Statistical Working Paper on Imputation Methodology for the FAOSTAT Production Domain

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Abstract

This paper proposes a new imputation method for the FAOSTAT domains based on linear mixed model and ensemble learning.

The proposal provides a resolution to many of the shortcomings of the current approach, and offers a flexible and robust framework to incorporate further information to improve performance.

A detailed account of the methodologies is provided. The linear cross-commodity information; meanwhile, the ensemble learning approach provides flexiblity and robustness where traditional imputation methods of applying a single model may have failed.

Keywords: Imputation, Linear Mixed Model, Ensemble Learning.

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1. Introduction

Missing values are commonplace in the agricultural production domain, stemming from non-response in surveys or a lack of capacity by the reporting entity to provide measurement. Yet a consistent and non-sparse production domain is of critical importance to FAO and to compute Food Balance Sheets (FBS). Thus accurate and reliable imputation is essential and a necessary requisite for continuing work. This paper addresses several shortcomings of the current work and a new methodology is proposed in order to resolve these issues and to increase the accuracy of imputation.

The primary objective of imputation is to incorporate all available and reliable information in order to provide best estimates of food supply in the SUA/FBS.

Presented in table 1 is a description of the existing flags in the current Statistical Working System (SWS). In this exercise, estimated and previously imputed data are marked as either **E** or **I** and are the target values to be imputed.

The set of values considered reliable enough to be considered *protected* is identified by the *flagValidTable*. The following table reports only those flag combination thare will not be overwritten by the module(**protected figures**):

Table 1: Description of the flags in the Statistical Working System

Flags	Description
	Official data reported on FAO Questionnaires from countries
I	Imputed figure
T	Unofficial
M	Not reported by country
E	Expert sources from FAO (including other divisions)

```
   flagObservationStatus flagMethod Valid Protected
 1:                    E          c  TRUE      TRUE
 2:                    E          f  TRUE      TRUE
 3:                    E          h  TRUE      TRUE
 4:                    I          c  TRUE      TRUE
 5:                    M          -  TRUE      TRUE
 6:                    T          -  TRUE      TRUE
 7:                    T          c  TRUE      TRUE
 8:                    T          h  TRUE      TRUE
 9:                    T          p  TRUE      TRUE
10:                               -  TRUE      TRUE
11:                               c  TRUE      TRUE
12:                               h  TRUE      TRUE
13:                               p  TRUE      TRUE
14:                               q  TRUE      TRUE
```

1.1. Yield

The next graph depicts the yield for *wheat*.

From the graph, we can first observe that there is a general increasing trend in the yield accross almost allcountries . Similar stories are observed in almost all commodities that have been studied during the development of the methodology. This is a result of continuous advancement in both technology and agricultural practice driven by research & development. Improved irrigation provides crop with sufficient and uninterrupted water source, while tailored compound feed provide the precise nutrient requirements ensuring the livestocks consume the optimal diet for growth. Regardless whether these practices are sustainable or beneficial, there are strong evidence of increased productivity over time.

