# Persistent Homology Of Randomized Simplicial Complexes

Masterarbeit

vorgelegt von


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Berlin, den –.–.—-

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig und eigenhändig sowie ohne unerlaubte fremde Hilfe und ausschließlich unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

Berlin, den –.–.—-

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### Zusammenfassung

Janz Abstracte Jeschichte.

# Contents

# 1 Foundations

This first section is committed to the introduction of the mathematical concepts we will need throughout this work. The goal is not an in depth build up, but rather a concise collection of basic definitions and theorems.

## 1.1 Simplicial Complexes

In this section we will introduce some notions of discrete geometry. For the following definitions let $S = \{s_1, ..., s_m\} \subseteq \mathbb{R}^n$ be a finite set of points.

**Definition 1.1.1** (Affine combination)**.** Let $\lambda_i \in \mathbb{R}$, $i = 1, ..., m$ be scalars, which satisfy $\sum\limits_{n=1}^{m} \lambda_i = 1$. Then:

$$\sum_{n=1}^{m} \lambda_i s_i$$

is called an ***affine combination*** of the points $s_i$. [2, Definition 2.11]

**Definition 1.1.2** (Affine hull, affinely independent)**.** The set of all affine combinations of points in $S$ is called the ***affine hull***. We say that the points in $S$ are ***affinely independent*** if they generate a subspace of dimension $m - 1$. [2, Definition 2.11]

An alternative definition is given in [1, III.1, paragraph 3]. The points in $S$ are called ***affinely independent*** if any two affine combinations $x = \sum\limits_{n=1}^{m} \lambda_i s_i$ and $y = \sum\limits_{n=1}^{m} \mu_i s_i$ are the same, iff $\mu_i = \lambda_i$ for $i = 1, ..., m$.

**Definition 1.1.3** (Convex combination)**.** Let $S = \{s_1, ..., s_m\} \subseteq \mathbb{R}^n$, $1 \leq m \in \mathbb{N}$. Let $0 \leq \lambda_i \in \mathbb{R}$, $i = 1, ..., m$ be scalars, which satisfy $\sum\limits_{n=1}^{m} \lambda_i = 1$. Then the sum:

$$\sum_{n=1}^{m} \lambda_i s_i$$

is called a ***convex combination*** of the points $s_i$. [2, Definition 2.12]

In other words, a convex combination is an affine combination, in which the $\lambda_i \geq 0$. Now we can analogously define the convex hull.

**Definition 1.1.4** (Convex hull)**.** The set of all convex combinations of points in $S$ is the ***convex hull*** of $S$.

We are particularly interested in the following special class of convex hulls.

**Definition 1.1.5** (k-Simplex)**.** A ***k-Simplex*** is the convex hull of $k + 1$ affinely independent points.

# Bibliography


[1] Herbert Edelsbrunner and John Harer. *Computational Topology: An Introduction.* 01 2010.

[2] Michael Joswig and Thorsten Theobald. *Polyhedral and algebraic methods in computational geometry.* Universitext. Springer, London, 2013. Revised and updated translation of the 2008 German original.