular, Hargrove et al. (2013) demonstrated improved dual-actuated prosthesis control through the use of reinnervated residual thigh muscles. The results found the classification accuracy of prosthesis user's attempted movements was higher for TMR amputees compared to non-TMR amputees, and that TMR amputees also completed virtual movements much faster.

The second example of reengineering neural pathways is an agonist–antagonist myoneural interface (AMI) (Clites et al., 2018). This technique surgically connects two muscle–tendons in series such that the contraction of one muscle stretches the other. The purpose of this surgical connection is to provide proprioceptive information to the human subject. Additionally, Clites et al. (2018) developed a bidirectional efferent–afferent neural control architecture by affixing bipolar surface electrodes adjacent to the two muscles comprising each AMI. This approach demonstrated improved volitional control of a 2DOF ankle–foot prosthesis. Additionally, the subject reported that the prosthesis aligned more closely with their perceived motion of their phantom limb. However, it should be noted that as with most forms of myoelectric control, this control method necessitates extensive subject-specific tuning that needs to be repeated each time electrodes are placed. The parameters that require tuning include activation threshold for each muscle, relative torque-producing capacity of each virtual muscle, and the minimum co-activation for increasing joint impedance.

The last example of neural engineering is the restoration of sensory feedback for lower-limb amputees. For example, researchers have demonstrated how residual neural pathways can be stimulated using implanted intraneural stimulation electrodes for the purpose of reducing phantom limb pain Boretius et al. (2010). A case study on two transfemoral amputees found that the neural stimulation not only reduced

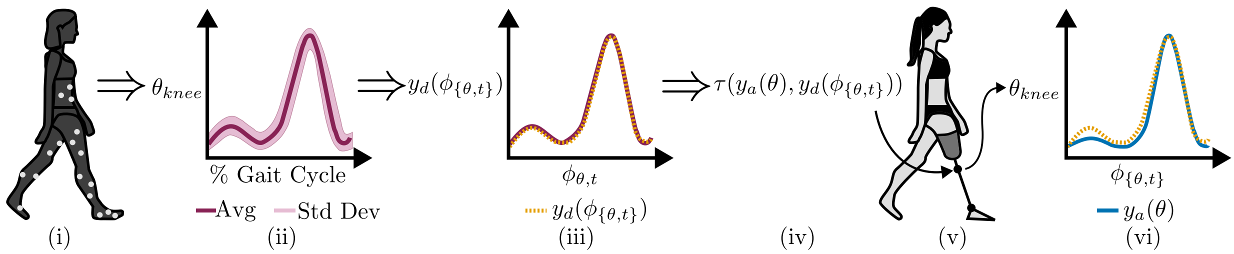

Fig. 5. Common process for mid-level control via kinematic reference trajectories. (i) Motion capture data is collected from able-bodied subject walking. (ii) The average and standard deviation is computed of each joint trajectory, such as the trajectory of the knee angle, θ_{knee}, with respect to the percent gait cycle. (iii) A function, $y_d(\phi_{\{\theta,t\}})$, is fit to the average data, parameterized by some state- or time-based phase variable $\phi_{\{\theta,t\}}$. (iv) A torque, τ, is computed based on this desired trajectory and the actual trajectory, $y_a(\theta)$, of the prosthesis. (v) This torque is commanded of the prosthesis actuators. (vi) The resultant joint trajectory of the prosthesis, $y_a(\theta)$, is close to the desired trajectory, $y_d(\phi_{\{\theta,t\}})$. Steps (i)–(iii) are part of data-driven approaches to generate reference trajectories, described in Section 7.1, and steps (iv)–(vi) are part of tracking controller methods, described in Section 7.2.

phantom limb pain but also increased walking speed and self-reported confidence while decreasing mental and physical fatigue (Petrini et al., 2019). These results suggest that sensory restoration is a promising novel technology for improving prosthesis walking.

7. Mid-level control via kinematic reference trajectory tracking

Instead of indirectly aiming to realize human kinematic trajectories through shaping a torque model based on the system states, phase, and task, other mid-level control strategies design a torque law to track a specific reference trajectory. These approaches first generate a desired kinematic reference trajectory for the prosthesis joints. The control objective is then to align the actual trajectory of the system with the desired trajectory. These desired responses of the system are often referred to as *outputs*, or virtual constraints (Westervelt et al., 2018), which we denote with y and can be represented by the following,

$$y(\theta) = y_a(\theta) - y_d(\phi_{\{\theta,t\}}), \tag{8}$$

where $y_a : \mathbb{R} \to \mathbb{R}$ is the actual measured output of the system, $y_d : \mathbb{R} \to \mathbb{R}$ is the desired trajectory that is a function of the progression variable $\phi_{\{\theta,t\}}$. While outputs can be functions of multiple joints (Gregg, Rouse, Hargrove and Sensinger, 2014; Zhao, Horn et al., 2017), generally for prosthesis control, outputs are prescribed for individual joints. For simplicity, we write the outputs in this manner, as a function of an individual joint, i.e. $y_a(\theta)$. Based on these outputs, a torque is computed to track these trajectories, directly prescribing a desired kinematic motion for the given behavior and phase. A common approach to mid-level control with reference tracking is illustrated in Fig. 5.

7.1. Reference trajectory generators

This subsection will discuss the various methods used to generate prosthesis reference trajectories.

7.1.1. Minimum jerk

Flash and Hogan (1985) developed a minimum jerk control method that generates trajectories optimized to minimize the rate of change of acceleration, or jerk, and found these patterns to resemble human motion. The total jerk, $\mathcal{J}$, in a desired trajectory $y_d(t)$ from time t_0 to t_f can be represented by,

$$\mathcal{J} = \int_{t_0}^{t_f} \dddot{y}_d(t)^2 dt. \tag{9}$$

Modifying the desired trajectory y_d to minimize this cost function provides a way to maximize smoothness of a trajectory over the given time span.

Minimum jerk provides a means to generate a smooth trajectory between two points given desired initial and final positions and a desired time duration. By selecting a time duration proportional to the duration of the previous stance phase, an adaptive response to the human's behavior can be encoded (Lenzi et al., 2014). Additional constraints can be included, such as maximum knee flexion angle to dictate foot clearance and desired acceleration at specific points such that these can be optimized based on able-bodied data (Lenzi et al., 2014).

Minimum jerk trajectory generators can also be used to continuously optimize a trajectory for a changing desired final position (Mendez et al., 2020) or simply be included as a component in other more complex trajectory generation methods. For example, Embry, Villarreal, Macaluso, and Gregg (2018) minimized jerk when developing continuous functions that model human kinematics to improve the smoothness of the functions. (This method will be discussed further in 7.1.4.)

Overall, minimum jerk strategies pose a benefit in their ability to yield smooth trajectories. Additionally, being able to prescribe positions and time duration in this approach addresses the limitation of impedance control methods with regards to speed adaptation; namely, impedance control parameters do not linearly relate to walking speed, forcing amputees to rely on compensatory strategies to walk at different speeds. Specifying a desired swing duration with minimum jerk control has also been shown to improve temporal symmetry between a prosthesis leg and the user's intact leg (Lenzi et al., 2014). However, minimum jerk control alone does not explicitly specify target angles and duration, which are features that affect the optimality of the determined trajectory with respect to other metrics.

7.1.2. Heuristic algorithms

To modify desired reference angles, heuristic algorithms have been developed with handcrafted function forms and hand-selected parameters that define the function. These functions can be based on current angles, velocities, and accelerations of the prosthesis joints and residual thigh (Hood et al., 2022; Hunt et al., 2021; Mendez et al., 2020). These algorithms aim to emulate the biomechanical relationships between joints observed in human locomotion.

Additionally, heuristic algorithms have been used to allow for improved volition with minimum jerk swing control by continuously optimizing the swing trajectory for updated desired maximum knee flexion angles based on the user's residual thigh movement (Hood et al., 2022; Mendez et al., 2020). Notably, the approach of Mendez et al. (2020) did not require user-specific tuning and allowed users to volitionally modulate foot clearance to smoothly traverse over obstacles without explicit classification of the environment obstacle. Similarly, the approach of Hood et al. (2022) allowed the swing trajectory to vary between stair heights and gait patterns (step-by-step, step-over-step, two-step), providing sufficient foot clearance and proper foot placement for tested stair heights and climbing patterns without explicit classification of the environment and gait pattern, and allowed the user to change their cadence when climbing different stair heights or using different climbing patterns.

Although these methods do not use a phase variable, they still are able to continuously modulate the prosthesis behavior based on the current behavior of the prosthesis-side limb, as with phase-based control approaches. These heuristic algorithms provide some volitional control to the human in a way that generalizes between subjects and does not pose the challenges EMG control does, as described in 6.4. One limitation of these methods is their heuristic nature. Specifically, it is unclear how to systematically develop these algorithms to realize optimal behavior across various devices or locomotion modes.

7.1.3. Geometric control

An interesting observation of human walking is that the CoP trajectory throughout a gait cycle is invariant across different subject weights (Hansen & Childress, 2010) and gait speeds (Hansen, Childress and Knox, 2004). This can be leveraged to encode the effective shape into prosthesis output functions (Gregg, Rouse et al., 2014). Then, through control, the distance from the CoP to the center of rotation of the knee and thigh can be constrained to a constant radius of curvature. An advantage of this continuous control approach for stance is that the effective radii and rotation centers (parameters used to determine the effective shape) are defined with respect to the user's height, and therefore do not require tuning. Specifically, in Gregg, Rouse et al. (2014) only five gains required hand-tuning, which is significantly less than that of impedance and neuromuscular model approaches. This reduction in tuning is in part due to the approach's elimination of control switches. Additionally, a good initial guess for these gains could be determined by normalizing them with a subject's body mass. However, the assumptions made for this example of geometric control introduce some limitations. First, the effective shape was modeled with constant curvature, while in human walking the curvature is typically non-constant. Additionally, the foot contact was modeled as holonomic, while in reality it is nonholonomic due to foot rolling. A final limitation is that the prosthesis can only measure and enforce effective shape during the stance phase, thus necessitating a different controller for the swing phase.

7.1.4. Data-driven kinematics

An alternative approach to encode human-like motion into prostheses is to design desired trajectories that match human joint kinematic trajectories, typically parameterized by a state-based phase variable. This provides a direct and continuous means to replicate human motion on prosthesis joints. One example of how to construct these output functions is by creating linear transformations of able-bodied data with discrete Fourier transforms, parameterized by the phase of the prosthesis-side hip (Quintero, Villarreal et al., 2018). Using this approach, different types of locomotive modes can be enabled by developing different sets of desired trajectories for various level-ground walking speeds and walking inclines. With only minimal tuning, this methodology has been shown to yield relationships between ankle work and walking speeds that are closer to that of biomechanical trends (Quintero, Villarreal et al., 2018).

An alternative approach to using discrete sets of outputs for different speeds and inclines is to construct continuous models of human kinematic data. For example, Embry et al. (2018) developed a kinematic predictive model $y_d^{\text{kin}} : \mathbb{R}\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}$ with continuous functions of gait phase $\phi_{\{\theta,t\}}$, a subject's walking speed v, and ground incline angle β. These functions were defined by a summation of N basis functions $B_i : \mathbb{R} \to \mathbb{R}$, weighted by task functions $C_i : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$,

$$y_d^{\text{kin}}(\phi_{\{\theta,t\}}, v, \beta) = \sum_{i=1}^{N} B_i(\phi_{\{\theta,t\}}) C_i(v, \beta) \approx \theta_h(\phi_{\{\theta,t\}}, v, \beta). \tag{10}$$

In this work, the basis model functions were generated using a convex optimization designed to make the basis model fit human kinematic data $\theta_h : \mathbb{R}\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}$ while minimizing jerk and constraining range of motion. Later work also used this model as a reference trajectory generator of a knee–ankle prosthesis, specifying ground incline, speed, and phase estimates (Best et al., 2021).

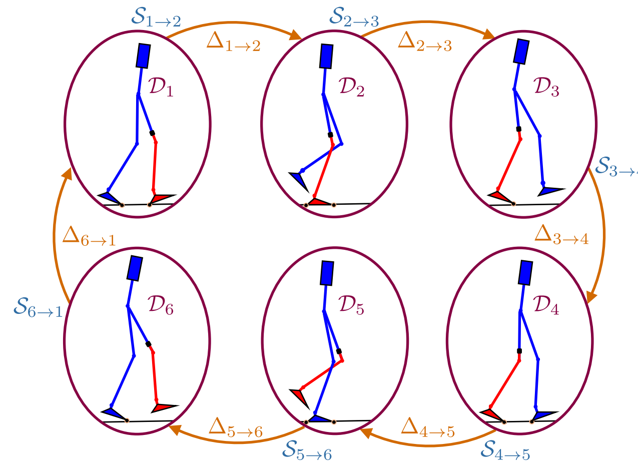

Fig. 6. Hybrid domain cycle. Multi-domain structure to model the changing contact points of the heel–toe roll motion occurring in human walking. The domains $\{\mathcal{D}_i\}_{i=1\cdots6}$ end when they reach their respective guard $\{\mathcal{S}_{i\to i^+}\}_{i=1\cdots6}$, where i^+ signifies the next index in the cycle. The impact map $\{\Delta_{i\to i^+}\}_{i=1\cdots6}$ maps the dynamics to the next domain. The blue portion of the biped represents an amputee and the red portion represents a prosthesis. The dots on the feet represent the ground contact points present in the respective domain. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

A second continuous model approach uses sparse Gaussian process (GP) regression models to form control surfaces of joint angles, velocities, and torques with respect to gait phase based on able-bodied data (Thatte, Shah et al., 2019). Given the phase variable and phase velocity as inputs, desired angles, velocities, and feedforward torques can be determined through these control surfaces.

