Statistical Analysis of the First Assignment in RT1

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1 Introduction

This report presents a statistical analysis comparing the execution time of two different implementations of a program. The program simulates the placement of golden and silver tokens by a robot on two concentric circles. The silver tokens are positioned along the inner circle, while the golden tokens are placed on the outer circle. The goal of the simulation is to pair each silver token with its corresponding golden token.
The analysis focuses on evaluating the performance of the two program implementations in terms of execution time. To assess this, we employ the T-Test, a commonly used statistical test. In our evaluation, we vary the angle offset of the golden tokens on the outer circumference, which determines their arrangement along the circle. Both implementations involve moving 6 silver tokens from the inner circle to their respective golden tokens on the outer circle.

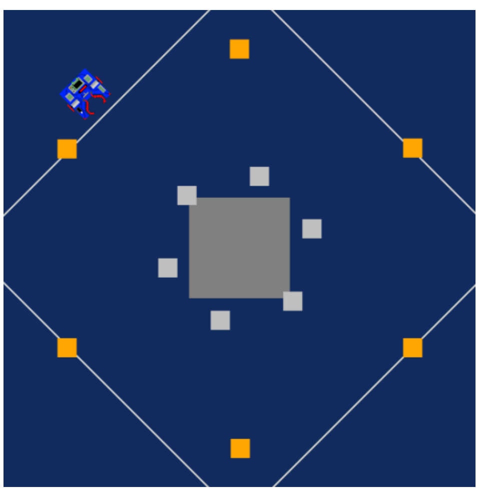

Figure 1: Initial configuration

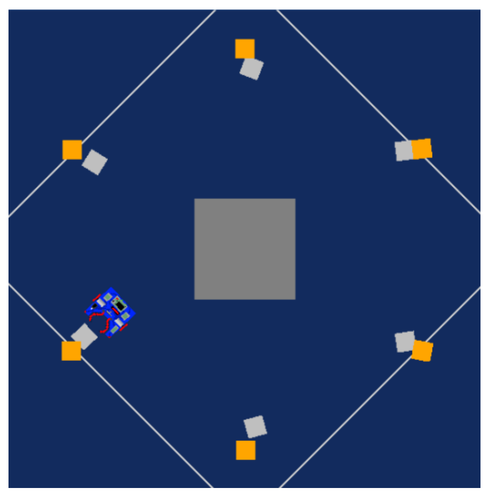

Figure 2: Final configuration

2 Hypotheses made

In the context of statistical analysis using the **T-Test**, it is crucial to formulate a research hypothesis that guides the investigation and allows for the evaluation of significant differences between two samples. This section of the report will provide a detailed description of the hypothesis formulation process, focusing on the null hypothesis, which asserts the absence of significant differences between the examined samples.
In my case, the hypotheses made are:

- H_0: $\mu_1 = \mu_2$, there are no substantial differences between the execution times of my implementation compared to the other one.
- H_1: $\mu_1 \neq \mu_2$, there are substantial differences between the execution times of my implementation compared to the other one.

The H_0 hypothesis above is the **Null Hypothesis**. In the context of statistical analysis using the **T-Test**, the null hypothesis states that there is no significant difference between the two considered samples. On the other hand, the H_1 hypothesis above is the **Alternative Hypothesis**, used as an alternative when the null hypothesis is not supported. Therefore, in this case, we will test the null hypothesis using the **Two-Sided T-Test**, which is used to determine if there are significant differences between the two samples without specifying the direction of these differences. If the null hypothesis is not supported, then the alternative hypothesis will be supported.

3 Data Collection

To conduct the statistical analysis, based on the hypotheses made, data collection was carried out by executing both program implementations 30 times. Consistent angle offset values were used for each execution, where the angle offset is utilized to vary the angular separation between the tokens. The execution time, measured in seconds, was recorded for each run.
The table below illustrates the angle offsets employed and their corresponding execution times, for both implementations:

N° execution	Angle offset	My Implementation Times	Other Implementation Times
1	-2,060424597	193,788177967	121,3933911
2	1,285208181	137,229959011	112,321095
3	2,012904269	144,727675915	133,918118
4	-2,677467967	133,202096939	124,3382721
5	1,800200872	138,712718010	134,3297551
6	1,280954395	136,746514082	117,894413
7	-1,191339204	136,207404137	138,9849582
8	-1,696353823	140,225404978	120,406168
9	-2,484088381	136,207803011	125,4198809
10	1,474160518	136,723016977	120,3659189
11	-2,93018648	156,299800873	127,3605928
12	-1,156747467	135,228929996	121,911978
13	-0,171594251	136,721837044	121,8825049
14	0,130268327	148,747511148	125,3756559
15	-0,36826719	135,236062050	117,3565621
16	-0,856482791	136,221502066	118,3809879
17	-1,4887443	137,229980946	123,85625
18	1,663733295	131,202381849	123,853457
19	0,490334709	141,253770113	134,9258821
20	-3,052077598	138,805794954	113,340066
21	-1,694369179	137,817510128	120,3308229
22	0,061899363	140,823760033	107,3766999
23	1,422226214	123,773239136	104,902539
24	2,612489442	129,802911997	179,451473
25	0,300823182	135,733092070	115,380271
26	2,826926283	140,250714064	121,890105
27	-2,468290453	156,445524931	160,520401
28	1,342997203	142,277531862	149,050493
29	-1,677858795	134,241894960	118,863039
30	1,500000000	131,734001160	128,913923

4 Formulas and Calculation Results

We will now proceed with the formulas used to calculate the necessary parameters for determining which of the two hypotheses is supported:

- N_1 represents the **number of observations in the first sample**.
- N_2 represents the **number of observations in the second sample**.
- $DoF = N_1 + N_2 - 2$ is the formula for calculating the **total degrees of freedom**. Specifically:
  - The **number of degrees of freedom** is equal to $30 + 30 - 2 = 58$.
- $\bar{X} = \frac{\Sigma^{i=1} i = nX_i}{n}$ is the formula for calculating the **mean** of each time column. Specifically:
  - The **mean of my implementation times** is equal to 140.121.
  - The **mean of the other implementation times** is equal to 126.143.
- $s = \sqrt{\frac{\Sigma i = 1^n (x_i - \mu)}{n}}$ is the formula for calculating the **standard deviation** of each time column. Specifically:
  - The **standard deviation of my implementation times** is equal to 11.958.
  - The **standard deviation of the other implementation times** is equal to 14.890.
- $\hat{\sigma}^2_{pooled} = \frac{(N_1-1)s_1^2 + (N_2-1)s_2^2}{N_1+N_2-2}$ is the formula for calculating the **pooled variance**. Specifically:
  - The **pooled variance** is equal to 182.348.
- $\hat{\sigma}\bar{x_1} - \bar{x_2} = \sqrt{\hat{\sigma}^2 pooled(\frac{1}{N_1} + \frac{1}{N_2})}$ is the formula for calculating the **pooled standard deviation of the difference of means**. Specifically:
  - The **pooled standard deviation of the difference of means** is equal to 3.487.
- $t_{\bar{x_1} - \bar{x_2}} = \frac{\bar{x}_1 - \bar{x}2}{\hat{\sigma}\bar{x_1} - \bar{x_2}}$ is the formula for calculating the **t-value**. Specifically:
  - The **t-value** is equal to 4.008.

5 Discussion of Results

The statistical analysis using the **t-test** provides important insights into the presence of significant differences between the two examined samples. By comparing the calculated **t-value** with the **critical value from the t-distribution table**, we can determine which hypothesis is supported. In this case, **the calculated t-value of 4.008 exceeds the critical value associated with the desired level of significance approximately 2** (as shown in the table below), indicating the presence of statistically significant differences between the samples.

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.61
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	1.44	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.93	4.318
13	1.35	1.771	2.16	2.65	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.12	2.583	2.921	3.686	4.015
17	1.333	1.74	2.11	2.567	2.898	3.646	3.965
18	1.33	1.734	2.101	2.552	2.878	3.61	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373
1000	1.282	1.646	1.962	2.33	2.581	3.098	3.3
Inf	1.282	1.645	1.96	2.326	2.576	3.091	3.291

Figure 3: t-distribution table used

This result supports the alternative hypothesis H_1, which suggests substantial differences in execution times between the two implementations of the program. It implies that there is evidence to suggest that my implementation performs differently than the other one in terms of execution time.

6 Conclusions

Based on the statistical analysis, we can draw meaningful conclusions regarding the performance comparison of the two implementations. The rejection of the null hypothesis H_0 suggests that **there are significant differences in execution times between my implementation and the other one**. This finding highlights the importance of considering different implementations when assessing program performance.

The results obtained indicate that my implementation has a higher average execution time (140.121 seconds) compared to the other implementation (126.143 seconds). Additionally, the standard deviation of my implementation (11.958) is smaller than the standard deviation of the other implementation (14.890), suggesting that my implementation has more consistent execution times.

In summary, **the statistical analysis provides evidence to support the alternative hypothesis, indicating significant differences in execution times between the two implementations**. These findings contribute to a better understanding of the performance characteristics of the program and can guide future development and optimization efforts.