Д/з № 13. Производная сложной функции. Семке Даниил 16.05.20

ИК-102, Мен-190207

№3285

$u = f(x, xy, xyz)$

$$\frac{\partial u}{\partial x} = f_1' + y f_2' + yz f_3'; \quad \frac{\partial u}{\partial y} = x f_2' + xz f_3'; \quad \frac{\partial u}{\partial z} = xy \cdot f_3'$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(f_1' + y f_2' + yz f_3'\right) = f_{11}'' + y \cdot f_{12}'' + yz f_{13}'' + y\left(f_{21}'' + y f_{22}'' + yz f_{23}''\right) + yz\left(f_{31}'' + y f_{32}'' + yz f_{33}''\right) =$$

$$= f_{11}'' + y^2 f_{22}'' + y^2z^2 f_{33}'' + 2y f_{12}'' + 2yz f_{13}'' + 2y^2 z f_{23}''; \quad f_{ij}'' = f_{ji}'' \quad i,j = \overline{1,3}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(x f_2' + xz f_3'\right) = x^2 f_{22}'' + x^2 z f_{23}'' + x^2 z f_{32}'' + x^2 z^2 f_{33}'' = x^2 f_{22}'' + 2x^2 z f_{23}'' + x^2 z^2 f_{33}''$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(xy f_3'\right) = x^2 y^2 f_{33}''$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial y}\left(f_1' + y f_2' + yz f_3'\right) = x f_{12}'' + xz f_{13}'' + f_2' + xy f_{22}'' + xyz f_{23}'' + xyz f_{32}'' + xyz^2 f_{33}'' +$$

$$+ z f_3' = xy f_{22}'' + xyz^2 f_{33}'' + x f_{12}'' + xz f_{13}'' + 2xyz f_{23}'' + f_2' + z f_3'$$

$$\frac{\partial^2 u}{\partial x \partial z} = xy f_{13}'' + xy^2 f_{23}'' + xy^2 z f_{33}'' + y \cdot f_3'$$

$$\frac{\partial^2 u}{\partial y \partial z} = x^2 y f_{23}'' + x^2 yz f_{33}'' + x f_3'$$

№3295 $u = f(\xi, \eta)$, где $\xi = xy$, $\eta = \frac{x}{y}$ $\quad du = f_1' d(\xi) + f_2' d\eta$

$$du = f_1' \cdot (y dx + x dy) + f_2'\left(\frac{y dx - x dy}{y^2}\right) \qquad d^2u = f_{11}'' d\xi^2 + 2 f_{12}'' d\xi d\eta + f_{22}'' d\eta^2$$

$$d^2u = f''_{11}(ydx+xdy)^2 + f''_{22}\left(\frac{(ydx-xdy)^2}{y^4}\right) + 2f''_{12}\left(\frac{y^2dx^2-x^2dy^2}{y^2}\right) + 2f'_1 dx dy - 2f'_2 \times$$

$$\times\left(\frac{y(dx-xdy)dy}{y^3}\right)$$

№3309 $u = \frac{1}{2a\sqrt{\pi t}} e^{-\frac{(x-b)^2}{4a^2t}}$ удовл. ур-ию теплопр-ти

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial t} = \frac{e^{-\frac{(x-b)^2}{4a^2t}}\left(\left(-\frac{(x-b)^2}{4a^2t}\right)' 2a\sqrt{\pi t} - \left(2a\sqrt{\pi t}\right)'\right)}{4a^2\pi t} = \frac{e^{-\frac{(x-b)^2}{4a^2t}}\left(\frac{(x-b)^2}{4a^2t^2}\cdot 2a\sqrt{\pi t} - \frac{a\pi}{\sqrt{\pi t}}\right)}{4a^2\pi t} =$$

$$= \frac{e^{-\frac{(x-b)^2}{4a^2t}}}{4a^2\pi t}\left(\frac{(x-b)^2}{2at\sqrt{t}}\cdot\sqrt{\pi} - \frac{a\sqrt{\pi}}{\sqrt{t}}\right) = \frac{e^{-\frac{(x-b)^2}{4a^2t}}\left((x-b)^2 - 2a^2t\right)}{8a^3t^2\sqrt{\pi t}}$$

$$\frac{\partial u}{\partial x} = -\frac{(x-b)\cdot e^{-\frac{(x-b)^2}{4a^2t}}}{4a^3t\sqrt{\pi t}}, \quad \frac{\partial^2 u}{\partial x^2} = \frac{1}{8a^5t^2\sqrt{\pi t}} e^{-\frac{(x-b)^2}{4a^2t}}\left((x-b)^2 - 2a^2t\right)$$

