

# Documentación Legendary Grandpupils

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## 1. Plantilla

```
1. #include <bits/stdc++.h>
2. #include <algorithm>
3. #include <bitset>
4. #include <cmath>
5. #include <cstdio>
6. #include <cstring>
7. #include <deque>
8. #include <fstream>
9. #include <functional>
10. #include <iomanip>
11. #include <iostream>
12. #include <limits.h>
13. #include <map>
14. #include <math.h>
15. #include <numeric>
16. #include <queue>
17. #include <set>
18. #include <sstream>
19. #include <stack>
20. #include <stdio.h>
21. #include <stdlib.h>
22. #include <string>
23. #include <utility>
24. #include <vector>
25.
26. #define PI 3.14159265358979323846
27. #define EPS 1e-6
28. #define INF 1000000000
29.
30. #define _ ios_base::sync_with_stdio(0), cin.tie(0), cin.tie(0), cout.tie(0), cout.precision(15);
31. #define FOR(i, a, b) for(int i=int(a); i<int(b); i++)
32. #define RFOR(i, a, b) for(int i=int(a)-1; i>=int(b); i--)
33. #define FORC(cont, it) for(decltype((cont).begin()) it = (cont).begin(); it != (cont).end(); it++)
34. #define RFORC(cont, it) for(decltype((cont).rbegin()) it = (cont).rbegin(); it != (cont).rend(); it++)
35. #define pb push_back
36.
37. using namespace std;
38.
39. typedef long long ll;
40. typedef pair<int, int> ii;
41. typedef vector<int> vi;
42.
43. #define MAXN 10
44. #define MOD 1000000007
45.
46. int main() { _
47.
48.     return 0;
49. }
```

## 2. DP

### NK

```
1.  ll fact[MAXN], inv[MAXN], pascal[MAXN][MAXN];
2.
3.  ll ppow(ll n, ll p) {
4.      ll ret = 1;
5.      while (p) {
6.          if (p & 1) ret = (ret*n) % MOD;
7.          n = (n*n) % MOD;
8.          p>>=1;
9.      }
10.     return ret;
11. }
12.
13. void generaFact() {
14.     fact[0] = 1;
15.     FOR(i, 1, MAXN)
16.         fact[i] = (fact[i-1] * i) % MOD;
17.
18.     inv[MAXN-1] = ppow(fact[MAXN-1], MOD - 2);
19.     for(int i = MAXN-1; i>0; i--)
20.         inv[i-1] = inv[i] * i % MOD;
21. }
22.
23. ll NK(ll n, ll k) {
24.     return fact[n] * inv[k] % MOD * inv[n-k] % MOD;
25. }
26.
27. // pascal i,j corresponde a la formula de combinatoria i!/(j!*(i-j)!), AKA. (N, K)
28. void trianguloPascal() {
29.     pascal[0][0] = 1;
30.     FOR(i, 1, MAXN) {
31.         pascal[i][0] = 1;
32.         FOR(j, 1, i+1) {
33.             pascal[i][j] = (pascal[i - 1][j - 1] + pascal[i - 1][j]) % MOD;
34.         }
35.     }
36. }
37.
38. int main(){
39.     ll n, k;
40.     generaFact();
41.     trianguloPascal();
42.     while (scanf("%lld %lld", &n, &k) != EOF) {
43.         printf("%lld\n", NK(n, k));
44.         printf("%lld\n", pascal[n][k]);
45.     }
46.
47.     return 0;
48. }
```

## Longest Common Subsequence

```
1.  #define MAXN 1000005
2.  #define MOD 1000000007
3.
4.  /* Returns Length of LCS for X[0..n-1], Y[0..m-1] */
5.  int lcs(string s1, string s2) {
6.      int n = s1.length();
7.      int m = s2.length();
8.      int L[n + 1][m + 1];
9.      memset(L, 0, sizeof(L));
10.
11.      /* Following steps build L[n+1][m+1] in bottom up fashion. Note
12.      that L[i][j] contains Length of LCS of X[0..i-1] and Y[0..j-1] */
13.      FOR(i, 1, n+1) {
14.          FOR(j, 1, m+1) {
15.              if (s1[i-1] == s2[j-1])
16.                  L[i][j] = L[i-1][j-1] + 1;
17.              else
18.                  L[i][j] = max(L[i-1][j], L[i][j-1]);
19.          }
20.      }
21.
22.      /* L[n][m] contains Length of LCS for X[0..n-1] and Y[0..m-1] */
23.      return L[n][m];
24.  }
25.
26.  int main() {
27.      string s1, s2;
28.      while (cin >> s1 >> s2)
29.          cout << lcs(s1, s2) << endl;
30.  }
```

## Longest Common Substring

```
1.  #define MAXN 10
2.  #define MOD 1000000007
3.
4.  /* Returns Length of Longest common substring of X[0..n-1] and Y[0..m-1] */
5.  // Create a table to store lengths of longest common suffixes of
6.  // substrings. Notethat LCSuff[i][j] contains length of longest
7.  // common suffix of X[0..i-1] and Y[0..j-1]. The first row and
8.  // first column entries have no logical meaning, they are used only
9.  // for simplicity of program
10. int LCSuff(string X, string Y) {
11.     int n = X.length();
12.     int m = Y.length();
13.
14.     int lcs[n+1][m+1];
15.     memset(lcs, 0, sizeof(lcs));
16.     int result = 0; // To store length of the longest common substring
17.
18.     /* Following steps build lcs[n+1][m+1] in bottom up fashion. */
19.     FOR(i, 1, n+1) {
20.         FOR(j, 1, m+1) {
21.             if (X[i-1] == Y[j-1]) {
22.                 lcs[i][j] = lcs[i-1][j-1] + 1;
23.                 result = max(result, lcs[i][j]);
24.             }
25.             else {
26.                 lcs[i][j] = 0;
27.             }
28.         }
29.     }
30.
31.     return result;
32. }
33.
34. int main() {
35.     string X, Y;
36.     cin >> X >> Y;
37.     cout << LCSuff(X, Y) << endl;
38.
39.     return 0;
40. }
```

## TSP

```
1. #define MAXN 25
2.
3. int N;
4. int dptsp[MAXN][1 << MAXN], costViajero[MAXN][MAXN];
5. int maskInicial;
6.
7. // Calcula costos de todos los nodos a todos los nodos.
8. void floyd() {
9.     FOR(k, 0, N) {
10.         FOR(i, 0, N) {
11.             FOR(j, 0, N) {
12.                 costViajero[i][j] = min(costViajero[i][j],
13. costViajero[i][k] + costViajero[k][j]);
14.             }
15.         }
16.     }
17.
18. // Bits prendidos significa ciudad que aun no se visita.
19. int tsp(int n, int mask) {
20.     if (!mask) return 0;
21.     int &c = dptsp[n][mask];
22.     if (~c) return c;
23.     c = INF;
24.     FOR(i, 0, N) {
25.         if (mask & 1 << i) {
26.             c = min(c, tsp(i, mask ^ 1 << i) + costViajero[n][i]);
27.         }
28.     }
29.     return c;
30. }
31.
32. int main() {
33.     int ans;
34.
35.     floyd();
36.     maskInicial = (1 << N) - 1, ans = INF;
37.     FOR(i, 0, N) {
38.         ans = min(ans, tsp(i, maskInicial ^ 1 << i));
39.     }
40.
41.     return 0;
42. }
```

### 3. Geometry

```
1. typedef long long ll;
2.
3. double degToRad(double theta){
4.     return theta * PI / 180.0;
5. }
6.
7. double radToDeg(double theta){
8.     return theta * 180 / PI;
9. }
10.
11. // ***** Struct: Punto *****
12. struct point {
13.     double x, y;
14.
15.     point() {}
16.     point(double xx, double yy) {
17.         x = xx;
18.         y = yy;
19.     }
20.     point inf() {
21.         x = INF;
22.         y = INF;
23.     }
24.
25.     double dist(point p) {
26.         return hypot(x - p.x, y - p.y);
27.     }
28.
29.     point rotate(point pivot, point p, double ang) {
30.         double s = sin(ang);
31.         double c = cos(ang);
32.
33.         // translate point back to origin:
34.         p.x -= pivot.x;
35.         p.y -= pivot.y;
36.
37.         // rotate point
38.         double xaux = p.x * c - p.y * s;
39.         double yaux = p.x * s + p.y * c;
40.
41.         // translate point back:
42.         p.x = xaux + pivot.x;
43.         p.y = yaux + pivot.y;
44.         return p;
45.     }
46.
47.     double getAngle(point pivot, point p) {
48.         return atan2(p.y - pivot.y, p.x - pivot.x);
49.     }
50.
51.     void swap() {
52.         double aux = x;
53.         x = y;
54.         y = aux;
55.     }
56.
57.     void swap(point &a, point &b) {
58.         point aux = a;
59.         a = b;
60.         b = aux;
61.     }
62.
63.     point punto(point const &p) const {
64.         return point(x * p.x, y * p.y);
65.     }
66.
67.     double cruz(point const &p) const {
68.         return x * p.y - y * p.x;
69.     }
70.
71.     double cruz(point const &p1, point const &p2) const {
72.         return (p1.x - x) * (p2.y - y) - (p1.y - y) * (p2.x - x);
```



```

73.     }
74.
75.     point operator +(point const &p) const {
76.         return point(x + p.x, y + p.y);
77.     }
78.
79.     point operator -(point const &p) const {
80.         return point(x + p.x, y + p.y);
81.     }
82.
83.     point operator /(double d) const {
84.         return point(x / d, y / d);
85.     }
86.
87.     bool operator <(point const &p) const {
88.         return (x - p.x) > EPS && x < p.x ||
89.             fabs(x - p.x) < EPS && (y - p.y) > EPS && y < p.y;
90.     }
91.
92.     bool operator ==(point p) const {
93.         return fabs(x - p.x) < EPS && fabs(y - p.y) < EPS;
94.     }
95.
96.     void print() {
97.         cout << x << "\t" << y << endl;
98.     }
99. };
100. // *****
101.
102. // ***** Struct: line *****
103. struct line {
104.     point p1, p2;
105.     double a, b, c;
106.
107.     line() {
108.         p1.inf();
109.         p2.inf();
110.     }
111.     line(double A, double B, double C) {
112.         a = A;
113.         b = B;
114.         c = C;
115.
116.         p1.inf();
117.         p2.inf();
118.     }
119.     line(point pp1, point pp2) {
120.         p1 = pp1;
121.         p2 = pp2;
122.
123.         if (p2 < p1)
124.             swap(p1, p2);
125.
126.         if(fabs(p1.x - p2.x) < EPS) {
127.             a = 1.0;
128.             b = 0.0;
129.             c = -p1.x;
130.         }
131.         else {
132.             a = -(double)(p1.y - p2.y) / (p1.x - p2.x);
133.             b = 1.0;
134.             c = -(double)(a * p1.x) - (b * p1.y);
135.         }
136.     }
137.
138.     line perpendicular(point p) {
139.         double c = -(a * p.x - b * p.y);
140.         return line(-b, a, c);
141.     }
142.
143.     bool sonParalelas(line l1, line l2){
144.         return fabs(l1.a - l2.a) < EPS &&
145.             fabs(l1.b - l2.b) < EPS;
146.     }

```

```

147.
148.     bool sonIguales(line l1, line l2){
149.         return sonParalelas(l1, l2) &&
150.             fabs(l1.c - l2.c) < EPS;
151.     }
152.
153.     bool insideLine(point p) {
154.         return ((p1.x - EPS <= p.x && p.x <= p2.x + EPS ||
155.             p2.x - EPS <= p.x && p.x <= p1.x + EPS) &&
156.
157.             (p1.y - EPS <= p.y && p.y <= p2.y + EPS ||
158.             p2.y - EPS <= p.y && p.y <= p1.y + EPS));
159.     }
160.
161.     bool intercectan(line l1, line l2, point &p){
162.         if (sonParalelas(l1, l2))
163.             return false;
164.
165.         p.x = (l2.b * l1.c - l1.b * l2.c) /
166.             (l2.a * l1.b - l1.a * l2.b);
167.
168.         if(fabs(l1.b) > EPS)
169.             p.y = -(l1.a * p.x + l1.c) / l1.b;
170.         else
171.             p.y = -(l2.a * p.x + l2.c) / l2.b;
172.
173.         return l2.insideLine(p);
174.     }
175. };
176. // *****
177.
178. // ***** Struct: triangle *****
179. struct triangle {
180.     point p1, p2, p3;
181.     double a, b, c;
182.     double A, B, C;
183.     double area, perimetro;
184.     int type; // equilatero=1, isoseles=2, equilatero=3
185.     bool rect; // triangulo rectangulo o no
186.
187.     triangle() {}
188.     triangle(double aa, double bb, double cc) {
189.         a = aa;
190.         b = bb;
191.         c = cc;
192.         sort();
193.
194.         act();
195.     }
196.
197.     triangle(point pp1, point pp2, point pp3) {
198.         p1 = pp1;
199.         p2 = pp2;
200.         p3 = pp3;
201.
202.         a = p1.dist(p2);
203.         b = p1.dist(p3);
204.         c = p2.dist(p3);
205.         sort();
206.
207.         act();
208.     }
209.
210.     double innerCircleRadio(){
211.         return area / (perimetro * 2.0);
212.     }
213.
214.     double outterCircleRadio(){
215.         return a * b * c / (4 * area);
216.     }
217.
218.     int getType() {
219.         if(a==b && b==c) return 1;
220.         else if(a==b || b==c) return 2;

```

```

221.         return 3;
222.     }
223.
224.     bool isRight(){
225.         return (a*a + b*b - c*c < EPS);
226.     }
227.
228.     void sort() {
229.         if (a > b)
230.             swap(a, b);
231.         if (a > c)
232.             swap(a, c);
233.         if (b > c)
234.             swap(b, c);
235.     }
236.
237.     double getPerimetro() {
238.         return a + b + c;
239.     }
240.
241.     double getArea() {
242.         double s = perimetro / 2.0;
243.         return sqrt(s * (s-a) * (s-b) * (s-c));
244.     }
245.
246.     double getAngles() {
247.         double aa = a * a;
248.         double bb = b * b;
249.         double cc = c * c;
250.
251.         A = acos(bb + cc - aa) / (2 * b * c);
252.         B = acos(aa + cc - bb) / (2 * a * c);
253.         C = acos(aa + bb - cc) / (2 * a * b);
254.     }
255.
256.     void act() {
257.         type = getType();
258.         rect = isRight();
259.         perimetro = getPerimetro();
260.         area = getArea();
261.         getAngles();
262.     }
263. };
264. // *****
265.
266.
267. // ***** Struct: circle *****
268. struct circle {
269.     point c;
270.     double r, circ, area;
271.
272.     circle() {
273.         c.x = c.y = 0;
274.         r=1;
275.
276.         act();
277.     }
278.     circle(point p, double rr){
279.         c = p;
280.         r = rr;
281.
282.         act();
283.     }
284.
285.     int insideCircle(point p){// 0-Dentro, 1-Borde, 2-Fuera
286.         int dx = p.x - c.x, dy = p.y - c.y;
287.         int Euc = dx*dx + dy*dy, rSq=r*r;
288.         double dist = c.dist(p);
289.         if (fabs(dist - r) < EPS) return 1;
290.         if (dist < r) return 0;
291.         return 2;
292.     }
293.
294.     double getArc(double deg) {

```

```

295.     return circ * deg / 360.0;
296. }
297. double getChord(double deg) {
298.     return 2.0 * r * r * (1 - cos(degToRad(deg)));
299. }
300. double getSector(double deg) {
301.     return area * deg / 360.0;
302. }
303. double getSegment(double deg) {
304.     triangle t(r, r, getChord(deg));
305.     return getSector(deg) - t.getArea();
306. }
307.
308. // 0 - No intersectan
309. // 1 - Intersecta 1 punto
310. // 2 - Intersecta 2 puntos
311. // 3 - Mismo circulo
312. // 4 - Circulo dentro del otro
313. int intersectCirc(circle const &cir, point &p1, point &p2) {
314.     double d = c.dist(cir.c);
315.
316.     if (d > r + cir.r) // No se tocan
317.         return 0;
318.     if ((r - cir.r) < EPS && c == cir.c) // Mismo circulo
319.         return 3;
320.     if (d + min(r, cir.r) < max(r, cir.r)) // Circulo contiene otro
321.         return 4;
322.
323.     double a = (r * r - cir.r * cir.r + d * d) / (2.0 * d);
324.     double h = sqrt(r * r - a * a);
325.
326.     // find p2
327.     point pp1(c.x + (a * (cir.c.x - c.x)) / d, c.y + (a * (cir.c.y - c.y)) / d);
328.
329.     // find intersection points p3
330.     p1 = point(pp1.x + (h * (cir.c.y - c.y) / d),
331.               pp1.y - (h * (cir.c.x - c.x) / d));
332.
333.     p2 = point(pp1.x - (h * (cir.c.y - c.y) / d),
334.               pp1.y + (h * (cir.c.x - c.x) / d));
335.
336.     if(fabs(d - r - cir.r) < EPS)
337.         return 1;
338.     return 2;
339. }
340.
341. double getArea() {
342.     return PI * r * r;
343. }
344.
345. double getCirc() {
346.     return PI * 2. * r;
347. }
348.
349. bool operator ==(circle const &cir) const{
350.     return c == cir.c && fabs(r - cir.r) < EPS;
351. }
352.
353. void act() {
354.     area = getArea();
355.     circ = getCirc();
356. }
357. };
358.
359. // *****
360.
361. typedef vector<point> vp;
362. typedef vector<line> vl;
363. typedef vector<triangle> vt;
364. typedef vector<circle> vc;
365.
366. // ***** Convex Hull *****
367. point p0;
368.

