3. Zad a)

Trokut - $\Delta abc$

Zraka - $e+td$

$e+td = \alpha a + \beta b + \gamma c = a + \beta(b-a) + \gamma(c-a)$

Sjecište će se desiti na $e+td$ ako je $\beta, \gamma \geq 0$ i $\beta + \gamma < 1$.

$$\left.\begin{aligned} x_e + t x_d &= x_a + \beta(x_b - x_a) + \gamma(x_c - x_a) \\ y_e + t y_d &= y_a + \beta(y_b - y_a) + \gamma(y_c - y_a) \\ z_e + t z_d &= z_a + \beta(z_b - z_a) + \gamma(z_c - z_a) \end{aligned}\right\}$$ Možemo zapisati kao matričnu jednadžbu.

$$\underbrace{\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix}}_{A} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$ Možemo riješiti Cramerovom metodom

$$\Rightarrow \beta = \frac{\begin{vmatrix} x_a - x_e & x_a - x_c & x_d \\ y_a - y_e & y_a - y_c & y_d \\ z_a - z_e & z_a - z_c & z_d \end{vmatrix}}{|A|}, \quad \gamma = \frac{\begin{vmatrix} x_a - x_b & x_a - x_e & x_d \\ y_a - y_b & y_a - y_e & y_d \\ z_a - z_b & z_a - z_e & z_d \end{vmatrix}}{|A|}, \quad t = \frac{\begin{vmatrix} x_a - x_b & x_a - x_c & x_a - x_e \\ y_a - y_b & y_a - y_c & y_a - y_e \\ z_a - z_b & z_a - z_c & z_a - z_e \end{vmatrix}}{|A|} \quad //$$

Sfera - centar $(x_c, y_c, z_c)$, $R$ - radijus $\Rightarrow \underbrace{(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 - R^2}_{f(p)} = 0$

$\Rightarrow$ u vektorskoj formi

$(p-c)(p-c) - R^2 = 0$

Zraka - $p(t) = e + td$

Trebamo riješiti $f(p(t)) = 0$

$$(e+td-c)(e+td-c) - R^2 = 0$$

$$e^2 + etd - ec + etd + (td)^2 - ctd - ec - ctd + c^2 - R^2 = 0$$

$$d \cdot d \cdot t^2 + 2d(e-c)t + (e-c)(e-c) - R^2 = 0$$

$$\Rightarrow t_{1,2} = \frac{-d(e-c) \pm \sqrt{(d \cdot (e-c))^2 - (d \cdot d)((e-c)\cdot(e-c) - R^2)}}{(d \cdot d)} \quad //$$

Zad. 3. b)

$$\frac{x^2}{c^2} + \frac{y^2}{s^2} = 1 \quad , \quad e + \lambda d \; , \; e = (e_1, e_2, e_3) \, , \; d = (d_1, d_2, d_3)$$

$$e_1 + e_2 + e_3 + \lambda d_1 + \lambda d_2 + \lambda d_3 = (e_1 + \lambda d_1) + (e_2 + \lambda d_2) + (e_3 + \lambda d_3)$$

$$\frac{(e_1+\lambda d_1)^2}{c^2} + \frac{(e_2+\lambda d_2)^2}{s^2} = 1 \qquad 0 \leq e_3 + \lambda d_3 \leq h$$

$$(e_1 + \lambda d_1)^2 \cdot s^2 + (e_2 - \lambda d_2)^2 \cdot c^2 - c^2 s^2 = 0$$

$$s^2 e_1^2 + 2 d_1 e_1 \lambda s^2 + \lambda^2 d_1^2 \cdot s^2 + c^2 e_2^2 - 2 d_2 e_2 c^2 \cdot \lambda + d_2^2 \cdot c^2 \cdot \lambda^2 - c^2 s^2 = 0$$

$$\lambda^2 (d_1^2 \cdot s^2 + d_2^2 c^2) + \lambda (2 d_1 e_1 s^2 - 2 d_2 c^2 \cdot e_2) + (s^2 e_1^2 + c^2 e_2^2 - c^2 s^2) = 0$$

$$\lambda_{1,2} = \frac{-(2 d_1 e_1 s^2 - 2 d_2 c^2 - e_2) \pm \sqrt{(2 d_1 e_1 s^2 - 2 d_2 c^2 \cdot e_2)^2 - 4 (d_1^2 \cdot s^2 + d_2^2 c^2)(s^2 e_1^2 + c^2 e_2^2 - c^2 s^2)}}{2 (d_1^2 \cdot s^2 + d_2^2 c^2)}$$

Zraka siječe cilindar za vrijednost $\lambda$ ako i samo ako $0 \leq e_3 + \lambda d_3 \leq h$