Overall, these biologically-inspired approaches provide smooth kinematic models with respect to gait phase and task variables allowing prostheses to replicate human kinematic trajectories for varying conditions. However, while these kinematic trajectories may be what humans have deemed optimal for walking with their biological limbs, it is unclear whether matching kinematic trajectories of able-bodied individuals improves physiological outcomes for individuals with amputation. Additionally, it remains unknown how optimal trajectories for an artificial limb may vary for devices with differing physical properties.

7.1.5. Hybrid zero dynamics

An approach for synthesizing prosthesis reference trajectories that does account for the physical parameters of the device and satisfies some optimality criteria is the hybrid zero dynamics framework. This method, originally developed in the field of bipedal robotics (Ames, 2014; Westervelt et al., 2003), systematically generates human-like periodic walking motions with formal guarantees of stability. An overarching benefit of the HZD method is that it models the discrete jumps in velocities that occur at foot impact, captured by the discrete dynamics Δ, by modeling walking as a hybrid system consisting of both continuous domains $\mathcal{D}$ and discrete impact events $\mathcal{S}$. Furthermore, by generating trajectories that satisfy the following constraint on the underactuated dynamics $\mathcal{Z}$,

$$\Delta(\mathcal{S} \cap \mathcal{Z}) \subset \mathcal{Z}, \tag{11}$$

this framework certifies that the desired outputs remain impact-invariant, i.e. the impacts caused by foot strike do not cause instability in the outputs. Thus, by synthesizing a trajectory that is stable, both in the underactuated and actuated coordinates, stability of the full hybrid system is guaranteed.

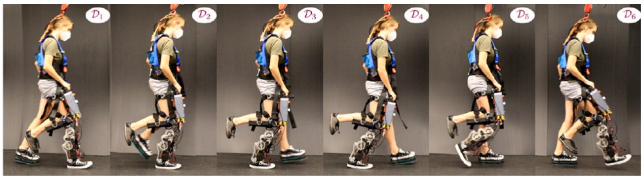

Fig. 7. Gait tiles of human-prosthesis multi-contact walking. Experimental results from Gehlhar and Ames (2023) showing the resultant multi-contact behavior obtained by developing trajectories with a multi-domain hybrid system. Labels correlate with domains in Fig. 6.

This approach has been demonstrated to realize stable bipedal robotic walking in various works (Ames, 2014; Ames, Galloway, Sreenath, & Grizzle, 2014; Ma, Kolathaya, Ambrose, Hubicki, & Ames, 2017; Reher, Hereid, Kolathaya, Hubicki, & Ames, 2020; Sreenath, Park, Poulakakis, & Grizzle, 2013). The work of Zhao, Horn et al. (2017) applied HZD in the context of prosthesis control, generating desired trajectories for the prosthesis system that satisfied these formal guarantees of stability for the human-prosthesis model. The HZD method has also been extended to multi-domain hybrid systems, as shown in Fig. 6, to model the changing contact points that occur in walking and realize human heel–toe roll behavior on a knee–ankle prosthesis (Gehlhar & Ames, 2023; Zhao, Horn et al., 2016), illustrated by the gait tiles in Fig. 7.

In general, the HZD method provides a systematic approach towards generating reference trajectories that resemble able-bodied kinematic trajectories while also satisfying formal guarantees of stability for the specific subject and prosthesis device under consideration. This model-based optimization framework naturally allows constraints related to human-likeness, comfort, and physical limitations to be incorporated in trajectory design. Additionally, these trajectories can be optimally chosen to reduce the mechanical cost of transport of the robotic human-prosthesis model. This strategy also generalizes to realize different behaviors, such as stair-climbing (Zhao, Reher, Horn, Paredes, & Ames, 2015). A challenge with this approach is the dependency on the human model—including the expected motion prescribed to the human in the optimization. The validity of the stability guarantees depends on this model. One other requirement to maintain these stability guarantees is a tracking controller with a sufficient convergence rate to ensure adherence to the stable periodic orbits. The next section will discuss different types of tracking controllers, including a control Lyapunov function approach which provides such a convergence certificate.

7.2. Tracking controllers

After a reference trajectory is determined for a particular task and gait phase, the controller will compute a torque to track these trajectories.

7.2.1. PID control with feedforward terms

A standard control method for trajectory tracking is proportional–integral–derivative (PID) control. This control law is defined by the positional errors, the integral of the positional errors, and the derivative of the positional errors (i.e. the velocity errors) and a tuned gain on each form of error to provide a corrective action to the system. For prosthesis reference trajectory tracking, a form of a PID controller is most commonly chosen, sometimes with the addition of a feedforward term $\tau_{\mathrm{ff}} : \mathbb{R} \to \mathbb{R}$:

$$\begin{aligned}\tau(\theta,\dot{\theta}) =& k_{\mathrm{P}}(y_a(\theta) - y_d(\phi_{\{\theta,t\}})) \\ &+ k_{\mathrm{I}} \int_0^t (y_a(\theta) - y_d(\phi_{\{\theta,t\}}))dt \\ &+ k_{\mathrm{D}}(\dot{y}_a(\theta,\dot{\theta}) - \dot{y}_d(\phi_{\{\theta,t\}})) + \tau_{\mathrm{ff}}(\phi_{\{\theta,t\}}).\end{aligned} \tag{12}$$

Here k_{P} is the proportional gain, k_{I} is integral gain, and k_{D} is the derivative gain.

While PID control is a notoriously simple control scheme, it has been applied in many different ways towards prosthesis control. For example, the integral term can be omitted, resulting in a PD controller which has been applied towards approximating a partial feedback linearizing controller to enforce effective shape outputs (Gregg, Lenzi et al., 2014) or towards tracking output trajectories (Quintero, Villarreal et al., 2018; Rezazadeh et al., 2019). Additionally, a damping term can be added to the actual velocity of the joints, as opposed to the velocity output tracking error, to create a smoother behavior (Quintero, Villarreal et al., 2018). Lastly, a PID controller was implemented in Best et al. (2021) to track the speed and incline-specific reference trajectories. This approach only required a dozen tunable parameters that were tuned once during level ground walking, and resulted in the prosthesis joint trajectories resembling human kinematic trends for continuously varying speeds and inclines.

A feedforward term τ_{ff} can also be added to a PD controller based on human data or the physical parameters of the prosthesis to reduce tracking error. For example, Thatte, Shah et al. (2019) added a feedforward torque based on the human torque control surface to a PD controller created with the desired joint angles and velocities from control surfaces formed with human kinematics. Additionally, Lenzi et al. (2014) added a feedforward term based on inertia, gravity, and friction to a PD controller tracking the generated minimum jerk swing trajectory.

Overall, PID control provides a simple way to realize a desired trajectory. There are typically fewer gains to tune than an FSM with multiple impedance laws and these gains can be used for multiple behaviors and subjects. While PID control achieves good tracking for the ballistic motion of prosthesis swing phases, tracking errors are generally present in stance phases. During stance phase, a prosthesis bears the weight of the user and a PID controller would require high gains to produce a large enough torque to compensate for the moment produced by the user's dynamic load. However, when the prosthesis is no longer supporting the human's weight in subsequent gait phases, these high gains could cause oscillations and aggressive motions. This motivates the need for more sophisticated controllers, such as control Lyapunov functions, which are discussed next.

7.2.2. Control Lyapunov functions

Model-based control methods used for bipedal robots, such as control Lyapunov functions (CLFs) (Ames et al., 2014), determine torque based on the full-order system dynamics, instead of relying only on tuned gains. CLFs also provide a sufficient convergence rate to guarantee stability to hybrid periodic orbits generated through HZD. A CLF, $V : \mathbb{R}^{2m} \to \mathbb{R}$, is a positive definite function that can be formed with the vector of m outputs $\boldsymbol{y} \in \mathbb{R}^m$ and their derivatives $\dot{\boldsymbol{y}} \in \mathbb{R}^m$. Using $\boldsymbol{\eta} = (\boldsymbol{y}^T, \dot{\boldsymbol{y}}^T)^T$, we have,

$$c_1\|\boldsymbol{\eta}\|^2 \le V(\boldsymbol{\eta}) \le c_2\|\boldsymbol{\eta}\|^2 \tag{13}$$

$$\dot{V}(\boldsymbol{\eta},\boldsymbol{\mu}) \le -c_3 V(\boldsymbol{\eta}), \tag{14}$$

where c_1, c_2, c_3 are positive constants. The derivative of the CLF $V(\boldsymbol{\eta})$ is a function of the output dynamics, which includes an auxiliary control input $\boldsymbol{\mu}$. Through this auxiliary control input the CLF derivative $\dot{V}(\boldsymbol{\eta},\boldsymbol{\mu})$ can be ensured to be less than the quantity $-c_3 V(\boldsymbol{\eta})$ to guarantee system stability. This auxiliary input is formed by,

$$\boldsymbol{\mu} = \dot{J}_y(\theta,\dot{\theta})\dot{\theta} + J_y(\theta)\ddot{\theta}, \tag{15}$$

where $\theta, \dot{\theta}, \ddot{\theta} \in \mathbb{R}^n$ respectively denote the positions, velocities, and accelerations of the system, $J_y : \mathbb{R}^n \to \mathbb{R}^{2\ m\times n}$ and $\dot{J}_y : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{2\ m\times n}$ are partial derivatives of the outputs $\boldsymbol{y}$. For details, see Gehlhar and Ames (2023). Here, $\boldsymbol{\mu}$ is a function of the accelerations of the system, $\ddot{\theta}$, determined through the prosthesis modeled dynamics, given by the Euler–Lagrange equation (Murray, Sastry, & Zexiang, 1994),

$$\ddot{\theta} = D^{-1}(\theta)(-H(\theta,\dot{\theta}) + J_c^T(\theta)\lambda_c + J_f^T(\theta)F_f + B\boldsymbol{\tau}). \tag{16}$$

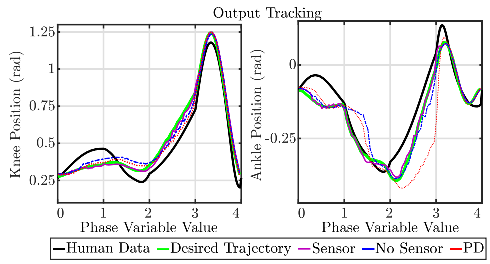

Fig. 8. Example of tracking results of different tracking controllers. A desired trajectory (green) was generated to match human motion capture data (black) and three different tracking controllers were tested in Gehlhar and Ames (2023). The "Sensor" tracking result (magenta) shows the performance of a model-based CLF-QP that uses real-time force sensing in the loop. Without the use of force sensors, the performance of the CLF-QP degrades (blue). For comparison, a PD controller (red) is tested which also has a larger tracking error than the force-sensing CLF-QP. The data is plotted with respect to the value of the phase variable. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Here, $D : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the inertia matrix; $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ contains the Coriolis, centrifugal, and gravity terms; $J_c : \mathbb{R}^n \to \mathbb{R}^{n_c \times n}$ is the partial derivative of the holonomic ground contact constraints that are enforced through the n_c constraint wrench forces and moments $\lambda_c \in \mathbb{R}^{n_c}$; $J_f : \mathbb{R}^n \to \mathbb{R}^{n_f \times n}$ projects the n_f interaction forces $F_f \in \mathbb{R}^{n_f}$ between the human and the prosthesis measured by a force sensor; and $B \in \mathbb{R}^{n\times m}$ is the actuation matrix that projects the torque vector $\boldsymbol{\tau} \in \mathbb{R}^m$. Through this chain of equations, one can see the CLF stability condition (14) is a function of the control input torque vector $\boldsymbol{\tau}$. Often, this CLF stability condition is placed as a constraint in a quadratic program (QP). The QP then determines torques that satisfy this constraint and minimize a given cost.

These CLFs rely on a model of the full-order system, and hence cannot be directly synthesized for prostheses since the human dynamics are unknown. One way to handle this lack of model information is to construct a model-independent CLF based solely on the outputs (Zhao, Horn et al., 2017). However, this approach does not gain the benefits of model-based control. Alternatively, CLFs can be constructed solely based on the prosthesis dynamics when the interaction force between the human and prosthesis and the rotations and velocities at this interface are known. In practice, these quantities can be measured with a load cell and IMU at the socket interface. Constructions for a prosthesis CLF were first developed with a feedback linearizing control approach (Gehlhar, Reher, & Ames, 2019; Gregg, Lenzi et al., 2014; Martin & Gregg, 2017). While this form is difficult to implement on hardware, Gehlhar and Ames (2021) built off of this theory work to develop a general class of stabilizing controllers using CLFs, such that a hardware implementable form can be constructed that provides formally-based stability guarantees of the human-prosthesis system.

On hardware, this controller, implemented with a QP, has been demonstrated to improve tracking performance over a traditional PD controller, a result shown to be invariant across terrains (Gehlhar et al., 2022) and subjects (Gehlhar & Ames, 2023), without requiring tuning between different conditions. A comparison between the tracking performance of a CLF-QP with and without force sensing capabilities and a PD controller is shown in Fig. 8.

As opposed to PID control, this model-based strategy allows the prosthesis to dynamically sense the human's load, enabling the prosthesis to leverage these dynamics or compensate for this load to achieve its desired motion. Additionally, this CLF approach provides a convergence certificate, that when paired with stable hybrid periodic orbits generated with HZD, provides formal guarantees of stability for the whole system. The optimization-based implementation of CLFs allows additional constraints and optimality criteria to be incorporated in the framework.