```

```

369. int orientation(point p, point q, point r) {
370.     int val = (q.y - p.y) * (r.x - q.x) -
371.               (q.x - p.x) * (r.y - q.y);
372.
373.     if (val == 0) return 0; // colinear
374.     return (val > 0)? 1: 2; // clock or counterclock wise
375. }
376.
377. bool compAng(const point p1, const point p2) {
378.     int o = orientation(p0, p1, p2);
379.
380.     return (o == 0 &&
381.            hypot(p0.x - p2.x, p0.y - p2.y) >= hypot(p0.x - p1.x, p0.y - p1.y)) ||
382.            (o == 2);
383. }
384.
385. bool compY(const point p1, const point p2) {
386.     return p1.y < p2.y || (p1.y == p2.y && p1.x < p2.x);
387. }
388.
389. vp convexHull(vp v) {
390.     vp c = v;
391.     int n = v.size(), mini = 0;
392.     p0 = c[0];
393.
394.     FOR(i, 1, n) {
395.         int y = c[i].y;
396.
397.         if (compY(c[i], p0)) {
398.             p0 = c[i];
399.             mini = i;
400.         }
401.     }
402.
403.     swap(c[0], c[mini]);
404.
405.     sort(c.begin() + 1, c.end(), compAng);
406.
407.     vp ans;
408.     ans.pb(c[0]);
409.     ans.pb(c[1]);
410.     ans.pb(c[2]);
411.
412.     FOR (i, 3, n) {
413.         while (orientation(ans[ans.size()-2], ans[ans.size()-1], c[i]) != 2) {
414.             ans.erase(ans.end());
415.         }
416.         ans.pb(c[i]);
417.     }
418.     ans.pb(ans[0]);
419.
420.     return ans;
421. }
422. // *****
423.
424.
425. // ***** Convex Hull v2 *****
426. double cross(const point &O, const point &A, const point &B) {
427.     return (A.x - O.x) * (B.y - O.y) - (A.y - O.y) * (B.x - O.x);
428. }
429.
430. vp convexHull(vp &P) {
431.     int n = P.size(), k = 0;
432.     vp H(2*n);
433.     sort(P.begin(), P.end());
434.     FOR(i, 0, n) {
435.         while (k >= 2 && cross(H[k-2], H[k-1], P[i]) <= 0) k--;
436.         H[k++] = P[i];
437.     }
438.     for (int i = n-2, t = k+1; i >= 0; i--) {
439.         while (k >= t && cross(H[k-2], H[k-1], P[i]) <= 0) k--;
440.         H[k++] = P[i];
441.     }
442.     H.resize(k);

```

```

443.     return H;
444. }
445. // *****
446.
447.
448.
449. // ***** Revisa si es CW *****
450. bool cw(vp v) {
451.     double ret = 0;
452.     FOR(i, 1, v.size()) {
453.         ret += v[i].x * v[i-1].y - v[i-1].x * v[i].y;
454.     }
455.     return ret < 0;
456. }
457. // *****
458.
459.
460. // ***** Revisa si un punto esta dentro en un poligono *****
461. bool PointInPolygon(vp v, point p) {
462.     bool ret = 0;
463.     int j = 0;
464.     FOR(i, 1, v.size()) {
465.         if(((v[i].y > p.y) != (v[j].y > p.y)) &&
466.            (p.x < (v[j].x - v[i].x) * (p.y - v[i].y) / (v[j].y - v[i].y) + v[i].x)
467.            )
468.             ret = !ret;
469.         j++;
470.     }
471.     return ret;
472. }
473. // *****
474.
475.
476. // ***** Calcular perimetro de un poligono *****
477. double perimetro(vp v) {
478.     double ret = 0;
479.     FOR(i, 0, v.size() - 1) {
480.         ret += v[i].dist(v[i + 1]);
481.     }
482.     return ret;
483. }
484. // *****
485.
486.
487. // ***** Calcular area de un poligono *****
488. double area(vp v) {
489.     double ret = 0;
490.     FOR(i, 1, v.size()) {
491.         ret += v[i].x * v[i-1].y - v[i-1].x * v[i].y;
492.     }
493.     return fabs(ret) / 2.0;
494. }
495. // *****
496.
497.
498.
499.
500. int main () { _
501.     point p(1, 2), p2(2,3);
502.     cout << p.x << " " << p.y << endl;
503.
504.     double ang = p.getAngle(p, p2);
505.     cout << p.getAngle(p, p2) * 180 / PI << endl;
506.     p.rotate(p, p2, -ang).print();
507. }

```

## 4. Graph's Theory

### DSU on Trees

```
1. const int maxN = 1e5 + 9, MOD = 1e9 + 7, INF = 2e7;
2. int n, comp[maxN], k, edges[maxN][2], sz[maxN], bg[maxN], answer[maxN], comps;
3. vector<int> adj[maxN];
4.
5. void precalc(int nd){
6.     //In this function, we calculate size and index of biggest child, sz and bg, resp
7.     sz[nd] = 1;
8.     bg[nd] = -1;
9.     for(int sn: adj[nd]){
10.         precalc(sn);
11.         if(bg[nd] == -1 || sz[bg[nd]] < sz[sn]) bg[nd] = sn;
12.         sz[nd] += sz[sn];
13.     }
14. }
15.
16. //THESE TWO FUNCTIONS DEPEND ON THE OPERATION OF THE PROBLEM. IN THIS EXAMPLE, IT IS
    ABOUT NODE DSU
17. int find(int u){
18.     if(comp[u] == u) return u;
19.     return comp[u] = find(comp[u]);
20. }
21.
22. void unite(int u, int v){
23.     comp[u] = find(v);
24.     comps--;
25. }
26.
27. void consider(int nd){
28.     //CONSIDERING OPERATION OF NODE ND
29.     if(find(edges[nd][0]) != find(edges[nd][1]))
30.         unite(find(edges[nd][0]), find(edges[nd][1]));
31.     //CONSIDERING OPERATION OF NODE ND
32. }
33.
34. void bruta(int nd){
35.     consider(nd);
36.     for(int sn: adj[nd])
37.         bruta(sn);
38. }
39.
40. void limpia(int nd){
41.
42.     //UNCONSIDERING OPERATION OF NODE ND
43.     comp[edges[nd][0]] = edges[nd][0];
44.     comp[edges[nd][1]] = edges[nd][1];
45.     //UNCONSIDERING OPERATION OF NODE ND
46.
47.     for(int sn: adj[nd])
48.         limpia(sn);
49. }
50.
51. void dfs(int nd, bool keep){
52.     //WHERE WE ACTUALLY CALCULATE THE ANSWER.
53.
54.     //SOLVE RECURSIVELY O(NLGN) FOR ALL CHILDREN EXCEPT BIGGEST NODES, DELETING
    AFTERWARDS
55.     for(int sn: adj[nd])
56.         if(sn != bg[nd])
57.             dfs(sn, 0);
58.
59.     //SOVE REC. FOR BIGGEST NODES O(NLGN), NOT DELETING AFTERWARDS
```

```

60.     if(bg[nd] != -1) dfs(bg[nd], 1);
61.
62.     //RECONSIDER CHILDREN IN O(N)
63.     for(int sn: adj[nd])
64.         if(sn != bg[nd])
65.             bruta(sn);
66.
67.     //CONSIDER THIS NODE
68.     consider(nd);
69.
70.     answer[nd] = comps;
71.
72.     //IF DELETION IS REQUESTED, DELETE. NOTE LIMPIA(ND) IS THE KEY, COMPS = K IS
SPECIFICALLY ABOUT THE PROBLEM
73.     if(!keep) limpia(nd), comps = k;
74. }
75.
76. int main(){
77.     ios::sync_with_stdio(false);
78.     cin.tie(0);
79.     cout.tie(0);
80.
81.     cin >> n >> k;
82.
83.     for(int i = 1, pa; i < n; i++){
84.         cin >> pa;
85.         pa--;
86.         adj[pa].push_back(i);
87.     }
88.
89.     for(int i = 0; i < k; i++)
90.         comp[i] = i;
91.
92.     comps = k;
93.
94.     for(int i = 0, u, v; i < n; i++){
95.         cin >> u >> v;
96.         u--;
97.         v--;
98.         if(u == -1) u = v = k;
99.         else edges[i][0] = u, edges[i][1] = v;
100.
101.     }
102.
103.     precalc(0);
104.
105.     dfs(0, 1);
106.
107.     for(int i = 0; i < n; i++) cout << answer[i] << '\n';
108.
109. }

```



## Centroid decomposition

```
1. //CENTROID DECOMPOSITION BY ADANCITO
2.
3. //NOTES:
4. /*
5.  -The consider() method contains an algorithm specifically thought of to count pairs
   in trees which fulfill a certain characteristic (for further details go to method).
   However, since this method will be called for all centroids, it can be changed to
   anything the problem needs to do.
6.  -If consider()'s efficiency is  $O(F(n))$ , then the overall complexity is  $O(F(n) * \lg n)$ .
7.  */
8. const int maxN = 5e5 + 6;
9. vector<int> adj[maxN];
10. int n, sz[maxN], arr[maxN];
11.
12. bool dead[maxN];
13.
14. void precalc(int nd, int an){
15.     sz[nd] = 1;
16.     for(int sn: adj[nd]){
17.         if(sn != an && !dead[sn]){
18.             precalc(sn, nd);
19.             sz[nd] += sz[sn];
20.         }
21.     }
22. }
23.
24. int getCentroid(int nd, int an, int tot){
25.     for(int sn: adj[nd]){
26.         if(sn != an && !dead[sn] && sz[sn] * 2 > tot){
27.             return getCentroid(sn, nd, tot);
28.         }
29.     }
30.     return nd;
31. }
32.
33. ll ans = 0;
34.
35. int arr1[maxN], arr2[maxN], curs, ots;
36.
37. void dfs(int nd, int an, int lev){
38.     // example:
39.     while(curs <= lev) arr2[curs++] = 0;
40.     arr2[lev]++;
41.     //bucket in arr2[lev] whatever nd represents. Afterwards, just do the same with
    children
42.
43.     for(int sn: adj[nd]){
44.         if(sn != an && !dead[sn]){
45.             dfs(sn, nd, lev);
46.         }
47.     }
48.
49. }
50.
51. void mergeVec(){
52.     //Accumulating arrays to keep the consider() method linear
53.     while(ots < curs) arr1[ots++] = 0;
54.     for(int i = 0; i < curs; i++){
55.         arr1[i] += arr2[i];
56.     }
57. }
58.
59. /*
```

60. *The most popular CD problem type is pair-counting in trees. This sample is a basic technique that uses bucketing and accumulating. Today, we will count pairs of trees whose path length is odd.*

```
61.  */
62. void consider(int nd){
63.
64.     //ots is the size for arr1, curs is the size for arr2
65.
66.     for(int sn: adj[nd]){
67.         if(!dead[sn]){
68.             curs = 0;
69.             dfs(sn, nd, 0);
70.
71.             //example:
72.
73.             for(int j = 0; j < curs; j++){
74.                 ans += ll(arr1[j]) * arr2[j];
75.             }
76.             //iterate through arr2, and multiply by frequency in arr1 at the bucket
place it corresponds
77.
78.             mergeVec();
79.
80.         }
81.     }
82. }
83.
84. void solve(int nd){
85.     precalc(nd, -1);
86.     nd = getCentroid(nd, -1, sz[nd]);
87.
88.     consider(nd);
89.     dead[nd] = 1;
90.
91.     for(int sn: adj[nd]){
92.         if(!dead[sn]){
93.             solve(sn);
94.         }
95.     }
96. }
97.
98. int main() {
99.     ios::sync_with_stdio(false);
100.    cin.tie(0), cout.tie(0);
101.
102.    cin >> n;
103.
104.    for(int i = 0; i < n; i++){
105.        cin >> arr[i];
106.    }
107.
108.
109.    for(int i = 1; i < n; i++){
110.        int u, v;
111.        cin >> u >> v, u--, v--;
112.        adj[u].push_back(v);
113.        adj[v].push_back(u);
114.    }
115.
116.    solve(0);
117.
118. }
```

## HLD

```

1.  const int maxN = 1e4 + 3, MOD = 1e9 + 9, LG = 21;
2.  //NOTE: maxN>2^LG must be held, remember maxN is twice ur biggest polymom
3.
4.  int he[maxN], par[maxN], ind[maxN], cs[maxN], heavy[maxN], head[maxN], n,
    T[maxN << 1];
5.  vector<int> adj[maxN];
6.  int ed[maxN];
7.
8.  int dfs(int nd){
9.      int ret = 1, heaS = 0, cr;
10.     for(int sn: adj[nd]) if(sn != par[nd]){
11.         par[sn] = nd, he[sn] = he[nd] + 1, cr = dfs(sn), ret += cr;
12.         if(heaS < cr) heaS = cr, heavy[nd] = sn;
13.     }
14.     return ret;
15. }
16.
17. void setPaths(){
18.     memset(heavy, -1, sizeof(heavy)), par[0] = -1, he[0] = 0, dfs(0);
19.     for(int i = 0, ct = 0; i < n; i++){
20.         if(par[i] == -1 || heavy[par[i]] != i)
21.             for(int j = i; j != -1; j = heavy[j])
22.                 head[j] = i, ind[j] = ct++;
23.     }
24.
25. int query(int l, int r){
26.     int ret = 0;
27.     for(l += n, r += n; l < r; l >>= 1, r >>= 1){
28.         if(l & 1) ret = max(ret, T[l++]);
29.         if(r & 1) ret = max(ret, T[--r]);
30.     }
31.     return ret;
32. }
33.
34. void update(int w, int x){
35.     w += n, T[w] = x;
36.     while(w > 1)
37.         w >>= 1, T[w] = max(T[w << 1], T[w << 1 | 1]);
38. }
39.
40. int ans(int u, int v){
41.     int ret = 0;
42.     for(; head[u] != head[v]; v = par[head[v]]){
43.         if(he[head[u]] > he[head[v]]) swap(u, v);
44.         ret = max(ret, query(ind[head[v]], ind[v] + 1));
45.     }
46.     if(he[u] > he[v]) swap(u, v);
47.     return max(ret, query(ind[u] + 1, ind[v] + 1));
48. }
49.
50. int main(){
51.     ios::sync_with_stdio(false);
52.     cin.tie(0), cout.tie(0);
53.
54.     int t;
55.     cin >> t;
56.     while(t--){
57.         cin >> n;
58.
59.         for(int i = 0; i < n; i++) adj[i].clear();
60.
61.         for(int i = 1, u, v; i < n; i++){
62.             cin >> u >> v >> cs[i];

```

```

63.         u--, v--;
64.         adj[u].push_back(v), adj[v].push_back(u);
65.         ed[i] = ii(u, v);
66.     }
67.
68.     setPaths();
69.
70.     for(int i = 1; i < n; i++){
71.         if(he[ed[i].fi] < he[ed[i].se]) swap(ed[i].fi, ed[i].se);
72.         update(ind[ed[i].fi], cs[i]);
73.     }
74.
75.     while(true){
76.         string inp;
77.         int a, b;
78.         cin >> inp;
79.         if(inp == "DONE") break;
80.         cin >> a >> b;
81.         if(inp == "CHANGE"){
82.             update(ind[ed[a].fi], b);
83.         }
84.         else{
85.             cout << ans(--a, --b) << '\n';
86.         }
87.     }
88.
89. }
90.
91. }

```

## Isomorphic Rooted Tree Test

```
1.  const int maxN = 1e5 + 3, maxV = 1e6;
2.
3.  int n, rta, rtb, mla, mlb;
4.
5.  struct El{
6.      int uu;
7.      vector<int> vc;
8.      El(int uu): uu(uu), vc(vector<int>()){}
9.      bool operator!=(const El &ot) const{
10.         return vc != ot.vc;
11.     }
12.     bool operator<(const El &ot) const{
13.         return vc < ot.vc;
14.     }
15. };
16. bool notequal(const vector<El> &a, const vector<El> &b){
17.     if(a.size() != b.size()) return true;
18.     for(int i = 0; i < a.size(); i++){
19.         if(a[i] != b[i]) return true;
20.     }
21.     return false;
22. }
23. vector<int> one[maxN], two[maxN];
24. vector<El> vcs[2][maxN];
25. int le[maxN];
26.
27. void getinp(vector<int> *adj, int &rt){
28.     for(int i = 0, pa; i < n; i++){
29.         cin >> pa;
30.         if(!pa) rt = i;
31.         else adj[--pa].push_back(i);
32.     }
33. }
34.
35. void dfs(vector<int> *adj, vector<El> *vcc, int nd, int ct, int &bes){
36.     bes = max(bes, ct);
37.     for(int sn: adj[nd]){
38.         vcc[ct + 1].push_back(El(int(vcc[ct].size() - 1)));
39.         dfs(adj, vcc, sn, ct + 1, bes);
40.     }
41. }
42.
43. bool isom(){
44.     for(int r = max(mla, mlb); r >= 0; r--){
45.         for(int u = 0; u < 2; u++) sort(vcs[u][r].begin(), vcs[u][r].end());
46.         //cout << "good " << r << '\n';
47.         if(notequal(vcs[0][r], vcs[1][r])) return false;
48.         for(int u = 0; u < 2 && r; u++){
49.             for(int i = 0, j = 0; i < vcs[u][r].size(); i = j){
50.                 for(;j<vcs[u][r].size() && vcs[u][r][i].vc == vcs[u][r][j].vc; j++){
51.                     int uu = vcs[u][r][j].uu;
52.                     vcs[u][r - 1][uu].vc.push_back(i);
53.                 }
54.             }
55.         }
56.     }
57.     return true;
58. }
59.
60. int main(){
61.     ios::sync_with_stdio(false);
62.     cin.tie(0), cout.tie(0);
63. }
```

```

64.     int t;
65.     cin >> t;
66.
67.     while(t--){
68.         cin >> n;
69.
70.         mlb = mla = 0;
71.         for(int i = 0; i < n; i++)
72.             one[i].clear(), two[i].clear(), vcs[0][i].clear(), vcs[1][i].clear();
73.
74.         getinp(one, rta), getinp(two, rtb);
75.
76.         vcs[0][0].push_back(E1(-1));
77.         vcs[1][0].push_back(E1(-1));
78.         dfs(one, vcs[0], rta, 0, mla), dfs(two, vcs[1], rtb, 0, mlb);
79.
80.         cout << isom() << '\n';
81.
82.     }
83.
84. }

```

## Dijkstra

```
1.  typedef long long ll;
2.  typedef pair<ll, ll> ii;
3.  typedef pair<ii, ll> iii;
4.  #define FIRST first.first
5.  #define SECOND first.second
6.  #define THIRD second
7.  typedef vector<ll> vi;
8.  typedef vector<ii> vii;
9.
10. #define MAXN 100000
11. #define MOD 1000000007
12.
13. vii edges[MAXN]; // first = cost, second = where
14. ii arr[MAXN]; // shortest paths to all nodes will be stored here
15. ll n; // size of graph
16.
17. void dijkstra(ll from, ll to) {
18.     for(int i = 0; i < n; i++) arr[i].first = INT_MAX;
19.     priority_queue<iii, vector<iii>, greater<iii> > pq;
20.     pq.push(iii(ii(0, -1), from));
21.     while(!pq.empty()){
22.         iii cur = pq.top();
23.         pq.pop();
24.         if(arr[cur.THIRD].first == INT_MAX)
25.             arr[cur.THIRD] = cur.first;
26.         else
27.             continue;
28.         for(int i = 0; i < edges[cur.THIRD].size(); i++){
29.             ll cost = edges[cur.THIRD][i].first,
30.             where = edges[cur.THIRD][i].second;
31.             if(arr[where].first == INT_MAX)
32.                 pq.push(iii(ii(cur.FIRST + cost, cur.THIRD), where));
33.         }
34.     }
35. }
```

## DSU

```
1.  #define MAXN 10
2.  #define MOD 1000000007
3.
4.  struct DSU {
5.      int bel[MAXN];
6.      vi s[MAXN];
7.
8.      void reset() {
9.          FOR(i, 0, MAXN)
10.             bel[i] = 0, s[i].clear();
11.     }
12.
13.     void unir(int a, int b) {
14.         if (a > b) swap(a, b);
15.
16.         FOR(i, 0, s[a].size()) {
17.             s[b].pb(s[a][i]);
18.             bel[a] = b;
19.         }
20.         s[a].clear();
21.     }
22.
23.     int belongs(int x) {
24.         return bel[x];
25.     }
26. };
27.
28. struct DSU2 {
29.     int numSets = 0;
30.     int setSize[MAXN];
31.     int parent[MAXN];
32.     int rank[MAXN];
33.
34.     UnionFind(int n) {
35.         numSets = n;
36.         FOR(i, 0, n) parent[i] = i, setSize[i] = rank[i] = 0;
37.     }
38.
39.     void make_set(int i) {
40.         parent[i] = i;
41.         rank[i] = 0;
42.     }
43.
44.     int find_set(int i) {
45.         if (i != parent[i])
46.             parent[i] = find_set(parent[i]);
47.         return parent[i];
48.     }
49.
50.     void unionSet(int i, int j) {
51.         if (!isSameSet(i, j)) {
52.             numSets--;
53.             int x = find_set(i), y = find_set(j);
54.
55.             if (rank[x] > rank[y]) {
56.                 parent[y] = x;
57.                 setSize[x] += setSize[y];
58.             }
59.             else {
60.                 parent[x] = y;
61.                 setSize[y] += setSize[x];
62.
63.                 if (rank[x] == rank[y]) rank[y]++;
64.             }
65.         }
66.     }
67.
68.     bool isSameSet(int i, int j) {
69.         return find_set(i) == find_set(j);
70.     }
}
```



71. };

## Find Cycles

```
1.  #define maxN 100005
2.
3.  vi edges[maxN];
4.  bool cur[maxN], visit[maxN];
5.  stack<int> ans;
6.
7.  int findCycle(int n) {
8.      if (cur[n]) return n;
9.      if (visit[n]) return -1;
10.
11.     cur[n] = true;
12.     visit[n] = true;
13.
14.     FOR(i, 0, edges[n].size()) {
15.         if(ans.size()) break;
16.
17.         int v = findCycle(edges[n][i]);
18.         if (v != -1) {
19.             cur[n] = false;
20.             ans.push(n);
21.             if(v == n) {
22.                 return -1;
23.             }
24.             return v;
25.         }
26.     }
27.     cur[n] = false;
28.     return -1;
29. }
30.
31. int main() {
32.     int n, m, a, b;
33.     while (cin >> n >> m) {
34.         memset(cur, false, sizeof(cur));
35.         memset(visit, false, sizeof(visit));
36.
37.         FOR(i, 0, m) {
38.             cin >> a >> b;
39.             edges[a].pb(b);
40.         }
41.
42.         FOR(i, 0, n) {
43.             findCycle(i);
44.         }
45.
46.         if(! ans.empty()) {
47.             cout << "YES\n";
48.             while(! ans.empty()) {
49.                 int val = ans.top();
50.                 ans.pop();
51.                 cout << val << " \n"[ans.empty()];
52.             }
53.         }
54.         else {
55.             cout << "NO\n";
56.         }
57.     }
58.     return 0;
59. }
```

