3. Robot Control

Part 3.1: Forward Kinematics

The coordinate of the end effector can be obtained by the part of the Transformation Matrix (*T(q)*) for the Robot joints.

To obtain the T(q), first D-H table have to be constructed (q =$[\theta_1, \theta_2, \theta_3, \theta_4]$).

	α	a	d	θ
Link 1	$\frac{\pi}{2}$	0	2.5	$\theta_1 + \frac{\pi}{2}$
Link 2	$\frac{\pi}{2}$	0	0	$\theta_2 + \frac{\pi}{2}$
Link 3	$-\frac{\pi}{2}$	3.5	0	θ_3
Link 4	0	3	0	θ_4

D-H Table for Robot

Next, from D-H Table construct the transform matrix for each joint (frames).

$$A_i^{i-1} = R_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, R_{x,\alpha_i} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ therefore,

$$A_1^0 = \begin{bmatrix} \cos(\theta_1 + \frac{\pi}{2}) & 0 & \sin(\theta_1 + \frac{\pi}{2}) & 0 \\ \sin(\theta_1 + \frac{\pi}{2}) & 0 & -\cos(\theta_1 + \frac{\pi}{2}) & 0 \\ 0 & 1 & 0 & 2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_2^1 = \begin{bmatrix} \cos(\theta_2 + \frac{\pi}{2}) & 0 & \sin(\theta_2 + \frac{\pi}{2}) & 0 \\ \sin(\theta_2 + \frac{\pi}{2}) & 0 & -\cos(\theta_2 + \frac{\pi}{2}) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_3^2 = \begin{bmatrix} \cos(\theta_3) & 0 & -\sin(\theta_3) & 3.5\cos(\theta_3) \\ \sin(\theta_3) & 0 & \cos(\theta_3) & 3.5\sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_4^3 = \begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & 3\cos(\theta_4) \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 3\sin(\theta_4) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(q) = A_1^0 A_2^1 A_3^2 A_4^3 = \begin{bmatrix} \cos\left(\theta_1 + \frac{\pi}{2}\right) & 0 & \sin\left(\theta_1 + \frac{\pi}{2}\right) & 0 \\ \sin\left(\theta_1 + \frac{\pi}{2}\right) & 0 & -\cos\left(\theta_1 + \frac{\pi}{2}\right) & 0 \\ 0 & 1 & 0 & 2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\left(\theta_2 + \frac{\pi}{2}\right) & 0 & \sin\left(\theta_2 + \frac{\pi}{2}\right) & 0 \\ \sin\left(\theta_2 + \frac{\pi}{2}\right) & 0 & -\cos\left(\theta_2 + \frac{\pi}{2}\right) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_3) & 0 & -\sin(\theta_3) & 3.5\cos(\theta_3) \\ \sin(\theta_3) & 0 & \cos(\theta_3) & 3.5\sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & 3\cos(\theta_4) \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 3\sin(\theta_4) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & -\sin(\theta_1)\cos(\theta_2) & 0 \\ -\sin(\theta_2)\cos(\theta_1) & \sin(\theta_1) & \cos(\theta_1)\cos(\theta_2) & 0 \\ \cos(\theta_2) & 0 & \sin(\theta_2) & 2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_3)\cos(\theta_4) & -\cos(\theta_3)\sin(\theta_4) & -\sin(\theta_3) & 3.5\cos(\theta_3) + 3\cos(\theta_3)\cos(\theta_4) \\ \sin(\theta_3)\cos(\theta_4) & -\sin(\theta_3)\sin(\theta_4) & \cos(\theta_3) & 3.5\sin(\theta_3) + 3\sin(\theta_3)\cos(\theta_4) \\ -\sin(\theta_4) & -\cos(\theta_4) & 0 & -3\sin(\theta_4) \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2^0 A_4^2$$

Coordinate of the end-effector (x, y, z) is corresponding to the T(q)'s last element of 1st, 2nd and 3rd row. Which means to obtain the coordinate of end-effector, whole matrix T(q) does not have to be calculated. By using corresponding row and column of $A_2^0, and\ A_4^2$, the desired part of the T(q) can be calculated (corresponding row and column is colored in above).