8. User-specific customization

As discussed throughout the review so far, a particular challenge of prosthesis control is customizing the locomotion for every individual user in order to optimize for various metrics aimed at naturalness, efficiency, and responsiveness. While some methods of control require less customization (because they are able to generalize across users), experimental results have largely demonstrated that user-specific customization yields improved performance (Liu, Wu, Si, & Huang, 2022; Quintero, Reznick et al., 2018; Simon et al., 2014). This motivates the need for fast and intuitive methods of user-customization that can be applied to a wide variety of control techniques. To date, four methods of user-customization have been explored: heuristic tuning performed by a domain-expert; automatic parameter selection based on biological data; parameter optimization using simulation-based software; and systematic tuning via human-in-the-loop testing. These methods are illustrated in Fig. 9.

8.1. Heuristic tuning via trial & error

The most common method of user-customization for prosthesis control is to manually adjust various control parameters using the clinician's (or an expert operator's) perception of the walking (Simon et al., 2014; Sup IV, Varol, Mitchell, Withrow, & Goldfarb, 2009). While this method of heuristic tuning is straightforward, it requires onerous human-subject testing. For instance, the process of tuning impedance parameters and FSM thresholds has been shown to take several hours for each prosthesis user (Shultz et al., 2015; Simon et al., 2014). This long tuning time is widely referenced as one of the reasons why dual-actuated powered prostheses have not yet been commercialized for clinical use (Quintero, Reznick et al., 2018).

One potential solution for reducing the clinical time required for heuristic user customization is to make the tuning process more intuitive. For example, Quintero, Reznick et al. (2018) conducted a case study on the use of an intuitive clinical control interface. The results found that through the intuitive interface, the tuning process was reduced to 10 min.

Another solution is to reduce the number of parameters that require tuning. Notably, recent work found that only 2–3 coefficients required subject-specific tuning per ambulation mode (Bhakta, Camargo, Kunapuli, Childers and Young, 2020) for a powered knee–ankle prosthesis. This tuning process took approximately 30–40 min per subject.

8.2. Biologically-inspired parameter selection

Another method for reducing the time required for user customization is to set certain control parameters using physiological values measured from biological systems. For example, previous research has discovered formulae to automatically tune several parameters of impedance control (Simon et al., 2014). One such formula, taken from Rouse, Hargrove, Perreault, and Kuiken (2012), determined the impedance stiffness parameter for the ankle joint k_{ankle} to be a function of the user's body mass W and the angle of the ankle joint θ_{ankle}:

$$k_{ankle}(\theta_{ankle}) = W(0.237\theta_{ankle} + 0.028). \quad (17)$$

This stiffness expression was obtained by characterizing the impedance of the ankle in a perturbation study. Similarly, quasi-stiffness control, as discussed in Section 6.1.1, uses the relationship between joint torque and joint angle in human walking to determine impedance parameters that emulate the quasi-stiffness of a biological joint on a prosthesis joint (Pfeifer, Vallery, Hardegger, Riener, & Perreault, 2012; Rouse, Hargrove et al., 2014).

Lastly, to design variable impedance control laws, human joint torque profiles have been leveraged (Best et al., 2023). Specifically, this approach optimizes coefficients for phase-based polynomials to define an impedance control model that fits the able-bodied dataset.

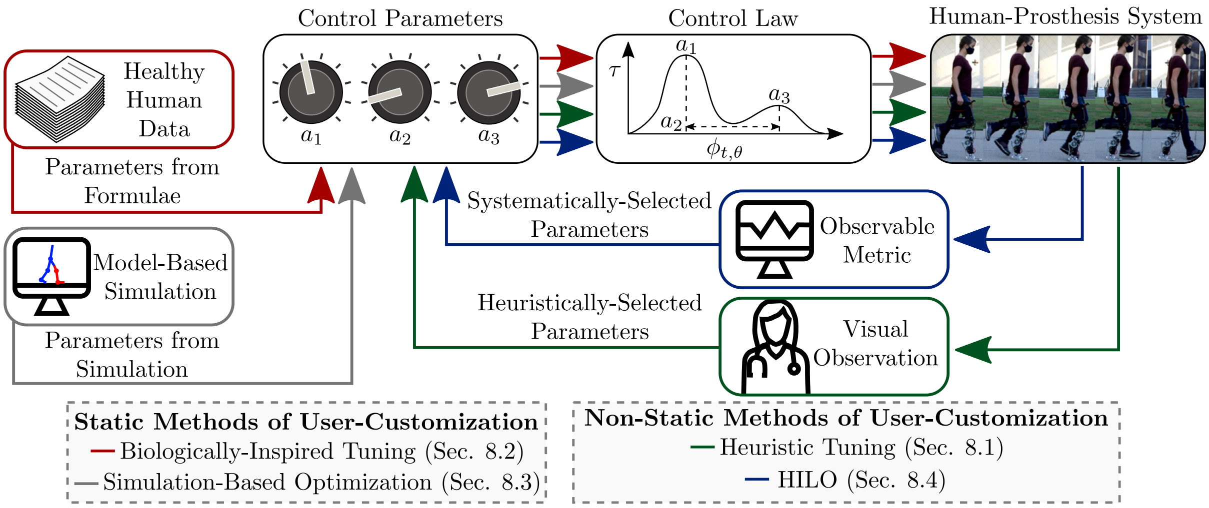

Fig. 9. Methods of user-specific customization. We present four methods of customization, each illustrated using a different color: (red) biologically-inspired tuning which leverages relationships derived from biological systems and data; (gray) simulation-based tuning which leverages computer-based environments to predict which parameters would result in optimal behavior; (green) heuristic tuning which selects new parameters in each experimental iteration based on a clinician or user's observations; and (blue) human-in-the-loop tuning which assigns numerical metrics to the experimentally-obtained prosthesis walking and uses algorithmic tools to predict new parameters to try in the subsequent iteration. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

8.3. Simulation-based optimization

The third method of user customization is to optimize the control parameters using results from simulation environments. A simulation environment commonly used in lower-limb prosthesis control is OpenSim (Camargo et al., 2022; Delp et al., 2007). This simulation environment allows operators to study human biomechanics resulting from prosthesis control. However, a known limitation is that it does not accurately capture human adaptation or disturbances.

Other simulation environments have also been created using musculoskeletal model dynamics. For example, Thatte and Geyer (2015) created an amputee neuromuscular model, modeling the residual thigh as a severed limb. This amputee model was attached to a rigid-body model of a prosthesis with series-elastic-actuators, constructed with known physical parameters. To determine control parameters, this model was used within a 2-stage optimization that first aimed to minimize cost of transport and in the second stage, aimed to maximize distance traveled over uneven terrain.

Lastly, simulation environments can be created using the rigid body dynamics of a human-prosthesis model constructed with average human data and physical parameters of a given prosthesis device. For example, Aghasadeghi et al. (2013) used this type of model within an ODE parameter estimation algorithm to determine impedance control parameters that yielded healthy human kinematic trajectories. Similarly, Gehlhar and Ames (2023) and Zhao, Hereid et al. (2016) used this model type within an HZD gait generation framework. As discussed in Section 7.1.5, the benefit of this approach is that it provides an automatic way to generate kinematic reference trajectories for a given prosthesis device that are specific to the height and weight of the user. Recent work extended this approach by including muscle model constraints within the gait optimization to yield more natural locomotion (Li et al., 2022).

8.4. Human-in-the-loop tuning

The final method of user customization leverages human-in-the-loop optimization (HILO) to systematically optimize control parameters for specific objective metrics. One commonly utilized HILO framework is reinforcement learning with human-subject testing in the loop (Gao, Si, Wen, Li, & Huang, 2021; Li, Wen, Gao, Si, & Huang, 2021; Wen, Li, Si, & Huang, 2020). For example, Wen, Si, Brandt, Gao, and Huang (2019) developed a reinforcement learning (RL) framework to automatically tune 12 impedance control parameters (3 parameters in each of the 4 FSM states) with the goal of generating normative target knee kinematics. Similarly, Wu et al. (2022) demonstrated reinforcement learning towards tuning 12 impedance control parameters, but instead the tuning objective was to mimic the motion of the intact knee. Another HILO approach is to utilize a covariance matrix adaptation evolution strategy (CMA-ES) which is a sample-efficient method towards finding optimal solutions in high-dimensional parameter spaces. Notably, this optimization strategy is more robust to human adaptation over time compared to Bayesian optimization. CMA-ES was demonstrated in Welker et al. (2021) towards optimizing metabolic rate of an ankle prosthesis by modifying parameters of the torque profile directly.

One advantage of HILO is the systematic tuning of non-intuitive control parameters. However, the main limitation is that these methods typically rely on subjectively determining the corresponding performance measures. This is especially challenging since it is not clear which numerical metrics are most important for prosthetic walking. To address this challenge, some researchers have utilized inverse reinforcement learning to first automatically obtain performance measures, which can then be used in a reinforcement learning framework (Liu et al., 2022).

Another solution is to utilize HILO frameworks with subjective human feedback. In particular, Thatte, Duan, and Geyer (2018) framed the HILO problem as a dueling bandits problem to automatically select which parameter set (out of 9 potential parameter sets) a prosthesis subject most preferred. Notably, this strategy addressed the problem of high-dimensional parameter spaces by using an offline optimization step (conducted using the Covariance Matrix Adaptation Strategy Hansen, 2006) to optimize 43 neuromuscular control policy parameters using existing gait data from 9 human subjects. In total, this tuning procedure consisted of approximately 45 min of walking, with 15 min of hand-tuning required prior to the Bayesian optimization procedure. Notably, this work is one of the first to formally incorporate subjective assessments of user preference for lower-limb prostheses using HILO.

9. Discussion

As prosthetic controllers become more complex, so does the task of user-specific tuning. Thus, future research is needed towards creating control architectures that can generalize across prosthesis users and conditions, as well as towards creating intuitive methods of user customization. To bring these devices into the real world, there remain several items that need to be addressed. In this section, we will highlight and discuss a few of these items, separated into open problems and practical considerations.

9.1. Open problems

In this review, we identify four *open problems* that we consider to be the most critical in terms of translating powered prostheses to clinical settings.

9.1.1. Open problem 1: Creating prosthesis controllers that generalize across subjects and tasks

As discussed, achieving natural, efficient, and responsive prosthesis performance with state-of-the-art control methods requires extensive tuning, which can be especially arduous for multi-joint prostheses. Hence, there is a discerning need for control methods that generalize easily across tasks and users. Such controllers would significantly reduce the amount of user-customization required in a clinical setting, improving the commercial viability of powered prostheses.

One existing approach for developing prosthesis task and gait phase estimators that generalize across users is the development of user-independent machine learning classifiers (Bhakta, Camargo, Donovan et al., 2020; Young & Hargrove, 2015). However, to expand these classifiers to a wider variety of locomotion modes and users, the collection of larger and more diverse data sets is required (Camargo, Ramanathan, Flanagan and Young, 2021). While this entails an extensive amount of human-subject experimentation and data collection, the obtained data sets would be extremely valuable training data for classifiers.

Second, to realize mid-level controllers that retain clinical benefits across a variety of users, one approach is to develop controllers that account for subject-specific differences and adapt to the human's real-time motion. Model-based approaches, such as HZD and neuromuscular model control, utilize analytical models of the human-prosthesis system. These models can account for subject-specific differences such as weight, height, and limb length. If these models were improved to capture additional subject-specific differences, such as muscle strength, and to better predict a human's motion, they could be leveraged to systematically generate prosthesis controllers in a subject-specific way. Moreover, control strategies that utilize external sensors, such as phase-based approaches and heuristic algorithms, are able to adapt to an individual's real-time motion. Thus, exploring ways to integrate real-time human sensing into model-based control strategies could build adaptive capabilities into these systematic methods.

Ideally, by developing both user-independent task estimation and subject-specific mid-level controllers, no additional tuning would be required. However, if parameters require tuning, it is important that these parameters be designed to either be able to be automatically tuned using self-tuning algorithms, or be clinically friendly. Examples of clinically intuitive parameters are the amount of knee flexion and ankle push-off as well as the transition point for ankle push-off, as used in Quintero Maldonado et al. (2018). Overall, in order to translate the achievements realized in research settings into clinical settings, it is important to develop prosthesis controllers that both generalize between users as well as contain clinically intuitive tunable parameters.

9.1.2. Open problem 2: Ensuring user safety

Beyond realizing the current achievements in the real-world, an open problem in prosthesis control is ensuring user safety through fault prevention. This relies on both understanding conditions of failure to enable quick fault detection as well as the development of active control strategies for mitigating faults. Compared to passive prostheses, powered prostheses have a unique capability for active fault prevention. This includes preventing stumbles due to obstacles or external perturbations. However, as powered prostheses become more complex, this also introduces additional opportunities for internal faults. Thus, powered prostheses also have a growing need for control schemes that are robust to control faults.

The first component of fault prevention research includes studying the biomechanic response of human-prosthesis balance recovery. Existing research towards this includes simulating uneven terrains (Sinitski et al., 2021; Sturk et al., 2019), introducing obstacles during prosthesis locomotion (Barnett, Polman, & Vanicek, 2014; Vrieling et al., 2007) and applying external forces to the human pelvis (Major, Serba, & Gordon, 2020; Olenšek, Zadravec, Burger, & Matjačić, 2021). Notably, King et al. (2019) developed a predictive targeting algorithm that determines the timing of disturbances to the swing foot caused from obstacles, allowing for more systematic studies of stumble recovery with respect to the timing of the perturbation during swing phase.

