Instituto Tecnológico de Aeronáutica

Programa de Pós-Graduação em Engenharia de Infraestrutura Aeronáutica
Programa de Pós-Graduação em Engenharia Aeronáutica e Mecânica

Prova de Seleção – 1º semestre de 2021 – Questões de Matemática

16 de novembro de 2020

Nome do Candidato

Observações

1. Duração da prova: 90 minutos (uma hora e meia)

2. Não é permitido o uso de calculadoras ou outros dispositivos eletrônicos

3. Cada pergunta admite uma única resposta

4. Marque a alternativa que considerar correta no formulário Google enviado por e-mail

Questões em Inglês

1. Mr. Bertlmann is a physicist that has the habit of wearing socks of different colors. Despite this fact, he always put in his travel suitcase pairs of identical socks, in the sense that the interchange of the socks of a pair would not be noticed. If he attends to a congress lecture with four pairs of socks with four different colors, how many distinct ways can he chose to wear different colors in each foot?

    (a) 4
    (b) 6
    (c) 8
    (d) 12
    (e) 16

2. In a library, ten numbered books of equal size should fit in a bookshelf, but due to an error of a carpenter, only eight of those books fit in there. If a librarian should chose eight out of those ten books and put it in the shelf following the numbering order of those chosen books, in how many ways could he do it?

   (a) 19

   (b) 45

   (c) 90

   (d) 8!

   (e) 16

3. What is the best approximation of $\dfrac{1}{\sqrt{3} - \sqrt{2}}$?

   (a) 5/2

   (b) 2.8

   (c) 3

   (d) π

   (e) 7/2

4. What is the solution of $\dfrac{1}{x^{\ln(x)+2}} = e$?

   (a) 1/e

   (b) 1

   (c) 2

   (d) This equation has no real solution.

   (e) This equation has more than one solution.

5. In the triangle ABC shown in Figure 1, the side $AC = 12$ is tangent to the circle while $CD = 8$. IF $AD = 9$, what should be the length of AB?

   (a) 12

   (b) 12.5

   (c) 13

   (d) 13.5

   (e) Additional information is needed in order to calculate AB.

6. Let $a_n = 4, 1, 1/4, 1/16 \ldots$ be a geometric progression. What is the value of the infinite sum $a_0 - a_1 + a_2 - a_3 \ldots$?

   (a) 16/5

   (b) $2\sqrt{3}$

   (c) 3

   (d) 16/3

   (e) None of the above values.
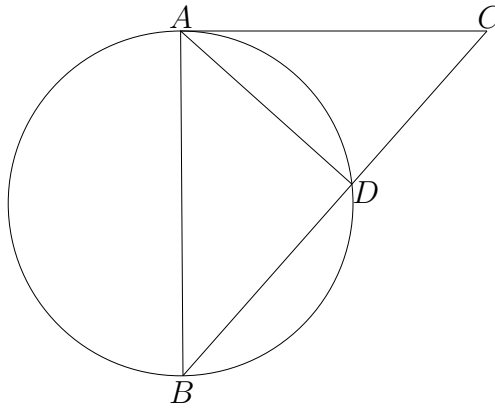
Figure 1: Circle and triangle

7. In order to determine if r is greater than zero, two statements are given:

    I. $rt = 18$

    II. $r + t = 11$,

    One can say that

    (a) statement I *alone* is sufficient, but statement II *alone* is *not* sufficient to answer the question asked;

    (b) statement II *alone* is sufficient, but statement I *alone* is *not* sufficient to answer the question asked;

    (c) *both* statements I and II *together* are sufficient to answer the question asked, but *neither* statement *alone* is sufficient;

    (d) *each* statement *alone* is sufficient to answer the question asked;

    (e) statements I and II *together* are *not* sufficient to answer the question asked, and additional data specific to the problem are needed.

8. If $\sqrt{3-x} = \sqrt{x} + 1$, then $x^2 =$?

    (a) 1

    (b) 2

    (c) $2 - x$

    (d) $2x - 2$

    (e) $3x - 1$

9. Each week, a shoes salesperson receives a commission equal to 15% of the first $800 in sales and 20% of all additional sales that week. What was the total sales he made, if his commission for the week was $ 320 ?