## Floyd-Warshall

```
1.  #define MAXN 10005
2.  #define MOD 1000000007
3.
4.  int graph[MAXN][MAXN];
5.  int dist[MAXN][MAXN];
6.
7.  void floydWarshall(int n) { // O(n^3)
8.      FOR(i, 0, n) FOR(j, 0, n) dist[i][j] = graph[i][j];
9.
10.     FOR(k, 0, n) {
11.         FOR(i, 0, n) {
12.             FOR(j, 0, n) {
13.                 dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
14.             }
15.         }
16.     }
17. }
18.
19. int main() {
20.     FOR(i, 0, MAXN) FOR(j, 0, MAXN) dist[i][j] = INF;
21.
22.     int n, m;
23.     cin >> n >> m;
24.     FOR(i, 0, m) {
25.         cin >> a >> b >> c;
26.         graph[a][b] = c;
27.     }
28.
29.     floydWarshall(n);
30.     return 0;
31. }
```

## Kruskal

```
1.  #define MAXN 10005
2.  #define MAXM 10005
3.  #define MOD 1000000007
4.
5.  struct Edge {
6.      int from, to, w;
7.      Edge() {}
8.      Edge(int a, int b, int c) : from(a), to(b), w(c) {}
9.      bool operator <(Edge const& e) const {
10.         return w < e.w;
11.     }
12. };
13.
14. struct UnionFind {
15.     int numSets = 0;
16.     int setSize[MAXN];
17.     int parent[MAXN];
18.     int rank[MAXN];
19.
20.     UnionFind(int n) {
21.         numSets = n;
22.         FOR(i, 0, n) parent[i] = i;
23.     }
24.
25.     void make_set(int i) {
26.         parent[i] = i;
27.         rank[i] = 0;
28.     }
29.
30.     int find_set(int i) {
31.         if (i != parent[i])
32.             parent[i] = find_set(parent[i]);
33.         return parent[i];
34.     }
35.
36.     void unionSet(int i, int j) {
37.         if (!isSameSet(i, j)) {
38.             numSets--;
39.             int x = find_set(i), y = find_set(j);
40.
41.             if (rank[x] > rank[y]) {
42.                 parent[y] = x;
43.                 setSize[x] += setSize[y];
44.             }
45.             else {
46.                 parent[x] = y;
47.                 setSize[y] += setSize[x];
48.
49.                 if (rank[x] == rank[y]) rank[y]++;
50.             }
51.         }
52.     }
53.
54.     bool isSameSet(int i, int j) {
55.         return find_set(i) == find_set(j);
56.     }
57. };
58.
59. vector<Edge> v;
60. vector<Edge> tree;
61.
62. int kruskal(int n, int m) { // O(E log E + E log V)
63.     sort(v.begin(), v.end());
64.
65.     int mst_w = 0;
66.     UnionFind UF(n);
67.
68.     FOR(i, 0, m) {
69.         Edge e = v[i];
70.
71.         if (!UF.isSameSet(e.from, e.to)) {
72.             mst_w += e.w;
```

```

73.         UF.unionSet(e.from, e.to);
74.         tree.pb(Edge(e.from, e.to, e.w));
75.     }
76. }
77.
78.     return mst_w;
79. }
80.
81. int main() {
82.     int n, m;
83.     int a, b, c;
84.     cin >> n >> m;
85.     FOR(i, 0, m) {
86.         cin >> a >> b >> c;
87.         v.pb(Edge(a, b, c));
88.     }
89.
90.     cout << kruskal(n, m) << endl;
91.
92.     return 0;
93. }

```

## Longest Path in DAG

```
1.  #define MAXN 10
2.  #define MOD 1000000007
3.
4.  vii graph[MAXN];
5.  stack<int> s;
6.  bool v[MAXN];
7.  int dist[MAXN], n, m;
8.
9.  void topologicalSortUtil(int act) {
10.     v[act] = 1;
11.
12.     FOR(i, 0, graph[act].size())
13.         if (!v[graph[act][i].second]) topologicalSortUtil(graph[act][i].second);
14.
15.     s.push(act);
16. }
17.
18. void longestPath(int source) {
19.     memset(v, 0, sizeof(v));
20.
21.     FOR(i, 0, n)
22.         if (!v[i]) topologicalSortUtil(i);
23.
24.     fill(dist, dist + n, -INF);
25.     dist[source] = 0;
26.
27.     while (!s.empty()) {
28.         int u = s.top();
29.         s.pop();
30.
31.         if (dist[u] != -INF) {
32.             FOR(i, 0, graph[u].size()) {
33.                 dist[graph[u][i].second] = max(dist[graph[u][i].second],
34.                 dist[u] + graph[u][i].first);
35.             }
36.         }
37.
38.         FOR(i, 0, n)
39.             if (dist[i] == -INF) cout << "INF" << endl;
40.             else cout << dist[i] << endl;
41.     }
42.
43.
44. int main() { _
45.     int a, b, c;
46.     while(cin >> n >> m) {
47.         FOR(i, 0, m) {
48.             cin >> a >> b >> c;
49.             graph[a].pb(ii(c, b));
50.         }
51.
52.         int source;
53.         cin >> source;
54.         longestPath(source);
55.     }
56.
57.     return 0;
58. }
```

## Maximum Diameter in a Graph

```
1. #define MAXN 10
2. #define MOD 1000000007
3.
4. // v es visitados localmente.
5. // v2 es todos los que ya he visitado (para grafos no totalmente conectados)
6. bool v[MAXN], v2[MAXN];
7. vi edges[MAXN];
8.
9. ii bfs(int x, bool b) {
10.     queue<ii> q;
11.     q.push(ii(x, 0));
12.
13.     v[x] = !b;
14.     v2[x] |= !b;
15.     ii last;
16.     while(! q.empty()) {
17.         last = q.front();
18.         q.pop();
19.
20.         FOR(i, 0, edges[last.first].size()) {
21.             if (v[edges[last.first][i]] == b) {
22.                 q.push(ii(edges[last.first][i], last.second + 1));
23.                 v[edges[last.first][i]] = !b;
24.                 v2[edges[last.first][i]] |= !b;
25.             }
26.         }
27.     }
28.
29.     return last;
30. }
31.
32. int maxDiameter(int x) {
33.     ii ret = bfs(x, false);
34.     return bfs(ret.first, true).second;
35. }
```

## Maximum Bipartite Matching

```
1.  #define MAXN 10005
2.  #define MOD 1000000007
3.
4.  int n, m;
5.  int matchL[MAXN], matchR[MAXN];
6.  vi edge[MAXN];
7.  bool v[MAXN];
8.
9.  bool dfs(int from) {
10.     if (v[from]) return 0;
11.     v[from] = 1;
12.
13.     FOR(i, 0, edge[from].size()) {
14.         int to = edge[from][i];
15.
16.         if (matchR[to] == -1 || dfs(matchR[to])) {
17.             matchL[from] = to;
18.             matchR[to] = from;
19.             return 1;
20.         }
21.     }
22.     return 0;
23. }
24.
25. ll MBM() {
26.     ll ans = 0;
27.     bool b = 1;
28.
29.     memset(matchL, -1, sizeof(matchL));
30.     memset(matchR, -1, sizeof(matchR));
31.
32.     while (b) {
33.         b = 0;
34.         memset(v, 0, sizeof(v));
35.         FOR(i, 0, n) {
36.             if (matchL[i] == -1 && dfs(i)) {
37.                 ans ++;
38.                 b = 1;
39.             }
40.         }
41.     }
42.     return ans;
43. }
44.
45. int main() { _
46.     int a, b;
47.     cin >> n >> m;
48.     FOR(i, 0, m) {
49.         cin >> a >> b;
50.         edge[a].pb(b);
51.     }
52.     cout << MBM() << endl;
53.
54.     return 0;
55. }
```



## Strongly Connected Components

```
1.  #define MAXN 1000
2.  #define MOD 1000000007
3.
4.  int n, m;
5.
6.  vi edge[MAXN];
7.  int disc[MAXN], low[MAXN], belongs[MAXN];
8.  bool v[MAXN];
9.  stack<int> st;
10. int ttime, comp;
11.
12. void SCCUtil(int u) {
13.     disc[u] = low[u] = ttime ++;
14.     st.push(u);
15.     v[u] = true;
16.
17.     FOR(i, 0, edge[u].size()) {
18.         int to = edge[u][i];
19.
20.         if (disc[to] == -1)
21.             SCCUtil(to);
22.
23.         if (v[to])
24.             low[u] = min(low[u], low[to]);
25.     }
26.
27.     if (low[u] == disc[u]) {
28.         comp ++;
29.         while (1) {
30.             int t = st.top();
31.             st.pop();
32.             belongs[t] = comp;
33.             v[t] = false;
34.
35.             if (t == u) break;
36.         }
37.     }
38. }
39.
40. void SSC() {
41.     ttime = comp = 0;
42.     memset(disc, -1, sizeof(disc));
43.     memset(low, -1, sizeof(low));
44.     memset(v, 0, sizeof(v));
45.
46.     FOR(i, 0, n)
47.         if (disc[i] == -1)
48.             SCCUtil(i);
49. }
```

## Topological Sort

```
1.  #define MAXN 10005
2.  #define MOD 1000000007
3.
4.  vi edges[MAXN];
5.  stack<int> s;
6.  bool v[MAXN];
7.
8.  void topologicalSortUtil(int act) {
9.      v[act] = 1;
10.
11.      FOR(i, 0, edges[act].size())
12.          if (!v[edges[act][i]]) topologicalSortUtil(edges[act][i]);
13.
14.      s.push(act);
15.  }
16.
17.  void topologicalSort(int n) {
18.      memset(v, 0, sizeof(v));
19.
20.      FOR(i, 0, n)
21.          if (!v[i]) topologicalSortUtil(i);
22.
23.      while (!s.empty()) {
24.          int a = s.top();
25.          s.pop();
26.          cout << a << " \n"[s.empty()];
27.      }
28.  }
29.
30.  int main() {
31.      int n, m;
32.      int a, b;
33.      while (cin >> n >> m) {
34.          FOR(i, 0, MAXN) edges[i].clear();
35.
36.          FOR(i, 0, m) {
37.              cin >> a >> b;
38.              edges[a].pb(b);
39.          }
40.
41.          topologicalSort(n);
42.      }
43.
44.      return 0;
45.  }
```

## 5. Number Theory

### Fermat Little's theorem

```
1.  ll mulmod(ll a, ll b, ll c) { // returns (a * b) % c, and minimize overflow
2.      return (ll)((__int128)(a) * (b) % c);
3.  }
4.
5.  ll fastPow(ll x, ll n, ll c) { // returns (a ** b) % c, and minimize overflow
6.      ll ret = 1;
7.      while (n) {
8.          if (n & 1) ret = mulmod(ret, x, c);
9.          x = mulmod(x, x, c);
10.         n >>= 1;
11.     }
12.     return ret;
13. }
14.
15.
16.
17. /** return modular multiplicative of: a mod p, assuming p is prime */
18. ll modInverse(ll a, ll p) {
19.     return fastPow(a, p - 2, p);
20. }
21.
22. /** return C(n,k) mod p, assuming p is prime */
23. ll modBinomial(ll n, ll k) {
24.     ll numerator = 1; // n*(n-1)* ... * (n-k+1)
25.     FOR(i, 0, k)
26.         numerator = (numerator * (n - i)) % MOD;
27.
28.     ll denominator = 1; // k!
29.     FOR (i, 1, k+1)
30.         denominator = (denominator * i) % MOD;
31.
32.     ll res = modInverse(denominator, MOD);
33.     res = (res * numerator) % MOD;
34.     return res;
35. }
36.
37. ll extendedGCD(ll a, ll b, ll &inva, ll &invb) {
38.     if (b) {
39.         ll g = extendedGCD(b, a%b, invb, inva);
40.         invb = invb - a / b*inva;
41.         return g;
42.     }
43.     inva = 1, invb = 0;
44.     return a;
45. }
46.
47. int main() {
48.     ll inva=0, invb=0;
49.     int c = extendedGCD(62424, 13, inva, invb);
50.     cout << modInverse(62424, 13) << endl;
51.     cout << (inva + 7) % 7 << " " << (invb + 62424) % 13 << " " << c << endl;
52.     return 0;
53. }
```

## Fibonacci

```
1.  const long double PHI ((1.0 + sqrt(5.0)) / 2.0);
2.
3.  ll O1(ll n) {
4.      return (ll)(floor(pow(PHI, n) / sqrt(5.0) + 0.5));
5.  }
6.
7.  void mult(ll F[2][2], ll M[2][2]) {
8.      ll x = (F[0][0] * M[0][0]) % MOD + (F[0][1] * M[1][0]) % MOD;
9.      ll y = (F[0][0] * M[0][1]) % MOD + (F[0][1] * M[1][1]) % MOD;
10.     ll z = (F[1][0] * M[0][0]) % MOD + (F[1][1] * M[1][0]) % MOD;
11.     ll w = (F[1][0] * M[0][1]) % MOD + (F[1][1] * M[1][1]) % MOD;
12.     F[0][0] = x % MOD;
13.     F[0][1] = y % MOD;
14.     F[1][0] = z % MOD;
15.     F[1][1] = w % MOD;
16. }
17.
18. void power(ll F[2][2], ll n) {
19.     if (n == 0 || n == 1) return;
20.
21.     ll M[2][2] = {{1, 1}, {1, 0}};
22.     power(F, n / 2);
23.     mult(F, M);
24.
25.     if (n % 2)
26.         mult(F, M);
27. }
28.
29. ll OlnMat(ll n) {
30.     if (n == 0) return 0;
31.
32.     ll F[2][2] = {{1,1},{1,0}};
33.     power(F, n - 1);
34.
35.     return F[0][0];
36. }
37.
38. /** Suma de Los primeros n numeros de fibo **/
39. ll fiboSuma(ll n) {
40.     return (MOD + OlnMat(n + 2) - 1) % MOD;
41. }
42.
43. ll On(ll n) {
44.     if (n == 1) return 0;
45.
46.     ll a = 0, b = 1, c = 1;
47.
48.     FOR(i, 2, n) {
49.         c = b + a;
50.         a = b;
51.         b = c;
52.     }
53.     return c;
54. }
55.
56. ll fibo(ll n) {
57.     if (n < 72) return O1(n);
58.     if (n < 150) return On(n);
59.
60.     return OlnMat(n);
61. }
62.
63. int main(){
64.     ll n;
65.     cin >> n;
66.     cout << fibo(n) << endl;
```

## Prime Numbers

```
1. int pc = 0, f[MAXN], primes[MAXN / 10];
2.
3.
4. // ***** IS PRIME() *****
5. //Para TODOS los primos
6.
7. ll mulmod2(ll a, ll b, ll c) { // returns (a * b) % c, and minimize overflow
8.     ll x = 0, y = a % c;
9.     while (b) {
10.         if (b & 1) x = (x + y) % c;
11.         y = (y << 1) % c;
12.         b >>= 1;
13.     }
14.     return x % c;
15. }
16.
17. ll mulmod(ll a, ll b, ll c) {
18.     return (ll)((__int128)(a) * b % c);
19. }
20.
21. ll fastPow(ll x, ll n, ll c) { // returns (a ** b) % c, and minimize overflow
22.     ll ret = 1;
23.     while (n) {
24.         if (n & 1) ret = mulmod(ret, x, c);
25.         x = mulmod(x, x, c);
26.         n >>= 1;
27.     }
28.     return ret;
29. }
30.
31. // Miller-Rabin primality test
32. bool millerRabin(ll n) {
33.     ll d = n - 1;
34.     int s = 0;
35.     while (d % 2 == 0) {
36.         s++;
37.         d >>= 1;
38.     }
39.     // It's guaranteed that these values will work for any number smaller than 2^64
40.     int a[12] = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 };
41.     FOR(i, 0, 12) if(n == a[i]) return true;
42.
43.     FOR(i, 0, 12) {
44.         bool comp = fastPow(a[i], d, n) != 1;
45.         if(comp) FOR(j, 0, s) {
46.             ll fp = fastPow(a[i], (1LL << (ll)j)*d, n);
47.             if (fp == n - 1) {
48.                 comp = false;
49.                 break;
50.             }
51.         }
52.         if(comp) return false;
53.     }
54.     return true;
55. }
56.
57. // Miller-Rabin primality test
58. ll rN(){
59.     return maxR * rand() + rand();
60. }
61.
62. bool miller(ll n, ll d){
63.     ll a = 2 + rN() % (n - 4);
64.     ll x = fastPow(a, d, n);
65.     if(x == 1 || x == n - 1)
66.         return true;
67.     while(d < n - 1){
68.         x = __int128(x) * x % n;
69.         if(x == 1)
70.             return false;
71.         if(x == n - 1)
```

```

72.         return true;
73.         d <= 1;
74.     }
75.     return false;
76. }
77.
78. bool isPrime(ll n, int k){
79.     if(n == 3 || n == 2)
80.         return true;
81.     if(n == 1 || n % 2 == 0)
82.         return false;
83.     ll d = n - 1;
84.     while(d % 2 == 0)
85.         d >>= 1;
86.     while(k--){
87.         if(!miller(n, d))
88.             return false;
89.     }
90.     return true;
91. }
92. // *****
93.
94.
95. // ***** GET PRIME FACTORS() *****
96. /**
97.  Devuele Los factores primos de un numero. No olvidar hacer unique para quitar repetidos
98.  int tamano = unique(primeFactors.begin(), primeFactors.end()) - primeFactors.begin();
99.  */
100.
101. ll A, B;
102. vl primeFactors;
103.
104. ll funcRand(ll x, ll n) {
105.     return (mulmod(x, (x + A), n) + B) % n;
106. }
107.
108. void pollardRho(ll n) {
109.     if (n == 1) return;
110.     if (millerRabin(n)) {
111.         primeFactors.pb(n);
112.         return;
113.     }
114.     ll d = n, x, y;
115.     while(d == n) {
116.         d = 1;
117.         x = y = 2;
118.         A = 2 + rand() % (n - 2);
119.         B = 2 + rand() % (n - 2);
120.
121.         while (d == 1) {
122.             x = funcRand(x, n);
123.             y = funcRand(funcRand(y, n), n);
124.             d = __gcd(abs(x - y), n);
125.         }
126.     }
127.
128.     if (n / d != d)
129.         pollardRho(n / d);
130.     pollardRho(d);
131. }
132. // *****
133.
134. // ***** GET SMALLEST DIV PRIME() *****
135. // Obtener f[t], el primo mas chico que divide a f[t]
136.
137. int divPrime[MAXN];
138.
139. void getSmallestDivPrime() {
140.     FOR(i, 2, MAXN) {
141.         if (divPrime[i] == -1) {
142.             for (int j = 1; i*j < MAXN; j++) if (divPrime[i*j] == -1) divPrime[i*j] = i;
143.         }
144.     }