(вычисл. все не стал приводить, т.к. бумаги мало)

$$\frac{\partial u}{\partial t} = a^2\cdot\frac{1}{8a^5t^2\sqrt{\pi t}}e^{-\frac{(x-b)^2}{4a^2t}}\left((x-b)^2 - 2a^2t\right) = a^2\frac{\partial^2 u}{\partial x^2}$$

№ 3316 $u = \sec x \frac{\partial z}{\partial x} + \sec y \frac{\partial z}{\partial y}$

$z = \sin y + f(\sin x - \sin y)$, где f - дифф.

$$u = \frac{1}{\cos x}\frac{\partial z}{\partial x} + \frac{1}{\cos y}\frac{\partial z}{\partial y} =$$

$$\frac{\partial z}{\partial x} = f'\cdot\cos x$$

$$\frac{\partial z}{\partial y} = \cos y - f'\cdot\cos y$$

$$= \frac{1}{\cos x}\left(f'\cos x\right) + \frac{1}{\cos y}\left(\cos y - f'\cos y\right) =$$

$$= f' + 1 - f' = 1$$

Ответ: 1

№3317 $z = x^n f\left(\frac{y}{x^2}\right)$ $\qquad x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y} = nz$, f-дифф.

$$\frac{\partial z}{\partial x} = nx^{n-1} f\left(\frac{y}{x^2}\right) + x^n\cdot\left(-\frac{2}{x^3}y\right) f'\left(\frac{y}{x^2}\right)$$

$$\frac{\partial z}{\partial y} = \frac{x^n}{x^2} f'\left(\frac{y}{x^2}\right)$$

$$x\left(nx^{n-1} f\left(\frac{y}{x^2}\right) - x^{n-3}\cdot 2y\cdot f'\left(\frac{y}{x^2}\right)\right) + 2y\cdot\frac{x^n}{x^2} f'\left(\frac{y}{x^2}\right) =$$

$$= x^n\cdot n f\left(\frac{y}{x^2}\right) - \cancel{x^{n-2}\,2y\, f'\left(\frac{y}{x^2}\right)} + \cancel{2y\, x^{n-2} f'\left(\frac{y}{x^2}\right)} = x^n\cdot n f\left(\frac{y}{x^2}\right) = nz$$

№3324 $x\frac{\partial u}{\partial x} + \alpha y\frac{\partial u}{\partial y} + \beta z\frac{\partial u}{\partial z} = nu$ $\quad$ (*)

$$u = x^n \varphi\left(\frac{y}{x^\alpha}, \frac{z}{x^\beta}\right)$$

$$x\frac{\partial u}{\partial x} = x\left(x^{n-1} n \varphi + x^n \varphi'\left(\frac{y}{x^\alpha}, \frac{z}{x^\beta}\right) = x^{n-1}\varphi + x^n\left(\frac{-\alpha y}{x^{\alpha+1}}\varphi_1'\right) + x^n\left(\frac{-\beta z}{x^{\beta+1}}\varphi_2'\right)\right) =$$

$$= \varphi\cdot n\cdot x^n - x^{n-\alpha}\cdot\alpha y\varphi_1' - \beta z x^{n-\beta}\varphi_2'$$

$$\alpha y\frac{\partial u}{\partial y} = \alpha y x^{n-\alpha}\varphi_1'$$

$$(*) = \boxed{n\cdot x^n \varphi = nu}$$

$$\beta z\frac{\partial u}{\partial z} = \beta z x^{n-\beta}\varphi_2'$$