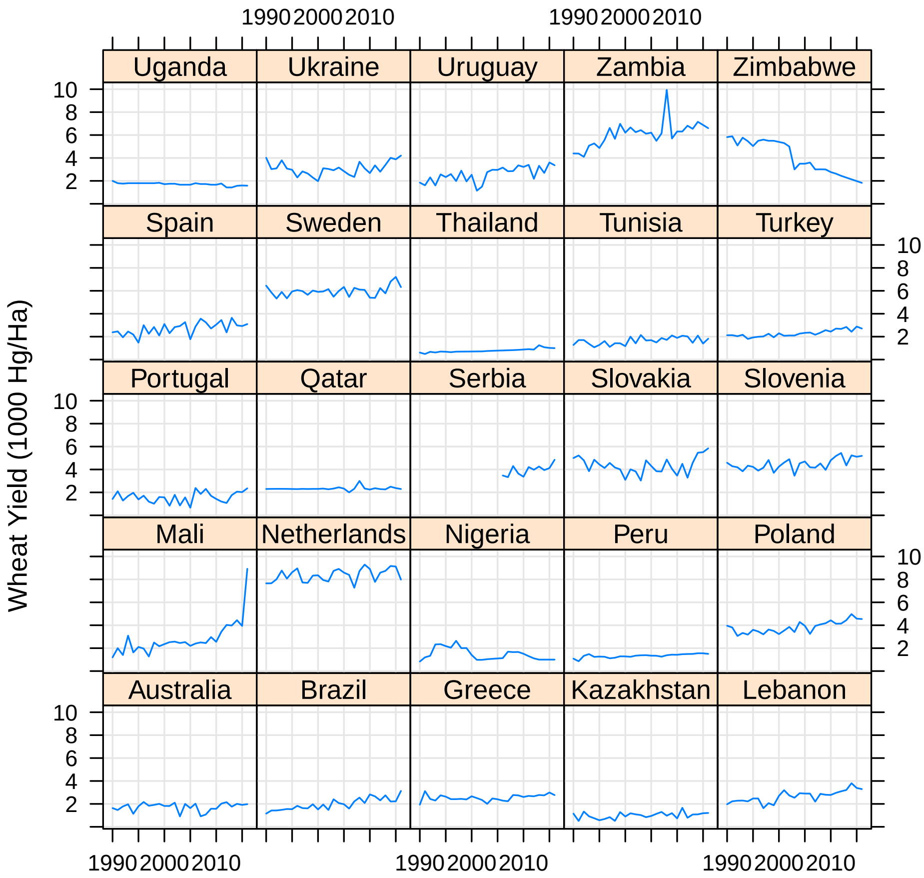
Nevertheless, just like all available technology such as internet, the distribution is far from perfect. The adoption of technology depends on the access which may be hindered by the presence of patents, or restriction imposed by service providers. Imperfect information and limit financial resources are also major obstacles for embracing the new developments, this is particularly true for countries where the majority of the producers are smallholders or subsistence farming.

Furthermore, producers face different constraints and cost. Countries such as Brazil and Russia which have large amounts of arable land do not bear the same cost for land acqusition in comparison to small states such as the Netherlands. The cost translates to different pressure to improve productivity and yield. Required innovations are also different for countries:

wheat breeding for the development of drought and disease-resistent varieties were crucial to withold Australia's dry climate.

Despite the differences among countries branching from various combination of technological advancement and economic condition, these factors all contribute towards a positive improvement in productitivity which can be estimated as an aggregated mixture effect.

Forces of nature also play a vital role in the determination of crop productivity. However, unlike technological advancement, precipitation and temperature are associated more closely to year-to-year variation.