Estimated end-effector coordinate (x, y, z) is:

$$x_e = \sin(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)\,(3\cos(\theta_4) + 3.5) + 3\cos(\theta_4)\sin(\theta_2)) + \cos(\theta_1)\sin(\theta_3)\,(3\cos(\theta_4) + 3.5)$$

$$y_e = \cos(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)\,(-3\cos(\theta_4) - 3.5) - 3\cos(\theta_2)\sin(\theta_4)) + \sin(\theta_1)\sin(\theta_3)\,(3\cos(\theta_4) + 3.5)$$

$$z_e = \cos(\theta_2)\cos(\theta_3)(3\cos(\theta_4) + 3.5) - 3sin(\theta_2)\,sin(\theta_4) + 2.5$$

Below Table is result of comparing estimated end-effector coordinate using Forward kinematics with the coordinate obtained by the computer vision implemented in previous part. The average error (difference in Euclidian distance between two points) is 0.81m. This is quite big. However, this does not mean the two point is distant through all the 10 experiments. Actually, the two points are sometimes very near, and sometimes very far away. By observing the image obtained from camera 1 and 2, it can be concluded that when the end-effector is distant from camera, it makes computer vision difficult to calculate the accurate coordinate of the end-effector. The maximum error of the z-coordinate is less than maximum error of the x and y coordinate. This also can be evidence of the conclusion considered above (z-coordinate will be less sensitive to the distance between end-effector and cameras).

$q = [\theta_1, \theta_2, \theta_3, \theta_4]$	Forward Kinematics xyz-coordinate	Computer Vision xyz-coordinate	$q = [\theta_1, \theta_2, \theta_3, \theta_4]$	Forward Kinematics xyz-coordinate	Computer Vision xyz-coordinate
[1.6, 1.6, 1.6, -1.6]	[-0.11, 3.41, 5.5]	[0.16 ,3.96, 5.28]	[0.8, 1.6, 1.6, -1.6]	[2.37, 2.46, 5.5]	[2.51, 3.36, 5.32]
[1.6, -1.6, 1.6, 1.6]	[-0.09, 3.41, 5.5]	[0.0, 3.96, 5.28]	[-0.8, -1.6, -1.6, 1.6]	[-2.39, 2.44, 5.5]	[-2.0, 2.68, 5.32]
[1.6, 1.6, -1.6, -1.6]	[0.09, -3.41, 5.5]	[0.32, -3.4, 5.32]	[1, 1, 1, -1]	[3.14, 3.1, 6.12]	[3.12, 4.32, 6.08]
[-1.6, 1.6, 1.6, -1.6]	[-0.09, -3.41, 5.5]	[0.04, -3.36, 5.32]	[1, 0.3, 1.8, 0.5]	[4.04, 4.51, 0.74]	[4.2, 6.6, 0.16]
[-1.6, 1.6, -1.6, -1.6]	[0.11, 3.41, 5.5]	[0.36, 4.04, 5.32]	[-1, -1, 0.5, 0.5]	[4.75, -0.45, 6.62]	[5.52, -0.44, 6.92]

End-effector position estimated by Forward kinematics and Computer vision with 10 different joints state

Part 3.2: Closed-loop Control

To obtain the formula for Velocity kinematics calculation, Jacobian Matrix have to be calculated first. Jacobian matrix J(q) can be calculated by following formula.

$$J(q) = \begin{bmatrix} \frac{\delta k_1(q)}{\delta\theta_1} & \frac{\delta k_1(q)}{\delta\theta_2} & \frac{\delta k_1(q)}{\delta\theta_3} & \frac{\delta k_1(q)}{\delta\theta_4} \\ \frac{\delta k_2(q)}{\delta\theta_1} & \frac{\delta k_2(q)}{\delta\theta_2} & \frac{\delta k_2(q)}{\delta\theta_3} & \frac{\delta k_2(q)}{\delta\theta_4} \\ \frac{\delta k_3(q)}{\delta\theta_1} & \frac{\delta k_3(q)}{\delta\theta_2} & \frac{\delta k_3(q)}{\delta\theta_3} & \frac{\delta k_3(q)}{\delta\theta_4} \end{bmatrix} \; where \begin{bmatrix} k_1(q) \\ k_2(q) \\ k_3(q) \end{bmatrix} = \begin{bmatrix} \sin(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)\,(3\cos(\theta_4)+3.5) + 3\cos(\theta_4)\sin(\theta_2)) + \cos(\theta_1)\sin(\theta_3)\,(3\cos(\theta_4)+3.5) \\ \cos(\theta_1)\,(\sin(\theta_2)\cos(\theta_3)\,(-3\cos(\theta_4)-3.5) - 3\cos(\theta_2)\sin(\theta_4)) + \sin(\theta_1)\sin(\theta_3)\,(3\cos(\theta_4)+3.5) \\ \cos(\theta_2)\cos(\theta_3)\,(3\cos(\theta_4)+3.5) - 3\sin(\theta_2)\sin(\theta_4) + 2.5 \end{bmatrix}$$