    (a) $ 1,000

    (b) $ 1,260

    (c) $ 1,300

    (d) $ 1,425

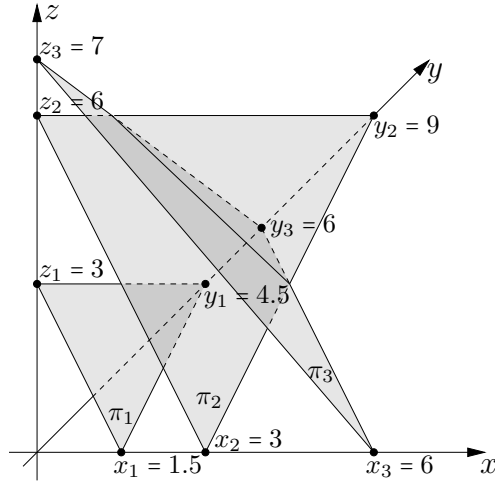   (e) $ 1,800

3

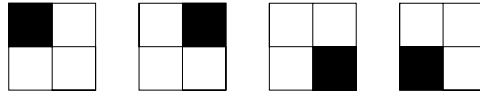Figure 2: First octant and planes π_1, π_2 and π_3

Figure 3: Four modes of painting one square

10. Figure 2 shows the first octant of the three-dimensional space ($x > 0, y > 0$ and $z > 0$) and the intercepts between three planes π_1, π_2, π_3 and the three cartesian axes. Each of these planes are associated to a linear equation in x, y and z and they form a system which

   (a) has no real solution.

   (b) has one real solution.

   (c) has two distinct real solutions.

   (d) has several distinct real solutions.

   (e) needs extra information in order to be discussed.

11. A block is composed by four squares, and each of them can be painted in black or white. Figure 3 shows the block with one square coloured in black. These four modes of painting which are not different, as they can be treated as only one mode of painting rotated by different angles. In how many different ways (which cannot be considered as rotations of the same modes) can we paint the four squares of the block?

   (a) 5

   (b) 6

   (c) 8

   (d) 12
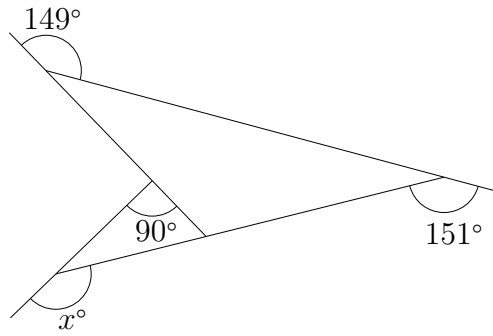
   (e) 16

4

Figure 4: Angles between lines

12. In Figure 4, what is the value of x?

    (a) 148

    (b) 149

    (c) 150

    (d) 151

    (e) 152

13. A company had two computers. The older one took 48 hours to develop a certain task of machine learning. The newer computer took 24 hours to develop de same task. The company acquired a newer computer, which took 16 hours to develop the same task. Supposing that the task can be completely subdivided in up to 96 equal and independent processes, what should be the minimal time necessary for the three computers to develop together this task?

    (a) 4 hours

    (b) 6 hours

    (c) 8 hours

    (d) 9 hours

    (e) 12 hours

14. Let r be the radius of a circle that has the same area of a regular hexagon with side l. What is the ratio r/l?

    (a) $\dfrac{\sqrt{27}}{2\pi}$

    (b) $\dfrac{\sqrt[4]{27}}{\sqrt{2\pi}}$

    (c) $\dfrac{3}{\pi}$

    (d) $\sqrt{\dfrac{3}{\pi}}$

    (e) 1

15. Which of the following polynomials has one common root with $x^2 - 5x + 6$?

   (a) $x^2 + 3x + 2$

   (b) $x^2 - x$

   (c) $x^2 + 5x + 6$

   (d) $x^2 - 7x + 12$

   (e) $x^2 - 9x + 20$

16. Which of the following numbers is prime?

   (a) 51

   (b) 91

   (c) 181

   (d) 289

   (e) 1001