Similarly, research towards transfemoral prosthesis fault prevention also includes identifying common conditions that contribute to user falls. For example, Thatte and Geyer (2022) identified two failure conditions: skipped transitions in the controller's FSM and foot scuffing in swing. Notably, unexpected transitions in an FSM have been shown to lead to sudden, large torque changes (Thatte, Shah et al., 2019). Additionally, Eveld, King, Zelik, and Goldfarb (2022) observed that falls were commonly caused when the sound limb failed to clear obstacles and when the prosthesis-side limb failed to initiate swing or failed to land after a swing step.

By understanding the conditions that cause user falls, researchers have recently had success towards detecting and mitigating failures. Towards detecting failures, there are two existing approaches: model-based and data-driven. Model-based approaches predict the response of the human-prosthesis system in order to detect abnormalities. However, a criticism of model-based methods is that models are likely not precise enough to accurately predict faults. Alternatively, data-driven methods use machine learning algorithms instead of models to identify abnormal behaviors. For example, Naseri, Liu, Lee, Liu, and Huang (2022) adopted novelty outlier detection (semi-supervised recognition) to identify abnormal interactions during level-ground walking. The results found that subject-specific models were required to characterize faulty interactions.

Towards mitigating failures, researchers have developed several failure-specific approaches. For example, to address missed transitions, Thatte, Shah et al. (2019) proposed using a continuous phase-variable approach. For foot scuffing, Gordon, Thatte, and Geyer (2019) developed a control approach that learns online to recognize a user's intent to avoid an obstacle and modifies the swing trajectory accordingly to avoid a trip. In Thatte, Srinivasan et al. (2019), they fused data from a laser range finder and an IMU to estimate the prosthetic limb pose, used sparse GPs to predict the user's future hip angles and heights, and updated prosthesis joint desired trajectories to avoid foot scuffing and early landing. Lastly, Eveld et al. (2022) proposed that the prosthesis should aid in ankle push-off (to assist the sound leg's ability to clear obstacles), initiate a swing step in response to a stumble, and provide more robust stance support.

Lastly, research on fault prevention also includes the study of *internal* faults, a growing concern as powered prostheses become more complex and incorporate additional sensors. Towards this, Lee et al. (2022) studied the biomechanics of a human-prosthesis system reacting to internal sensor errors and observed two balance recovery strategies: regulating trunk and intact leg angular momentum, and delaying the loading of body weight.

9.1.3. Open problem 3: Achieving volitional control without degrading healthy biomechanical performance

Another open problem towards improving the responsiveness of powered prostheses is enabling volitional control. While volitional control for single-actuated prostheses has been achieved using direct proportional EMG control, using EMG sensors introduces many complications as described in 3.3. Mainly, since the muscles used for intent detection of ankle joints are removed in a transfemoral amputation, alternative means are needed in order to enable volitional control on multi-joint prostheses. The current solution is to utilize external mechanical sensors as described in Sections 5.2 and 7.1.2. However, for state-based phase variable methods, limitations include degradation of volitional control. Specifically, when the prosthesis foot is flat on the ground, the global hip angle (which is used in the phase variables of Gehlhar and Ames (2023) and Rezazadeh et al. (2019)) is fully determined by the prosthesis knee and ankle joints. This results in the prosthesis user having little volitional control over the phase variable in this phase. Additionally, Best et al. (2023) and Rezazadeh et al. (2019) observed a slow ankle push-off when using a state-based phase variable. While this issue was mitigated by using feedforward phase progression, this solution inhibited volitional control. Thus, further investigation needs to be conducted between the trade-offs of state-based and time-based control in order to achieve the larger goal of creating powered prosthesis controllers that consistently feel responsive to users.

9.1.4. Open problem 4: Formulating device-level control objectives and improving human models to advance human-prosthesis-level performance

Throughout this survey, we have discussed various methods of prosthesis control aimed at the control objectives presented in Section 2: naturalness, efficiency, and responsiveness. As discussed, the ability of a control strategy to satisfy these objectives is typically evaluated using specific performance outcome measures such as reductions in metabolic expenditure (Ledoux & Goldfarb, 2017), improved gait symmetry (both kinematic and kinetic) (Wen et al., 2020), and total mechanical work of intact joints (Camargo, Bhakta, Herrin, & Young, 2023; Farris & Sawicki, 2011). However, it is important to note that it is still unclear which performance measures most closely align with *healthy* human-prosthesis walking as evaluated by a clinician. Thus, more research is needed towards identifying clear and agreed-upon performance outcome measures corresponding to clinically healthy prosthesis walking. Moreover, once such performance measures are identified, systematic methods are needed towards optimizing these measures directly. In this review, we identify two approaches. Importantly, these approaches differ from those presented in Section 8 since they would eliminate the time-intensive need for heuristic tuning or human-in-the-loop testing.

The first approach is to design device-level control objectives that correspond to optimal performance outcomes. While there exist device-level metrics that are currently targeted in research settings, these metrics do not necessarily lead to improved human-prosthesis walking. For example, a commonly used device metric is profile matching which compares the difference between the kinematic and kinetic behavior of the prosthetic limb to an able-bodied leg. Even though matching kinematic and kinetic trends provides a sensible first-pass evaluation of how the prosthetic leg compares to able-bodied behavior, there are physical reasons this may not accurately emulate a human leg. Mainly, given a prosthesis with different physical properties than those of an intact leg, matching torque trends will not result in similar kinematics, and vice versa. Additionally, as detailed in Gehlhar et al. (2019) and Martin and Gregg (2017), the prosthesis only affects the human through the interaction forces at the socket. As such, different prosthetic devices following the same kinematic trajectories or torque profiles will produce different socket forces than each other and a biological leg, hence affecting the human dynamics in different manners. For these reasons, prostheses that only aim to emulate the kinematic and kinetic behavior of human legs will have a different dynamic effect on the human than a biological leg and the overall walking will differ from able-bodied walking. While bearing some visual resemblance to biological walking seems important for social reasons, purely imitating kinematic or kinetic behavior seems misaimed for restoring emulative function of a lower-limb to an amputee. Another device-level metric is the magnitude of ankle push-off. However, while it was long believed that prosthesis ankle push-off was correlated with metabolic reduction of the human user (Caputo & Collins, 2014a), this does not always hold true (Quesada et al., 2016). Overall, these mismatches in device-level metrics used in the field of prosthesis control and their resulting effect on a prosthesis user highlights that a better understanding is needed regarding the coupling between the dynamics of the prosthesis joints and the intact joints. If clear relationships between device-level outcomes and human-prosthesis performance metrics could be identified, then it would be possible to target the design of prosthesis controllers to achieve these device-level control objectives.

The second approach for directly optimizing for performance measures is to develop more accurate human-prosthesis system models. This would allow for a more systematic approach towards offline optimization. While developing model-based optimization strategies is already an active area of research (Gehlhar & Ames, 2023; Thatte & Geyer, 2015; Zhao, Horn et al., 2016), discrepancies currently exist between these human models and a real human user. As such, the existing methods do not necessarily result in optimal behavior because of differences between the predicted human behavior and their actual behavior. Alternatively, a human-prosthesis model that more accurately accounts for both subject-specific differences and the physical parameters of the device could predict how the prosthesis motion will influence the overall human-prosthesis motion. Then, by encoding the human-prosthesis performance metrics as cost functions in simulation, prosthesis control parameters could be optimized with respect to this metric. In summary, building more sophisticated human models would minimize differences between predicted performance measures and real-world behavior, allowing prosthesis controllers to be directly optimized for human locomotion objectives.

9.2. Practical considerations

Finally, we will discuss *practical considerations* of translating powered prostheses from research settings into the real world. In contrast to open problems, these considerations are not things which remain to be *achieved* but instead are simply important to *consider* when developing and evaluating powered prostheses.

9.2.1. Practical consideration 1: Prosthesis design plays a critical role in translating powered prostheses to the real world

While we have not discussed prosthetic design in this review, it is important to note that design considerations play a key role in the development and realization of prosthesis controllers. From a design perspective, a large limiting factor in developing commercial dual-actuated powered prostheses is the increased weight and size that accompanies actuation and power requirements. Part of this challenge could be addressed through control. By developing control strategies that minimize energy usage, like the effort in Zhao, Horn et al. (2017), smaller actuators and power sources could be used, reducing the size and weight of a device and extending battery life.

Other ways to improve the energy efficiency of the device could be addressed through the mechanical design. For example, incorporating passive components, such as dampers and springs, could provide some compliance for energy absorption as well as capture and recover energy inputted to the device, as done in Bartlett, King, Goldfarb, and Lawson (2021), Culver et al. (2022), Lee and Goldfarb (2020, 2021) and Rouse, Mooney and Herr (2014). However, incorporating this compliance into the system would increase controller complexity and the chance for model uncertainty in model-based control methods. Additionally, series-elastic-actuators (such as those used in Au & Herr, 2008; Azocar et al., 2020) decrease the bandwidth of the controller but provide a means to obtain torque feedback enabling closed-loop torque control. A different approach towards improving energy efficiency through design is by selecting low impedance actuators such that a prosthesis leg could swing forward with less or no actuation in swing phase (Bartlett et al., 2022; Elery et al., 2020). These low impedance actuators increase the passive responsiveness of a device to a user's residual limb, thus also improving volitional control. Another benefit of this design is that the low impedance minimizes the effect of unmodeled actuator dynamics such that without torque feedback (i.e. with open-loop torque control) and without arduous tuning, desired impedance behaviors can be achieved.

Aside from energy efficiency, prosthesis design choices also influence the performance and measured success of a given control method. For example, the location of the center of mass of a powered prosthesis has been shown to influence the metabolic efficiency of an amputee (Lehmann et al., 1998). Since metabolic cost is considered as a control objective, the influence design plays in this measure will affect the evaluation of a given control method, preventing direct observation of the controller's performance based on this metric.

To decouple the mechanical design factor from the control problem, testbeds or "emulators" have been developed that provide external assistance to a human, as a prosthesis would, without the use of a portable prototype (Caputo & Collins, 2014b; Flowers & Mann, 1977). This reduces the time and expertise generally required to develop a specialized prosthesis prototype while allowing experimentation of various control algorithms. The power and actuation for the system are kept off board, such that these typical design factors for compact devices do not limit the control capabilities. This freedom in control design allows evaluation of control strategies before making compromises because of design limitations. While the challenge of design and its influence on control capabilities must be confronted to realize powered prostheses in the real world, developing and evaluating control approaches in ideal conditions could shape what design features to prioritize in device development as well as uncover underlying principles that influence a prosthesis controller's efficacy.

9.2.2. Practical consideration 2: To realize the benefits of powered prostheses, prosthesis user training is likely required

Lastly, it is important to consider the level of training each prosthesis user has obtained when evaluating the performance of prosthesis controllers. As noted in Simon et al. (2014), teaching amputees how to ambulate on powered prostheses is critical to achieving positive outcomes. For example, Armannsdottir et al. (2018) and Jayaraman et al. (2018) required several training sessions before being able to demonstrate improvements in biomechanical gait of amputees using a powered prosthesis. Additionally, Fylstra, Lee, Li, Lewek, and Huang (2022) showed that prior to training, prosthesis users' walking gait was asymmetric. However, by using visual feedback for the human users during the gait tuning process, all subjects significantly increased their prosthesis-side stance time and decreased their stance time asymmetry. Hence, training amputees to walk with prosthesis devices in a biomechanically healthy manner is likely required in order to achieve an optimal walking pattern and fully exploit the benefits powered prostheses can offer.

One important aspect of this consideration is the tradeoff between short-term user comfort and long-term biomechanic advantages. Specifically, while it is well-understood that user preference is an important consideration in prosthesis control (i.e., user preference has been shown to correlate with positive reception of passive prostheses by consumers Hafner, Sanders, Czerniecki, & Fergason, 2002), following user preference does not necessarily converge to biomechanically optimal walking. For example, transfemoral amputees tend to avoid stance knee flexion with their prosthesis because it feels like buckling. In other words, they have been conditioned to keep their prosthetic leg straight to not feel like they are falling. From biomechanics though, we know that knee flexion is important for weight acceptance and to reduce the impact effects from foot strike (Blumentritt et al., 1997). Here, a user's initial preference leads to a behavior that is biomechanically disadvantageous and longer term training may be required to adapt to control schemes that support healthier biomechanical patterns. To this end, moving forward it is important to distinguish between when to optimize prosthesis controllers for user preferences and when to temporarily ignore user preference in order to achieve biomechanically optimal walking.

10. Conclusion

Overall, powered prostheses have promising potential to achieve natural, efficient, and responsive locomotion for individuals with lower-limb amputations. Throughout this survey, we introduce the state-of-the-art control strategies across levels of control: high-level task estimation and mid-level desired torque computation. These methods have demonstrated various improvements to prosthesis performance with respect to naturalness of motion, efficiency of gait, and responsiveness to the user. Developing control methods that generalize across users and can be customized for users in a straight-forward manner could enable realization of these benefits for a wider population of users. This widespread applicability in addition to improvements in ensuring user safety, human volitional control, and the overall health of the resultant human-prosthesis gait could make these devices clinically viable to be used by people with lower-limb amputations in everyday life, motivating pursuit of future research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article

Acknowledgments

This material is based upon work supported by Wandercraft, France under Award No. WANDERCRAFT.21, NIH Director's New Innovator, USA Award DP2-HD111709, and NSF, USA Awards 1923239 and 1924526. This review paper was adapted for a chapter in the first author's PhD thesis (Gehlhar, 2023).

References

Aghasadeghi, N., Zhao, H., Hargrove, L. J., Ames, A. D., Perreault, E. J., & Bretl, T. (2013). Learning impedance controller parameters for lower-limb prostheses. In *Intelligent robots and systems (IROS), 2013 IEEE/RSJ international conference on* (pp. 4268–4274). IEEE.

Ames, A. D. (2014). Human-inspired control of bipedal walking robots. *IEEE Transactions on Automatic Control*, *59*(5), 1115–1130.

