$\sqrt{2} \ln \left| \sqrt{2}(x+1) \right| + \sqrt{(x+1)^2}$

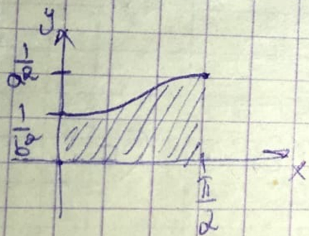
$= -\frac{1}{\sqrt{2}} \ln \frac{1}{\sqrt{2}}$.

№2215 $\quad ab \neq 0$

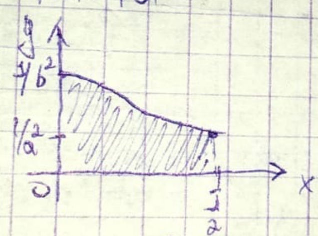
$$I = \int_0^{\frac{\pi}{2}} \frac{dx}{a^2\sin^2 x + b^2\cos^2 x} = \frac{1}{ab} \cdot \operatorname{arctg}\left(\frac{a}{b} \operatorname{tg} x\right) \Bigg|_0^{\frac{\pi}{2}-0} = \frac{1}{|ab|} \cdot \frac{\pi}{2}$$

Графики:

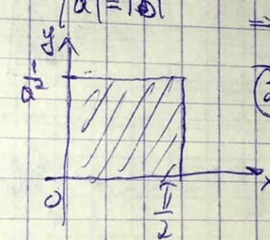
$|a| < |b|$



$|a| > |b|$



$|a| = |b|$



① $f(x) = \frac{1}{a^2\sin^2 x + b^2\cos^2 x}$ непрерывна на $[0; \frac{\pi}{2}] \Rightarrow$

$\Rightarrow$ f - интегр. на $[0; \frac{\pi}{2}] \Rightarrow I$ существует

② $f(x)$ - непр. и интегр. $\Rightarrow g(t)$ - непр. на $[0; \frac{\pi}{2}] \Rightarrow$

$$\Rightarrow \lim_{t \to \frac{\pi}{2}-0} \frac{1}{ab} \operatorname{arctg}\left(\frac{a}{b} \operatorname{tg} t\right) = \int_0^{\frac{\pi}{2}} \frac{dx}{f(x)} = \frac{1}{|ab|} \cdot \frac{\pi}{2}$$

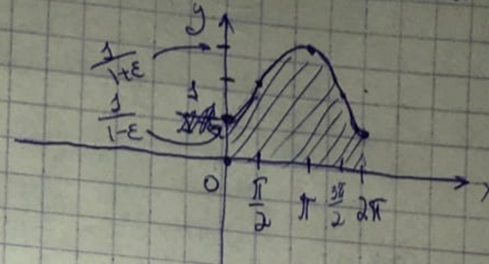
По сути, это несобственный интеграл второго рода. из определения которого вытекает обобщенная формула Н-Л для несобств. интегралов от функций, неограниченных в точке: $\int_a^b f(x)dx = \lim_{x \to b-0} F(x) - F(a)$

...ровая фу-ция.

Домашняя работа № 6

Семке Даниил, КН-102
МЕН-190207

№2213 $\displaystyle\int_0^{2\pi}\frac{dx}{1+\varepsilon\cos x}$, $(0\le\varepsilon<1)$



1) $\varepsilon=0$: $\displaystyle\int_0^{2\pi}1\,dx=2\pi$

2) $\varepsilon\in(0;1)$ $\displaystyle\int_0^{2\pi}\frac{dx}{1+\varepsilon\cos x}=\left[\begin{array}{l}\text{т.к. } F(x) \text{ имеет разрыв}\\ \text{в } x=\pi \Rightarrow \text{разобьем на 2 интегр.}\end{array}\right]=\int_0^{\pi}\frac{dx}{1+\varepsilon\cos x}+\int_{\pi}^{2\pi}\frac{dx}{1+\varepsilon\cos x}\ (=)$

$$F(x)=\int\frac{dx}{1+\varepsilon\cos x}=2\int\frac{d\,tg\frac{x}{2}}{tg^2\frac{x}{2}(1-\varepsilon)+(1+\varepsilon)}=\frac{2}{\sqrt{1-\varepsilon^2}}\,arctg\left(tg\frac{x}{2}\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\right)+C$$

$$(=)\ \frac{2}{\sqrt{1-\varepsilon^2}}\,arctg\left(tg\frac{x}{2}\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\right)\Bigg|_0^{\pi-0}+\frac{2}{\sqrt{1-\varepsilon^2}}\,arctg\left(tg\frac{x}{2}\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\right)\Bigg|_{\pi+0}^{2\pi}=\frac{\pi}{\sqrt{1-\varepsilon^2}}+\frac{\pi}{\sqrt{1-\varepsilon^2}}=$$

