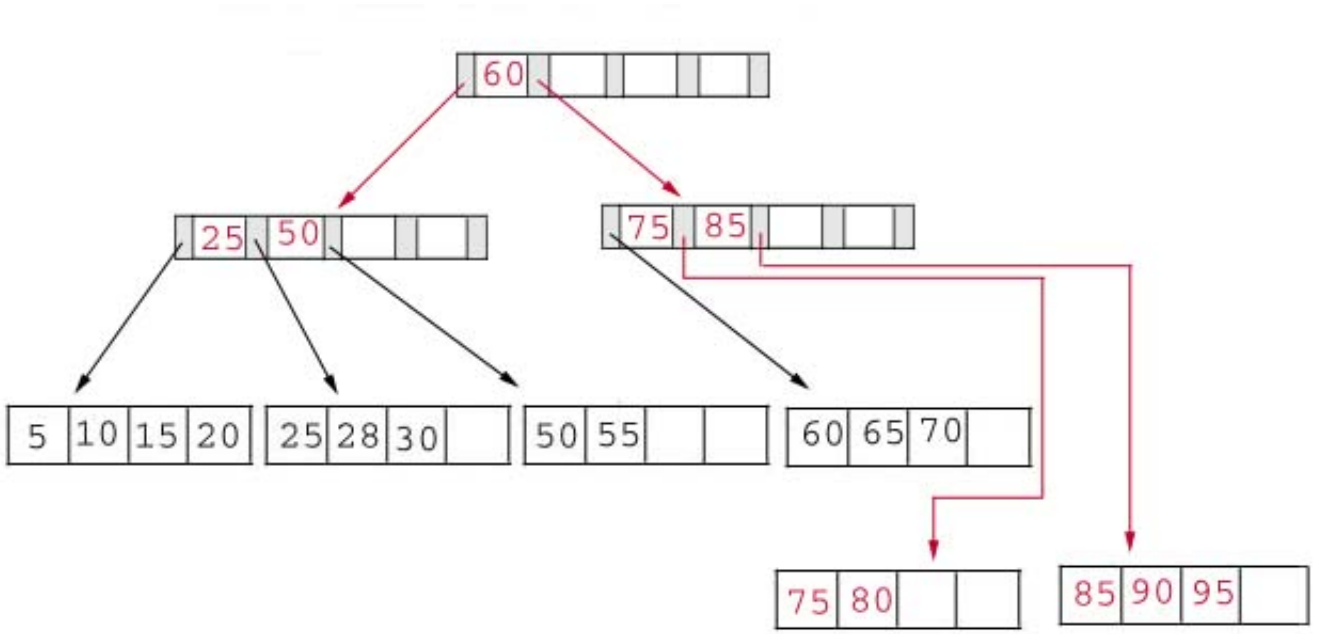
Data Structures and Algorithms

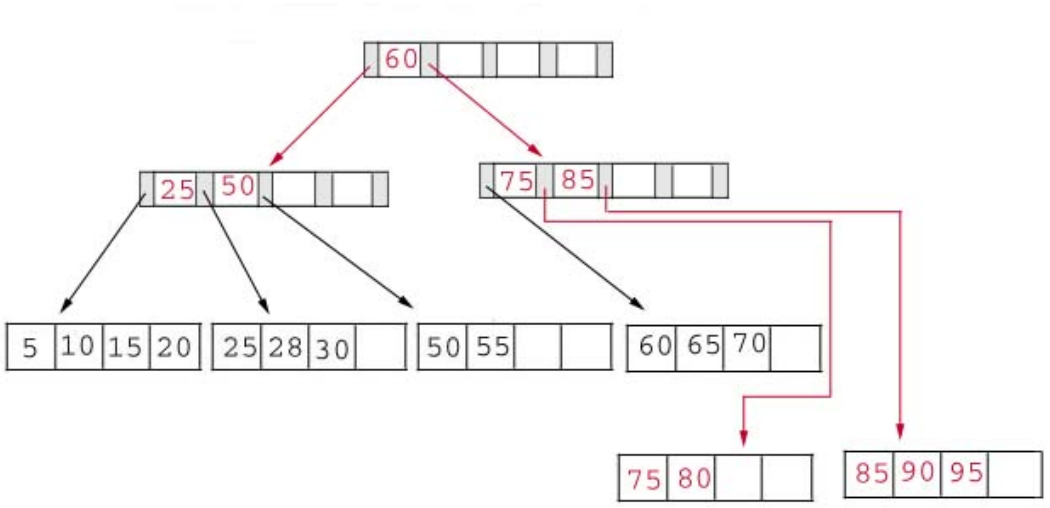
Lesson 7: *Sometimes It Pays Not to Be Binary*