```

```

145.}
146.// *****
147.
148.
149.// ***** GET DIVISORS() *****
150.vll factorsOfN;
151.vl divisors;
152.
153.// Usar cuando se necesita sacar muchos numeros
154.void getFactorsOfN2(int n) {
155.    int t = n;
156.    int last = -1;
157.    while (divPrime[t] != -1) {
158.        if (divPrime[t] == last)    factorsOfN[factorsOfN.size() - 1].second ++;
159.        else                        factorsOfN.pb(ii(divPrime[t], 1));
160.        last = divPrime[t];
161.        t /= divPrime[t];
162.    }
163.}
164.
165.// Usar cuando es numeros muy grandes
166.void getFactorsOfN(ll n) {
167.    pollardRho(n);
168.    sort(primeFactors.begin(), primeFactors.end());
169.    int ss = unique(primeFactors.begin(), primeFactors.end()) - primeFactors.begin();
170.    int s = 0;
171.    ll num = n;
172.    FOR(i, 0, ss) {
173.        bool b = false;
174.        while (num % primeFactors[i] == 0) {
175.            if (b)    factorsOfN[s].second ++;
176.            else      factorsOfN.pb(pll(primeFactors[i], 1)), b = 1;
177.
178.            num /= primeFactors[i];
179.        }
180.        s ++;
181.    }
182.}
183.
184.void getDivisorsOfN(ll n) {
185.    primeFactors.clear();
186.    factorsOfN.clear();
187.    divisors.clear();
188.
189.    getFactorsOfN(n);
190.    divisors.push_back(1);
191.
192.    FOR(i, 0, factorsOfN.size()) {
193.        int s = divisors.size(), num = 1;
194.
195.        FOR(j, 0, factorsOfN[i].second) {
196.            num *= factorsOfN[i].first;
197.            FOR(k, 0, s) divisors.push_back(divisors[k] * num);
198.        }
199.    }
200.    sort(divisors.begin(), divisors.end());
201.    FOR(i, 0, divisors.size())    cout << divisors[i] << endl;
202.}
203.// *****
204.
205.
206.// ***** Criba de Eratosthenes *****
207.void Eratosthenes() {
208.    bool isPrime[MAXN];
209.    memset(isPrime, 0, sizeof(isPrime));
210.    FOR(i, 2, MAXN) {
211.        if (!isPrime[i]) {
212.            primes[pc ++] = i;
213.            for (int j = 2; j*i < MAXN; j++) {
214.                isPrime[i*j] = true;
215.            }
216.        }
217.    }

```

```

218.}
219.// *****
220.
221.
222.// ***** Criba de Eratosthenes O(N) *****
223.void EratosthenesON(){
224.    FOR(i, 2, MAXN) {
225.        if (! f[i]) primes[pc++] = i, f[i] = i;
226.        for (int j = 0; j < pc && 1LL*i*primes[j] < MAXN && primes[j] <= f[i]; j++)
227.            f[i*primes[j]] = primes[j];
228.    }
229.}
230.// *****
231.
232.// ***** Criba de Atkin *****
233.void Atkin() {
234.    ll sqrtArraySize = sqrt(MAXN);
235.    ll n;
236.
237.    bool isPrime[MAXN];
238.    memset(isPrime, false, sizeof(isPrime));
239.
240.    ll pp[MAXN];
241.    FOR(i, 0, sqrtArraySize + 5) pp[i] = i * i;
242.
243.    FOR(i, 1, sqrtArraySize + 1) {
244.        FOR(j, 1, sqrtArraySize + 1) {
245.            n = 4 * pp[i] + pp[j];
246.
247.            if (n <= MAXN && (n % 12 == 1 || n % 12 == 5))
248.                isPrime[n] = !isPrime[n];
249.
250.            n = 3 * pp[i] + pp[j];
251.
252.            if (n <= MAXN && (n % 12 == 7))
253.                isPrime[n] = !isPrime[n];
254.
255.            if (i > j) {
256.                n = 3 * pp[i] - pp[j];
257.                if(n <= MAXN && n % 12 == 11)
258.                    isPrime[n] = !isPrime[n];
259.            }
260.        }
261.    }
262.
263.    FOR(i, 5, sqrtArraySize + 1) {
264.        if (isPrime[i]) {
265.            ll omit = pp[i];
266.
267.            for (ll j = omit; j <= MAXN; j += omit) {
268.                isPrime[j] = false;
269.            }
270.        }
271.    }
272.
273.    if (MAXN >= 2) {
274.        primes[pc++] = 2;
275.        if (MAXN >= 3) {
276.            primes[pc++] = 3;
277.        }
278.    }
279.
280.    FOR(i, 2, MAXN) {
281.        if (isPrime[i]) {
282.            primes[pc++] = i;
283.        }
284.    }
285.}
286.// *****
287.
288.// ***** Euler Totem *****
289.// REGRESA PINCHE CANTIDAD DE COPRIMOS DE N
290.// Para sacar la puta suma de coprimos es n*phi(n) / 2

```



```
291.11 eulerTotient(11 n) {
292.    factorsOfN.clear();
293.    getFactorsOfN(n);
294.    FOR(i, 0, factorsOfN.size())
295.        n = n / factorsOfN[i].first * (factorsOfN[i].first - 1);
296.    return n;
297.}
```

## Triangle of Mahonian

```
1. #define MAXN 35
2. #define MAXM 5005
3.
4. ll arr[MAXN][MAXM];
5.
6. int main() {
7.     memset(arr, 0, sizeof(arr));
8.     arr[0][0] = arr[1][0] = 1;
9.
10.    FOR(i, 0, MAXN) {
11.        for(int m = 0; m < MAXM - 5 && arr[i][m] != 0; m++) {
12.            while(m < 5000 && ) {
13.                FOR(j, 0, i+1) {
14.                    arr[i+1][j+m] += arr[i][m];
15.                }
16.            }
17.        }
18.    }
19.
20.    return 0;
21. }
```

## 6. Segment Tree

### Lazy Propagation

```
1.  const int maxN = 1e5, MOD = 1E9 + 7;
2.  int HEIGHT, n;
3.  ll d[maxN], t[maxN << 1];
4.
5.  void apply(int p, ll value) {
6.      t[p] = value;
7.      if (p < n) d[p] = value;
8.  }
9.
10. void build(int p) {
11.     while (p > 1) p >>= 1, t[p] = max(max(t[p<<1], t[p<<1|1]), d[p]);
12. }
13.
14. void push(int p) {
15.     for (int s = HEIGHT; s > 0; --s) {
16.         int i = p >> s;
17.         if (d[i] != 0) {
18.             apply(i<<1, d[i]);
19.             apply(i<<1|1, d[i]);
20.             d[i] = 0;
21.         }
22.     }
23. }
24.
25. void change(int l, int r, ll value) {
26.     l += n, r += n;
27.     int l0 = l, r0 = r;
28.     for (; l < r; l >>= 1, r >>= 1) {
29.         if (l&1) apply(l++, value);
30.         if (r&1) apply(--r, value);
31.     }
32.     build(l0);
33.     build(r0 - 1);
34. }
35.
36. ll query(int l, int r) {
37.     l += n, r += n;
38.     push(l);
39.     push(r - 1);
40.     ll res = LLONG_MIN;
41.     for (; l < r; l >>= 1, r >>= 1) {
42.         if (l&1) res = max(res, t[l++]);
43.         if (r&1) res = max(t[--r], res);
44.     }
45.     return res;
46. }
```

## Persistent Segment Tree

```
1.  const int maxN = 1e6;
2.
3.  struct Node{
4.      Node *l, *r;
5.      int count;
6.      Node(int count, Node *l, Node *r):count(count), l(l), r(r){}
7.      Node * insert(int, int, int);
8.  };
9.  Node * segTrees[maxN + 1];
10.
11. Node * Node::insert(int left, int right, int pos){
12.     if(pos < left || pos >= right)
13.         return this;
14.     if(left + 1 == right)
15.         return new Node(count + 1, segTrees[0], segTrees[0]);
16.     int m = (left + right) >> 1;
17.     return new Node(count + 1, l -> insert(left, m, pos), r -> insert(m, right, pos));
18. }
19. int solve(Node * last, Node * first, int left, int right, int k){
20.     if(left + 1 == right)
21.         return left;
22.     int m = (left + right) >> 1, dif = last -> l -> count - first -> l -> count;
23.     if(dif < k)
24.         return solve(last -> r, first -> r, m, right, k - dif);
25.     return solve(last -> l, first -> l, left, m, k);
26. }
27.
28. int main() {
29.     ios::sync_with_stdio(false);
30.     cin.tie(0);
31.     cout.tie(0);
32.     int n, q, x, y, k, arr[maxN], vals[maxN];
33.     segTrees[0] = new Node(0, NULL, NULL);
34.     segTrees[0] -> l = segTrees[0] -> r = segTrees[0];
35.     while(cin >> n >> q){
36.         map<int, int> myMap;
37.         for(int i = 0; i < n; i++){
38.             cin >> arr[i];
39.             myMap[arr[i]];
40.         }
41.         int valSize = 0;
42.         for(map<int, int>::iterator it = myMap.begin(); it != myMap.end(); it++){
43.             it -> second = valSize;
44.             vals[valSize++] = it -> first;
45.         }
46.         for(int i = 1; i <= n; i++)
47.             segTrees[i] = segTrees[i - 1] -> insert(0, valSize, myMap[arr[i - 1]]);
48.         while(q--){
49.             cin >> x >> y >> k;
50.             cout << vals[solve(segTrees[y], segTrees[x - 1], 0, valSize, k)] << '\n';
51.         }
52.     }
53. }
```

## 7. Treap

```
8. const int maxN = 1e5 + 3, MOD = 1e9 + 7, AL = 10;
9.
10. ll fc[maxN][2];
11.
12. ll perm(int *bu){
13.     ll den = 1, tot = 0, imp = 0;
14.     for(int i = 0; i < AL; i++){
15.         //cout << bu[i] << ' ';
16.         imp += bu[i] & 1;
17.         den = den * fc[bu[i] >> 1][1] % MOD;
18.         tot += bu[i] >> 1;
19.     }
20.     //cout << '\n';
21.     if(imp > 1) return 0;
22.     return fc[tot][0] * den % MOD;
23. }
24.
25. struct Node{
26.     int ca, pri, bu[AL], cnt;
27.     Node *l, *r;
28.     bool rev;
29.     Node(int ca): ca(ca), l(NULL), r(NULL), rev(false){
30.         memset(bu, 0, sizeof(bu)), pri = (rand() << 15) + rand();
31.     }
32. };
33. typedef Node* pnode;
34.
35. int cnt(pnode nd){
36.     return nd? nd->cnt: 0;
37. }
38.
39. void upd(pnode nd){
40.     if(!nd) return;
41.     nd->cnt = 1 + cnt(nd->l) + cnt(nd->r);
42.
43.     for(int i = 0; i < AL; i++)
44.         nd->bu[i] = (nd->l? nd->l->bu[i]: 0) + (nd->r? nd->r->bu[i]: 0);
45.     nd->bu[nd->ca]++;
46. }
47.
48. void apply(pnode nd){
49.     if(nd) nd->rev ^= 1;
50. }
51.
52. void push(pnode nd){
53.     if(nd && nd->rev)
54.         swap(nd->l, nd->r), apply(nd->l), apply(nd->r), nd->rev = 0;
55. }
56.
57. void split(pnode t, pnode &l, pnode &r, int key, int add){
58.     if(!t)
59.         return void(l = r = NULL);
60.     push(t);
61.     int mykey = add + cnt(t->l) + 1;
62.     if(mykey <= key)
63.         split(t->r, t->r, r, key, mykey), l = t;
64.     else
65.         split(t->l, l, t->l, key, add), r = t;
66.     upd(t);
67. }
68.
69. void merge(pnode &t, pnode l, pnode r){
70.     push(l), push(r);
71.     if(!l || !r)
```

```

72.         t = l? l: r;
73.     else if(l->pri > r->pri)
74.         merge(l->r, l->r, r), t = l;
75.     else
76.         merge(r->l, l, r->l), t = r;
77.     upd(t);
78. }
79.
80. void insert(pnode &t, pnode it){
81.     if(!t)
82.         t = it;
83.     else if(it->pri > t->pri)
84.         it->l = t, t = it;
85.     else
86.         insert(t->r, it);
87.     upd(t);
88. }
89.
90. int n, m;
91. string st;
92.
93. ll fastPow(ll a, ll b){
94.     ll ret = 1;
95.     while(b){
96.         if(b & 1)
97.             ret = ret * a % MOD;
98.         a = a * a % MOD, b >>= 1;
99.     }
100.    return ret;
101. }
102.
103. int main(){
104.     ios::sync_with_stdio(false);
105.     cin.tie(0), cout.tie(0);
106.
107.     srand(8000000);
108.
109.     fc[0][0] = fc[0][1] = 1;
110.     for(ll i = 1; i < maxN; i++)
111.         fc[i][0] = fc[i - 1][0] * i % MOD, fc[i][1] = fastPow(fc[i][0],
MOD - 2);
112.
113.     cin >> n >> m >> st;
114.
115.     pnode head = NULL;
116.     for(char c: st) insert(head, new Node(c - 'a'));
117.
118.     while(m--){
119.         int ty, l, r;
120.         cin >> ty >> l >> r;
121.         pnode two, thr;
122.         split(head, head, two, l - 1, 0);
123.         split(two, two, thr, r - l + 1, 0);
124.         if(ty == 1)
125.             apply(two);
126.         else
127.             cout << perm(two->bu) << '\n';
128.         merge(head, head, two);
129.         merge(head, head, thr);
130.     }
131.
132. }

```

## 8. Strings

### Aho-Corasick

```
1. const int maxN = 1e3 + 3, MOD = 1e9 + 9, LG = 21;
2. //NOTE: maxN > 2^LG must be held, remember maxN is twice ur biggest polynom
3. const int dr[8][2] = {{1, 0}, {0, 1}, {-1, 0}, {0, -1}, {1, 1}, {-1, -1}, {-1, 1}, {1, -1}};
4. const string chr = "ECAGDHBH";
5.
6. string mat[maxN], dic[maxN], ans[maxN];
7. int n, m, w;
8. bool v[maxN][maxN];
9.
10. vector<ii> ini[8];
11.
12. bool ins(int x, int y){
13.     return x >= 0 && x < n && y >= 0 && y < m;
14. }
15.
16. void findInits(int d, int r, int c){
17.     int x = r - dr[d][0], y = c - dr[d][1];
18.
19.     v[r][c] = 1;
20.
21.     if(!ins(x, y)) ini[d].push_back(ii(r, c));
22.     else if(!v[x][y]) findInits(d, x, y);
23. }
24.
25. int tr[maxN * maxN][26], dp[maxN * maxN], lf[maxN * maxN], tc, fWord[maxN * maxN],
    lastInd[maxN], sz[maxN];
26.
27. int ins(int par, char c){
28.     if(tr[par][c - 'A']) return tr[par][c - 'A'];
29.     int k, ret = tr[par][c - 'A'] = tc++;
30.
31.     for(k = dp[par]; k && !tr[k][c - 'A']; k = dp[k]);
32.
33.     if(k != par) k = tr[k][c - 'A'];
34.
35.     if(fWord[k] != -1)
36.         lf[ret] = k;
37.     else
38.         lf[ret] = lf[k];
39.
40.     dp[ret] = k;
41.
42.     return ret;
43. }
44.
45. string bld(int sz, int r, int c, int u){
46.     r -= dr[u][0] * sz, c -= dr[u][1] * sz;
47.     return to_string(r) + " " + to_string(c) + " " + chr[u];
48. }
49.
50. int main(){
51.     ios::sync_with_stdio(false);
52.     cin.tie(0), cout.tie(0);
53.
54.     int t;
55.     cin >> t;
56.     while(t--){
57.         cin >> n >> m >> w;
58.
59.         queue<int> q;
```

```

60.
61.     for(int i = 0; i < n; i++) cin >> mat[i];
62.     for(int i = 0; i < w; i++) {
63.         cin >> dic[i];
64.         sz[i] = int(dic[i].size());
65.         reverse(dic[i].begin(), dic[i].end());
66.         q.push(i);
67.     }
68.
69.     for(int u = 0; u < 8; u++){
70.         memset(v, 0, sizeof(v)), ini[u].clear();
71.         for(int i = 0; i < n; i++)
72.             for(int j = 0; j < m; j++)
73.                 if(!v[i][j])
74.                     findInits(u, i, j);
75.     }
76.
77.     memset(tr, 0, tc * sizeof(tr[0])), memset(fWord, -1, sizeof(fWord));
78.     memset(lastInd, 0, sizeof(lastInd));
79.     tc = 1;
80.
81.     while(q.size()){
82.         int c = q.front(); q.pop();
83.
84.         lastInd[c] = ins(lastInd[c], dic[c].back());
85.
86.         //cout << c << ' ' << dic[c].back() << '\n';
87.
88.         dic[c].pop_back();
89.
90.         if(!dic[c].empty())
91.             q.push(c);
92.         else
93.             fWord[lastInd[c]] = c;
94.     }
95.
96.     //for(int i = 0; i < 26; i++) cout << tr[0][i] << ' '; cout << '\n';
97.     map<int, string> myMap;
98.
99.     for(int u = 0; u < 8; u++){
100.         for(ii el: ini[u]){
101.             int c = 0;
102.
103.             for(; ins(el.fi, el.se); el.fi += dr[u][0],
104.                 el.se += dr[u][1]){
105.                 while(c && !tr[c][mat[el.fi][el.se] - 'A'])
106.                     c = dp[c];
107.                 c = tr[c][mat[el.fi][el.se] - 'A'];
108.
109.                 //cout << c << '\n';
110.
111.                 for(int i = c; i; i = lf[i])
112.                     if(fWord[i] != -1)
113.                         myMap[i] = bld(sz[fWord[i]]-1, el.fi, el.se, u);
114.             }
115.         }
116.
117.         for(int i = 0; i < w; i++)
118.             cout << myMap[lastInd[i]] << '\n';
119.         cout << '\n';
120.     }
121. }

```



## KMP

```
1.  int lps[MAXN];
2.  vi v;
3.
4.  void computeLPSArray(string pat, int M) {
5.      int len = 0;
6.      int i = 1;
7.      lps[0] = 0;
8.      while (i < M) {
9.          if (pat[i] == pat[len])
10.             lps[i++] = ++ len;
11.          else {
12.              if (len != 0)    len = lps[len - 1];
13.              else            lps[i++] = 0;
14.          }
15.      }
16.  }
17.  void KMPSearch(string pat, string txt) {
18.      int M = pat.length();
19.      int N = txt.length();
20.      int i = 0;
21.      int j = 0;
22.
23.      computeLPSArray(pat, M);
24.
25.      while (i < N) {
26.          if (pat[j] == txt[i]) {
27.              j++;
28.              i++;
29.          }
30.
31.          if (j == M) {
32.              v.pb(i - j);
33.              j = lps[j - 1];
34.          }
35.          else if (i < N && pat[j] != txt[i]) {
36.              if (j != 0)    j = lps[j - 1];
37.              else          i = i + 1;
38.          }
39.      }
40.  }
41.
42.  int main() {
43.      string txt = "ABABDABACDABABCABAB";
44.      string pat = "AB";
45.      KMPSearch(pat, txt);
46.
47.      FOR(i, 0, v.size())
48.          cout << v[i] << " \n"[i==v.size()-1];
49.
50.      return 0;
51.  }
```