$$=\frac{2\pi}{\sqrt{1-\varepsilon^2}}$$

Ответ: $\dfrac{2\pi}{\sqrt{1-\varepsilon^2}}$

№2276 $\displaystyle\int_0^{2\pi}\frac{dx}{\sin^4x+\cos^4x}=\left[\begin{array}{l}\text{т.к. } F(x) \text{ имеет разрыв в}\\ x=\frac{\pi}{4}+\frac{\pi k}{2},\ k\in\{0;1,2,3\}\Rightarrow\\ \Rightarrow \text{разобьем на 5 интегр.}\end{array}\right]=$

$$F(x)=\int\frac{dx}{\sin^4x+\cos^4x}=\int\frac{dx}{\left(\frac{1-\cos2x}{2}\right)^2+\left(\frac{1+\cos2x}{2}\right)^2}=\int\frac{dx}{\frac{1+\cos^22x}{2}}=2\int\frac{dx}{1+\cos^22x}=$$

$$=2\int\frac{dx}{\sin^22x+2\cos^22x}=\int\frac{d\,tg2x}{tg^22x+2}=\frac{1}{\sqrt2}\,arctg\left(\frac{1}{\sqrt2}tg2x\right)+C$$

$$\int_0^{2\pi}\frac{dx}{\sin^4x+\cos^4x}=\int_0^{\frac{\pi}{4}}f(x)dx+\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}f(x)dx+\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}}f(x)dx+\int_{\frac{5\pi}{4}}^{\frac{7\pi}{4}}f(x)dx+\int_{\frac{7\pi}{4}}^{2\pi}f(x)dx=\frac{1}{\sqrt2}\cdot\frac{\pi}{2}+\left(\frac{\pi}{2}\cdot\frac{1}{\sqrt2}+\frac{\pi}{2}\cdot\frac{1}{\sqrt2}\right)\cdot3+$$

$$+\frac{1}{\sqrt2}\cdot\frac{\pi}{2}=\frac{\pi}{2\sqrt2}+\frac{3\cdot\pi}{\sqrt2}+\frac{\pi}{2\sqrt2}=\frac{4\pi}{\sqrt2}=\pi\cdot2\sqrt2$$

Ответ: $\pi\cdot2\sqrt2$

№2242 $\displaystyle\int_{1/e}^{e}|\ln x|dx=-\int_{1/e}^{1}\ln x\,dx+\int_1^e\ln x\,dx=-(x(\ln x-1))\Big|_{\frac1e}^{1}+(x(\ln x-1))\Big|_1^e\ (=)$

$$(=)\ 1-\frac{2}{e}+1=2-\frac{2}{e}=2\left(1-\frac1e\right)$$

$$\int|\ln x|dx=x\ln x-\int1\,dx=x\ln x-x=x(\ln x-1)$$

Ответ: $2\left(1-\frac1e\right)$

Посчитать: $\int^t$, $t\in(0;\frac{\pi}{2})$ и найти предел $\int^{t}$ ...

№2243 $\displaystyle\int_0^1 \arccos x\,dx = \underbrace{x\arccos x\Big|_0^1}_{0} + \int_0^1 \frac{x\,dx}{\sqrt{1-x^2}} = \int_0^1 \frac{x\,dx}{\sqrt{1-x^2}} = -\int_0^1 \frac{d(1-x^2)}{\sqrt{1-x^2}} =$

$= -\sqrt{1-x^2}\Big|_0^1 = 1$

Ответ: 1

№2244. $\displaystyle\int_0^{\sqrt3} x\,\mathrm{arctg}\,x\,dx = \int_0^{\sqrt3} \mathrm{arctg}\,x\,d\frac{x^2}{2} = \frac{x^2}{2}\mathrm{arctg}\,x\Big|_0^{\sqrt3} - \int_0^{\sqrt3}\frac{x^2\,dx}{2(1+x^2)} = \frac{\pi}{2} - \frac{1}{2}\int_0^{\sqrt3}\frac{x^2\,dx}{1+x^2} =$

$\displaystyle= \frac{\pi}{2} - \frac{1}{2}\int_0^{\sqrt3}\left(1-\frac{1}{1+x^2}\right)dx = \frac{\pi}{2} - \frac{1}{2}\int_0^{\sqrt3} 1\,dx + \frac{1}{2}\int_0^{\sqrt3}\frac{1}{1+x^2}\,dx = \frac{\pi}{2} - \frac{1}{2}(x-\mathrm{arctg}\,x)\Big|_0^{\sqrt3} =$

$\displaystyle= \frac{\pi}{2} - \frac{1}{2}\left(\sqrt3 - \frac{\pi}{3}\right) = \frac{2\pi}{3} - \frac{\sqrt3}{2}$