Ames, A. D., Galloway, K., Sreenath, K., & Grizzle, J. W. (2014). Rapidly exponentially stabilizing control Lyapunov functions and hybrid zero dynamics. *IEEE Transactions on Automatic Control*, *59*(4), 876–891.

Aminian, K., Najafi, B., Büla, C., Leyvraz, P.-F., & Robert, P. (2002). Spatio-temporal parameters of gait measured by an ambulatory system using miniature gyroscopes. *Journal of Biomechanics*, *35*(5), 689–699.

Armannsdottir, A., Tranberg, R., Halldorsdottir, G., & Briem, K. (2018). Frontal plane pelvis and hip kinematics of transfemoral amputee gait. Effect of a prosthetic foot with active ankle dorsiflexion and individualized training – a case study. *Disability and Rehabilitation: Assistive Technology*, *13*(4), 388–393, PMID: 28974119.

Au, S., Berniker, M., & Herr, H. (2008). Powered ankle-foot prosthesis to assist level-ground and stair-descent gaits. *Neural Networks*, *21*(4), 654–666, Robotics and Neuroscience.

Au, S. K., & Herr, H. M. (2008). Powered ankle-foot prosthesis. *IEEE Robotics & Automation Magazine*, *15*(3), 52–59.

Au, S. K., Weber, J., & Herr, H. (2009). Powered ankle–foot prosthesis improves walking metabolic economy. *IEEE Transactions on Robotics*, *25*(1), 51–66.

Azocar, A. F., Mooney, L. M., Duval, J.-F., Simon, A. M., Hargrove, L. J., & Rouse, E. J. (2020). Design and clinical implementation of an open-source bionic leg. *Nature Biomedical Engineering*, *4*, 941–953.

Bae, J., & Tomizuka, M. (2011). Gait phase analysis based on a hidden Markov model. *Mechatronics*, *21*(6), 961–970.

Barnett, C. T., Polman, R. C., & Vanicek, N. (2014). Longitudinal kinematic and kinetic adaptations to obstacle crossing in recent lower limb amputees. *Prosthetics and Orthotics International*, *38*(6), 437–446.

Bartlett, H. L., King, S. T., Goldfarb, M., & Lawson, B. E. (2021). A semi-powered ankle prosthesis and unified controller for level and sloped walking. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *29*, 320–329.

Bartlett, H. L., King, S. T., Goldfarb, M., & Lawson, B. E. (2022). Design and assist-as-needed control of a lightly powered prosthetic knee. *IEEE Transactions on Medical Robotics and Bionics*, *4*(2), 490–501.

Baud, R., Manzoori, A. R., Ijspeert, A., & Bouri, M. (2021). Review of control strategies for lower-limb exoskeletons to assist gait. *Journal of NeuroEngineering and Rehabilitation*, *18*(1), 1–34.

Best, T. K., Embry, K. R., Rouse, E. J., & Gregg, R. D. (2021). Phase-variable control of a powered knee-ankle prosthesis over continuously varying speeds and inclines. In *2021 IEEE/RSJ international conference on intelligent robots and systems* IROS, (pp. 6182–6189). IEEE.

Best, T. K., Welker, C. G., Rouse, E. J., & Gregg, R. D. (2023). Data-driven variable impedance control of a powered knee–ankle prosthesis for adaptive speed and incline walking. *IEEE Transactions on Robotics*, 1–19. http://dx.doi.org/10.1109/TRO.2022.3226887.

Bhakta, K., Camargo, J., Compton, W., Herrin, K., & Young, A. (2021). Evaluation of continuous walking speed determination algorithms and embedded sensors for a powered knee & ankle prosthesis. *IEEE Robotics and Automation Letters*, *6*(3), 4820–4826.

Bhakta, K., Camargo, J., Donovan, L., Herrin, K., & Young, A. (2020). Machine learning model comparisons of user independent & dependent intent recognition systems for powered prostheses. *IEEE Robotics and Automation Letters*, *5*(4), 5393–5400.

Bhakta, K., Camargo, J., Kunapuli, P., Childers, L., & Young, A. (2020). Impedance control strategies for enhancing sloped and level walking capabilities for individuals with transfemoral amputation using a powered multi-joint prosthesis. *Military Medicine*, *185*(Supplement_1), 490–499.

Blumentritt, S., Scherer, H. W., Wellershaws, U., & Michael, J. W. (1997). Design principles, biomeclianical data and clinical experience with a polycentric knee offering controlled stance phase knee flexion: A preliminary report. *JPO Journal of Prosthetics and Orthotics*, *9*, 18–24.

Bohannon, R. (1997). Comfortable and maximum walking speed of adults aged 20-79 years: Reference values and determinants. *Age and Ageing*, *26*, 15–19.

Boonstra, A., Schrama, J., Fidler, V., & Eisma, W. (1995). The gait of unilateral transfemoral amputees. *Scandinavian Journal of Rehabilitation Medicine*, *26*, 217–223.

Boretius, T., Badia, J., Pascual-Font, A., Schuettler, M., Navarro, X., Yoshida, K., et al. (2010). A transverse intrafascicular multichannel electrode (TIME) to interface with the peripheral nerve. *Biosensors and Bioelectronics*, *26*(1), 62–69.

Brandt, A., & Huang, H. H. (2019). Effects of extended stance time on a powered knee prosthesis and gait symmetry on the lateral control of balance during walking in individuals with unilateral amputation. *Journal of Neuroengineering and Rehabilitation*, *16*(1), 1–11.

Brockway, J. (1987). Derivation of formulae used to calculate energy expenditure in man. *Human Nutrition. Clinical Nutrition*, *41*(6), 463–471.

Browning, R., Modica, J., Kram, R., & Goswami, A. (2007). The effects of adding mass to the legs on the energetics and biomechanics of walking. *Medicine and Science in Sports and Exercise*, *39*, 515–525.

Bukowski, E. L. (2006). Atlas of amputations and limb deficiencies: Surgical, prosthetic, and rehabilitation principles, ed 3. *Physical Therapy*, *86*(4), 595–596.

Camargo, J., Bhakta, K., Herrin, K., & Young, A. (2023). Biomechanical evaluation of stair ambulation using impedance control on an active prosthesis. *Journal of Biomechanical Engineering*, *145*(2), 1–9.

Camargo, J., Bhakta, K., Maldonado-Contreras, J., Zhou, S., Herrin, K., & Young, A. (2022). OpenSim model for biomechanical analysis with the open-source bionic leg. In *2022 international symposium on medical robotics* ISMR, (pp. 1–6). IEEE.

Camargo, J., Flanagan, W., Csomay-Shanklin, N., Kanwar, B., & Young, A. (2021). A machine learning strategy for locomotion classification and parameter estimation using fusion of wearable sensors. *IEEE Transactions on Biomedical Engineering*, *68*(5), 1569–1578.

Camargo, J., Ramanathan, A., Flanagan, W., & Young, A. (2021). A comprehensive, open-source dataset of lower limb biomechanics in multiple conditions of stairs, ramps, and level-ground ambulation and transitions. *Journal of Biomechanics*, *119*, Article 110320.

Caputo, J. M., & Collins, S. H. (2014a). Prosthetic ankle push-off work reduces metabolic rate but not collision work in non-amputee walking. *Scientific Reports*, *4*(1), 1–9.

Caputo, J. M., & Collins, S. H. (2014b). A universal ankle–foot prosthesis emulator for human locomotion experiments. *Journal of Biomechanical Engineering*, *136*(3).

Cempini, M., Hargrove, L. J., & Lenzi, T. (2017). Design, development, and bench-top testing of a powered polycentric ankle prosthesis. In *2017 IEEE/RSJ international conference on intelligent robots and systems* IROS, (pp. 1064–1069).

Ceseracciu, E., Reggiani, M., Sawacha, Z., Sartori, M., Spolaor, F., Cobelli, C., et al. (2010). SVM classification of locomotion modes using surface electromyography for applications in rehabilitation robotics. In *19th international symposium in robot and human interactive communication* (pp. 165–170). IEEE.

Clites, T. R., Carty, M. J., Ullauri, J. B., Carney, M. E., Mooney, L. M., Duval, J.-F., et al. (2018). Proprioception from a neurally controlled lower-extremity prosthesis. *Science Translational Medicine*, *10*(443), eaap8373.

Culver, S., Bartlett, H., Shultz, A., & Goldfarb, M. (2018). A stair ascent and descent controller for a powered ankle prosthesis. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *26*(5), 993–1002.

Culver, S. C., Vailati, L. G., & Goldfarb, M. (2022). A primarily-passive knee prosthesis with powered stance and swing assistance. In *2022 international conference on rehabilitation robotics* ICORR, (pp. 1–6).

Delp, S. L., Anderson, F. C., Arnold, A. S., Loan, P., Habib, A., John, C. T., et al. (2007). OpenSim: open-source software to create and analyze dynamic simulations of movement. *IEEE Transactions on Biomedical Engineering*, *54*(11), 1940–1950.

DeVita, P., Helseth, J., & Hortobagyi, T. (2007). Muscles do more positive than negative work in human locomotion. *The Journal of Experimental Biology*, *210*, 3361–3373.

Dewald, H. A., Lukyanenko, P., Lambrecht, J. M., Anderson, J. R., Tyler, D. J., Kirsch, R. F., et al. (2019). Stable, three degree-of-freedom myoelectric prosthetic control via chronic bipolar intramuscular electrodes: a case study. *Journal of NeuroEngineering and Rehabilitation*, *16*(1), 1–13.

Diaz, J. P., da Silva, R. L., Zhong, B., Huang, H. H., & Lobaton, E. (2018). Visual terrain identification and surface inclination estimation for improving human locomotion with a lower-limb prosthetic. In *2018 40th annual international conference of the IEEE engineering in medicine and biology society* EMBC, (pp. 1817–1820). IEEE.

Eilenberg, M. F., Geyer, H., & Herr, H. (2010). Control of a powered ankle–foot prosthesis based on a neuromuscular model. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *18*(2), 164–173.

Elery, T., Rezazadeh, S., Nesler, C., & Gregg, R. D. (2020). Design and validation of a powered knee–ankle prosthesis with high-torque, low-impedance actuators. *IEEE Transactions on Robotics*, *36*(6), 1649–1668.

Embry, K. R., Villarreal, D. J., Macaluso, R. L., & Gregg, R. D. (2018). Modeling the kinematics of human locomotion over continuously varying speeds and inclines. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *26*(12), 2342–2350.

Engsberg, J., Lee, A., Tedford, K., & Harder, J. (1993). Normative ground reaction force data for able-bodied and below-knee-amputee children during walking. *Journal of Pediatric Orthopedics*, *13*, 169–173.

Esposito, E. R., Rodriguez, K. M., Ràbago, C. A., & Wilken, J. M. (2014). Does unilateral transtibial amputation lead to greater metabolic demand during walking. *Journal of Rehabilitation Research and Development*, *51*(8), 1287–1296.

Evans, R. L., & Arvind, D. (2014). Detection of gait phases using orient specks for mobile clinical gait analysis. In *2014 11th international conference on wearable and implantable body sensor networks* (pp. 149–154). IEEE.

Eveld, M., King, S., Zelik, K., & Goldfarb, M. (2022). Factors leading to falls in transfemoral prosthesis users: a case series of sound-side stumble recovery responses. *Journal of NeuroEngineering and Rehabilitation*, *19*.

Farris, D., & Sawicki, G. (2011). The mechanics and energetics of human walking and running: A joint level perspective. *Journal of the Royal Society, Interface / The Royal Society*, *9*, 110–118.

Fey, N. P., Simon, A. M., Young, A. J., & Hargrove, L. J. (2014). Controlling knee swing initiation and ankle plantarflexion with an active prosthesis on level and inclined surfaces at variable walking speeds. *IEEE Journal of Translational Engineering in Health and Medicine*, *2*, 1–12.

Flash, T., & Hogan, N. (1985). The coordination of arm movements: an experimentally confirmed mathematical model. *Journal of Neuroscience*, *5*(7), 1688–1703.

Fleming, A., Stafford, N., Huang, S., Hu, X., Ferris, D. P., & Huang, H. H. (2021). Myoelectric control of robotic lower limb prostheses: a review of electromyography interfaces, control paradigms, challenges and future directions. *Journal of Neural Engineering*, *18*(4), Article 041004.

Flowers, W., & Mann, R. (1977). An electrohydraulic knee-torque controller for a prosthesis simulator. *Journal of Biomechanical Engineering*, *99*(1), 3–8.

Fluit, R., Prinsen, E., Wang, S., & Kooij, H. (2019). A comparison of control strategies in commercial and research knee prostheses. *IEEE Transactions on Bio-Medical Engineering*, *PP*.

Fritz, S., & Lusardi, M. (2009). Walking speed: The sixth vital sign. *Journal of Geriatric Physical Therapy*, *32*, 46–49.

Fylstra, B. L., Lee, I.-C., Li, M., Lewek, M., & Huang, H. (2022). Human-prosthesis cooperation: Combining adaptive prosthesis control with visual feedback guided gait.

Gadaleta, M., Cisotto, G., Rossi, M., Rehman, R. Z. U., Rochester, L., & Del Din, S. (2019). Deep learning techniques for improving digital gait segmentation. In *2019 41st annual international conference of the IEEE engineering in medicine and biology society* EMBC, (pp. 1834–1837). IEEE.

Gailey, R. (2008). Review of secondary physical conditions associated with lower-limb amputation and long-term prosthesis use. *The Journal of Rehabilitation Research and Development*, *45*(1), 15–30.