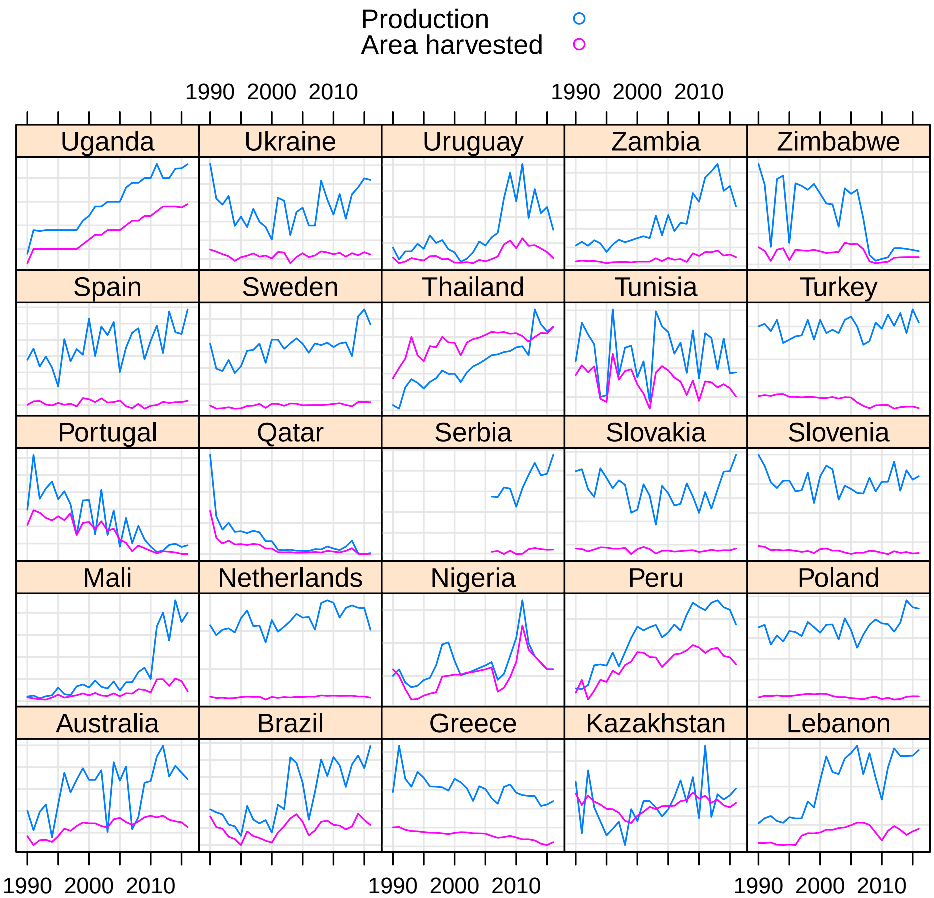
1.2. Production and Area Harvested

Although yield plays a vital role in the production process, the actual quantity of production is mainly dictated by the area sown and harvested. The illustration in this section show that the crop-production series is usually dominated by how much area was planted and harvested.

In contrast to the simple mechanism of yield where all dominant factors contribute towards improving the productivity, the mechanism of production is much more unpredictable.

Production is determined by area harvested and hence sown in the previous period by the farmer, which ultimately depends on the perception of information and subjective judgement of the producer. Production can increase or decrease as a response to the state of the market, where wheat field can be substitue to sorghum if prices are expected to be high. Further, individual entities faces different risk profiles, even under the assumption of all producers are profit-seeking the risk profile may alter the portfolio of products held by the producer. Markets have been known to be difficult to forecast, let alone the prediction of human judgement is just shy of impossible under curren state of understanding.

Only in cases where the commodity is a major staple or exporting item, we can observe simple trend explained by the continuous increase in demand. On the other hand, commodities which are of relative lesser importance, the pattern of the production may display unpredictable erratic behaviour.



2. Proposed Methodology

The imputation of missing observations is traditionally done via a model. For example, we may consider a simple global mean model where we compute the mean of all available observations and use that value to impute missing observations. Alternatively, we could use a more complex model (i.e. linear/exponential/logistic regression, spline, etc.), fit the model to the available data, and then estimate missing values using this model. However, this approach has two problems. First, we may choose a poor model and thus obtain poor estimates. Second, we have to specify which model to use for each set of data, and this could be very tedious if we have many time-series to impute. To avoid this problems, we consider ensemble models.

2.1. Ensemble Imputation

Ensemble learning refers to the process of building a collection of simple base models or learners which are later combined to obtain a composite model or prediction. One of the most famous applications of ensemble learning was the prediction of movie ratings held by Netflix in which the top two performers both used an ensemble of different models. Ensembles are very popular in the data-mining community because of their ability to combine multiple models and come up with an estimate that is better than any of the individual models.

The method consist of two steps:

1. Building multiple models/learners.
2. Combining the models or predictions.

The ensemble method reduces the risk of choosing a poor model as we are averaging multiple models. Thus we reduce the risk of implementing a single model which may produce poor imputations for a certain subset of data. Moreover, model selection is unecessary, since all model are included in the final ensemble.