Ответ: $\frac{2\pi}{3} - \frac{\sqrt3}{2}$

№2246

$[x = a\sin t]$

$\displaystyle\int_0^a x^2\sqrt{a^2-x^2}\,dx = \begin{bmatrix} x\in[0;a],\ t\in[0;\underset{=\frac{\pi}{2}}{\arcsin 1}] \\ x = a\sin t \\ dx = a\cos t\,dt \end{bmatrix} = \int_0^{\frac{\pi}{2}} a^2\sin^2 t\cdot a\cos t\cdot a\cos t\,dt = a^4\int_0^{\frac{\pi}{2}}\sin^2 t\cos^2 t\,dt =$

$\displaystyle= \frac{a^4}{4}\int_0^{\frac{\pi}{2}}\sin^2 2t\,dt = \frac{a^4}{8}\int_0^{\frac{\pi}{2}}\sin^2 2t\,d2t = \cancel{\frac{a^4}{8}\int_0^{\frac{\pi}{2}}\frac{1-\cos^2 2t}{2}\,d2t} = \cancel{\frac{a^4}{16}\int_0^{\frac{\pi}{2}}(1-\cos^2 2t)\,dt} =$

$\displaystyle= \cancel{\frac{a^4}{16}\,2t} = \frac{a^4}{16}\int_0^{\frac{\pi}{2}} 1-\cos 4t\,d2t = \frac{a^4}{8}\left(t-\frac{\sin 4t}{4}\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi\cdot a^4}{16}$

Ответ: $\left(\frac{a}{2}\right)^4\pi$

№2247 $\left[x+1 = \frac{1}{t}\right]$

$\displaystyle\int_0^{\frac{3}{4}}\frac{dx}{(x+1)(\sqrt{x^2+1})} = \begin{bmatrix} x+1 = \frac{1}{t} \\ dx = -\frac{1}{t^2} \end{bmatrix} = \int_0^{3/4}$

$F(\ )\ \int\frac{dx}{}$ $\left[x+1 = \frac{1}{}\right]$

№2247

$$\int_0^{\frac{3}{4}} \frac{dx}{(x+1)\sqrt{x^2+1}} = \left[\begin{array}{l} t = \frac{1}{1+x} \\ x+1 = \frac{1}{t} \\ x^2 = \frac{1}{t^2} - \frac{2}{t} + 1 \end{array}\right] = \int_{0}^{\frac{3}{4}} =$$

$$\int \frac{dx}{(x+1)\sqrt{x^2+1}} = \left[\quad\right] = -\int \frac{dt}{\sqrt{t^2-2t+2}} = -\int \frac{d}{}$$

$$\int \frac{dx}{(x+1)\sqrt{x^2+1}} = -\int \frac{dt}{t^2 \cdot \frac{1}{t}\sqrt{\frac{1}{t^2}-\frac{2}{t}+2}} = -\int \frac{dt}{t\sqrt{\frac{1}{t^2}-\frac{2}{t}+2}} = -\int \frac{dt}{\sqrt{2t^2-2t+1}} =$$

$$= -\int \frac{dt}{\sqrt{\left(\sqrt{2}t - \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}}} = -\frac{1}{\sqrt{2}} \ln\left|\sqrt{2}t - \frac{1}{\sqrt{2}} + \sqrt{2t^2-2t+1}\right| =$$

$$-\frac{1}{\sqrt{2}} \ln \frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \ln 2^{-\frac{1}{2}} =$$

$$= -\frac{1}{\sqrt{2}} \ln\left|\frac{1-x}{\sqrt{2}(x+1)} + \sqrt{\frac{2-2x-2+(x+1)^2}{(x+1)^2}}\right| = -\frac{1}{\sqrt{2}} \ln\left(\frac{1-x}{\sqrt{2}(x+1)} + \sqrt{\frac{x^2+1}{(x+1)^2}}\right) -$$

$$= -\frac{1}{\sqrt{2}} \ln \frac{1}{\sqrt{2}}$$

№2247

$$\int_0^{3/4}\frac{dx}{(x+1)\sqrt{x^2+1}} = \begin{bmatrix} x+1=\frac{1}{t} \\ dx=-\frac{1}{t^2}dt \\ x\in[0;3/4] \\ t\in[\frac{4}{7};1]\end{bmatrix} = \int_{4/7}^{1}\frac{dt}{\sqrt{2t^2-2t+1}} = \frac{1}{\sqrt2}\ln =$$

