Классификация квадрик на плоскости.

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№12.2.19(а,б)

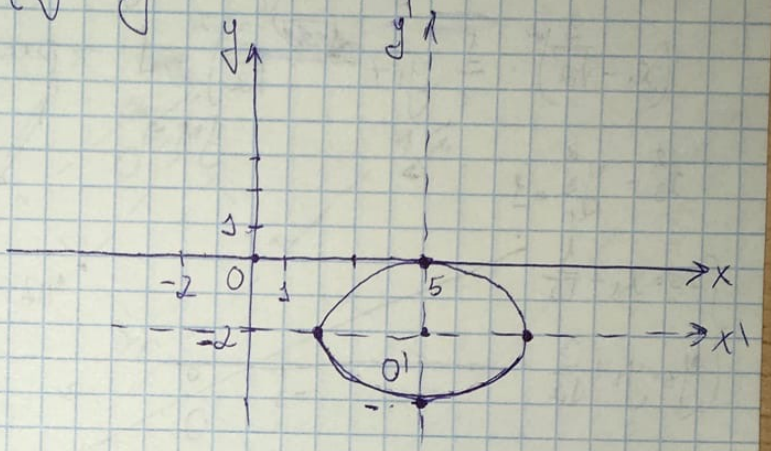
а) $4x^2+9y^2-40x+36y+100=0$

$4(x-5)^2+9(y+2)^2=36 \mid :36$

$\frac{(x-5)^2}{9}+\frac{(y+2)^2}{4}=1$

$\begin{cases} x-5=x_1 \\ y+2=y_1 \end{cases}$

$\frac{x_1^2}{9}+\frac{y_1^2}{4}=1$ – эллипс



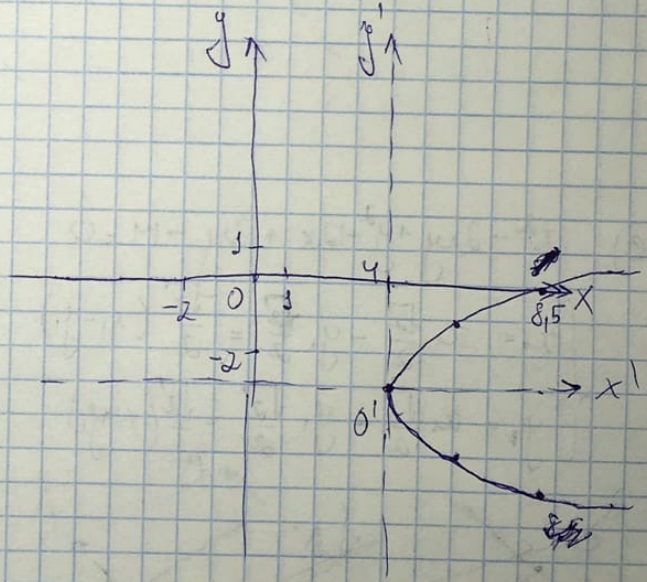
б) $y^2-2x+6y+17=0$

$(y+3)^2+8-2x=0$

$(y+3)^2=2x-8=2(x-4)$

$\begin{cases} y+3=y_1 \\ p=1 \\ x-4=x_1 \end{cases}$ $y_1^2=2px_1=2x_1$

Парабола



№12.2.20(б) и (в)

б) $3x^2+10xy+3y^2-2x-14y-13=0$

Повернем на угол α: $\operatorname{ctg}2\alpha=\frac{a_{11}-a_{22}}{2a_{12}}=\frac{3-3}{2a_{12}}=\frac{0}{2a_{12}}=0 \Rightarrow 2\alpha=\frac{\pi}{2}+2\pi k, k\in\mathbb{Z}$

$\alpha=\frac{\pi}{4}+\pi k, k\in\mathbb{Z} \Rightarrow k=0$

$\boxed{\alpha=\frac{\pi}{4}}$

$\begin{cases} x=x_1\frac{\sqrt{2}}{2}-y_1\frac{\sqrt{2}}{2} \\ y=x_1\frac{\sqrt{2}}{2}+y_1\frac{\sqrt{2}}{2} \end{cases}$ Подставим в форму:

$$\frac{3}{2}(x_1-y_1)^2+5(x_1^2-y_1^2)+\frac{3}{2}(x_1+y_1)^2-\sqrt{2}(x_1-y_1)-7\sqrt{2}(x_1+y_1)-13=0$$

$$\frac{3}{2}x_1^2+\frac{3}{2}y_1^2+5x_1^2-5y_1^2+\frac{3}{2}x_1^2+\frac{3}{2}y_1^2-\sqrt{2}x_1+\sqrt{2}y_1-7\sqrt{2}x_1-7\sqrt{2}y_1-13=0$$