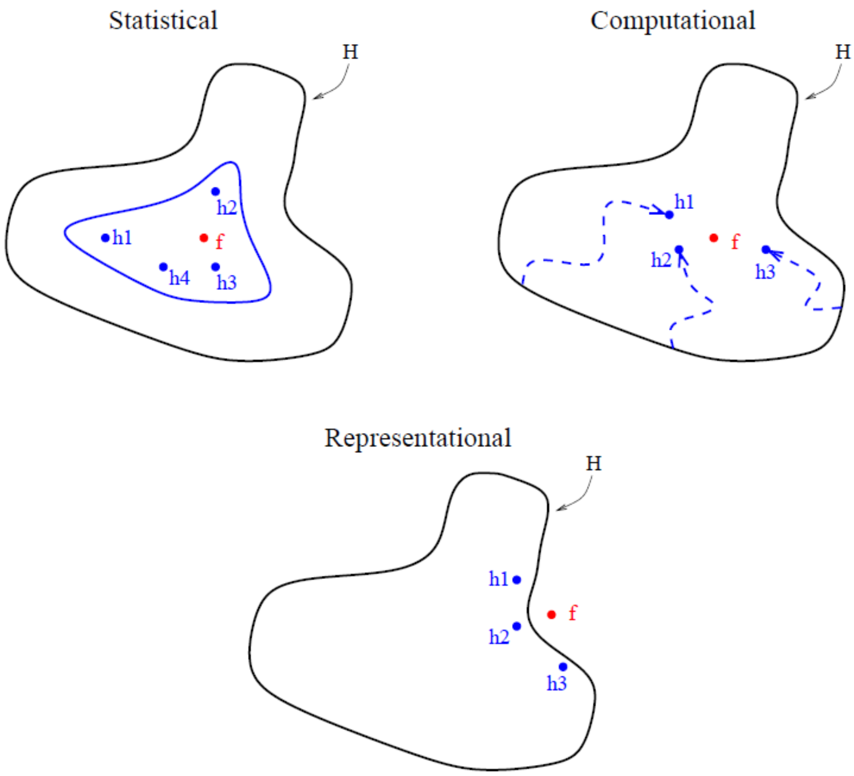
Thomas Dietterich describes several problems in machine learning in his paper "Ensemble Methods in Machine Learning" (seehttp://www.eecs.wsu.edu/~holder/courses/CptS570/fall07/papers/Dietterich00.pdf), and he also discusses how using an ensemble can reduce the errors from the following three issues.

Statistical: A lack of data may allow multiple models to fit the training set well.

Computational: Optimization procedures occasionally converge to local solutions instead of the global solution.

Representational: It may not be possible to model the true phenomenon with a known model.

The **statistical problem** refers to the lack of data to support a particular hypothesis. The problem can be formulated as finding the best hypothesis among competing models in the space $\mathcal{H}$. In the top left graph of the depiction from Dietterich we see a blue boundary, and the idea is that all models within this boundary will give the same fit to the training data. Thus, there is insufficient information to determine which one is better. By combining the models, we reduce the risk of choosing a terrible model. For example, if we only observe two data points for a country, then fitting a linear line or a log curve can both give the same



accuracy on the training data and we may have no information to distinguish between the two.

The second problem is that some models are fit by **optimizing** some cost function. These numerical algorithms can often converge to local solutions instead of the global solution. The top right graph from Dietterich represents this problem, with points h1, h2, and h3 representing the local solutions and f the true global solution. Thus, combining the multiple fits should get us closer to the true optimum f. At the time of writing this vignette, no models which use this numerical optimization are present in the default methodology; however, we could introduce such models in the future (for example, a neural network).

The final problem, **representational**, refers to the fact that the true function f can not be represented by any of the individual models. However, by combining the models we may expand the space of representable functions and more closely approximate the true function f. For example, if the production of a country has been growing at a linear rate in the distant past but has expanded rapidly recently, then neither a linear or exponential model will provide a satisfactory result. However, an ensemble combining a linear and exponential model will provide a better solution by capturing different characteristics of the data.

From an implementation point of view, the algorithm is adaptive and will not need constant updating. For example, If the data generating mechanism changes in the future, the next fit of the ensemble will shift weights to models which better represent the data and thus it will not be necessary to constantly monitor and update the methodologies/models manually.

2.2. Description of Models

This section describes the different base learners for the ensemble methodology, and they are listed in increasing order of complexity. An effective ensemble will have base models as diverse as possible. If there is no diversity and all models generate similar results, then little is gained by combining these models and the ensemble model will not be much of an improvement from an individual model.

- **Mean**: Mean of all observations
- **Median**: Median of all observations
- **Linear**: Linear Regression
- **Exponential**: Exponential function
- **Logistic**: Logistic function
- **Naive**: Linear interpolation followed by last observation carried forward and first observation carried backward.
- **ARIMA**: Autoregressive Integrated Moving Average model selected based on the AICC, and imputation via Kalman Filter.
- **LOESS**: Local regression with linear models and model window varying based on sample size.
- **Splines**: Cubic spline interpolation.
- **MARS**: Multivariate Adaptive Regression Spline
- **Mixed Model**: Linear mixed model with time as a fixed effect and country as the random effect.