## Suffix Array

```
1.  typedef pair<int, int> ii;
2.  typedef pair<ii, int> iii;
3.  typedef long long ll;
4.  typedef vector<ll> vd;
5.  typedef vector<vd> Matrix;
6.  const int maxN = 1e6;
7.
8.  iii sa[maxN], aux[maxN];
9.  int inv[maxN], lcp[maxN + 1], values[maxN + 1], n;
10. string st;
11.
12. #define a(i) (fir? sa[i].FIRST: sa[i].SECOND)
13.
14. void radiixSort(bool fir){
15.     memset(values, 0, sizeof(values));
16.     for(int i = 0; i < n; i++)
17.         values[a(i) + 1]++;
18.     for(int i = 1; i <= n; i++)
19.         values[i] += values[i - 1];
20.     for(int i = n - 1; i >= 0; i--)
21.         aux[--values[a(i) + 1]] = sa[i];
22.     for(int i = 0; i < n; i++)
23.         sa[i] = aux[i];
24. }
25.
26. void createSuffixArray(){
27.     for(int i = 0; i < n; i++)
28.         sa[i] = iii(ii(st[i], 0), i);
29.     sort(sa, sa + n);
30.     for(int cnt = 1; cnt <= n; cnt <= 1){
31.         for(int i = 0, j = 0; i < n; i = j)
32.             for(ii cur = sa[i].first; j < n && sa[j].first == cur; j++)
33.                 sa[j].FIRST = inv[sa[j].THIRD] = i;
34.         for(int i = 0; i < n; i++)
35.             sa[i].SECOND = (sa[i].THIRD + cnt < n)? sa[inv[sa[i].THIRD + cnt]].FIRST: -1;
36.         radiixSort(false);
37.         radiixSort(true);
38.     }
39. }
40.
41. void createLCPArray(){
42.     for(int i = 0; i < n; i++)
43.         inv[sa[i].THIRD] = i;
44.     for(int i = 0, k = 0; i < n; i++, k? k--: k++){
45.         if(inv[i] + 1 == n){
46.             k = lcp[n - 1] = 0;
47.             continue;
48.         }
49.         int cur = sa[inv[i]].THIRD, next = sa[inv[i] + 1].THIRD;
50.         for(; max(cur, next) + k < n && st[cur + k] == st[next + k]; k++);
51.         lcp[inv[i] + 1] = k;
52.     }
53. }
54.
55. int main() {
56.     ios::sync_with_stdio(false), cin.tie(0), cout.tie(0);
57.     cin >> n >> st;
58.     createSuffixArray();
59.     createLCPArray();
60. }
```

## Z Function

```
1. // Devuelve el arreglo Z
2. vector<int> z_function(string &s){
3.     int L = 0, R = 0, n = s.length();
4.     vector<int> z(n);
5.     for(int i = 1; i < n; i++){
6.         if(i <= R)
7.             z[i] = min(z[i-L], R - i + 1);
8.         while(i + z[i] < n && s[i + z[i]] == s[z[i]])
9.             z[i]++;
10.        if(i + z[i] - 1 > R){
11.            L = i;
12.            R = i + z[i] - 1;
13.        }
14.    }
15.    return z;
16. }
17.
18. int main(){ io
19.     string A, B;
20.     cin >> A >> B;
21.     string z_Arg = B + '$' + A;
22.     vector<int> z = z_function(z_Arg);
23. }
```

## 9. Flows

### Dinic

```
1.  /*
2.       $O(V^2E)$ 
3.      On unit networks  $O(E\sqrt{V})$ 
4.      Unit network = edges with capacity = 1
5.  */
6.  struct FlowEdge {
7.      int v, u;
8.      long long cap, flow = 0;
9.      FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap) {}
10. };
11.
12. struct Dinic {
13.     const long long flow_inf = 1e18;
14.     vector<FlowEdge> edges;
15.     vector<vector<int>> adj;
16.     int n, m = 0;
17.     int s, t;
18.     vector<int> level, ptr;
19.     queue<int> q;
20.
21.     Dinic(int n, int s, int t) : n(n), s(s), t(t) {
22.         adj.resize(n);
23.         level.resize(n);
24.         ptr.resize(n);
25.     }
26.
27.     void add_edge(int v, int u, long long cap) {
28.         edges.push_back(FlowEdge(v, u, cap));
29.         edges.push_back(FlowEdge(u, v, 0));
30.         adj[v].push_back(m);
31.         adj[u].push_back(m + 1);
32.         m += 2;
33.     }
34.
35.     bool bfs() {
36.         while (!q.empty()) {
37.             int v = q.front();
38.             q.pop();
39.             for (int id : adj[v]) {
40.                 if (edges[id].cap - edges[id].flow < 1)
41.                     continue;
42.                 if (level[edges[id].u] != -1)
43.                     continue;
44.                 level[edges[id].u] = level[v] + 1;
45.                 q.push(edges[id].u);
46.             }
47.         }
48.         return level[t] != -1;
49.     }
50.
51.     long long dfs(int v, long long pushed) {
52.         if (pushed == 0)
53.             return 0;
54.         if (v == t)
55.             return pushed;
56.         for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
57.             int id = adj[v][cid];
58.             int u = edges[id].u;
59.             if (level[v] + 1 != level[u] || edges[id].cap - edges[id].flow < 1)
60.                 continue;
61.             long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
```

```

62.         if (tr == 0)
63.             continue;
64.         edges[id].flow += tr;
65.         edges[id ^ 1].flow -= tr;
66.         return tr;
67.     }
68.     return 0;
69. }
70.
71. long long flow() {
72.     long long f = 0;
73.     while (true) {
74.         fill(level.begin(), level.end(), -1);
75.         level[s] = 0;
76.         q.push(s);
77.         if (!bfs())
78.             break;
79.         fill(ptr.begin(), ptr.end(), 0);
80.         while (long long pushed = dfs(s, flow_inf)) {
81.             f += pushed;
82.         }
83.     }
84.     return f;
85. }
86. };

```

## Edmons-Karp

```
1.  /*
2.  Edmonds-Karp algorithm  $O(VE^2)$ 
3.  Max flow
4.  The matrix capacity stores the capacity for every pair of vertices.
5.  adj is the adjacency list of the undirected graph,
6.  since we have also to use the reversed of directed edges when we are looking for
    augmenting paths.
7.
8.  The function maxflow will return the value of the maximal flow.
9.  During the algorithm the matrix capacity will actually store the residual capacity of
    the network.
10. The value of the flow in each edge will actually no stored,
11. but it is easy to extent the implementation - by using an additional matrix - to also
    store the flow and return it.
12. */
13.
14. int n;
15. vector<vector<int>> capacity;
16. vector<vector<int>> adj;
17.
18. int bfs(int s, int t, vector<int>& parent) {
19.     fill(parent.begin(), parent.end(), -1);
20.     parent[s] = -2;
21.     queue<pair<int, int>> q;
22.     q.push({s, INF});
23.
24.     while (!q.empty()) {
25.         int cur = q.front().first;
26.         int flow = q.front().second;
27.         q.pop();
28.
29.         for (int next : adj[cur]) {
30.             if (parent[next] == -1 && capacity[cur][next]) {
31.                 parent[next] = cur;
32.                 int new_flow = min(flow, capacity[cur][next]);
33.                 if (next == t)
34.                     return new_flow;
35.                 q.push({next, new_flow});
36.             }
37.         }
38.     }
39.
40.     return 0;
41. }
42.
43. int maxflow(int s, int t) {
44.     int flow = 0;
45.     vector<int> parent(n);
46.     int new_flow;
47.
48.     while (new_flow = bfs(s, t, parent)) {
49.         flow += new_flow;
50.         int cur = t;
51.         while (cur != s) {
52.             int prev = parent[cur];
53.             capacity[prev][cur] -= new_flow;
54.             capacity[cur][prev] += new_flow;
55.             cur = prev;
56.         }
57.     }
58.
59.     return flow;
60. }
```

## Hungarian

```

1.  const int maxN = 50 + 9, MOD = 1e9 + 7, INF = 2e7;
2.
3.  int n, m, a[maxN][maxN];
4.
5.  /*
6.
7.  HUNGARIAN ALGORITHM
8.
9.  -efficiency  $O(n^2 * m)$ 
10. -input matrix a[1..n][1..m] ONE BASED!
11. -n <= m
12. -need to fix an INF value!
13. -it can handle negative values :0
14. -returns MINIMUM cost (just *= -1 to change it to maximum cost)
15.
16. Such an incredible implementation demands credit for the author. Thanks again, Andrei
    Lopatin, we remember your genius every time we look at this beautiful piece of code!
17.
18. "This implementation was actually developed by Andrei Lopatin several years ago. It
    is distinguished by an amazing brevity: the whole algorithm is placed in 30 lines of
    code ."
19.
20. */
21.
22. int hungarian(){
23.     vector<int> u (n+1), v (m+1), p (m+1), way (m+1);
24.     for (int i=1; i<=n; ++i) {
25.         p[0] = i;
26.         int j0 = 0;
27.         vector<int> minv (m+1, INF);
28.         vector<char> used (m+1, false);
29.         do {
30.             used[j0] = true;
31.             int i0 = p[j0], delta = INF, j1;
32.             for (int j=1; j<=m; ++j)
33.                 if (!used[j]) {
34.                     int cur = a[i0][j]-u[i0]-v[j];
35.                     if (cur < minv[j])
36.                         minv[j] = cur, way[j] = j0;
37.                     if (minv[j] < delta)
38.                         delta = minv[j], j1 = j;
39.                 }
40.             for (int j=0; j<=m; ++j)
41.                 if (used[j])
42.                     u[p[j]] += delta, v[j] -= delta;
43.             else
44.                 minv[j] -= delta;
45.             j0 = j1;
46.         } while (p[j0] != 0);
47.         do {
48.             int j1 = way[j0];
49.             p[j0] = p[j1];
50.             j0 = j1;
51.         } while (j0);
52.     }
53.
54.     /*
55.     Recovery of the answer in a more usual form, i.e. finding for each row the i = 1
    \ldots n number of the column selected in it ans [i] is done as follows:*/
56.     vector<int> ans (n+1);
57.     for (int j=1; j<=m; ++j)
58.         ans[p[j]] = j;
59.

```

```

60.      /*
61.
62.      The value of the matching found can simply be taken as the potential of the zero
        column (taken with the opposite sign). In fact, how easy it is to trace the code -v
        [0] contains a sum of all quantities delta, i.e. total potential change. Although at
        each potential change several values could change at once u [i] and v [j], the total
        change in the potential value is exactly the same delta, since as long as there is no
        increasing chain, the number of reachable rows is exactly one more than the number of
        reachable columns (only the current row does not have a pair in the form of a visited
        column )*/
63.
64.      return -v[0];
65.  }
66.
67.  int main(){
68.      ios::sync_with_stdio(false);
69.      cin.tie(0);
70.      cout.tie(0);
71.
72.      cin >> n >> m;
73.
74.      for(int i = 1; i <= n; i++)
75.          for(int j = 1; j <= m; j++)
76.              cin >> a[i][j];
77.
78.      cout << hungarian();
79.  }

```



## Min-cost flow

```
1.  /*
2.  Minimum-cost flow
3.   $O(n^3m)$ 
4.  */
5.
6.  struct Edge
7.  {
8.      int from, to, capacity, cost;
9.  };
10.
11. vector<vector<int>> adj, cost, capacity;
12.
13. const int INF = 1e9;
14.
15. void shortest_paths(int n, int v0, vector<int>& d, vector<int>& p) {
16.     d.assign(n, INF);
17.     d[v0] = 0;
18.     vector<int> m(n, 2);
19.     deque<int> q;
20.     q.push_back(v0);
21.     p.assign(n, -1);
22.
23.     while (!q.empty()) {
24.         int u = q.front();
25.         q.pop_front();
26.         m[u] = 0;
27.         for (int v : adj[u]) {
28.             if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
29.                 d[v] = d[u] + cost[u][v];
30.                 p[v] = u;
31.                 if (m[v] == 2) {
32.                     m[v] = 1;
33.                     q.push_back(v);
34.                 } else if (m[v] == 0) {
35.                     m[v] = 1;
36.                     q.push_front(v);
37.                 }
38.             }
39.         }
40.     }
41. }
42.
43. int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t) {
44.     adj.assign(N, vector<int>());
45.     cost.assign(N, vector<int>(N, 0));
46.     capacity.assign(N, vector<int>(N, 0));
47.     for (Edge e : edges) {
48.         adj[e.from].push_back(e.to);
49.         adj[e.to].push_back(e.from);
50.         cost[e.from][e.to] = e.cost;
51.         cost[e.to][e.from] = -e.cost;
52.         capacity[e.from][e.to] = e.capacity;
53.     }
54.
55.     int flow = 0;
56.     int cost = 0;
57.     vector<int> d, p;
58.     while (flow < K) {
59.         shortest_paths(N, s, d, p);
60.         if (d[t] == INF)
61.             break;
62.
63.         // find max flow on that path
```

```

64.     int f = K - flow;
65.     int cur = t;
66.     while (cur != s) {
67.         f = min(f, capacity[p[cur]][cur]);
68.         cur = p[cur];
69.     }
70.
71.     // apply flow
72.     flow += f;
73.     cost += f * d[t];
74.     cur = t;
75.     while (cur != s) {
76.         capacity[p[cur]][cur] -= f;
77.         capacity[cur][p[cur]] += f;
78.         cur = p[cur];
79.     }
80. }
81.
82. if (flow < K)
83.     return -1;
84. else
85.     return cost;
86. }

```

## 10. Variuos

### Gauss-Jordan

```
1. // Regresa el vector con los valores de cada variable.
2. // Si no es posible obtener solucion, regresa un vector vacio.
3. vector<double> gaussJordan(vector<vector<double> > v) {
4.     int n = v.size();
5.     double val;
6.     FOR(i, 0, n) {
7.         if (v[i][i] == 0) {
8.             bool b = 1;
9.             FOR(j, i+1, n) {
10.                 if (v[i][j] != 0) {
11.                     swap(v[i], v[j]);
12.                     b = 0;
13.                     break;
14.                 }
15.             }
16.             if (b) return vector<double>();
17.         }
18.
19.         val = v[i][i];
20.         FOR(j, i, n+1) {
21.             v[i][j] /= val;
22.         }
23.
24.         FOR(k, 0, n) {
25.             if (i == k) continue;
26.             val = -v[k][i];
27.             FOR(j, i, n+1) {
28.                 v[k][j] += v[i][j] * val;
29.             }
30.         }
31.     }
32.
33.     vector<double> vd(n);
34.     FOR(i, 0, n) vd[i] = v[i][n];
35.     return vd;
36. }
37.
38. int main() { _
39.     vector<double> v(4);
40.     vector<vector<double> > vv;
41.     v[0] = 1;
42.     v[1] = 1;
43.     v[2] = 1;
44.     v[3] = 5;
45.     vv.pb(v);
46.
47.     v[0] = 2;
48.     v[1] = 3;
49.     v[2] = 5;
50.     v[3] = 8;
51.     vv.pb(v);
52.
53.     v[0] = 4;
54.     v[1] = 0;
55.     v[2] = 5;
56.     v[3] = 2;
57.     vv.pb(v);
58.
59.     FOR(i, 0, 3) {
60.         FOR(j, 0, 4)
61.             cout << vv[i][j] << " ";
```

```
62.         cout << endl;
63.     }
64.     cout << endl << endl;
65.
66.     vector<double> vd = gaussJordan(vv);
67.     printArr(vd, 0, 3);
68.
69.     return 0;
70. }
```

## 2-Sat

```
1.  #define MAXN 100005
2.  #define MOD 1000000007
3.
4.  struct TwoSAT{
5.      int n;
6.      vector<int> G[MAXN*2];
7.      int S[MAXN*2], c;
8.      bool mark[MAXN*2];
9.
10.     bool dfs(int x){
11.         if(mark[x^1]) return false;
12.         if(mark[x]) return true;
13.         mark[x] = true;
14.         S[c++] = x;
15.
16.         FOR(i, 0, G[x].size())
17.             if(!dfs(G[x][i])) return false;
18.         return true;
19.     }
20.
21.     void init(int n){
22.         this->n = n;
23.         FOR(i, 0, 2*n) G[i].clear();
24.         memset(mark, 0, sizeof(mark));
25.     }
26.
27.     /*
28.      * Each clause is in the form x or y
29.      * x is abs(x) - 1 and xval is x < 0 ? 1 : 0
30.      */
31.     void add_clause(int x, int xval, int y, int yval){
32.         x = x*2 + xval;
33.         y = y*2 + yval;
34.         G[x^1].push_back(y);
35.         G[y^1].push_back(x);
36.     }
37.
38.     bool solve(){
39.         for(int i = 0; i < 2*n; i += 2){
40.             if(!mark[i] && !mark[i+1]){
41.                 c = 0;
42.                 if(!dfs(i)){
43.                     while(c > 0) mark[S[--c]] = false;
44.                     if(!dfs(i + 1)) return false;
45.                 }
46.             }
47.         }
48.         return true;
49.     }
50. }TS;
51.
52. int main(){
53.     int n, m;
54.     while(scanf("%d %d", &n, &m) != EOF){
55.         TS.init(n);
56.         // scan m clauses
57.         FOR(i, 0, m){
58.             int a, b;
59.             scanf("%d %d", &a, &b);
60.             TS.add_clause(abs(a) - 1, a < 0 ? 1 : 0, abs(b) - 1, b < 0 ? 1 : 0);
61.         }
62.         printf("%d\n", TS.solve()? 1 : 0);
63.     }
64.     return 0;
65. }
```

## Bits

```
1. typedef unsigned int ui;
2. typedef unsigned long long ull;
3.
4. ull BitCount(ull u) {
5.     ull uCount = u - ((u >> 1) & 033333333333) - ((u >> 2) & 011111111111);
6.     return ((uCount + (uCount >> 3)) & 030707070707) % 63;
7. }
8.
9. ull flipBits(ull n, int k) {
10.    ull mask = (1 << k) - 1;
11.    return ~n & mask;
12. }
13.
14. ull flipBits(ull n) {
15.    return ~n;
16. }
17.
18. ull differentBits(ull a, ull b) {
19.    return BitCount(a ^ b);
20. }
21.
22. void getEvenOddBits(ull n) {
23.    // Para ui, hacerlo con 8 A's o 5's
24.    ull evenBits = n & 0xAAAAAAAAAAAAAA;
25.    ull oddBits = n & 0x5555555555555555;
26. }
27.
28. ull rotateBits(ull n, int d) {
29.    // d negative for left rotation, positive for right.
30.    d %= 64;
31.    return (n >> d) | (n << (64 - d));
32. }
33.
34. string toBinary(ull n) {
35.    string s = "";
36.    while(n) {
37.        if (n & 1) s = "1" + s;
38.        else s = "0" + s;
39.        n >>= 1;
40.    }
41.    return s;
42. }
43.
44. ui getSetBitsFromOneToN(ull n){
45.    ui two = 2, ans = 0;
46.    ull N = n;
47.    while(n) {
48.        ans += (N / two) * (two >> 1);
49.        if ((N & (two - 1)) > (two >> 1) - 1)
50.            ans += (N & (two - 1)) - (two >> 1) + 1;
51.        two <<= 1;
52.        n >>= 1;
53.    }
54.    return ans;
55. }
```

## Fast IO

```
1.  const int BUFFSIZE = 10240;
2.  char BUFF[BUFFSIZE + 1], *ppp = BUFF;
3.  int RR, CHAR, SIGN, BYTES = 0;
4.  #define GETCHAR(c) { \
5.      if(ppp-BUFF==BYTES && (BYTES==0 || BYTES==BUFFSIZE)) { BYTES = fread(BUFF,1,BUFFSIZE,stdin);
      ppp=BUFF; } \
6.      if(ppp-BUFF==BYTES && (BYTES>0 && BYTES<BUFFSIZE)) { BUFF[0] = 0; ppp=BUFF; }\
7.      c = *ppp++; \
8.  }
9.  #define DIGIT(c) (((c) >= '0') && ((c) <= '9'))
10. #define MINUS(c) ((c)=='-')
11. #define GETNUMBER(n) { \
12.     n = 0; SIGN = 1; do { GETCHAR(CHAR); } while(!(DIGIT(CHAR) || MINUS(CHAR))); \
13.     if(MINUS(CHAR)) { SIGN = -1; GETCHAR(CHAR); } \
14.     while(DIGIT(CHAR)) { n = ((n<<3) + (n << 1)) + CHAR-'0'; GETCHAR(CHAR); } if(SIGN == -1) { n =
      -n; } \
15. }
```