B-Trees, Tries, Suffix Trees

Outline

1. **B-Trees**
2. Tries
3. Suffix Trees



B-Trees

- **B-trees** are balanced trees for fast search, finding successors and predecessors, insert, delete, maximum, minimum, etc.
- Not to be confused with binary trees
- They are designed to keep related data close to each other in (disk) memory to minimise retrieval time
- Important when working with large amount of data that is stored on secondary storage (e.g. disks)
- Used extensively in databases

When Big-O Doesn't Work

- An underlying assumption of Big-O is that all elementary operations take roughly the same amount of time
- This just isn't true of disk look-up
  - ⋆ The typical time of an elementary operation on a modern processor is 10^{-9} seconds
  - ⋆ But a typical hard disk might do 120 revolutions per second
  - ⋆ The typical time it takes to locate a record is around 10ms or 10^7 **times slower** than an elementary operation

Accessing Data from Disk

- When accessing data from disk minimising the number of disk accesses is critical for good performance
- In database applications we want to store data as large sets
- Storing data in binary trees is disastrous as we typically need around $\log_2(n)$ disk accesses before we locate our data
- It is not unusual in databases for $n = 10\,000\,000$ so that $\log_2(n) \approx 24$
- Using binary trees it would often take several seconds to find a record

Multiway-Trees

- To remedy this we can use M-way trees (i.e. trees where each non-leaf node has M children) so that the access time is

$$\log_M(n) = \frac{\log_2(n)}{\log_2(M)}$$

- In practice we might use $M \approx 200 \approx 2^8$ so we can reduce the depth of the tree by around a factor of 8
- The basic data structure for doing this is the B-tree
- There are many variants of B-trees, all trying to squeeze a bit more performance from the basic structure

$\mathbf{B}^+$ Tree

- A basic implementation would obey the following rules

1. The data items are stored at leaves
2. The non-leaf nodes store up to M-1 keys to guide the search: key i represents the smallest key in subtree $i + 1$
3. The root is either a leaf or has between 2 and M children
4. All non-leaf nodes except the root have between $\lceil M/2 \rceil$ and M children
5. All leaves except the root are at the same depth and have between $\lceil L/2 \rceil$ and L data entries

Choosing M and L

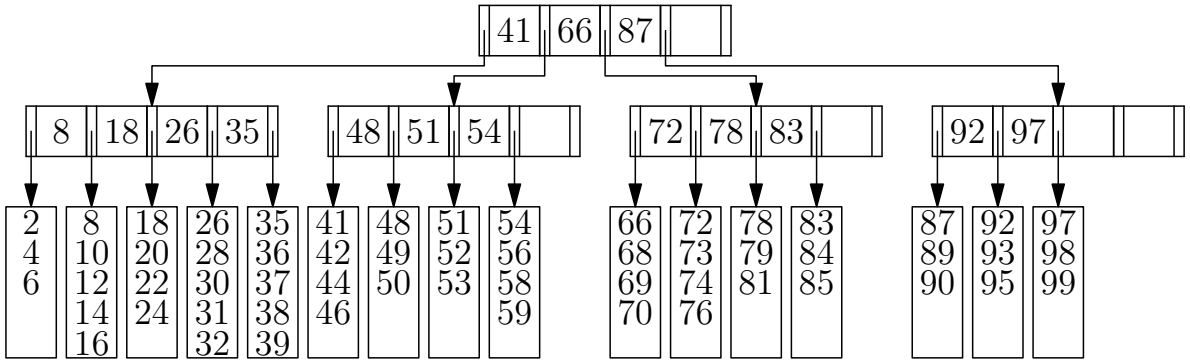
- The choice of M and L depends on the block size (the information read in one go from disk)
- It also depends on the type of data that is being stored (integer, reals, strings, etc.)
- M and L might be in the hundreds or thousands
- In the examples below we consider tiny $M = L = 5$ which is unrealistic, but drawable