Gao, X., Si, J., Wen, Y., Li, M., & Huang, H. (2021). Reinforcement learning control of robotic knee with human-in-the-loop by flexible policy iteration. *IEEE Transactions on Neural Networks and Learning Systems*.

Gehlhar, R. (2023). *Model-based lower-limb powered prosthesis control: Developing and realizing nonlinear subsystem control methods for generalizable prosthesis control* (Ph.D. thesis), USA: California Institute of Technology.

Gehlhar, R., & Ames, A. D. (2021). Separable control Lyapunov functions with application to prostheses. *IEEE Control Systems Letters*, *5*(2), 559–564.

Gehlhar, R., & Ames, A. D. (2023). Emulating human kinematic behavior on lower-limb prostheses via multi-contact models and force-based nonlinear control. In *2023 IEEE international conference on robotics and automation*. ICRA, IEEE.

Gehlhar, R., Reher, J., & Ames, A. D. (2019). Control of separable subsystems with application to prostheses. arXiv preprint arXiv:1909.03102.

Gehlhar, R., Yang, J.-h., & Ames, A. D. (2022). Powered prosthesis locomotion on varying terrains: Model-dependent control with real-time force sensing. *IEEE Robotics and Automation Letters*, *7*(2), 5151–5158.

Geyer, H., & Herr, H. (2010). A muscle-reflex model that encodes principles of legged mechanics produces human walking dynamics and muscle activities. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *18*(3), 263–273.

Gordon, M., Thatte, N., & Geyer, H. (2019). Online learning for proactive obstacle avoidance with powered transfemoral prostheses. In *2019 international conference on robotics and automation* ICRA, (pp. 7920–7925).

Gregg, R. D., Lenzi, T., Hargrove, L. J., & Sensinger, J. W. (2014). Virtual constraint control of a powered prosthetic leg: From simulation to experiments with transfemoral amputees. *IEEE Transactions on Robotics*, *30*(6), 1455–1471.

Gregg, R. D., Rouse, E. J., Hargrove, L. J., & Sensinger, J. W. (2014). Evidence for a time-invariant phase variable in human ankle control. *PLOS ONE*, *9*(2), 1–13.

Hafner, B. J., Sanders, J. E., Czerniecki, J., & Fergason, J. (2002). Energy storage and return prostheses: does patient perception correlate with biomechanical analysis? *Clinical Biomechanics*, *17*(5), 325–344.

Hansen, N. (2006). The CMA evolution strategy: a comparing review. In *Towards a new evolutionary computation* (pp. 75–102). Springer.

Hansen, A., & Childress, D. (2010). Investigations of roll-over shape: Implications for design, alignment, and evaluation of ankle-foot prostheses and orthoses. *Disability and Rehabilitation*, *32*, 2201–2209.

Hansen, A., Childress, D., & Knox, E. (2004). Roll-over shapes of human locomotor systems: Effects of walking speed. *Clinical Biomechanics (Bristol, Avon)*, *19*, 407–414.

Hansen, A. H., Childress, D. S., Miff, S. C., Gard, S. A., & Mesplay, K. P. (2004). The human ankle during walking: implications for design of biomimetic ankle prostheses. *Journal of Biomechanics*, *37*(10), 1467–1474.

Hargrove, L. J., Englehart, K., & Hudgins, B. (2007). A comparison of surface and intramuscular myoelectric signal classification. *IEEE Transactions on Biomedical Engineering*, *54*(5), 847–853.

Hargrove, L. J., Simon, A. M., Young, A. J., Lipschutz, R. D., Finucane, S. B., Smith, D. G., et al. (2013). Robotic leg control with EMG decoding in an amputee with nerve transfers. *New England Journal of Medicine*, *369*(13), 1237–1242.

Hargrove, L. J., Young, A. J., Simon, A. M., Fey, N. P., Lipschutz, R. D., Finucane, S. B., et al. (2015). Intuitive control of a powered prosthetic leg during ambulation: A randomized clinical trial. *JAMA*, *313*(22), 2244–2252.

He, Z., & Zhang, W. (2011). Estimation of walking speed using accelerometer and artificial neural networks. In *International workshop on computer science for environmental engineering and ecoinformatics* (pp. 42–47). Springer.

Herr, H., & Grabowski, A. (2011). Bionic ankle-foot prosthesis normalizes walking gait for persons with leg amputation. *Proceedings. Biological Sciences / The Royal Society*, *279*, 457–464.

Hijjawi, J. B., Kuiken, T. A., Lipschutz, R. D., Miller, L. A., Stubblefield, K. A., & Dumanian, G. A. (2006). Improved myoelectric prosthesis control accomplished using multiple nerve transfers. *Plastic and Reconstructive Surgery*, *118*(7), 1573–1578.

Hobara, H., Kobayashi, Y., Nakamura, T., Yamasaki, N., Nakazawa, K., Akai, M., et al. (2011). Lower extremity joint kinematics of stair ascent in transfemoral amputees. *Prosthetics and Orthotics International*, *35*, 467–472.

Hogan, N. (1984a). Adaptive control of mechanical impedance by coactivation of antagonist muscles. *IEEE Transactions on Automatic Control*, *29*(8), 681–690.

Hogan, N. (1984b). Impedance control: An approach to manipulation. In *1984 American control conference* (pp. 304–313).

Holgate, M. A., Sugar, T. G., & Bohler, A. W. (2009). A novel control algorithm for wearable robotics using phase plane invariants. In *2009 IEEE international conference on robotics and automation* (pp. 3845–3850).

Hong, J. H., & Mun, M. S. (2005). Relationship between socket pressure and EMG of two muscles in trans-femoral stumps during gait. *Prosthetics and Orthotics International*, *29*(1), 59–72.

Hong, W., Paredes, V., Chao, K., Patrick, S., & Hur, P. (2019). Consolidated control framework to control a powered transfemoral prosthesis over inclined terrain conditions. In *2019 international conference on robotics and automation* ICRA, (pp. 2838–2844).

Hood, S., Gabert, L., & Lenzi, T. (2022). Powered knee and ankle prosthesis with adaptive control enables climbing stairs with different stair heights, cadences, and gait patterns. *IEEE Transactions on Robotics*, *38*(3), 1430–1441.

Hu, J.-S., Sun, K.-C., & Cheng, C.-Y. (2013). A kinematic human-walking model for the normal-gait-speed estimation using tri-axial acceleration signals at waist location. *IEEE Transactions on Biomedical Engineering*, *60*(8), 2271–2279.

Huang, S., & Ferris, D. P. (2012). Muscle activation patterns during walking from transtibial amputees recorded within the residual limb-prosthetic interface. *Journal of Neuroengineering and Rehabilitation*, *9*(1), 1–16.

Huang, H., Kuiken, T. A., Lipschutz, R. D., et al. (2008). A strategy for identifying locomotion modes using surface electromyography. *IEEE Transactions on Biomedical Engineering*, *56*(1), 65–73.

Huang, H. H., Si, J., Brandt, A., & Li, M. (2021). Taking both sides: seeking symbiosis between intelligent prostheses and human motor control during locomotion. *Current Opinion in Biomedical Engineering*, *20*, Article 100314.

Huang, S., Wensman, J. P., & Ferris, D. P. (2014). An experimental powered lower limb prosthesis using proportional myoelectric control. *Journal of Medical Devices*, *8*(2).

Huang, H., Zhang, F., Hargrove, L. J., Dou, Z., Rogers, D. R., & Englehart, K. B. (2011). Continuous locomotion-mode identification for prosthetic legs based on neuromuscular–mechanical fusion. *IEEE Transactions on Biomedical Engineering*, *58*(10), 2867–2875.

Hunt, G. R., Hood, S., & Lenzi, T. (2021). Stand-up, squat, lunge, and walk with a robotic knee and ankle prosthesis under shared neural control. *IEEE Open Journal of Engineering in Medicine and Biology*, *2*, 267–277.

Hurley, G. R. B., McKenney, R., Robinson, M., Zadravec, M., & Pierrynowski, M. R. (1990). The role of the contralateral limb in below-knee amputee gait. *Prosthetics and Orthotics International*, *14*(1), 33–42, PMID: 2192355.

Hussain, T., Iqbal, N., Maqbool, H. F., Khan, M., Awad, M. I., & Dehghani-Sanij, A. A. (2020). Intent based recognition of walking and ramp activities for amputee using sEMG based lower limb prostheses. *Biocybernetics and Biomedical Engineering*, *40*(3), 1110–1123.

Inman, V. T. (1966). Human locomotion. *Canadian Medical Association Journal*, *94 20*, 1047–1054.

Isakov, E., Burger, H., Krajnik, J., Gregorič, M., & Marinček, Č. (1997). Double-limb support and step-length asymmetry in below-knee amputees. *Scandinavian Journal of Rehabilitation Medicine*, *29 2*, 75–79.

Jaegers, S. M., Arendzen, J. H., & de Jongh, H. J. (1995). Prosthetic gait of unilateral transfemoral amputees: a kinematic study. *Archives of Physical Medicine and Rehabilitation*, *76 8*, 736–743.

Jayaraman, C., Hoppe-Ludwig, S., Deems-Dluhy, S., McGuire, M., Mummidisetty, C., Siegal, R., et al. (2018). Impact of powered Knee-Ankle prosthesis on low back muscle mechanics in transfemoral amputees: A case series. *Frontiers in Neuroscience*, *12*.

Jiang, S., Partrick, S., Zhao, H., & Ames, A. D. (2012). Outputs of human walking for bipedal robotic controller design. In *2012 American control conference (ACC)* (pp. 4843–4848).

Jin, D., Yang, J., Zhang, R., Wang, R., & Zhang, J. (2006). Terrain identification for prosthetic knees based on electromyographic signal features. *Tsinghua Science and Technology*, *11*(1), 74–79.

King, S., Eveld, M., Martínez, A., Zelik, K., & Goldfarb, M. (2019). A novel system for introducing precisely-controlled, unanticipated gait perturbations for the study of stumble recovery. *Journal of NeuroEngineering and Rehabilitation*, *16*, 69.

Krausz, N. E., Lenzi, T., & Hargrove, L. J. (2015). Depth sensing for improved control of lower limb prostheses. *IEEE Transactions on Biomedical Engineering*, *62*(11), 2576–2587.

Kuiken, T. A., Dumanian, G., Lipschutz, R. D., Miller, L., & Stubblefield, K. (2004). The use of targeted muscle reinnervation for improved myoelectric prosthesis control in a bilateral shoulder disarticulation amputee. *Prosthetics and Orthotics International*, *28*(3), 245–253.

Kuiken, T. A., Li, G., Lock, B. A., Lipschutz, R. D., Miller, L. A., Stubblefield, K. A., et al. (2009). Targeted muscle reinnervation for real-time myoelectric control of multifunction artificial arms. *JAMA*, *301*(6), 619–628.

Kulkarni, J., Wright, S., Toole, C., Morris, J., & Hirons, R. (1996). Falls in patients with lower limb amputations: Prevalence and contributing factors. *Physiotherapy*, *82*(2), 130–136.

Kurbis, A. G., Laschowski, B., & Mihailidis, A. (2022). Stair recognition for robotic exoskeleton control using computer vision and deep learning. bioRxiv.

Latash, M., & Zatsiorsky, V. (1993). Joint stiffness: Myth or reality? *Human Movement Science*, *12*, 653–692.

Lawson, B. E., Ledoux, E. D., & Goldfarb, M. (2017). A robotic lower limb prosthesis for efficient bicycling. *IEEE Transactions on Robotics*, *33*(2), 432–445.

Lawson, B. E., Mitchell, J., Truex, D., Shultz, A., Ledoux, E., & Goldfarb, M. (2014). A robotic leg prosthesis: Design, control, and implementation. *IEEE Robotics & Automation Magazine*, *21*(4), 70–81.

Lawson, B. E., Ruhe, B., Shultz, A., & Goldfarb, M. (2015). A powered prosthetic intervention for bilateral transfemoral amputees. *IEEE Transactions on Biomedical Engineering*, *62*(4), 1042–1050.

Lawson, B. E., Varol, H. A., Huff, A., Erdemir, E., & Goldfarb, M. (2013). Control of stair ascent and descent with a powered transfemoral prosthesis. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *21*(3), 466–473.

Ledoux, E. D., & Goldfarb, M. (2017). Control and evaluation of a powered transfemoral prosthesis for stair ascent. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *25*(7), 917–924.

Lee, J., & Goldfarb, M. (2020). Swing-assist for enhancing stair ambulation in a primarily-passive knee prosthesis. In *2020 IEEE international conference on robotics and automation (ICRA)* (pp. 740–746).

Lee, J. T., & Goldfarb, M. (2021). Effect of a swing-assist knee prosthesis on stair ambulation. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *29*, 2046–2054.

Lee, H., & Hogan, N. (2014). Time-varying ankle mechanical impedance during human locomotion. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *23*.

Lee, I. C., Liu, M., Lewek, M. D., Hu, X., Filer, W. G., & Huang, H. (2022). Towards safe wearer-prosthesis interaction: Evaluation of gait stability and human compensation strategy under faults in robotic transfemoral prostheses. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*.

van Leeuwen, J. L. (1992). Muscle function in locomotion. (pp. 191–250). chapter 7.

Lehmann, J. F., Price, R., Okumura, R., Questad, K., de Lateur, B. J., & Négretot, A. (1998). Mass and mass distribution of below-knee prostheses: Effect on gait efficacy and self-selected walking speed. *Archives of Physical Medicine and Rehabilitation*, *79*(2), 162–168.

Lempereur, M., Rousseau, F., Rémy-Néris, O., Pons, C., Houx, L., Quellec, G., et al. (2020). A new deep learning-based method for the detection of gait events in children with gait disorders: Proof-of-concept and concurrent validity. *Journal of Biomechanics*, *98*, Article 109490.