## FFT 1

```

1. struct Complex{
2.     double real, imag;
3.     Complex(){}
4.     Complex(const complex<double> &a): real(a.real()), imag(a.imag()){}
5.     Complex(double real, double imag): real(real), imag(imag){}
6.     Complex operator*(const Complex &a){
7.         return Complex(real * a.real - imag * a.imag, real * a.imag + a.real * imag);
8.     }
9.     Complex operator+(const Complex &a){return Complex(real + a.real,
    imag + a.imag);}
10. };
11. Complex conj(const Complex &a){return Complex(a.real, -a.imag);}
12.
13. const int maxN = 1e6;
14. int maxL;
15.
16. int reverses[maxN << 2];
17. bool v[maxN << 2];
18. Complex steps[30], unityRoots[maxN << 2], first[maxN << 2], second[maxN << 2];
19. ll sums[maxN << 2];
20.
21. int reverse(int num, int exp){
22.     int reverse = 0, pot = 1, inv = 1 << (exp - 1);
23.     while(inv >= 1){
24.         if(num & pot) reverse |= inv;
25.         inv >>= 1, pot <<= 1;
26.     }
27.     return reverse;
28. }
29.
30. void sqRoot(Complex *vec, int len, int loog){
31.     for(int i = len - 1; i >= 0; i--){
32.         vec[i << 1] = vec[i];
33.         vec[(i << 1) + 1] = vec[i] * steps[loog];
34.     }
35. }
36.
37. void FFT(Complex *coef, int arrS){
38.     const int saiz = ceil(log2(arrS));
39.
40.     memset(v, false, sizeof(v));
41.     for(int i = 0; i < (1 << saiz); i++){
42.         if(!v[reverses[i] >> (maxL - saiz)])
43.             swap(coef[i], coef[reverses[i] >> (maxL - saiz)]), v[i] = true;
44.         unityRoots[0] = Complex(1, 0);
45.
46.         //DIVIDE AND CONQUER...
47.         for(int T = 1, u = 0; T < (1 << saiz); T <<= 1, u++){
48.             sqRoot(unityRoots, T, u);
49.             for(int i = 0; i < (1 << saiz); i += (T << 1)){
50.                 for(int j = 0; j < T; j++){
51.                     Complex lTmp = coef[i + j], rTmp = coef[i + j + T];
52.
53.                     coef[i + j] = lTmp + rTmp * unityRoots[j];
54.                     coef[i + j + T] = lTmp + rTmp * unityRoots[j + T];
55.                 }
56.             }
57.         }
58. }
59.
60. //one & two = coefficient arrays w/sizes, res is where results are stored
61. void polynomMultiplication(int *one, int *two, int *res, int oS, int tS){
62.     const int saiz = 1 << int(ceil(log2(oS + tS)));

```



```

63.
64.     for(int i = 0; i < saiz; i++)
65.         first[i] = Complex(i < oS? double(one[i]): 0.0, 0.0);
66.     for(int i = 0; i < saiz; i++)
67.         second[i] = Complex(i < tS? double(two[i]): 0.0, 0.0);
68.
69.     //FFT
70.     FFT(first, saiz), FFT(second, saiz);
71.
72.     //INVERSE FFT = FFT(conj(C1 * C2) / N)
73.     for(int i = 0; i < saiz; i++)
74.         first[i] = conj(first[i] * second[i]);
75.     FFT(first, saiz);
76.
77.     for(int i = 0; i < saiz; i++)
78.         res[i] = int(round(first[i].real / saiz));
79. }

```

## FFT 2

```

1. const int maxN = 2.1e6 + 3, MOD = 1e9 + 7, LG = 21;
2. //NOTE: maxN>2^LG must be held, remember maxN is twice ur biggest polynom
3.
4. cd ur[maxN], res[maxN], ors[maxN], cf[maxN];
5. //assumes sz and step are 2-powers
6. void FFT(int cfs, int step, int his, int rs, int sz, cd *res){
7.     if(cfs == 1)
8.         for(int i = 0; i < sz; i++) res[i] = cf[his];
9.     else{
10.
11.         FFT(ceil(cfs/2.0), step+1, his, rs<<1, sz>>1, res);
12.         FFT(cfs>>1, step+1, 1 << step | his, rs<<1, sz>>1, res + (sz>>1));
13.
14.         for(int i = 0, m = sz >> 1; i < m; i++){
15.             cd cr = res[i];
16.             res[i] = cr + res[i + m] * ur[i * rs];
17.             res[i + m] = cr + res[i + m] * ur[(i + m) * rs];
18.         }
19.     }
20. }
21.
22. void mu(const vector<ll> &a, const vector<ll> &b, vector<ll> &ans){
23.     int sz = 1<<int(ceil(log2(a.size() + b.size() - 1)));
24.
25.     for(int i = 0; i < a.size(); i++) cf[i] = a[i];
26.     FFT((int)a.size(), 0, 0, (1<<LG)/sz, sz, res);
27.
28.     for(int i = 0; i < b.size(); i++) cf[i] = b[i];
29.     FFT((int)b.size(), 0, 0, (1<<LG)/sz, sz, ors);
30.
31.     for(int i = 0; i < sz; i++) cf[i] = conj(res[i] * ors[i]);
32.     FFT(sz, 0, 0, (1<<LG)/sz, sz, res);
33.
34.
35.
36.     for(int i = 0; i < a.size() + b.size() - 1; i++)
37.         ans.push_back(ll(round(res[i].real() / sz)));
38.
39. }
40.
41. string st;
42. int main(){
43.     ios::sync_with_stdio(false);
44.     cin.tie(0), cout.tie(0);
45.
46.     ur[0] = 1, ur[1] = polar(1.0, M_PI * 2 / (1 << LG));
47.
48.     for(int i = 2; i < (1 << LG); i++)
49.         ur[i] = ur[i - 1] * ur[1];
50.
51.     cin >> st;
52.
53.     vector<ll> a, b, ans;
54.     for(int i = 0; i < st.size(); i++)
55.         a.push_back((st[i] == 'B')? 1: 0);
56.     for(int i = int(st.size() - 1); i >= 0; i--)
57.         b.push_back((st[i] == 'A')? 1: 0);
58.
59.     mu(a, b, ans);
60.     for(int i = int(st.size() - 2); i >= 0; i--) cout << ans[i] << '\n';
61.
62. }
63. //1010

```

```

64. //1010
65.
66. //coefficients can be handled with an integer for how many, and a pair of ints: step
    and history, which will tell the index of the coefficient in the base case (the only
    time it will be used)
67. //now, for the unity roots, use formula  $x * 2 \% sz$  to get square simply, in this way
    we can keep only one array of unity roots.
68. //for m current unity roots, when we square them, all indices will have at least one
    zero to left more in binary notation. So we just need to calculate squares of the
    upper side of argand's diagram. So its new size will be a power of two and the step
    unity will be twice bigger.
69. //args: cfs(how many coefficients), step(for coef), history(parities defined),
    rootstep(size of unity root step), rootam(amount of roots to consider)
70. //so process is simple now:
71. //base case: return coefficient rootam times
72. //regular case: rec(ceil(n/2), step+1, history, rootstep*2, next2pow(rootam))
73. //      rec(floor(n/2), step+1, 1<< st|history, rootstep*2, next2pow(rootam))
74.
75. //now where should we store them?
76. //create array that will store evaluations. So add new parameter Foo *pl, where we
    will store results of shit. When we call recursively, store at end all even- coef
    evals. Then put in current and forget, so use same place for unevens

```

## Fraction

```
1. struct Fraction {
2.     ll num, den;
3.
4.     Fraction() {num = den = 1;}
5.
6.     Fraction(ll numm, ll denn) : num(numm), den(denn) {
7.         simplify();
8.     }
9.
10.    void simplify() {
11.        bool signo = false;
12.        if (num < 0) signo = !signo;
13.        if (den < 0) signo = !signo;
14.        ll g = __gcd(num, den);
15.        if (g) {
16.            num /= g;
17.            den /= g;
18.        }
19.        if (signo) num = -num;
20.    }
21.
22.    Fraction operator + (const Fraction &r) const {
23.        Fraction ret;
24.        ret.num = num*r.den + r.num*den;
25.        ret.den = r.den*den;
26.        ret.simplify();
27.        return ret;
28.    }
29.
30.    Fraction operator + (const ll &r) const {
31.        Fraction ret;
32.        ret.num = num + r*den;
33.        ret.den = den;
34.        return ret;
35.    }
36.
37.    Fraction operator - (const Fraction &r) const {
38.        Fraction ret;
39.        ret.num = num*r.den - r.num*den;
40.        ret.den = r.den*den;
41.        ret.simplify();
42.        return ret;
43.    }
44.
45.    Fraction operator - (const ll &r) const {
46.        Fraction ret;
47.        ret.num = num - r * den;
48.        ret.den = den;
49.        return ret;
50.    }
51.
52.    Fraction operator * (const Fraction &r) const {
53.        Fraction ret;
54.        ret.num = num*r.num;
55.        ret.den = den*r.den;
56.        ret.simplify();
57.        return ret;
58.    }
59.
60.    Fraction operator * (const ll &r) const {
61.        Fraction ret;
62.        ret.num = num*r;
63.        ret.den = den;
64.        ret.simplify();
65.        return ret;
66.    }
67.
68.    Fraction operator / (const Fraction &r) const {
69.        Fraction ret;
70.        ret.num = num*r.den;
71.        ret.den = den*r.num;
```

```

72.         ret.simplify();
73.         return ret;
74.     }
75.
76.     Fraction operator / (const ll &r) const {
77.         Fraction ret;
78.         ret.num = num;
79.         ret.den = den*r;
80.         ret.simplify();
81.         return ret;
82.     }
83.
84.     Fraction pow(int n) const {
85.         Fraction ret(1, 1);
86.         Fraction x(num, den);
87.         while (n) {
88.             if (n & 1) ret = ret * x;
89.             x = x * x;
90.             n >>= 1;
91.         }
92.         return ret;
93.     }
94.
95.     bool operator <(const Fraction &r) const {
96.         return num * r.den < den * r.num;
97.     }
98.
99.     bool operator ==(const Fraction &r) const {
100.        return num == r.num && den == r.den;
101.    }
102.
103.    string to_str() {
104.        return to_string(num) + "/" + to_string(den);
105.    }
106. };
107.
108. int main() { _
109.     Fraction f;
110.     while(cin >> f.num >> f.den) {
111.         f.simplify();
112.         cout << f.num << "/" << f.den << endl;
113.     }
114.
115.     return 0;
116. }

```

## Hour

```
1. struct Hora {
2.     int h, m, s, t;
3.
4.     Hora() { t = h = m = s = 0; }
5.
6.     Hora(int hh, int mm, int ss) : h(hh), m(mm), s(ss) {
7.         simplify();
8.     }
9.
10.    Hora(string ss) {
11.        int f1 = ss.find(":");
12.        h = atoi(ss.substr(0, f1).c_str());
13.
14.        int f2 = ss.find(":", f1+1);
15.        if (f2 != -1) {
16.            m = atoi(ss.substr(f1+1, f2-f1).c_str());
17.            s = atoi(ss.substr(f2+1).c_str());
18.        }
19.        else {
20.            m = atoi(ss.substr(f1+1).c_str());
21.            s = 0;
22.        }
23.
24.        simplify();
25.    }
26.
27.    Hora operator +(const Hora &r) const {
28.        Hora ret;
29.
30.        ret.h = h + r.h;
31.        ret.m = m + r.m;
32.        ret.s = s + r.s;
33.
34.        ret.simplify();
35.
36.        return ret;
37.    }
38.
39.    Hora operator -(const Hora &r) const {
40.        Hora ret;
41.
42.        ret.h = h - r.h;
43.        ret.m = m - r.m;
44.        ret.s = s - r.s;
45.
46.        ret.simplify();
47.
48.        return ret;
49.    }
50.
51.    bool operator <(const Hora &r) const {
52.        return t < r.t;
53.    }
54.
55.    bool operator ==(const Hora &r) const {
56.        return t == r.t;
57.    }
58.
59.    void simplify() {
60.        t = 3600 * h + 60 * m + s;
61.        t = (t + 86400) % 86400;
62.
63.        int tt = t;
64.        h = tt / 3600;
65.        tt %= 3600;
66.        m = tt / 60;
67.        s = tt % 60;
68.    }
69.
70.    string to_str() {
71.        string ret = "";
72.
```

```

73.         if (h < 10) ret = "0";
74.         ret += std::to_string(h) + ":";
75.
76.         if (m < 10) ret += "0";
77.         ret += std::to_string(m) + ":";
78.
79.         if (s < 10) ret += "0";
80.         ret += std::to_string(s);
81.
82.         return ret;
83.     }
84.
85.     string to_str(const bool b) {
86.         string ret = "";
87.
88.         if (h < 10) ret = "0";
89.         ret += to_string(h) + ":";
90.
91.         if (m < 10) ret += "0";
92.         ret += to_string(m);
93.
94.         return ret;
95.     }
96. };
97.
98. int main() { _
99.     Hora h1, h2;
100.    string s1, s2;
101.
102.    while(cin >> s1 >> s2) {
103.        h1 = Hora(s1);
104.        h2 = Hora(s2);
105.
106.        cout << (h1 - h2).to_str() << endl;
107.    }
108.
109.    return 0;
110. }

```

## Matrix Exponentiation

```
1.  #define FIRST first
2.  #define SECOND second.first
3.  #define THIRD second.second
4.
5.  using namespace std;
6.  typedef pair<int, int> ii;
7.  typedef vector<double> vd;
8.  typedef vector<vd> Matrix;
9.  typedef long long ll;
10.
11. const int maxN = 100;
12.
13. Matrix operator*(const Matrix &first, const Matrix &second){
14.     Matrix ret(first.size(), vd(second[0].size(), 0.0));
15.     for(int i = 0; i < first.size(); i++)
16.         for(int j = 0; j < second[0].size(); j++)
17.             for(int k = 0; k < second.size(); k++)
18.                 ret[i][j] += first[i][k] * second[k][j];
19.     return ret;
20. }
21. Matrix operator^(Matrix mat, int coef){
22.     Matrix ret(mat.size(), vd(mat.size(), 0.0));
23.     for(int i = 0; i < mat.size(); i++) ret[i][i] = 1.0;
24.     while(coef){
25.         if(coef & 1)
26.             ret = ret * mat;
27.         coef >>= 1, mat = mat * mat;
28.     }
29.     return ret;
30. }
```



## Roman Numbers

```
1.  #define MAXN 10
2.  #define MOD 1000000007
3.
4.  int mil[MAXN];
5.
6.  string fill(char c, int n) {
7.      string s;
8.      while (n--) s += c;
9.      return s;
10. }
11.
12. string toRoman(ll n, int nivel) {
13.     if (n == 0) return "";
14.     if (n < 4) return fill('I', n);
15.     if (n < 6) return fill('I', 5 - n) + "V";
16.     if (n < 9) return string("V") + fill('I', n - 5);
17.     if (n < 11) return fill('I', 10 - n) + "X";
18.     if (n < 40) return fill('X', n / 10) + toRoman(n % 10, nivel);
19.     if (n < 60) return fill('X', 5 - n / 10) + 'L' + toRoman(n % 10, nivel);
20.     if (n < 90) return string("L") + fill('X', n / 10 - 5) + toRoman(n % 10, nivel);
21.     if (n < 110) return fill('X', 10 - n / 10) + "C" + toRoman(n % 10, nivel);
22.     if (n < 400) return fill('C', n / 100) + toRoman(n % 100, nivel);
23.     if (n < 600) return fill('C', 5 - n / 100) + 'D' + toRoman(n % 100, nivel);
24.     if (n < 900) return string("D") + fill('C', n / 100 - 5) + toRoman(n % 100, nivel);
25.     if (n < 1100) return fill('C', 10 - n / 100) + "M" + toRoman(n % 100, nivel);
26.     if (n < 4000) return fill('M', n / 1000) + toRoman(n % 1000, nivel);
27.
28.     string ret = toRoman(n / 1000, nivel + 1);
29.     string ret2 = toRoman(n % 1000, nivel);
30.     mil[nivel] = ret.length();
31.     if (ret2 == "") return ret;
32.     return ret + " " + toRoman(n % 1000, nivel);
33. }
34.
35. string toRoman(ll n) {
36.     string ret = toRoman(n, 0);
37.     for(int i = 0; mil[i]; i++) {
38.         ret = fill('_', mil[i]) + "\n" + ret;
39.     }
40.
41.     return ret;
42. }
43.
44. int main() { _
45.     ll n;
46.     while (cin >> n) {
47.         cout << toRoman(n) << endl;
48.     }
49.     return 0;
50. }
```

## Rotate Matrix

```
1. #define MAXN 10
2. #define MOD 1000000007
3.
4. int mat[MAXN][MAXN];
5.
6. void giraMat(int k, int n) {
7.     k %= 4;
8.
9.     FOR(p, 0, k) {
10.        FOR(i, 0, n/2) {
11.            FOR(j, i, n - i - 1) {
12.                ii p1(i, j);
13.                ii p2(j, n - i - 1);
14.                ii p3(n - i - 1, n - j - 1);
15.                ii p4(n - j - 1, i);
16.
17.                int aux = mat[p1.first][p1.second];
18.                mat[p1.first][p1.second] = mat[p4.first][p4.second];
19.                mat[p4.first][p4.second] = mat[p3.first][p3.second];
20.                mat[p3.first][p3.second] = mat[p2.first][p2.second];
21.                mat[p2.first][p2.second] = aux;
22.            }
23.        }
24.    }
25. }
26.
27. int main() { _
28.     int cant = 1;
29.     FOR(i, 0, MAXN) FOR(j, 0, MAXN) {
30.         mat[i][j] = cant;
31.         cant ++;
32.     }
33.
34.     giraMat(6, MAXN);
35.
36.     FOR(i, 0, MAXN) FOR(j, 0, MAXN)
37.         cout << mat[i][j] << "\t\n"[j == MAXN-1];
38.     return 0;
39. }
```

## Sorts

```
1. #define MAXN 15
2. #define MOD 1000000007
3.
4. int arr[MAXN];
5. int comp, inter;
6.
7. void print() {
8.     FOR(i, 0, MAXN) {
9.         cout << arr[i] << " \n"[i == MAXN-1];
10.    }
11.    cout << endl;
12. }
13.
14. void bubbleSort() {
15.     int n = MAXN;
16.     bool b = true;
17.
18.     for(int i = 0; i < n - 1 && b; i++){
19.         b = false;
20.         FOR(j, 0, n - 1 - i) {
21.             if (arr[j+1] < arr[j]) {
22.                 swap(arr[j], arr[j + 1]);
23.                 b = true;
24.
25.                 inter ++;
26.             }
27.             comp ++;
28.         }
29.     }
30. }
31.
32. void selectionSort() {
33.     int n = MAXN;
34.     FOR(i, 0, n - 1) {
35.         int mini = i;
36.         FOR(j, i + 1, n) {
37.             comp ++;
38.             if (arr[j] < arr[mini]) mini = j;
39.
40.         }
41.
42.         if (i != mini) {
43.             swap(arr[i], arr[mini]);
44.             inter ++;
45.         }
46.     }
47. }
48.
49. void intercambioSort() {
50.     int n = MAXN;
51.     FOR(i, 0, n - 1) {
52.         FOR(j, i + 1, n) {
53.             if (arr[i] > arr[j]) {
54.                 swap(arr[i], arr[j]);
55.                 inter++;
56.             }
57.             comp++;
58.         }
59.     }
60. }
61.
62. void insertionSort() {
63.     int j, key, n = MAXN;
64.     FOR(i, 1, n) {
65.         key = arr[i];
66.         j = i - 1;
67.         while (j >= 0 && arr[j] > key) {
68.             inter ++;
69.             arr[j+1] = arr[j];
70.             j = j - 1;
71.         }
72.     }
```

```

73.         comp = inter;
74.         if(j)    comp ++;
75.
76.         arr[j+1] = key;
77.     }
78. }
79.
80. void insertionBinarySort() {
81.     int n = MAXN;
82.     vi v;
83.     FOR(i, 0, n) {
84.         v.insert(upper_bound(v.begin(), v.end(), arr[i]), arr[i]);
85.     }
86.
87.     copy(v.begin(), v.end(), arr);
88. }
89.
90.
91. void Une(int ini, int mitad, int fin) {
92.     int aux[fin - ini + 1];
93.     int i = ini, j = mitad + 1, k = 0;
94.
95.     while(i <= mitad && j <= fin) {
96.         if (arr[i] < arr[j]) {
97.             aux[k] = arr[i];
98.             i ++;
99.         }
100.        else {
101.            aux[k] = arr[j];
102.            j ++;
103.        }
104.        k ++;
105.    }
106.
107.    for(; i <= mitad; i ++, k ++)    aux[k] = arr[i];
108.    for(; j <= fin; j ++, k ++)    aux[k] = arr[j];
109.
110.    for(k = 0; ini <= fin; ini ++, k ++)
111.        arr[ini] = aux[k];
112.}
113.
114.void mergeSort(int ini, int fin){
115.    if(ini < fin){
116.        int mitad = (ini + fin) / 2;
117.        mergeSort(ini, mitad);
118.        mergeSort(mitad + 1, fin);
119.        Une(ini, mitad, fin);
120.    }
121.}
122.
123.
124.void heapify(int n, int i) {
125.    int maxi = i;
126.    int l = 2*i + 1;
127.    int r = 2*i + 2;
128.
129.    if (l < n && arr[l] > arr[maxi])
130.        maxi = l;
131.
132.    if (r < n && arr[r] > arr[maxi])
133.        maxi = r;
134.
135.    if (maxi != i) {
136.        swap(arr[i], arr[maxi]);
137.        heapify(n, maxi);
138.    }
139.}
140.
141.void heapSort() {
142.    int n = MAXN;
143.    for (int i = n / 2 - 1; i >= 0; i--)
144.        heapify(n, i);
145.
146.    for (int i = n-1; i >= 0; i--) {