Then the formula for updating the joints angles in closed loop control is expressed by following equation.

$$\boldsymbol{q_{new} = q + (J^{+}(q) \cdot (K_p \cdot error^T + K_d \cdot \Delta error^T + K_i \cdot errors_i^T)) * dt}$$

$$where \quad \boldsymbol{J^{+}(q)} = pseudo\ inverse\ of\ J(q), \quad \boldsymbol{K_p} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad \boldsymbol{K_d} = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}, \quad \boldsymbol{K_i} = \begin{bmatrix} 0.00001 & 0 & 0 \\ 0 & 0.00001 & 0 \\ 0 & 0 & 0.00001 \end{bmatrix},$$

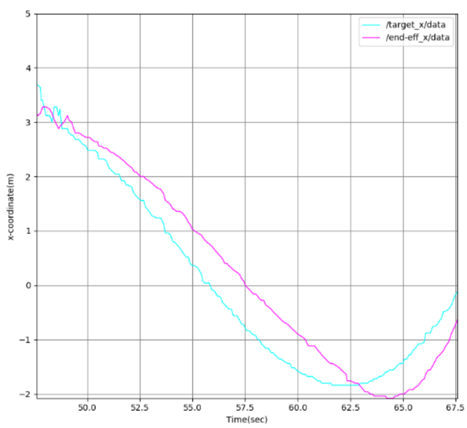
$$\textbf{error} = (\text{target}_{\text{pos}} - \text{end effector}_{\text{pos}}), \quad \boldsymbol{\Delta}\textbf{error}\ (\text{delivative of error}) = \text{error}_t - \text{error}_{t-1}, \quad \textbf{errors}_\textbf{i} = \begin{cases} \int_{t-10}^{t} \text{error dt} : & \text{if } 10 < \left\| \int_{t-10}^{t} \text{error dt} \right\| < 25 \\ [0,0,0]^T : & \text{otherwise} \end{cases}$$

$$t = current\ time\ step$$

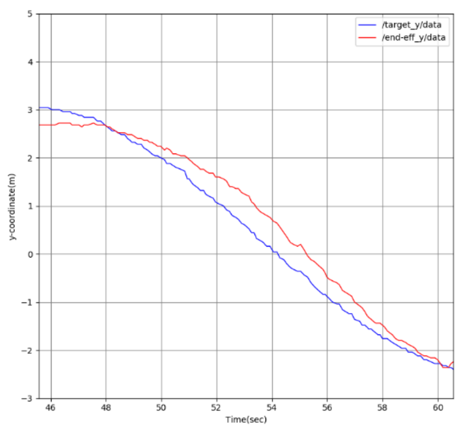
The position of $\text{target}_{\text{camera}}$ and $\text{end effector}_{\text{camera}}$ obtained from Computer vision are smoothed into $\text{target}_{\text{pos}}$ and $\text{end effector}_{\text{pos}}$ in following method

$$\textbf{target}_\textbf{pos} = \text{mean of target}_{\text{camera t}-3} \text{ to target}_{\text{camera t}}, \qquad \textbf{end effector}_\textbf{pos} = \text{mean of end effector}_{\text{camera t}-5} \text{ to end effector}_{\text{camera t}}$$

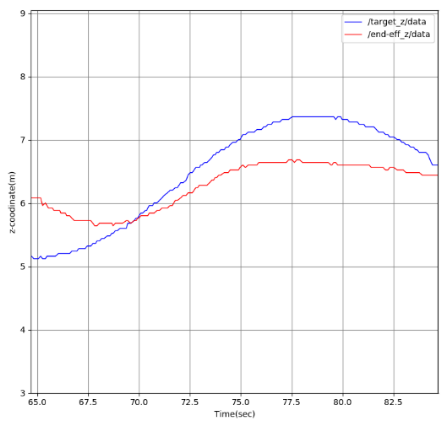
Three plots presented below is comparing x, y, z position of the robot end-effector with the x, y, z position of the target.



X-coordinate of the target and end-effector



Y-coordinate of the target and end-effector



Z-coordinate of the target and end-effector