B-Tree Example

- $M = 5$, $L = 5$

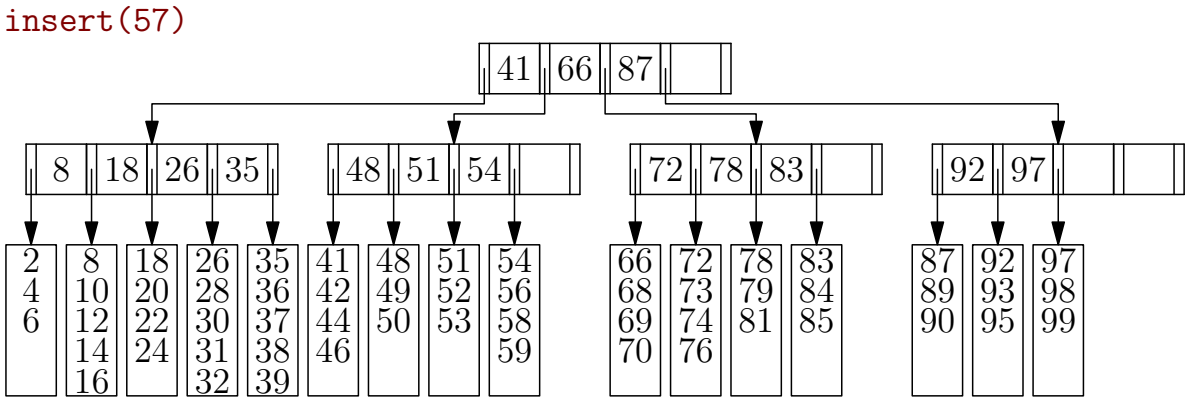
B-Tree Example

- $M = 5,\ L = 5$



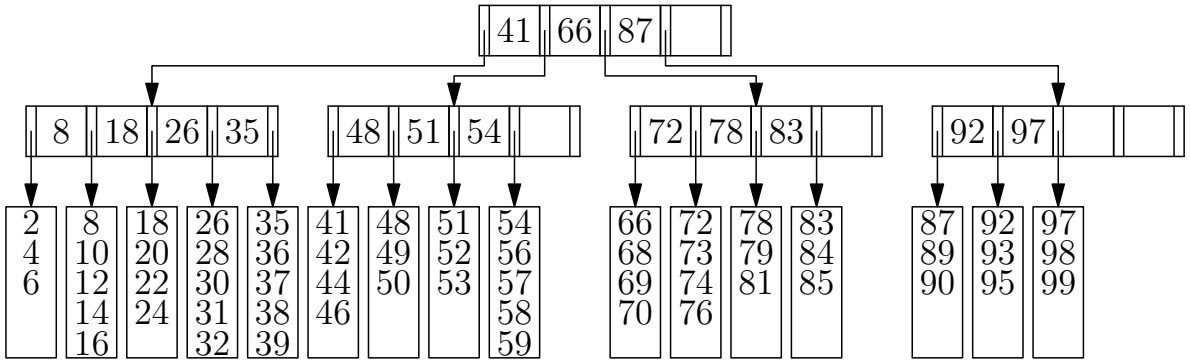
B-Tree Example

- $M = 5,\ L = 5$



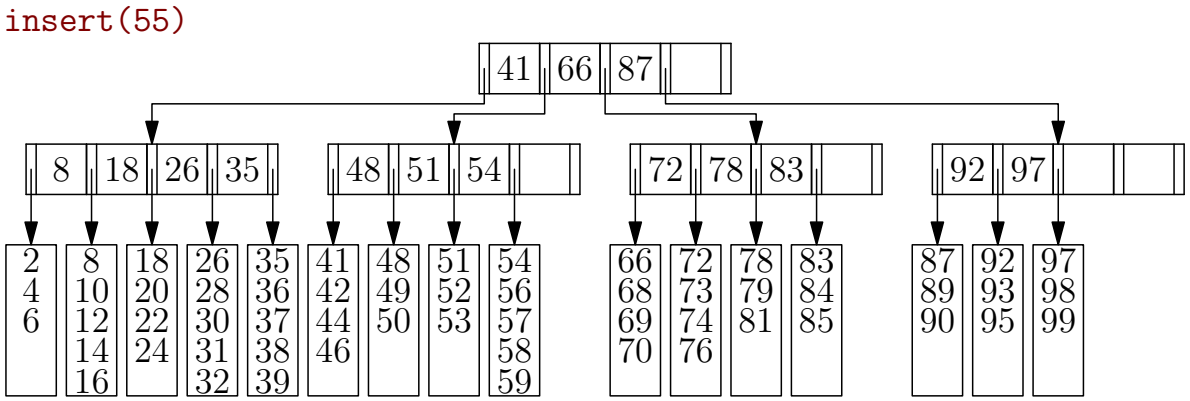
B-Tree Example

- $M = 5,\ L = 5$



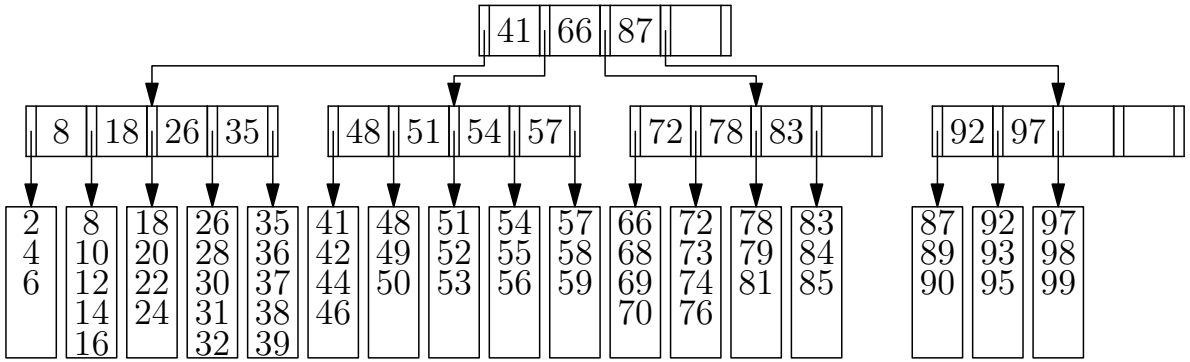
B-Tree Example

- $M = 5,\ L = 5$



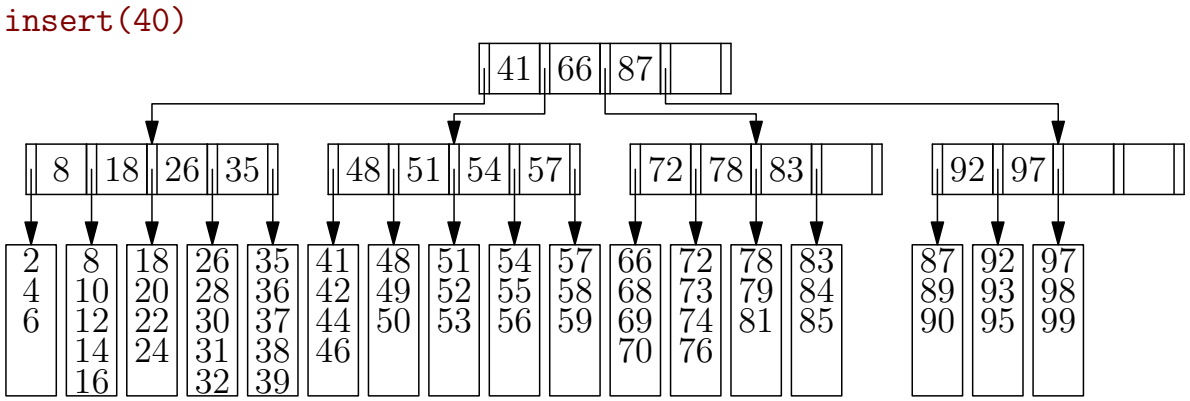
B-Tree Example

- $M = 5,\ L = 5$



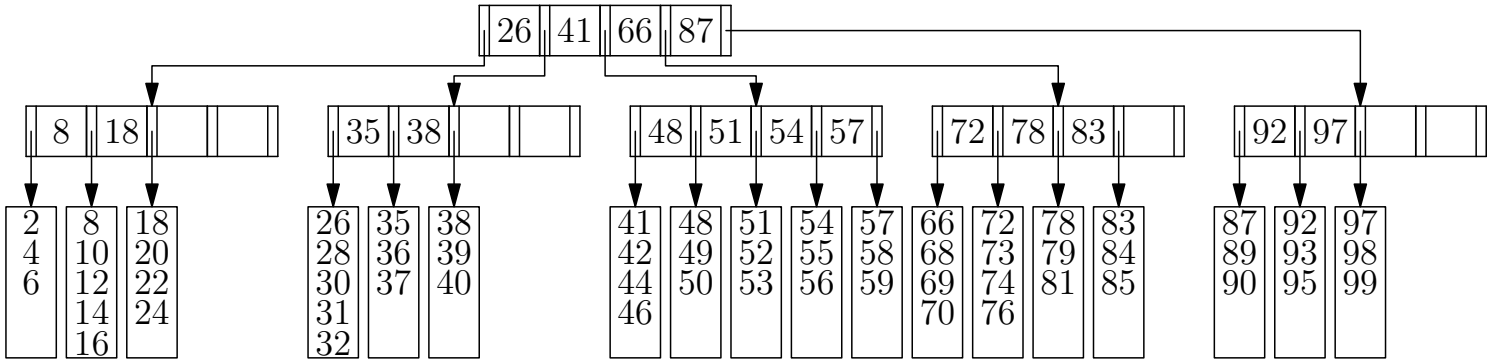
B-Tree Example

- $M = 5,\ L = 5$