```

```

147.         swap(arr[0], arr[i]);
148.         heapify(i, 0);
149.     }
150.}
151.
152.int part(int low, int high) {
153.    int pivot = arr[high];
154.    int i = low - 1;
155.
156.    FOR(j, low, high) {
157.        if (arr[j] <= pivot) {
158.            i++;
159.            swap(arr[i], arr[j]);
160.        }
161.    }
162.    swap(arr[i + 1], arr[high]);
163.    return (i + 1);
164.}
165.
166.// QuickSort with less recursion calls.
167.void quickSortNoTail(int low, int high) {
168.    while (low < high) {
169.        int pi = part(low, high);
170.
171.        if (pi - low < high - pi) {
172.            quickSortNoTail(low, pi - 1);
173.            low = pi + 1;
174.        }
175.        else {
176.            quickSortNoTail(pi + 1, high);
177.            high = pi - 1;
178.        }
179.    }
180.}
181.
182.void quickSort(int low, int high) {
183.    if (low < high) {
184.        int pi = part(low, high);
185.        quickSort(low, pi - 1);
186.        quickSort(pi + 1, high);
187.    }
188.}

```

## 11. Otros

### Series

$$\sum_{i=0}^n 1 = n$$

$$\sum_{i=0}^n i = \frac{n * (n + 1)}{2}$$

$$\sum_{i=0}^n i^2 = \frac{n * (n + 1) * (2n + 1)}{6}$$

$$\sum_{i=0}^n i^3 = \frac{n^2 * (n + 1)^2}{4}$$

$$\sum_{i=0}^n i^k \approx \frac{1}{k + 1} * n^{k+1}$$

$$\sum_{i=0}^n a^i = \frac{a^{n+1} - 1}{a - 1}$$

$$\sum_{i=0}^n 2^i = 2^{n+1} - 1$$

$$\sum_{i=0}^n \binom{i}{n} = 2^n$$

$$\log x^y = y \log x$$

$$\log xy = \log x + \log y$$

$$\log \frac{x}{y} = \log x - \log y$$

## First 5000 digits of PI

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808  
6513282306647093844609550582231725359408128481117450284102701938521105559644622948954930381964428810975665933  
4461284756482337867831652712019091456485669234603486104543266482133936072602491412737245870066063155881748815  
2092096282925409171536436789259036001133053054882046652138414695194151160943305727036575959195309218611738193  
2611793105118548074462379962749567351885752724891227938183011949129833673362440656643086021394946395224737190  
7021798609437027705392171762931767523846748184676694051320005681271452635608277857713427577896091736371787214  
6844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134  
999998372978049951059731732816096318595024459455346908302642522308253344685035261931188171010003137838752886  
5875332083814206171776691473035982534904287554687311595628638823537875937519577818577805321712268066130019278  
7661119590921642019893809525720106548586327886593615338182796823030195203530185296899577362259941389124972177  
5283479131515574857242454150695950829533116861727855889075098381754637464939319255060400927701671139009848824  
0128583616035637076601047101819429555961989467678374494482553797747268471040475346462080466842590694912933136  
7702898915210475216205696602405803815019351125338243003558764024749647326391419927260426992279678235478163600  
9341721641219924586315030286182974555706749838505494588586926995690927210797509302955321165344987202755960236  
480665499119881834797753566369807426542527862551818417574672890977727938000816470600161452491921732172147723  
5014144197356854816136115735255213347574184946843852332390739414333454776241686251898356948556209921922218427  
2550254256887671790494601653466804988627232791786085784383827967976681454100953883786360950680064225125205117  
3929848960841284886269456042419652850222106611863067442786220391949450471237137869609563643719172874677646575  
7396241389086583264599581339047802759009946576407895126946839835259570982582262052248940772671947826848260147  
6990902640136394437455305068203496252451749399651431429809190659250937221696461515709858387410597885959772975  
4989301617539284681382686838689427741559918559252459539594310499725246808459872736446958486538367362226260991  
2460805124388439045124413654976278079771569143599770012961608944169486855584840635342207222582848864815845602  
8506016842739452267467678895252138522549954666727823986456596116354886230577456498035593634568174324112515076  
0694794510965960940252288797108931456691368672287489405601015033086179286809208747609178249385890097149096759  
8526136554978189312978482168299894872265880485756401427047755513237964145152374623436454285844479526586782105  
1141354735739523113427166102135969536231442952484937187110145765403590279934403742007310578539062198387447808  
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## First 150 Fibonacci numbers

0 : 0  
1 : 1  
2 : 1  
3 : 2  
4 : 3  
5 : 5  
6 :  $8 = 2^3$   
7 : 13  
8 :  $21 = 3 \times 7$   
9 :  $34 = 2 \times 17$   
10 :  $55 = 5 \times 11$   
11 : 89  
12 :  $144 = 2^4 \times 3^2$   
13 : 233  
14 :  $377 = 13 \times 29$   
15 :  $610 = 2 \times 5 \times 61$   
16 :  $987 = 3 \times 7 \times 47$   
17 : 1597  
18 :  $2584 = 2^3 \times 17 \times 19$   
19 :  $4181 = 37 \times 113$   
20 :  $6765 = 3 \times 5 \times 11 \times 41$   
21 :  $10946 = 2 \times 13 \times 421$   
22 :  $17711 = 89 \times 199$   
23 : 28657  
24 :  $46368 = 2^5 \times 3^2 \times 7 \times 23$   
25 :  $75025 = 5^2 \times 3001$   
26 :  $121393 = 233 \times 521$   
27 :  $196418 = 2 \times 17 \times 53 \times 109$   
28 :  $317811 = 3 \times 13 \times 29 \times 281$   
29 : 514229  
30 :  $832040 = 2^3 \times 5 \times 11 \times 31 \times 61$   
31 :  $1346269 = 557 \times 2417$   
32 :  $2178309 = 3 \times 7 \times 47 \times 2207$   
33 :  $3524578 = 2 \times 89 \times 19801$   
34 :  $5702887 = 1597 \times 3571$   
35 :  $9227465 = 5 \times 13 \times 141961$   
36 :  $14930352 = 2^4 \times 3^3 \times 17 \times 19 \times 107$   
37 :  $24157817 = 73 \times 149 \times 2221$   
38 :  $39088169 = 37 \times 113 \times 9349$   
39 :  $63245986 = 2 \times 233 \times 135721$   
40 :  $102334155 = 3 \times 5 \times 7 \times 11 \times 41 \times 2161$   
41 :  $165580141 = 2789 \times 59369$   
42 :  $267914296 = 2^3 \times 13 \times 29 \times 211 \times 421$   
43 : 433494437  
44 :  $701408733 = 3 \times 43 \times 89 \times 199 \times 307$   
45 :  $1134903170 = 2 \times 5 \times 17 \times 61 \times 109441$   
46 :  $1836311903 = 139 \times 461 \times 28657$   
47 : 2971215073  
48 :  $4807526976 = 2^6 \times 3^2 \times 7 \times 23 \times 47 \times 1103$   
49 :  $7778742049 = 13 \times 97 \times 6168709$   
50 :  $12586269025 = 5^2 \times 11 \times 101 \times 151 \times 3001$   
51 :  $20365011074 = 2 \times 1597 \times 6376021$   
52 :  $32951280099 = 3 \times 233 \times 521 \times 90481$   
53 :  $53316291173 = 953 \times 55945741$   
54 :  $86267571272 = 2^3 \times 17 \times 19 \times 53 \times 109 \times 5779$   
55 :  $139583862445 = 5 \times 89 \times 661 \times 474541$   
56 :  $225851433717 = 3 \times 7^2 \times 13 \times 29 \times 281 \times 14503$   
57 :  $365435296162 = 2 \times 37 \times 113 \times 797 \times 54833$   
58 :  $591286729879 = 59 \times 19489 \times 514229$   
59 :  $956722026041 = 353 \times 2710260697$   
60 :  $1548008755920 = 2^4 \times 3^2 \times 5 \times 11 \times 31 \times 41 \times 61 \times 2521$   
61 :  $2504730781961 = 4513 \times 555003497$   
62 :  $4052739537881 = 557 \times 2417 \times 3010349$   
63 :  $6557470319842 = 2 \times 13 \times 17 \times 421 \times 35239681$   
64 :  $10610209857723 = 3 \times 7 \times 47 \times 1087 \times 2207 \times 4481$   
65 :  $17167680177565 = 5 \times 233 \times 14736206161$   
66 :  $27777890035288 = 2^3 \times 89 \times 199 \times 9901 \times 19801$   
67 :  $44945570212853 = 269 \times 116849 \times 1429913$   
68 :  $72723460248141 = 3 \times 67 \times 1597 \times 3571 \times 63443$   
69 :  $117669030460994 = 2 \times 137 \times 829 \times 18077 \times 28657$   
70 :  $190392490709135 = 5 \times 11 \times 13 \times 29 \times 71 \times 911 \times 141961$   
71 :  $308061521170129 = 6673 \times 46165371073$   
72 :  $498454011879264 = 2^5 \times 3^3 \times 7 \times 17 \times 19 \times 23 \times 107 \times 103681$   
73 :  $806515533049393 = 9375829 \times 86020717$



74 : 1304969544928657 = 73 x 149 x 2221 x 54018521  
75 : 2111485077978050 = 2 x 5<sup>2</sup> x 61 x 3001 x 230686501  
76 : 3416454622906707 = 3 x 37 x 113 x 9349 x 29134601  
77 : 5527939700884757 = 13 x 89 x 988681 x 4832521  
78 : 8944394323791464 = 2<sup>3</sup> x 79 x 233 x 521 x 859 x 135721  
79 : 14472334024676221 = 157 x 92180471494753  
80 : 23416728348467685 = 3 x 5 x 7 x 11 x 41 x 47 x 1601 x 2161 x 3041  
81 : 37889062373143906 = 2 x 17 x 53 x 109 x 2269 x 4373 x 19441  
82 : 61305790721611591 = 2789 x 59369 x 370248451  
83 : 99194853094755497  
84 : 160500643816367088 = 2<sup>4</sup> x 3<sup>2</sup> x 13 x 29 x 83 x 211 x 281 x 421 x 1427  
85 : 259695496911122585 = 5 x 1597 x 9521 x 3415914041  
86 : 420196140727489673 = 6709 x 144481 x 433494437  
87 : 679891637638612258 = 2 x 173 x 514229 x 3821263937  
88 : 1100087778366101931 = 3 x 7 x 43 x 89 x 199 x 263 x 307 x 881 x 967  
89 : 1779979416004714189 = 1069 x 1665088321800481  
90 : 2880067194370816120 = 2<sup>3</sup> x 5 x 11 x 17 x 19 x 31 x 61 x 181 x 541 x 109441  
91 : 4660046610375530309 = 13<sup>2</sup> x 233 x 741469 x 159607993  
92 : 7540113804746346429 = 3 x 139 x 461 x 4969 x 28657 x 275449  
93 : 12200160415121876738 = 2 x 557 x 2417 x 4531100550901  
94 : 19740274219868223167 = 2971215073 x 6643838879  
95 : 31940434634990099905 = 5 x 37 x 113 x 761 x 29641 x 67735001  
96 : 51680708854858323072 = 2<sup>7</sup> x 3<sup>2</sup> x 7 x 23 x 47 x 769 x 1103 x 2207 x 3167  
97 : 83621143489848422977 = 193 x 389 x 3084989 x 361040209  
98 : 135301852344706746049 = 13 x 29 x 97 x 6168709 x 599786069  
99 : 218922995834555169026 = 2 x 17 x 89 x 197 x 19801 x 18546805133  
100 : 354224848179261915075 = 3 x 5<sup>2</sup> x 11 x 41 x 101 x 151 x 401 x 3001 x 570601  
101 : 573147844013817084101 = 743519377 x 770857978613  
102 : 927372692193078999176 = 2<sup>3</sup> x 919 x 1597 x 3469 x 3571 x 6376021  
103 : 1500520536206896083277 = 519121 x 5644193 x 512119709  
104 : 2427893228399975082453 = 3 x 7 x 103 x 233 x 521 x 90481 x 102193207  
105 : 3928413764606871165730 = 2 x 5 x 13 x 61 x 421 x 141961 x 8288823481  
106 : 6356306993006846248183 = 953 x 55945741 x 119218851371  
107 : 10284720757613717413913 = 1247833 x 8242065050061761  
108 : 16641027750620563662096 = 2<sup>4</sup> x 3<sup>4</sup> x 17 x 19 x 53 x 107 x 109 x 5779 x 11128427  
109 : 26925748508234281076009 = 827728777 x 32529675488417  
110 : 43566776258854844738105 = 5 x 11<sup>2</sup> x 89 x 199 x 331 x 661 x 39161 x 474541  
111 : 70492524767089125814114 = 2 x 73 x 149 x 2221 x 1459000305513721  
112 : 114059301025943970552219 = 3 x 7<sup>2</sup> x 13 x 29 x 47 x 281 x 14503 x 10745088481  
113 : 184551825793033096366333 = 677 x 272602401466814027129  
114 : 298611126818977066918552 = 2<sup>3</sup> x 37 x 113 x 229 x 797 x 9349 x 54833 x 95419  
115 : 483162952612010163284885 = 5 x 1381 x 28657 x 2441738887963981  
116 : 781774079430987230203437 = 3 x 59 x 347 x 19489 x 514229 x 1270083883  
117 : 1264937032042997393488322 = 2 x 17 x 233 x 29717 x 135721 x 39589685693  
118 : 2046711111473984623691759 = 353 x 709 x 8969 x 336419 x 2710260697  
119 : 3311648143516982017180081 = 13 x 1597 x 159512939815855788121  
120 : 5358359254990966640871840 = 2<sup>5</sup> x 3<sup>2</sup> x 5 x 7 x 11 x 23 x 31 x 41 x 61 x 241 x 2161 x 2521 x 20641  
121 : 8670007398507948658051921 = 89 x 97415813466381445596089  
122 : 14028366653498915298923761 = 4513 x 555003497 x 5600748293801  
123 : 22698374052006863956975682 = 2 x 2789 x 59369 x 68541957733949701  
124 : 36726740705505779255899443 = 3 x 557 x 2417 x 3010349 x 3020733700601  
125 : 59425114757512643212875125 = 5<sup>3</sup> x 3001 x 158414167964045700001  
126 : 96151855463018422468774568 = 2<sup>3</sup> x 13 x 17 x 19 x 29 x 211 x 421 x 1009 x 31249 x 35239681  
127 : 155576970229531065681649693 = 27941 x 5568053048227732210073  
128 : 251728825683549488150424261 = 3 x 7 x 47 x 127 x 1087 x 2207 x 4481 x 186812208641  
129 : 40730579504080553832073954 = 2 x 257 x 5417 x 8513 x 39639893 x 433494437  
130 : 659034621587630041982498215 = 5 x 11 x 131 x 233 x 521 x 2081 x 24571 x 14736206161  
131 : 1066340417491710595814572169  
132 : 1725375039079340637797070384 = 2<sup>4</sup> x 3<sup>2</sup> x 43 x 89 x 199 x 307 x 9901 x 19801 x 261399601  
133 : 2791715456571051233611642553 = 13 x 37 x 113 x 3457 x 42293 x 351301301942501  
134 : 4517090495650391871408712937 = 269 x 4021 x 116849 x 1429913 x 24994118449  
135 : 7308805952221443105020355490 = 2 x 5 x 17 x 53 x 61 x 109 x 109441 x 1114769954367361  
136 : 11825896447871834976429068427 = 3 x 7 x 67 x 1597 x 3571 x 63443 x 23230657239121  
137 : 19134702400093278081449423917  
138 : 30960598847965113057878492344 = 2<sup>3</sup> x 137 x 139 x 461 x 691 x 829 x 18077 x 28657 x 1485571  
139 : 50095301248058391139327916261 = 277 x 2114537501 x 85526722937689093  
140 : 81055900096023504197206408605 = 3 x 5 x 11 x 13 x 29 x 41 x 71 x 281 x 911 x 141961 x 12317523121  
141 : 131151201344081895336534324866 = 2 x 108289 x 1435097 x 142017737 x 2971215073  
142 : 212207101440105399533740733471 = 6673 x 46165371073 x 688846502588399  
143 : 343358302784187294870275058337 = 89 x 233 x 8581 x 1929584153756850496621  
144 : 555565404224292694404015791808 = 2<sup>6</sup> x 3<sup>3</sup> x 7 x 17 x 19 x 23 x 47 x 107 x 1103 x 103681 x 10749957121  
145 : 898923707008479989274290850145 = 5 x 514229 x 349619996930737079890201  
146 : 1454489111232772683678306641953 = 151549 x 9375829 x 86020717 x 11899937029  
147 : 2353412818241252672952597492098 = 2 x 13 x 97 x 293 x 421 x 3529 x 6168709 x 347502052673  
148 : 3807901929474025356630904134051 = 3 x 73 x 149 x 2221 x 11987 x 54018521 x 81143477963  
149 : 6161314747715278029583501626149 = 110557 x 162709 x 4000949 x 85607646594577  
150 : 9969216677189303386214405760200 = 2<sup>3</sup> x 5<sup>2</sup> x 11 x 31 x 61 x 101 x 151 x 3001 x 12301 x 18451 x 23068650