$\frac{1}{t}-1=\frac{3}{4}$, $t=\frac{4}{7}$

№2248

$$\int_0^{\ln 2}\sqrt{e^x-1}\,dx = 2\int_0^1 1-\frac{1}{t^2+1}dt = 2(t-\operatorname{arctg} t)\Big|_0^1 = 2\left(1-\frac{\pi}{4}\right) = 2-\frac{\pi}{2}$$

$$\int\sqrt{e^x-1}\,dx = \begin{bmatrix}\sqrt{e^x-1}=t \\ e^x-1=t^2 \\ x=\ln(t^2+1) \\ dx=\frac{2t}{t^2+1}\end{bmatrix} = 2\int\frac{t^2}{t^2+1}dt = 2\int 1-\frac{1}{t^2+1}dt = 2(t-\operatorname{arctg}t)+C$$

$x\in[0;\ln 2]$

$t\in[0;1]$

Ответ: $2-\frac{\pi}{2}$

№2249

$$\int_0^1\frac{\arcsin\sqrt{x}}{\sqrt{x(1-x)}}dx = \begin{bmatrix}\sqrt{x}=t,\ x=t^2 \\ dx=2t\,dt \\ x\in[0;1]\Rightarrow t\in[0;1]\end{bmatrix} = \int_0^1\frac{\arcsin t\cdot 2t\,dt}{t\sqrt{1-t^2}} =$$

$$=2\int_0^1\arcsin t\,d\arcsin t = \arcsin^2 t\Big|_0^1 = \left(\frac{\pi}{2}\right)^2$$

Ответ: $\frac{\pi^2}{4}$

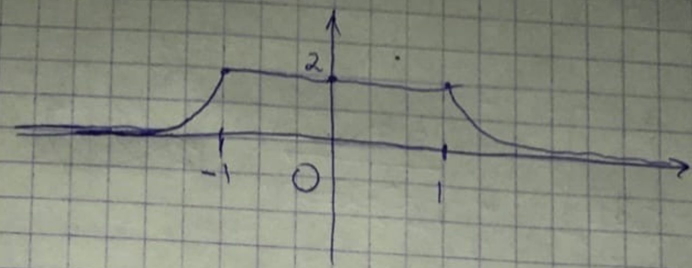
№2238(8)

$$I = \int_0^{\pi}\frac{\sin x\,dx}{\sqrt{1-2a\cos x+a^2}}$$

① $a=0$ $\quad I = \int_0^{\pi}\frac{\sin x\,dx}{1} = -\cos x\Big|_0^{\pi} = 2$

② $a\neq0$ $\quad I = \frac{1}{2a}\int_0^{\pi}\frac{d(1-2a\cos x+a^2)}{\sqrt{1-2a\cos x+a^2}} = \frac{2\cdot1}{2a}\sqrt{1-2a\cos x+a^2}\Big|_0^{\pi} = \frac{|a+1|-|1-a|}{a} = \begin{cases} 2, \text{если } a\in[-1;1] \\ \frac{2}{|a|},\ |a|>1\end{cases}$

Пример графика функции $I = I(a)$:



№2255.

$$\int_0^a x^3 f(x^2)\,dx = \frac{1}{2}\int_0^{a^2} x f(x)\,dx, \quad a>0$$

$$\int_0^a x^2\cdot x\, f(x^2)\,dx = \begin{bmatrix} x^2 = t \\ x=\sqrt{t} \\ x\in[0;a] \\ t\in[0;a^2] \end{bmatrix} = \int_0^{a^2} t f(t)\,dt =$$

$$\int_0^a x^2\cdot x\, f(x^2)\,dx = \int_0^{a^2} x f(x)\,dx$$

чтд

2261

$$\int_0^{2\pi} f(x)\cos x\,dx = \begin{bmatrix} \sin x = t \\ dt=\cos x\,dx \\ t\in[0;1] \end{bmatrix} = \int_0^{\frac{\pi}{2}} f(x)\cos x\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} f(x)\cos x\,dx + \int_{\frac{3\pi}{2}}^{2\pi} f(x)\cos x\,dx =$$

У нас функция $t = \sin x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow$ на промежутке $[0; 2\pi]$ не имеет обратной (критерий обратимости) $\Rightarrow$ разбиваем функцию на соотв. промежутки.

$$= \int_0^1 f(\arcsin t)\,dt + \int_1^{-1} f(\pi - \arcsin t)\,dt + \int_{-1}^0 f(2\pi + \arcsin t)\,dt = \int_0^1 f(\arcsin t)\,dt - \int_{-1}^1 f(\pi - \arcsin t)\,dt$$

$$+ \int_{-1}^0 f(2\pi + \arcsin t)$$

Ответ: $\int_0^1 \big(f(\arcsin t) - f(\pi - \arcsin t)\big)\,dt + \int_{-1}^0 \big(f(2\pi + \arcsin t) - f(\pi - \arcsin t)\big)\,dt$