Lenzi, T., Hargrove, L., & Sensinger, J. (2014). Speed-adaptation mechanism: Robotic prostheses can actively regulate joint torque. *IEEE Robotics & Automation Magazine*, *21*(4), 94–107.

Li, K., Tucker, M., Gehlhar, R., Yue, Y., & Ames, A. D. (2022). Natural multicontact walking for robotic assistive devices via musculoskeletal models and hybrid zero dynamics. *IEEE Robotics and Automation Letters*, *7*(2), 4283–4290.

Li, M., Wen, Y., Gao, X., Si, J., & Huang, H. (2021). Toward expedited impedance tuning of a robotic prosthesis for personalized gait assistance by reinforcement learning control. *IEEE Transactions on Robotics*, *38*(1), 407–420.

Li, Q., Young, M., Naing, V., & Donelan, J. (2010). Walking speed estimation using a shank-mounted inertial measurement unit. *Journal of Biomechanics*, *43*(8), 1640–1643.

Liu, M., Wang, D., & Huang, H. (2015). Development of an environment-aware locomotion mode recognition system for powered lower limb prostheses. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *24*(4), 434–443.

Liu, W., Wu, R., Si, J., & Huang, H. (2022). A new robotic knee impedance control parameter optimization method facilitated by inverse reinforcement learning. *IEEE Robotics and Automation Letters*, *7*(4), 10882–10889.

Ma, W. L., Kolathaya, S., Ambrose, E. R., Hubicki, C. M., & Ames, A. D. (2017). Bipedal robotic running with DURUS-2D: Bridging the gap between theory and experiment. In *Proceedings of the 20th international conference on hybrid systems: computation and control* HSCC '17, (pp. 265–274). New York, NY, USA: Association for Computing Machinery.

Major, M. J., Serba, C. K., & Gordon, K. E. (2020). Perturbation recovery during walking is impacted by knowledge of perturbation timing in below-knee prosthesis users and non-impaired participants. *PLoS One*, *15*(7), Article e0235686.

Mannini, A., Genovese, V., & Sabatini, A. M. (2013). Online decoding of hidden Markov models for gait event detection using foot-mounted gyroscopes. *IEEE Journal of Biomedical and Health Informatics*, *18*(4), 1122–1130.

Mariani, B., Hoskovec, C., Rochat, S., Büla, C., Penders, J., & Aminian, K. (2010). 3D gait assessment in young and elderly subjects using foot-worn inertial sensors. *Journal of Biomechanics*, *43*(15), 2999–3006.

Martin, A. E., & Gregg, R. D. (2017). Stable, robust hybrid zero dynamics control of powered lower-limb prostheses. *IEEE Transactions on Automatic Control*, *62*(8), 3930–3942.

Martinez-Villalpando, E. C., Mooney, L., Elliott, G., & Herr, H. (2011). Antagonistic active knee prosthesis. a metabolic cost of walking comparison with a variable-damping prosthetic knee. In *2011 Annual international conference of the ieee engineering in medicine and biology society* (pp. 8519–8522).

Mendez, J., Hood, S., Gunnel, A., & Lenzi, T. (2020). Powered knee and ankle prosthesis with indirect volitional swing control enables level-ground walking and crossing over obstacles. *Science Robotics*, *5*(44), eaba6635.

Miller, W. C., Deathe, A. B., Speechley, M., & Koval, J. (2001). The influence of falling, fear of falling, and balance confidence on prosthetic mobility and social activity among individuals with a lower extremity amputation. *Archives of Physical Medicine and Rehabilitation*, *82*(9), 1238–1244.

Miller, W. C., Speechley, M., & Deathe, B. (2001). The prevalence and risk factors of falling and fear of falling among lower extremity amputees. *Archives of Physical Medicine and Rehabilitation*, *82*(8), 1031–1037.

Molen, N. H. (2004). Energy/speed relation of below-knee amputees walking on a motor-driven treadmill. *Internationale Zeitschrift Für Angewandte Physiologie Einschließlich Arbeitsphysiologie*, *31*, 173–185.

Morgenroth, D. C., Segal, A. D., Zelik, K. E., Czerniecki, J. M., Klute, G. K., Adamczyk, P. G., et al. (2011). The effect of prosthetic foot push-off on mechanical loading associated with knee osteoarthritis in lower extremity amputees. *Gait & Posture*, *34*(4), 502–507.

Murray, R. M., Sastry, S. S., & Zexiang, L. (1994). *A mathematical introduction to robotic manipulation* (1st ed.). CRC Press, Inc..

Myers, A. C., Huang, H., & Zhu, Y. (2015). Wearable silver nanowire dry electrodes for electrophysiological sensing. *RSC Advances*, *5*(15), 11627–11632.

Myung, I. J. (2003). Tutorial on maximum likelihood estimation. *Journal of Mathematical Psychology*, *47*(1), 90–100.

Naseri, A., Liu, M., Lee, I.-C., Liu, W., & Huang, H. (2022). Characterizing prosthesis control fault during human-prosthesis interactive walking using intrinsic sensors. *IEEE Robotics and Automation Letters*, *7*(3), 8307–8314.

Nazmi, N., Rahman, M. A. A., Yamamoto, S.-I., & Ahmad, S. A. (2019). Walking gait event detection based on electromyography signals using artificial neural network. *Biomedical Signal Processing and Control*, *47*, 334–343.

Nefian, A. V., Liang, L., Pi, X., Liu, X., & Murphy, K. (2002). Dynamic Bayesian networks for audio-visual speech recognition. *EURASIP Journal on Advances in Signal Processing*, *2002*(11), 1–15.

Nolan, L., & Lees, A. (2000). The functional demands on the intact limb during walking for active trans-femoral and trans-tibial amputees. *Prosthetics and Orthotics International*, *24*, 117–125.

Nolan, L., Wit, A., Dudziñski, K., Lees, A., Lake, M., & Wychowanski, M. (2003). Adjustments in gait symmetry in trans-femoral and trans-tibial amputees. *Gait & Posture*, *17*, 142–151.

Nowroozi, F., Salvanelli, M. L., & Gerber, L. H. (1983). Energy expenditure in hip disarticulation and hemipelvectomy amputees. *Archives of Physical Medicine and Rehabilitation*, *64 7*, 300–303.

Olenšek, A., Zadravec, M., Burger, H., & Matjačić, Z. (2021). Dynamic balancing responses in unilateral transtibial amputees following outward-directed perturbations during slow treadmill walking differ considerably for amputated and non-amputated side. *Journal of NeuroEngineering and Rehabilitation*, *18*(1), 1–11.

Ortiz-Catalan, M., Brånemark, R., Håkansson, B., & Delbeke, J. (2012). On the viability of implantable electrodes for the natural control of artificial limbs: review and discussion. *Biomedical Engineering Online*, *11*(1), 1–24.

Ossur, Reykjavik, & Iceland (2018). The power knee.

Pasquina, P. F., Evangelista, M., Carvalho, A. J., Lockhart, J., Griffin, S., Nanos, G., et al. (2015). First-in-man demonstration of a fully implanted myoelectric sensors system to control an advanced electromechanical prosthetic hand. *Journal of Neuroscience Methods*, *244*, 85–93.

Peeraer, L., Aeyels, B., & Van der Perre, G. (1990). Development of EMG-based mode and intent recognition algorithms for a computer-controlled above-knee prosthesis. *Journal of Biomedical Engineering*, *12*(3), 178–182.

Petrini, F. M., Bumbasirevic, M., Valle, G., Ilic, V., Mijović, P., Čvančara, P., et al. (2019). Sensory feedback restoration in leg amputees improves walking speed, metabolic cost and phantom pain. *Nature Medicine*, *25*(9), 1356–1363.

Pfeifer, S., Vallery, H., Hardegger, M., Riener, R., & Perreault, E. J. (2012). Model-based estimation of knee stiffness. *IEEE Transactions on Biomedical Engineering*, *59*(9), 2604–2612.

Price, M. A., Beckerle, P., & Sup, F. C. (2019). Design optimization in lower limb prostheses: A review. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *27*(8), 1574–1588.

Quesada, R. E., Caputo, J. M., & Collins, S. H. (2016). Increasing ankle push-off work with a powered prosthesis does not necessarily reduce metabolic rate for transtibial amputees. *Journal of Biomechanics*, *49*(14), 3452–3459.

Quintero, D., Reznick, E., Lambert, D. J., Rezazadeh, S., Gray, L., & Gregg, R. D. (2018). Intuitive clinician control interface for a powered knee-ankle prosthesis: A case study. *IEEE Journal of Translational Engineering in Health and Medicine*, *6*, 1–9.

Quintero, D., Villarreal, D. J., Lambert, D. J., Kapp, S., & Gregg, R. D. (2018). Continuous-phase control of a powered knee–ankle prosthesis: Amputee experiments across speeds and inclines. *IEEE Transactions on Robotics*, *34*(3), 686–701.

Quintero Maldonado, D., Reznick, E., Lambert, D., Rezazadeh, S., Gray, L., & Gregg, R. (2018). Intuitive clinician control interface for a powered knee-ankle prosthesis: A case study. *IEEE Journal of Translational Engineering in Health and Medicine*, *PP*, 1.

Ralston, H. J. (2004). Energy-speed relation and optimal speed during level walking. *Internationale Zeitschrift FÜR Angewandte Physiologie Einschließlich Arbeitsphysiologie*, *17*, 277–283.

Reher, J. P., Hereid, A., Kolathaya, S., Hubicki, C. M., & Ames, A. D. (2020). Algorithmic foundations of realizing multi-contact locomotion on the humanoid robot DURUS. In *Algorithmic foundations of robotics XII* (pp. 400–415). Springer.

Rezazadeh, S., Quintero, D., Divekar, N., Reznick, E., Gray, L., & Gregg, R. D. (2019). A phase variable approach for improved rhythmic and non-rhythmic control of a powered Knee-Ankle prosthesis. *IEEE Access*, *7*, 109840–109855.

Riener, R., Rabuffetti, M., & Frigo, C. (2002). Stair ascent and descent at different inclinations. *Gait & Posture*, *15*(1), 32–44.

Rouse, E. J., Gregg, R. D., Hargrove, L. J., & Sensinger, J. W. (2013). The difference between stiffness and quasi-stiffness in the context of biomechanical modeling. *IEEE Transactions on Biomedical Engineering*, *60*(2), 562–568.

Rouse, E. J., Hargrove, L. J., Perreault, E. J., & Kuiken, T. A. (2012). Estimation of human ankle impedance during walking using the perturberator robot. In *2012 4th IEEE RAS & EMBS international conference on biomedical robotics and biomechatronics (BioRob)* (pp. 373–378).

Rouse, E. J., Hargrove, L. J., Perreault, E. J., & Kuiken, T. A. (2014). Estimation of human ankle impedance during the stance phase of walking. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *22*(4), 870–878.

Rouse, E. J., Mooney, L. M., & Herr, H. M. (2014). Clutchable series-elastic actuator: Implications for prosthetic knee design. *International Journal of Robotics Research*, *33*(13), 1611–1625.

Sabatini, A. M., Martelloni, C., Scapellato, S., & Cavallo, F. (2005). Assessment of walking features from foot inertial sensing. *IEEE Transactions on Biomedical Engineering*, *52*(3), 486–494.

Sánchez Manchola, M. D., Bernal, M. J. P., Munera, M., & Cifuentes, C. A. (2019). Gait phase detection for lower-limb exoskeletons using foot motion data from a single inertial measurement unit in hemiparetic individuals. *Sensors*, *19*(13), 2988.

Schmalz, T., Blumentritt, S., & Marx, B. (2007). Biomechanical analysis of stair ambulation in lower limb amputees. *Gait & Posture*, *25*(2), 267–278.

Schmid, M., Beltrami, G., Zambarbieri, D., & Verni, G. (2005). Centre of pressure displacements in trans-femoral amputees during gait. *Gait & Posture*, *21*, 255–262.