## First 1000 prime numbers

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47
53	59	61	67	71	73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173	179	181	191	193	197
199	211	223	227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359	367	373	379
383	389	397	401	409	419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541	547	557	563	569	571
577	587	593	599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743	751	757	761
769	773	787	797	809	811	821	823	827	829	839	853	857	859	863
877	881	883	887	907	911	919	929	937	941	947	953	967	971	977
983	991	997	1009	1013	1019	1021	1031	1033	1039	1049	1051	1061	1063	1069
1087	1091	1093	1097	1103	1109	1117	1123	1129	1151	1153	1163	1171	1181	1187
1193	1201	1213	1217	1223	1229	1231	1237	1249	1259	1277	1279	1283	1289	1291
1297	1301	1303	1307	1319	1321	1327	1361	1367	1373	1381	1399	1409	1423	1427
1429	1433	1439	1447	1451	1453	1459	1471	1481	1483	1487	1489	1493	1499	1511
1523	1531	1543	1549	1553	1559	1567	1571	1579	1583	1597	1601	1607	1609	1613
1619	1621	1627	1637	1657	1663	1667	1669	1693	1697	1699	1709	1721	1723	1733
1741	1747	1753	1759	1777	1783	1787	1789	1801	1811	1823	1831	1847	1861	1867
1871	1873	1877	1879	1889	1901	1907	1913	1931	1933	1949	1951	1973	1979	1987
1993	1997	1999	2003	2011	2017	2027	2029	2039	2053	2063	2069	2081	2083	2087
2089	2099	2111	2113	2129	2131	2137	2141	2143	2153	2161	2179	2203	2207	2213
2221	2237	2239	2243	2251	2267	2269	2273	2281	2287	2293	2297	2309	2311	2333
2339	2341	2347	2351	2357	2371	2377	2381	2383	2389	2393	2399	2411	2417	2423
2437	2441	2447	2459	2467	2473	2477	2503	2521	2531	2539	2543	2549	2551	2557
2579	2591	2593	2609	2617	2621	2633	2647	2657	2659	2663	2671	2677	2683	2687
2689	2693	2699	2707	2711	2713	2719	2729	2731	2741	2749	2753	2767	2777	2789
2791	2797	2801	2803	2819	2833	2837	2843	2851	2857	2861	2879	2887	2897	2903
2909	2917	2927	2939	2953	2957	2963	2969	2971	2999	3001	3011	3019	3023	3037
3041	3049	3061	3067	3079	3083	3089	3109	3119	3121	3137	3163	3167	3169	3181
3187	3191	3203	3209	3217	3221	3229	3251	3253	3257	3259	3271	3299	3301	3307
3313	3319	3323	3329	3331	3343	3347	3359	3361	3371	3373	3389	3391	3407	3413
3433	3449	3457	3461	3463	3467	3469	3491	3499	3511	3517	3527	3529	3533	3539
3541	3547	3557	3559	3571	3581	3583	3593	3607	3613	3617	3623	3631	3637	3643
3659	3671	3673	3677	3691	3697	3701	3709	3719	3727	3733	3739	3761	3767	3769
3779	3793	3797	3803	3821	3823	3833	3847	3851	3853	3863	3877	3881	3889	3907
3911	3917	3919	3923	3929	3931	3943	3947	3967	3989	4001	4003	4007	4013	4019
4021	4027	4049	4051	4057	4073	4079	4091	4093	4099	4111	4127	4129	4133	4139
4153	4157	4159	4177	4201	4211	4217	4219	4229	4231	4241	4243	4253	4259	4261
4271	4273	4283	4289	4297	4327	4337	4339	4349	4357	4363	4373	4391	4397	4409
4421	4423	4441	4447	4451	4457	4463	4481	4483	4493	4507	4513	4517	4519	4523
4547	4549	4561	4567	4583	4591	4597	4603	4621	4637	4639	4643	4649	4651	4657
4663	4673	4679	4691	4703	4721	4723	4729	4733	4751	4759	4783	4787	4789	4793
4799	4801	4813	4817	4831	4861	4871	4877	4889	4903	4909	4919	4931	4933	4937
4943	4951	4957	4967	4969	4973	4987	4993	4999	5003	5009	5011	5021	5023	5039
5051	5059	5077	5081	5087	5099	5101	5107	5113	5119	5147	5153	5167	5171	5179
5189	5197	5209	5227	5231	5233	5237	5261	5273	5279	5281	5297	5303	5309	5323
5333	5347	5351	5381	5387	5393	5399	5407	5413	5417	5419	5431	5437	5441	5443
5449	5471	5477	5479	5483	5501	5503	5507	5519	5521	5527	5531	5557	5563	5569
5573	5581	5591	5623	5639	5641	5647	5651	5653	5657	5659	5669	5683	5689	5693
5701	5711	5717	5737	5741	5743	5749	5779	5783	5791	5801	5807	5813	5821	5827
5839	5843	5849	5851	5857	5861	5867	5869	5879	5881	5897	5903	5923	5927	5939
5953	5981	5987	6007	6011	6029	6037	6043	6047	6053	6067	6073	6079	6089	6091
6101	6113	6121	6131	6133	6143	6151	6163	6173	6197	6199	6203	6211	6217	6221
6229	6247	6257	6263	6269	6271	6277	6287	6299	6301	6311	6317	6323	6329	6337
6343	6353	6359	6361	6367	6373	6379	6389	6397	6421	6427	6449	6451	6469	6473
6481	6491	6521	6529	6547	6551	6553	6563	6569	6571	6577	6581	6599	6607	6619
6637	6653	6659	6661	6673	6679	6689	6691	6701	6703	6709	6719	6733	6737	6761
6763	6779	6781	6791	6793	6803	6823	6827	6829	6833	6841	6857	6863	6869	6871
6883	6899	6907	6911	6917	6947	6949	6959	6961	6967	6971	6977	6983	6991	6997
7001	7013	7019	7027	7039	7043	7057	7069	7079	7103	7109	7121	7127	7129	7151
7159	7177	7187	7193	7207	7211	7213	7219	7229	7237	7243	7247	7253	7283	7297
7307	7309	7321	7331	7333	7349	7351	7369	7393	7411	7417	7433	7451	7457	7459
7477	7481	7487	7489	7499	7507	7517	7523	7529	7537	7541	7547	7549	7559	7561
7573	7577	7583	7589	7591	7603	7607	7621	7639	7643	7649	7669	7673	7681	7687
7691	7699	7703	7717	7723	7727	7741	7753	7757	7759	7789	7793	7817	7823	7829
7841	7853	7867	7873	7877	7879	7883	7901	7907	7919					

## First 25 Catalan numbers

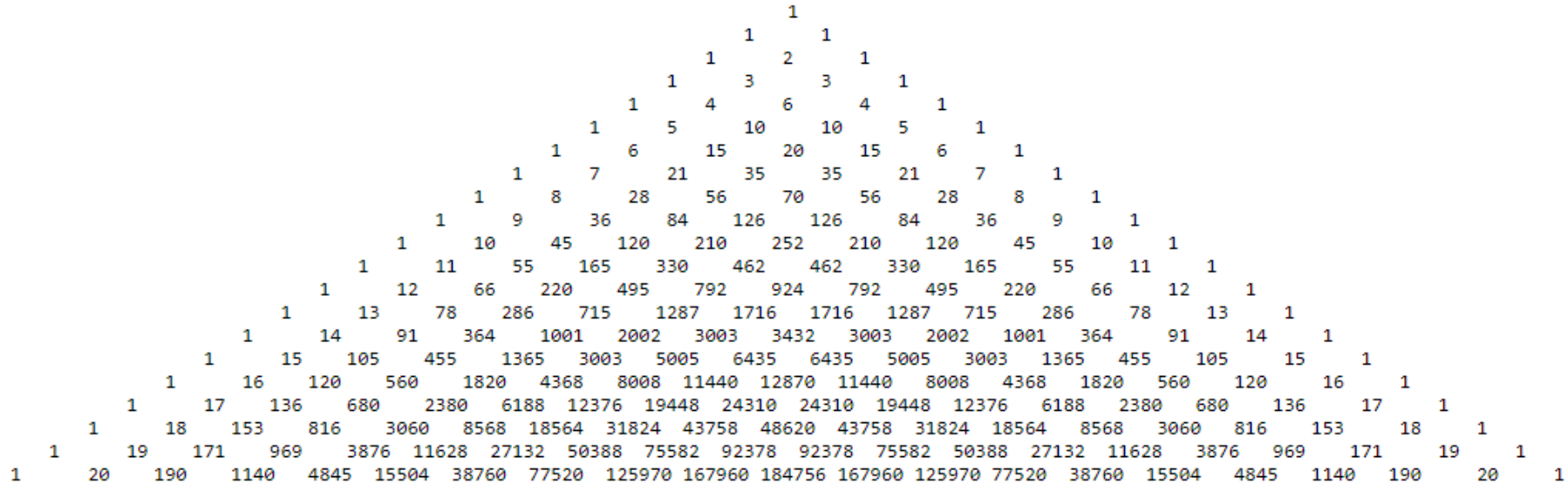
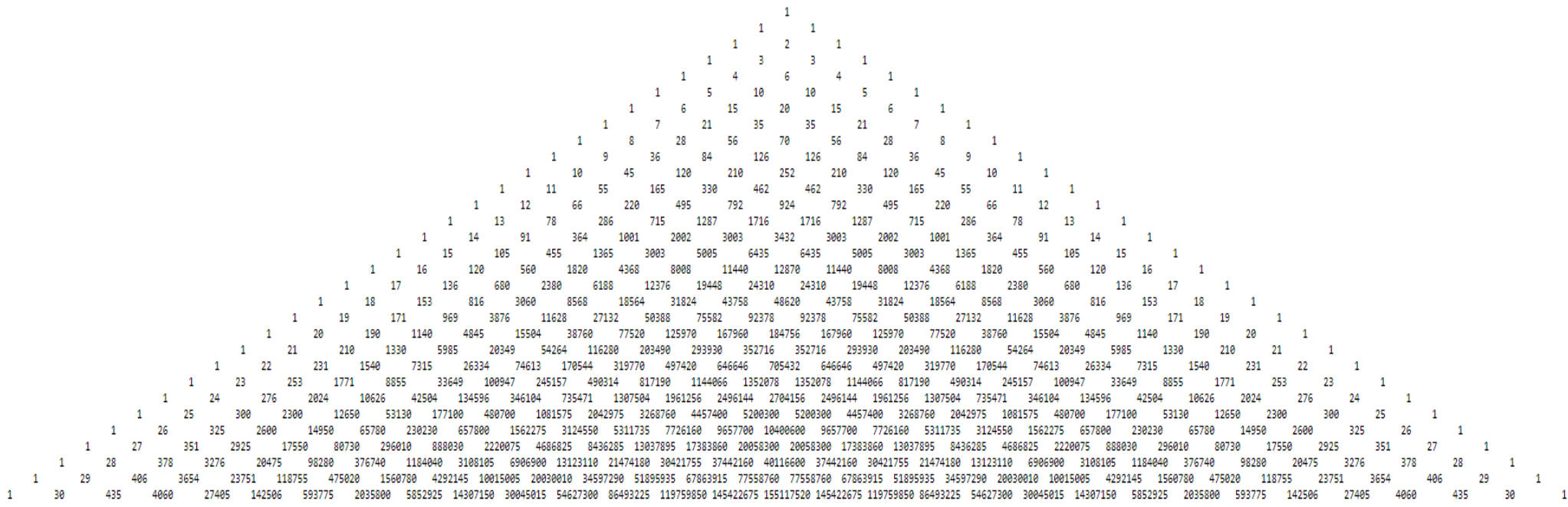
1	1
2	1
3	2
4	5
5	14
6	42
7	132
8	429
9	1430
10	4862
11	16796
12	58786
13	208012
14	742900
15	2674440
16	9694845
17	35357670
18	129644790
19	477638700
20	1767263190
21	6564120420
22	24466267020
23	91482563640
24	343059613650
25	1289904147324
26	4861946401452
27	18367353072152
28	69533550916004
29	263747951750360
30	1002242216651360
31	3814986502092300

## First 100 powers of 2

2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	512
2^10	1024
2^11	2048
2^12	4096
2^13	8192
2^14	16384
2^15	32768
2^16	65536
2^17	131072
2^18	262144
2^19	524288
2^20	1048576
2^21	2097152
2^22	4194304
2^23	8388608
2^24	16777216
2^25	33554432
2^26	67108864
2^27	134217728
2^28	268435456
2^29	536870912
2^30	1073741824
2^31	2147483648
2^32	4294967296
2^33	8589934592
2^34	17179869184
2^35	34359738368
2^36	68719476736
2^37	137438953472
2^38	274877906944
2^39	549755813888
2^40	1099511627776
2^41	2199023255552
2^42	4398046511104
2^43	8796093022208
2^44	17592186044416
2^45	35184372088832
2^46	70368744177664
2^47	140737488355328
2^48	281474976710656
2^49	562949953421312
2^50	1125899906842624
2^51	2251799813685248
2^52	4503599627370496
2^53	9007199254740992
2^54	18014398509481984
2^55	36028797018963968
2^56	72057594037927936
2^57	144115188075855872
2^58	288230376151711744
2^59	576460752303423488
2^60	1152921504606846976
2^61	2305843009213693952
2^62	4611686018427387904
2^63	9223372036854775808
2^64	18446744073709551616
2^65	36893488147419103232
2^66	73786976294838206464
2^67	147573952589676412928
2^68	295147905179352825856
2^69	590295810358705651712
2^70	1180591620717411303424
2^71	2361183241434822606848
2^72	4722366482869645213696
2^73	9444732965739290427392

2^74	18889465931478580854784
2^75	37778931862957161709568
2^76	75557863725914323419136
2^77	151115727451828646838272
2^78	302231454903657293676544
2^79	604462909807314587353088
2^80	1208925819614629174706176
2^81	2417851639229258349412352
2^82	4835703278458516698824704
2^83	9671406556917033397649408
2^84	19342813113834066795298816
2^85	38685626227668133590597632
2^86	77371252455336267181195264
2^87	154742504910672534362390528
2^88	309485009821345068724781056
2^89	618970019642690137449562112
2^90	1237940039285380274899124224
2^91	2475880078570760549798248448
2^92	4951760157141521099596496896
2^93	9903520314283042199192993792
2^94	19807040628566084398385987584
2^95	39614081257132168796771975168
2^96	79228162514264337593543950336
2^97	158456325028528675187087900672
2^98	316912650057057350374175801344
2^99	633825300114114700748351602688
2^100	1267650600228229401496703205376
2^101	2535301200456458802993406410752
2^102	5070602400912917605986812821504
2^103	10141204801825835211973625643008
2^104	20282409603651670423947251286016
2^105	40564819207303340847894502572032
2^106	81129638414606681695789005144064
2^107	162259276829213363391578010288128
2^108	324518553658426726783156020576256
2^109	649037107316853453566312041152512
2^110	1298074214633706907132624082305024
2^111	2596148429267413814265248164610048
2^112	5192296858534827628530496329220096
2^113	1038459371706965525706092658440192
2^114	20769187434139310514121985316880384
2^115	41538374868278621028243970633760768
2^116	83076749736557242056487941267521536
2^117	166153499473114484112975882535043072
2^118	332306998946228968225951765070086144
2^119	664613997892457936451903530140172288
2^120	1329227995784915872903807060280344576
2^121	2658455991569831745807614120560689152
2^122	5316911983139663491615228241121378304
2^123	10633823966279326983230456482242756608
2^124	21267647932558653966460912964485513216
2^125	42535295865117307932921825928971026432
2^126	85070591730234615865843651857942052864
2^127	170141183460469231731687303715884105728
2^128	340282366920938463463374607431768211456
2^129	680564733841876926926749214863536422912
2^130	1361129467683753853853498429727072845824
2^131	272258935367507707706996859454145691648
2^132	5444517870735015415413993718908291383296
2^133	10889035741470030830827987437816582766592
2^134	21778071482940061661655974875633165533184
2^135	43556142965880123323311949751266331066368
2^136	87112285931760246646623899502532662132736
2^137	174224571863520493293247799005065324265472
2^138	348449143727040986586495598010130648530944
2^139	696898287454081973172991196020261297061888
2^140	1393796574908163946345982392040522594123776
2^141	2787593149816327892691964784081045188247552
2^142	5575186299632655785383929568162090376495104
2^143	11150372599265311570767859136324180752990208
2^144	22300745198530623141535718272648361505980416
2^145	44601490397061246283071436545296723011960832
2^146	89202980794122492566142873090593446023921664
2^147	178405961588244985132285746181186892047843328
2^148	356811923176489970264571492362373784095686656
2^149	713623846352979940529142984724747568191373312
2^150	1427247692705959881058285969449495136382746624

1	1																				
1	2	1																			
1	3	3	1																		
1	4	6	4	1																	
1	5	10	10	5	1																
1	6	15	20	15	6	1															
1	7	21	35	35	21	7	1														
1	8	28	56	70	56	28	8	1													
1	9	36	84	126	126	84	36	9	1												
1	10	45	120	210	252	210	45	10	1												
1	11	55	165	330	462	462	330	165	55	11	1										
1	12	66	220	495	792	924	792	495	220	66	12	1									
1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1								
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1							
1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1						
1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1					
1	17	136	680	2380	6188	12376	19448	24310	24310	19448	12376	6188	2380	680	136	17	1				
1	18	153	816	3060	8568	18564	31824	43758	48620	43758	31824	18564	8568	3060	816	153	18	1			
1	19	171	969	3876	11628	27132	50388	92378	75582	92378	75582	50388	27132	11628	3876	969	171	19	1		
1	20	190	1140	4845	15504	38760	77520	125970	167960	184756	167960	125970	77520	38760	15504	4845	1140	190	20	1	
1	21	210	1330	5985	20349	54264	116280	203490	293930	352716	352716	293930	203490	116280	54264	20349	5985	1330	210	21	
1	22	231	1540	7315	26334	74613	170544	319770	497420	646646	705432	646646	497420	319770	170544	74613	26334	7315	1540	231	
22	1	23	253	1771	8855	33649	100947	245157	490314	817190	1144066	1352078	1352078	1144066	817190	490314	245157	100947	33649	8855	1771
253	23	1	276	2024	10626	42504	134596	346104	735471	1307504	1961256	2496144	2704156	2496144	1961256	1307504	735471	346104	134596	42504	10626
2024	276	24	1	2300	12650	53130	177100	480700	1081575	2042975	3268760	4457400	5200300	5200300	4457400	3268760	2042975	1081575	480700	177100	53130
12650	2300	300	25	1	65780	230230	657800	1562275	3124550	5311735	7726160	9657700	10400600	9657700	7726160	5311735	3124550	1562275	657800	230230	
65780	14950	2600	325	26	1	80730	296010	888030	2220075	4686825	8436285	1303									



## Código ASCII

ASCII	Hex	Symbol	ASCII	Hex	Symbol	ASCII	Hex	Symbol	ASCII	Hex	Symbol
0	0	NUL	32	20	(space)	64	40	@	96	60	`
1	1	SOH	33	21	!	65	41	A	97	61	a
2	2	STX	34	22	"	66	42	B	98	62	b
3	3	ETX	35	23	#	67	43	C	99	63	c
4	4	EOT	36	24	\$	68	44	D	100	64	d
5	5	ENQ	37	25	%	69	45	E	101	65	e
6	6	ACK	38	26	&	70	46	F	102	66	f
7	7	BEL	39	27	'	71	47	G	103	67	g
8	8	BS	40	28	(	72	48	H	104	68	h
9	9	TAB	41	29	)	73	49	I	105	69	i
10	A	LF	42	2A	*	74	4A	J	106	6A	j
11	B	VT	43	2B	+	75	4B	K	107	6B	k
12	C	FF	44	2C	,	76	4C	L	108	6C	l
13	D	CR	45	2D	-	77	4D	M	109	6D	m
14	E	SO	46	2E	.	78	4E	N	110	6E	n
15	F	SI	47	2F	/	79	4F	O	111	6F	o
16	10	DLE	48	30	0	80	50	P	112	70	p
17	11	DC1	49	31	1	81	51	Q	113	71	q
18	12	DC2	50	32	2	82	52	R	114	72	r
19	13	DC3	51	33	3	83	53	S	115	73	s
20	14	DC4	52	34	4	84	54	T	116	74	t
21	15	NAK	53	35	5	85	55	U	117	75	u
22	16	SYN	54	36	6	86	56	V	118	76	v
23	17	ETB	55	37	7	87	57	W	119	77	w
24	18	CAN	56	38	8	88	58	X	120	78	x
25	19	EM	57	39	9	89	59	Y	121	79	y
26	1A	SUB	58	3A	:	90	5A	Z	122	7A	z
27	1B	ESC	59	3B	;	91	5B	[	123	7B	{
28	1C	FS	60	3C	<	92	5C	\	124	7C	
29	1D	GS	61	3D	=	93	5D	]	125	7D	}
30	1E	RS	62	3E	>	94	5E	^	126	7E	~
31	1F	US	63	3F	?	95	5F	_	127	7F	␣

## Laws and facts

### Sum of squares

An integer greater than one can be written as a sum of two squares if and only if its prime decomposition contains no prime congruent to 3 (mod 4) raised to an odd power.

### Goldbach's conjecture

Every even integer greater than 2 can be expressed as the sum of two primes.

### Is Divisible by prime

Given a number  $M$ , we want to know if it is divisible by a prime  $P$ .

1. Find the smallest  $K$  for this condition  $(KP + 1) \bmod 10$ .
2.  $N = (KP + 1) / 10$
3. Multiply the last digit of  $M$  times  $N$  and add the result to  $M$  without the last digit. (You might use  $(N-P)$  as well for the multiplication instead of  $N$ , note that it would be a negative number).
4. Repeat until  $M$  is short enough.