2.3. Model "Level"

We wish to be able to construct models of varying levels of complexity, and in doing so we would like to be able to build very localized models as well as more general/global models. One way of doing this is by restricting the training dataset for each model constructed. For example, if we have fairly good data availability for a particular country/commodity, then we may wish to use only that country's data when building a model. However, if one country/commodity only has a few valid observations, then we may need to use global trends for that commodity to model that country/commodity more accurately.

In the current code, there are two "levels" for constructing a model: "local" and "global." The first, "local," means that the model will only use data for that specific slice of data (defined by the byKey parameter) in the fitting of the model. Thus, we will construct a different model for each different time series slice of the data. Most of the implemented models fall under this methodology: mean, linear, exponential, logistic, naive, loess, splines, and MARS.

The "global" level means that the model uses all the data available. The mixed model follows this approach in that all data is used to fit a mixed effects model (where year is considered a fixed effect and country a random effect).

2.4. Extrapolation

The ensembling process makes use of many different models, but we must be careful in considering what kinds of models to use in which scenarios. As a simple example, suppose a country has production values of 1, 2, and 4 in three consecutive years and then twenty years of missing data. If we fit an exponential model to this data, we'll be estimating production values over 4 million at the end of the twenty year period! In addition, other models (such as splines and LOESS) don't extrapolate well.

Thus, for each model, we have an extrapolation parameter. This value allows the user to control how far outside the range of the data a particular model can be used. Using this functionality allows us to prevent extrapolating with models that clearly shouldn't be used outside the range of the data while still making use of these models for interpolation purposes.

2.5. Computation of Weights

To construct an ensemble, one has to use a weighted average of all of the input models. However, one must determine a meaningful way for computing weights, as models which perform poorly shouldn't receive as much weight as models which fit the data well. A simple approach is to compare, for each valid observation, the model estimate and the true value. If we average the error between these two values across all valid observations, we get an estimate for the error of the model. Then, we can use this error to compute the model weights:

$$w_i = 1/e_i / \sum_{j=1}^{n} 1/e_j$$

where w_i is the weight of the ith model, e_i is the error of the ith model, and n is the total number of models. Thus, models with smaller errors receive more weight in the final ensemble, and the summation on the bottom of the above formula ensures that the weights sum up to one (ensuring that our weighted sum is in fact a weighted mean). This approach is possible in the current code by setting the errorType to "raw" in the imputation parameters list.

However, the above approach is not ideal. The problem lies in the fact that complex models generally will fit the training data better because they are more complex. In reality, we want to know if these models are more accurate at predicting unknown values, and thus we need a way to measure how effectively these models can predict on new observations. To accomplish this, one has to use cross-validation.

With cross-validation, the observed data are split into k different groups (often $k = 10$, and this is the default for this package as well). Then, for each group i, we build a model using all of the observed data except for those in group i and we measure how well this model estimates the data in group i. If we average this error across all k groups, we get a measure for how well this model predicts on our particular dataset. We perform this cross-validation error estimation for each of our different models, and then we compute model weights via

$$w_i = (1/e_i) / \sum_{j=1}^{n} \left(1/e_j\right)$$

The formula here is the same as the one above, but now the errors are the average cross-validation errors instead of the errors on the training set.

3. Models for Ensembling

This package implements many complex models that may not be familiar to the user, and so this section goes through the models and describes how the algorithm works as well as gives an example of the usage of that model. The order of this section is alphabetical, not by complexity.

3.1. defaultArima

The defaultArima model first fits an AutoRegressive, Integrated Moving Average (ARIMA) model to the time series provided, and it attempts to find the best model using the auto.arima function from the **forecast** package. If such a model is found, that model is used (along with KalmanSmooth) to generate new smoothed estimates of the time series.

```
> model = ensembleModel(model = defaultArima, extrapolationRange = Inf,
+                       level = "local")
> imputationParameters$yieldParams$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.2. defaultExp

This algorithm fits the following model: $\log(Y+1) = \beta_0 + \beta_1 t$ where Y is the dependent variable (i.e. production, seed rates, etc) t is time, and β_0, β_1 are the estimated coefficients. This model is equivalent to $Y + 1 = e^{\beta_0+\beta_1 t}$, hence the name exponential. The 1 in the formula ensures that $\log(Y+1)$ always exists (assuming $Y \geq 0$).