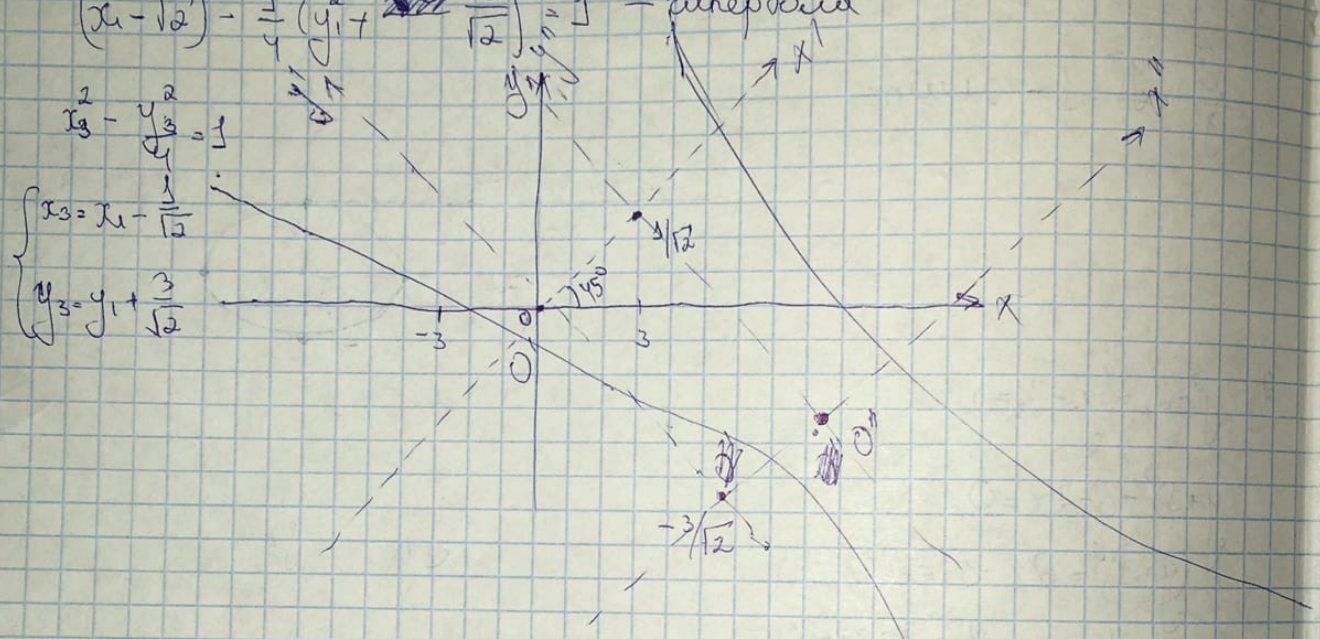
$$8x_1^2-8\sqrt{2}x_1-2y_1^2-6\sqrt{2}y_1-13=0$$

$$8\left(x_1^2-\sqrt{2}x_1+\frac{1}{2}\right)-2\left(y_1^2+3\sqrt{2}y_1+\frac{9}{2}\right)=8 \quad |:8$$

$$\left(x_1-\frac{1}{\sqrt{2}}\right)^2-\frac{1}{4}\left(y_1+\frac{3}{\sqrt{2}}\right)^2=1$$ — гипербола

$$x_3^2-\frac{y_3^2}{4}=1$$

$$\begin{cases} x_3=x_1-\frac{1}{\sqrt{2}} \\ y_3=y_1+\frac{3}{\sqrt{2}} \end{cases}$$



е) $x^2-2xy+y^2-12x+12y-14=0$

$$\begin{cases} x_1=x\frac{\sqrt{2}}{2}-y_1\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}(x_1-y_1) \\ y_1=x\frac{\sqrt{2}}{2}+y_1\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}(x_1+y_1) \end{cases}$$

$$\frac{1}{2}(x_1-y_1)^2-(x_1^2-y_1^2)+\frac{1}{2}(x_1+y_1)^2+12\sqrt{2}y_1-14=0$$

