# MDK assignment Ivan

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## 1 Algorithm

With the unit twists specified, the reference configuration chosen and the homogeneous matrices specified, all inputs are now ready to be put trough formulas to create the geometric Jacobian.

First of all, using the relation shown in equation 1 the homogeneous matrix H(q) from the first frame to any frame can be calculated with equation 2, only using the original inputs.

$$H_i^{(i-1)}(q_i) = e^{\hat{T}_i^{(i-1),(i-1)} q_(i)} H_i^{(i-1)}(0) \tag{1}$$

$$H_n^0(q) = e^{\hat{T}_1^{0,0} q_1} H_1^0(0) e^{\hat{T}_2^{1,1} q_2} H_2^1(0) ... e^{\hat{T}_n^{(n-1),(n-1)} q_n} H_n^{(n-1)}(0) \tag{2}$$

Using the adjoint of H and the unit twist the i-th twist in the Jacobian matrix can be calculated using equation 3.

$$T_i = Ad_{H_{(i-1)}^{(0)}} \hat{T}_i^{(i-1),(i-1)} \tag{3}$$

To put this into an algorithm:

1. define $J(q) = (T_1 T_2 ... T_n)$.
2. for each $T_i$ in $J(q)$ use equation 2 to calculate $H_i^0(q)$.
3. Use this result to define the adjoint in equation 3.
4. Add the result from equation 3 to $J(q)$.
5. continue until $i = n$.

The main benefit this algorithmic method has over the inspection method is that instead of having to calculate all twists one by one manually, only the unit twists and homogeneous matrices need to be defined, after which the algorithm takes over the calculations. The more joints the chain mechanism has, the faster the algorithmic method is compared to inspection.

## 2 Implementation

Input unit twists:

$$\hat{T}_1^{0,0} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \hat{T}_2^{1,1} = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \hat{T}_3^{2,2} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \hat{T}_4^{3,3} = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \hat{T}_5^{4,4} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \hat{T}_6^{5,5} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Input homogeneous matrices:

$$H_{(1)}^{(0)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} H_{(2)}^{(1)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -a \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} H_{(3)}^{(2)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_{(4)}^{(3)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} H_{(5)}^{(4)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} H_{(6)}^{(5)} = \begin{pmatrix} 1 & 0 & 0 & e \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

With a=90 b=425 c=392 d=93 e=95 f=82

# 3 Control

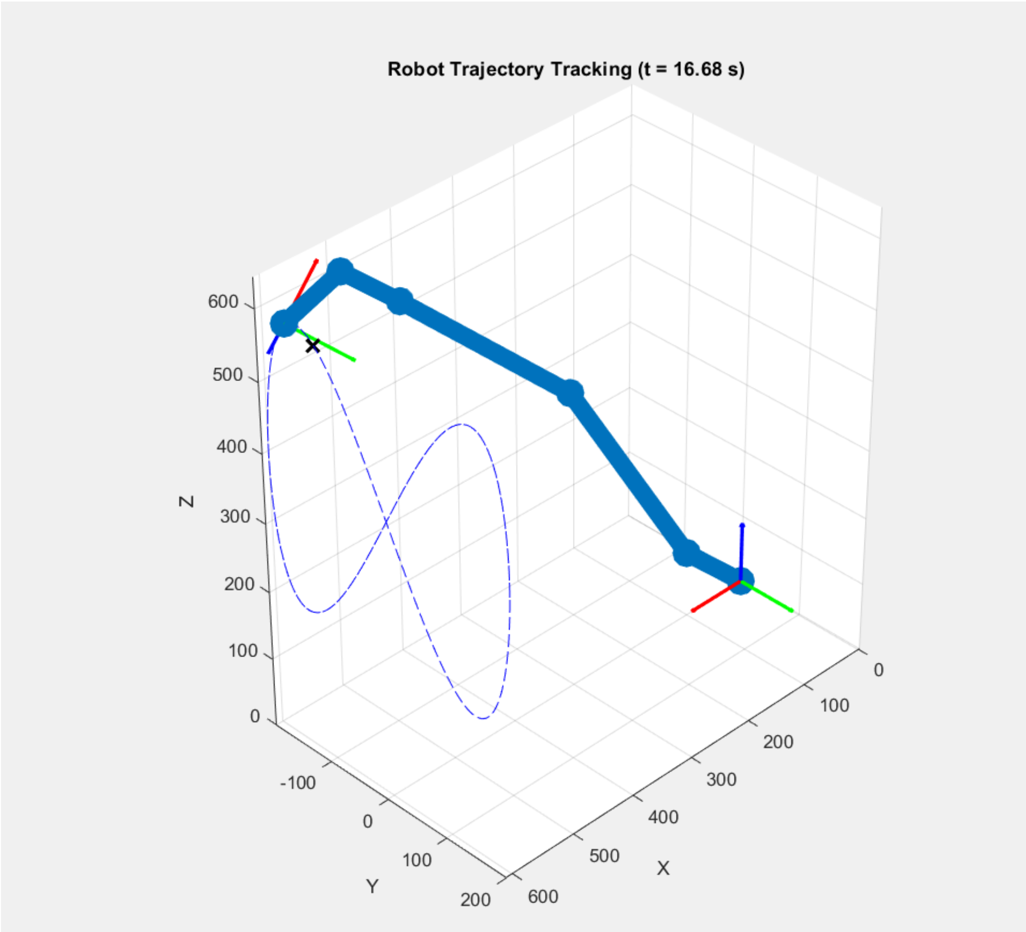


Figure 1: Completed robot creating the butterfly shape