```
> model = ensembleModel(model = defaultExp, extrapolationRange = 1,
+                       level = "local")
> imputationParameters$yieldParams$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.3. defaultGlobalMean

This model is quite simple: it computes the mean from all available observations and uses that value to impute any missing values. This model is not recommended for most domains; however, it may perform reasonably well when imputing rates or proportions, as the average may not vary drastically from country to country. A variable like production is very different, values can vary drastically in scale and so a global mean is not appropriate.

```
> model = ensembleModel(model = defaultGlobalMean, extrapolationRange = Inf,
+                       level = "global")
> imputationParameters$yieldParams$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

Note that in the above figure, the imputed values may appear to be different. However, this is simply due to the different scale in each of the grids; the imputed value is always about 8.

3.4. defaultGlobalMedian

The global median works exactly the same as the global mean, but computes the median instead of the mean. Again, this type of model should only be used when imputing rates or something similar (i.e. no drastic differences in scale across groups).

```
> model = ensembleModel(model = defaultGlobalMedian, extrapolationRange = Inf,
+                       level = "global")
> imputationParameters$yieldParams$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.5. defaultLm

The defaultLm model uses a simple linear regression model for imputation. It fits a model of the form: $Y = \beta_0 + \beta_1 t$, where Y is the value to impute, t is the time, and β_0, β_1 are estimated coefficients.

```
> model = ensembleModel(model = defaultLm, extrapolationRange = Inf,
+                       level = "local")
> imputationParameters$yieldParams$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.6. defaultLoess

The defaultLoess model works by fitting a "local" linear regression model at each point in the model space. The model is local in the sense that the fit at time t uses only nearby time points, say $t-k$ to $t+k$. Furthermore, points further away from t are given less weight in the regression model. This type of model has several tuning parameters such as the size of the neighborhood and the degree of model to fit (i.e. we could fit local linear models, quadratic, etc.). For simplicity, we use a local linear model, and we choose the smallest span possible to allow for the most flexible model. Addditionally, the local nature of the loess model means that it likely will not extrapolate well, so the recommended extrapolation range is 1.

```
> model = ensembleModel(model = defaultLoess, extrapolationRange = 1,
+                       level = "local")
> imputationParameters$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.7. defaultLogistic

Logistic curves are S-shaped curves of the form

$$f(x) = A + \frac{B}{1 + e^{-C(t-D)}}$$

. These types of functions make sense in scenarios where a variable is increasing but may have some upper bound (i.e. production may increase greatly as technology improves, but there is some maximum production level a country can obtain). This algorithm attempts to first fit all four parameters above via numerical least squares. If that approach fails, A is assumed to be 0 and numerical least squares are tried again. If that model also fails, B is assumed to be the largest value and model fitting proceeds via generalized least squares.

```
> model = ensembleModel(model = defaultLogistic, extrapolationRange = 1,
+                       level = "local")
> imputationParameters$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

Note: in the first example, the logistic regression decays rapidly to 0. This may not be very reasonable, and thus we recommend using a small extrapolation range for this model.

3.8. defaultMars

The defaultMars model uses a technique known as Multivariate Adaptive Regression Splines (MARS). This algorithm seeks to model the data using piecewise linear regression splines, and it determines the breakpoints of the splines using some optimization criterion. On our sample dataset, we don't see anything too interesting:

```
> model = ensembleModel(model = defaultMars, extrapolationRange = Inf,
+                       level = "local")
> imputationParameters$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.9. defaultMean

This model computes a mean on each subset of the data and uses that one value to impute any missing values.

```
> model = ensembleModel(model = defaultMean, extrapolationRange = Inf,
+                       level = "local")
> imputationParameters$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.10. defaultMedian

This model computes a median on each subset of the data and uses that one value to impute any missing values.