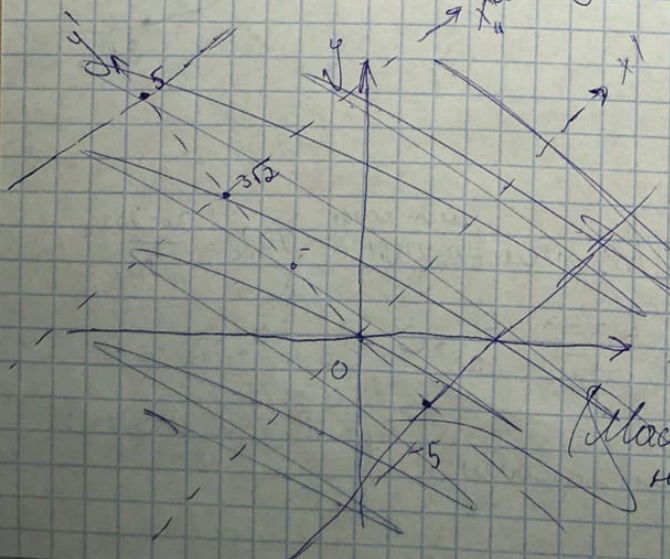
$$2y_1^2+12\sqrt{2}y_1-14=0$$

$$2\left(y_1+6\cdot 3\sqrt{2}\right)^2-25\cdot 2=0 \quad |:2$$

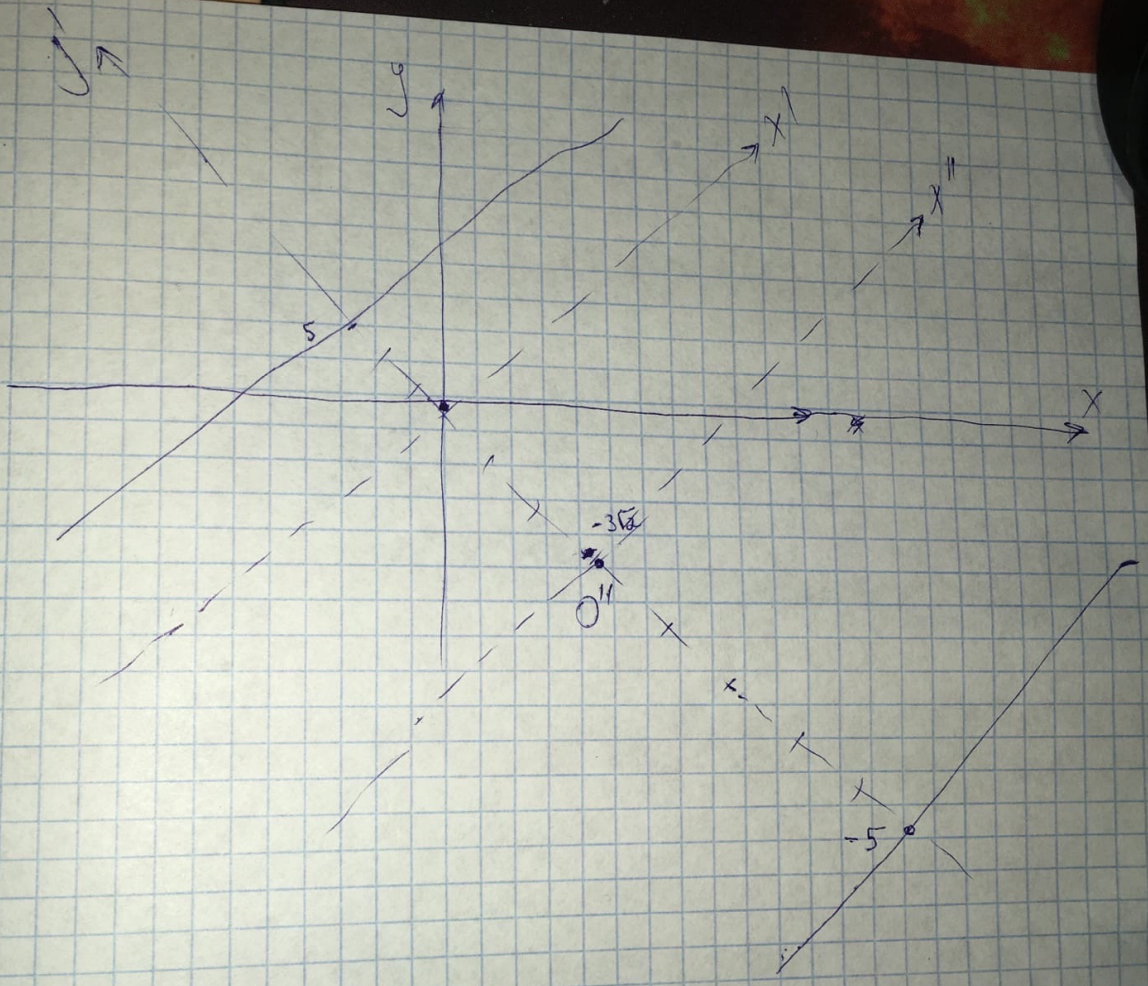
$$(y_1+3\sqrt{2})^2=25$$

$$y_1+3\sqrt{2}=y_2 \quad \boxed{y_2^2-25=0}$$ — пара прямых.

$$y_2^2=\pm 5$$



(масштаб немного неточный)


y
x'
x''
5
x
-3
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-5