Sensinger, J. W., Lock, B. A., & Kuiken, T. A. (2009). Adaptive pattern recognition of myoelectric signals: exploration of conceptual framework and practical algorithms. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *17*(3), 270–278.
Shorter, A. L., & Rouse, E. J. (2018). Mechanical impedance of the ankle during the terminal stance phase of walking. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *26*(1), 135–143.
Shultz, A. H., Lawson, B. E., & Goldfarb, M. (2015). Running with a powered knee and ankle prosthesis. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *23*(3), 403–412.
Sigurðardóttir, J. (2015). *EMG as a control parameter in lower limb prosthetics: surface vs. implanted electrodes* (Ph.D. thesis), Reykjavik University.
Simon, A. M., Fey, N. P., Ingraham, K. A., Finucane, S. B., Halsne, E. G., & Hargrove, L. J. (2016). Improved Weight-Bearing symmetry for transfemoral amputees during standing up and sitting down with a powered Knee-Ankle prosthesis. *Archives of Physical Medicine and Rehabilitation*, *97*(7), 1100–1106.
Simon, A. M., Ingraham, K. A., Fey, N. P., Finucane, S. B., Lipschutz, R. D., Young, A. J., et al. (2014). Configuring a powered knee and ankle prosthesis for transfemoral amputees within five specific ambulation modes. *PLoS One*, *9*(6), Article e99387.
Sinitski, E. H., Lemaire, E. D., Baddour, N., Besemann, M., Dudek, N., & Hebert, J. S. (2021). Maintaining stable transtibial amputee gait on level and simulated uneven conditions in a virtual environment. *Disability and Rehabilitation: Assistive Technology*, *16*(1), 40–48.
Sinnet, R. W., Zhao, H., & Ames, A. D. (2011). Simulating prosthetic devices with human-inspired hybrid control. In *2011 IEEE/RSJ International conference on intelligent robots and systems* (pp. 1723–1730). IEEE.
Sreenath, K., Park, H.-W., Poulakakis, I., & Grizzle, J. (2013). Embedding active force control within the compliant hybrid zero dynamics to achieve stable, fast running on MABEL. *International Journal of Robotics Research*, *32*(3), 324–345.
Sturk, J. A., Lemaire, E. D., Sinitski, E. H., Dudek, N. L., Besemann, M., Hebert, J. S., et al. (2019). Maintaining stable transfemoral amputee gait on level, sloped and simulated uneven conditions in a virtual environment. *Disability and Rehabilitation: Assistive Technology*, *14*(3), 226–235.
Sup, F., Bohara, A., & Goldfarb, M. (2008). Design and control of a powered transfemoral prosthesis. *International Journal of Robotics Research*, *27*(2), 263–273, PMID: 19898683.
Sup, F., Varol, H. A., & Goldfarb, M. (2011). Upslope walking with a powered knee and ankle prosthesis: Initial results with an amputee subject. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *19*(1), 71–78.
Sup IV, F., Varol, A., Mitchell, J., Withrow, T., & Goldfarb, M. (2009). Preliminary evaluations of a self-contained anthropomorphic transfemoral prosthesis. *IEEE/ASME Transactions on Mechatronics : A Joint Publication of the IEEE Industrial Electronics Society and the ASME Dynamic Systems and Control Division*, *14*, 667–676.
Suzuki, K. (1974). Force plate study on the artificial limb gait. *Journal of Japanese Orthopaedic Association*, *46*, 43–55.
Svensson, W., & Holmberg, U. (2005). Foot and ground measurement using portable sensors. In *9th International conference on rehabilitation robotics, 2005. ICORR 2005* (pp. 448–451).
Taborri, J., Palermo, E., Rossi, S., & Cappa, P. (2016). Gait partitioning methods: A systematic review. *Sensors*, *16*(1), 66.
Taborri, J., Scalona, E., Palermo, E., Rossi, S., & Cappa, P. (2015). Validation of inter-subject training for hidden Markov models applied to gait phase detection in children with cerebral palsy. *Sensors*, *15*(9), 24514–24529.
Teixeira-Salmela, L. F., Nadeau, S., Milot, M.-H., Gravel, D., & Requião, L. F. (2008). Effects of cadence on energy generation and absorption at lower extremity joints during gait. *Clinical Biomechanics*, *23*(6), 769–778.
Thatte, N. (2019). *Design and evaluation of robust control methods for robotic transfemoral prostheses* (Ph.D. thesis), Carnegie Mellon University, USA.
Thatte, N., Duan, H., & Geyer, H. (2018). A method for online optimization of lower limb assistive devices with high dimensional parameter spaces. In *2018 IEEE International conference on robotics and automation* ICRA, (pp. 5380–5385). IEEE.
Thatte, N., & Geyer, H. (2015). Toward balance recovery with leg prostheses using neuromuscular model control. *IEEE Transactions on Biomedical Engineering*, *63*(5), 904–913.
Thatte, N., & Geyer, H. (2022). Comparison of balance recovery among current control strategies for robotic leg prostheses. In J. C. Moreno, J. Masood, U. Schneider, C. Maufroy, & J. L. Pons (Eds.), *Wearable robotics: Challenges and trends* (pp. 63–67). Cham: Springer International Publishing.
Thatte, N., Shah, T., & Geyer, H. (2019). Robust and adaptive lower limb prosthesis stance control via extended Kalman filter-based gait phase estimation. *IEEE Robotics and Automation Letters*, *4*(4), 3129–3136.
Thatte, N., Srinivasan, N., & Geyer, H. (2019). Real-time reactive trip avoidance for powered transfemoral prostheses. *Robotics: Science and Systems XV*.
Torburn, L., Schweiger, G., Perry, J., & Powers, C. (1994). Below-knee amputee gait in stair ambulation: A comparison of stride characteristics using five different prosthetic feet. *Clinical Orthopaedics and Related Research*, 185–192.
Tran, M., Gabert, L., Cempini, M., & Lenzi, T. (2019). A lightweight, efficient fully powered knee prosthesis with actively variable transmission. *IEEE Robotics and Automation Letters*, *4*(2), 1186–1193.
Tucker, M., Olivier, J., Pagel, A., Bleuler, H., Bouri, M., Lambercy, O., et al. (2015). Control strategies for active lower extremity prosthetics and orthotics: A review. *Journal of NeuroEngineering and Rehabilitation*, *12*, 1.
Ulf Holmberg, W. S. (2006). An autonomous control system for a prosthetic foot ankle. *IFAC Proceedings Volumes*, *39*(16), 856–861, 4th IFAC Symposium on Mechatronic Systems.
Vack, H. J., Nielsen, D. H., & Shurp, D. G. (1999). Kinetic patterns during stair ascent in patients with transtibial amputations using three different prostheses. *Jpo Journal of Prosthetics and Orthotics*, *11*, 57–62.
Vaskov, A. K., Vu, P. P., North, N., Davis, A. J., Kung, T. A., Gates, D. H., et al. (2022). Surgically implanted electrodes enable real-time finger and grasp pattern recognition for prosthetic hands. *IEEE Transactions on Robotics*.
Vathsangam, H., Emken, A., Spruijt-Metz, D., & Sukhatme, G. S. (2010). Toward free-living walking speed estimation using Gaussian process-based regression with on-body accelerometers and gyroscopes. In *2010 4th International conference on pervasive computing technologies for healthcare* (pp. 1–8). IEEE.
Villarreal, D. J., & Gregg, R. D. (2014). A survey of phase variable candidates of human locomotion. In *2014 36th Annual international conference of the ieee engineering in medicine and biology society* (pp. 4017–4021).
Villarreal, D., & Gregg, R. (2016). Unified phase variables of relative degree two for human locomotion. In *2016 38th Annual international conference of the ieee engineering in medicine and biology society* EMBC, (pp. 6262–6267). IEEE.
Villarreal, D. J., Poonawala, H. A., & Gregg, R. D. (2017). A robust parameterization of human gait patterns across phase-shifting perturbations. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *25*(3), 265–278.
Voloshina, A. S., & Collins, S. H. (2020). Lower limb active prosthetic systems—overview. *Wearable Robotics*, 469–486.
Vrieling, A., Van Keeken, H., Schoppen, T., Otten, E., Halbertsma, J., Hof, A., et al. (2007). Obstacle crossing in lower limb amputees. *Gait & Posture*, *26*(4), 587–594.
Vu, H. T. T., Dong, D., Cao, H.-L., Verstraten, T., Lefeber, D., Vanderborght, B., et al. (2020). A review of gait phase detection algorithms for lower limb prostheses. *Sensors*, *20*(14), 3972.
Vu, H. T. T., Gomez, F., Cherelle, P., Lefeber, D., Nowé, A., & Vanderborght, B. (2018). ED-FNN: A new deep learning algorithm to detect percentage of the gait cycle for powered prostheses. *Sensors*, *18*(7), 2389.
Wang, J., Kannape, O. A., & Herr, H. M. (2013). Proportional EMG control of ankle plantar flexion in a powered transtibial prosthesis. In *2013 IEEE 13th International conference on rehabilitation robotics* ICORR, (pp. 1–5).
Waters, R. L., & Mulroy, S. (1999). The energy expenditure of normal and pathologic gait. *Gait & Posture*, *9*(3), 207–231.
Waters, R. L., Perry, J., Antonelli, D. J., & Hislop, H. J. (1976). Energy cost of walking of amputees: the influence of level of amputation.. *The Journal of Bone and Joint Surgery. American*, *58 1*, 42–46.
Welker, C. G., Voloshina, A. S., Chiu, V. L., & Collins, S. H. (2021). Shortcomings of human-in-the-loop optimization of an ankle-foot prosthesis emulator: a case series. *Royal Society Open Science*, *8*(5), Article 202020.
Wen, Y., Li, M., Si, J., & Huang, H. (2020). Wearer-prosthesis interaction for symmetrical gait: a study enabled by reinforcement learning prosthesis control. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *28*(4), 904–913.
Wen, Y., Si, J., Brandt, A., Gao, X., & Huang, H. H. (2019). Online reinforcement learning control for the personalization of a robotic knee prosthesis. *IEEE Transactions on Cybernetics*, *50*(6), 2346–2356.
Wentink, E. C., Prinsen, E. C., Rietman, J. S., & Veltink, P. H. (2013). Comparison of muscle activity patterns of transfemoral amputees and control subjects during walking. *Journal of Neuroengineering and Rehabilitation*, *10*(1), 1–11.
Westervelt, E. R., Grizzle, J. W., Chevallereau, C., Choi, J. H., & Morris, B. (2018). *Feedback control of dynamic bipedal robot locomotion*. CRC Press.
Westervelt, E. R., Grizzle, J. W., & Koditschek, D. E. (2003). Hybrid zero dynamics of planar biped walkers. *IEEE Transactions on Automatic Control*, *48*(1), 42–56.
Windrich, M., Grimmer, M., Christ, O., Rinderknecht, S., & Beckerle, P. (2016). Active lower limb prosthetics: A systematic review of design issues and solutions. *BioMedical Engineering OnLine*, *15*.
Winter, D. A. (1983). Energy generation and absorption at the ankle and knee during fast, natural, and slow cadences.. *Clinical Orthopaedics and Related Research*, *175*, 147–154.
Winter, D. (1991). *The biomechanics and motor control of human gait: normal, elderly and pathological* (2nd). University of Waterloo Press.
Winter, D. (2009). *Biomechanics and motor control of human movement*. Wiley.
Wong, C. K., Wilska, J., & Stern, M. (2012). Balance, balance confidence, and falls using nonmicroprocessor and microprocessor knee prostheses: a case study after vascular amputation with 12-month follow-up. *JPO: Journal of Prosthetics and Orthotics*, *24*(1), 16–18.
Wu, R., Li, M., Yao, Z., Liu, W., Si, J., & Huang, H. (2022). Reinforcement learning impedance control of a robotic prosthesis to coordinate with human intact knee motion. *IEEE Robotics and Automation Letters*, *7*(3), 7014–7020.
Yeon, S. H., Shu, T., Song, H., Hsieh, T.-H., Qiao, J., Rogers, E. A., et al. (2021). Acquisition of surface emg using flexible and low-profile electrodes for lower extremity neuroprosthetic control. *IEEE Transactions on Medical Robotics and Bionics*, *3*(3), 563–572.
Young, A. J., & Hargrove, L. J. (2015). A classification method for user-independent intent recognition for transfemoral amputees using powered lower limb prostheses. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, *24*(2), 217–225.

Young, A. J., Simon, A. M., Fey, N. P., & Hargrove, L. J. (2014). Intent recognition in a powered lower limb prosthesis using time history information. *Annals of Biomedical Engineering*, *42*(3), 631–641.

Zhang, J., Fiers, P., Witte, K. A., Jackson, R. W., Poggensee, K. L., Atkeson, C. G., et al. (2017). Human-in-the-loop optimization of exoskeleton assistance during walking. *Science*, *356*(6344), 1280–1284.

Zhang, F., & Huang, H. (2013). Source selection for real-time user intent recognition toward volitional control of artificial legs. *IEEE Journal of Biomedical and Health Informatics*, *17*(5), 907–914.

Zhang, X., & Huang, H. (2015). A real-time, practical sensor fault-tolerant module for robust emg pattern recognition. *Journal of Neuroengineering and Rehabilitation*, *12*(1), 1–16.

Zhao, H., Ambrose, E., & Ames, A. D. (2017). Preliminary results on energy efficient 3D prosthetic walking with a powered compliant transfemoral prosthesis. In *Robotics and automation (ICRA), 2017 IEEE international conference on* (pp. 1140–1147). IEEE.

Zhao, H., Hereid, A., Ambrose, E., & Ames, A. D. (2016). 3D multi-contact gait design for prostheses: Hybrid system models, virtual constraints and two-step direct collocation. In *Decision and control (CDC), 2016 IEEE 55th conference on* (pp. 3668–3674). IEEE.

Zhao, H., Horn, J., Reher, J., Paredes, V., & Ames, A. D. (2016). Multicontact locomotion on transfemoral prostheses via hybrid system models and optimization-based control. *IEEE Transactions on Automation Science and Engineering*, *13*(2), 502–513.

Zhao, H., Horn, J., Reher, J., Paredes, V., & Ames, A. D. (2017). First steps toward translating robotic walking to prostheses: a nonlinear optimization based control approach. *Autonomous Robots*, *41*(3), 725–742.

Zhao, H., Reher, J., Horn, J., Paredes, V., & Ames, A. D. (2015). Realization of stair ascent and motion transitions on prostheses utilizing optimization-based control and intent recognition. In *Rehabilitation robotics (ICORR), 2015 IEEE International conference on* (pp. 265–270). IEEE.

Zhao, H., Wang, Z., Qiu, S., Wang, J., Xu, F., Wang, Z., et al. (2019). Adaptive gait detection based on foot-mounted inertial sensors and multi-sensor fusion. *Information Fusion*, *52*, 157–166.

Ziegler-Graham, K., MacKenzie, E. J., Ephraim, P. L., Travison, T. G., & Brookmeyer, R. (2008). Estimating the prevalence of limb loss in the United States: 2005 to 2050. *Archives of Physical Medicine and Rehabilitation*, *89*(3), 422–429.

Zihajehzadeh, S., & Park, E. J. (2016). Regression model-based walking speed estimation using wrist-worn inertial sensor. *PLoS One*, *11*(10), Article e0165211.