```
> model = ensembleModel(model = defaultMedian, extrapolationRange = Inf,
+                       level = "local")
> imputationParameters$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.11. defaultMixedModel

The defaultMixedModel is a more complex model that's fit to global datasets (rather than each individual time-series). So, let's use a different dataset to run some analyses with this model:

```
> mixedModelData = extractedData[geographicAreaM49 < 100, ]
> # mixedModelData = processProductionDomain(mixedModelData,
> #          processingParameters = defaultProcessingParameters())
> updateMissingFlags = function(data, value, flag, missingFlag = "M"){
+     missingIndex = which(is.na(data[[value]]))
+     invisible(data[missingIndex, `:=`(c(flag), list(missingFlag))])
+ }
> updateMissingFlags(data = mixedModelData, value = "Value_measuredElement_5421",
+           flag = "flagObservationStatus_measuredElement_5421")
```

Now, let's run the ensemble imputation with the default imputation parameters. This example uses the **lme4** package to fit a mixed model.

```
> newParameters = defaultImputationParameters(5421)
> newParameters$newImputationColumn = "test"
> newParameters$estimateNoData = TRUE
```

```
> model = ensembleModel(model = defaultMixedModel, extrapolationRange = Inf,
+                       level = "global")
> newParameters$ensembleModels = list(model)
> imputeVariable(data = mixedModelData, imputationParameters = newParameters)
```

For most cases, we see the same results as we did with the linear regression: a simple least-squares curve is fit to the available data and then that model is used to impute the missing values. However, the mixed model fit to the data can also be used for estimation on time-series where very little data is available, for example on area 46 and 66.

More complex cases are also available: for example, we could fit a hierarchical model (also with the **lme4** package) that allows us to impute countries with missing data based on some hierarchy (for example, continents). In this example, we just made up arbitrary regions.

```
> invisible({
+     mixedModelData[geographicAreaM49 == "66", Value_measuredElement_5421 := NA]
+     mixedModelData[geographicAreaM49 == "66",
+                flagObservationStatus_measuredElement_5421 := "M"]
+     mixedModelData[,region := factor(ifelse(geographicAreaM49 < 15, 1,
+                                     ifelse(geographicAreaM49 < 50, 2, 3)))]
+ })
> formals(defaultMixedModel)$modelFormula = Value_measuredElement_5421 ~
+     timePointYears*region + (timePointYears|geographicAreaM49/region)
> hierarchical = ensembleModel(model = defaultMixedModel, level = "global",
+                              extrapolationRange = Inf)
> globalMean = ensembleModel(model = defaultGlobalMean, level = "global",
+                            extrapolationRange = Inf)
> globalMedian = ensembleModel(model = defaultGlobalMedian, level = "global",
+                              extrapolationRange = Inf)
> newParameters$ensembleModels = list(hierarchical = hierarchical,
+                                     globalMean = globalMean,
+                                     globalMedian = globalMedian)
> imputeVariable(data = mixedModelData, imputationParameters = newParameters)
```

In this complicated example, we can see several different types of imputation occurring. The most common case seems to be the hierarchical model, which simplifies to just a linear regression on countries with enough data to fit the model. The global mean and median don't seem to be as good of models, but in a few cases they provide some value.

Note that we are also able to impute values in region 66, where no data exists (because we deleted the one value that was present before we began). This imputation was performed by looking at the cross-validation error of the three models considered on all the other datasets and averaging the error across all available observations. Generally the mixed model performs best, and so it gets the most weight on this set of data where we have no available data.

3.12. defaultNaive

This model performs simple linear interpolation between available observations. However, if a missing value is outside the range of the data, then this model estimates that value by carrying back the first observation or carrying forward the last observation (depending on if the missing value is before the available data or after it). Because of this, this model is not recommended for extrapolation (and has a default extrapolationRange of 0 in allDefaultModels).

```
> model = ensembleModel(model = defaultNaive, extrapolationRange = 0,
+                       level = "local")
> imputationParameters$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

3.13. defaultSpline

The defaultSpline model uses the spline function from the stats package (part of base) to fit a spline to the available observations. Missing observations are then imputed by the spline estimate at that location.

```
> model = ensembleModel(model = defaultSpline, extrapolationRange = 0,
+                       level = "local")
> imputationParameters$ensembleModels = list(model)
> imputeVariable(data = exampleData, imputationParameters = imputationParameters)
```

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