B-Tree Example

- $M = 5,\ L = 5$



Other Changes

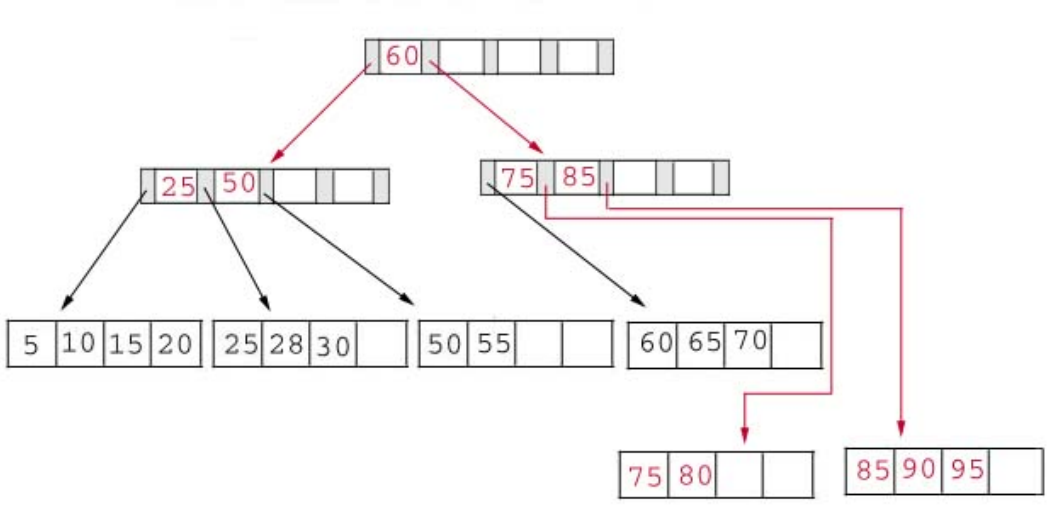
- If the root is full then it can be split into two and a new root created
- B-trees also have to allow the removal of records without losing their structure
- There are a number of strategies to further improve the performance (e.g. neighbouring nodes can adopt a child if the current node cannot expand any more)
- The actual implementation of B-trees is tricky because there are many special cases

B-Tree Summary

- B-trees are an important data structure for databases where reducing the number of disk searches is vital
- They tend to be much more complex than the other data structures we have seen
- The problem of disk access can be improved by replacing disk memory with solid-state drives (still slow compared to memory)
- For massive databases new data structures have been developed to allow faster (although less flexible) information access (e.g. NOSQL, MongoDB, Neo4j)

Outline

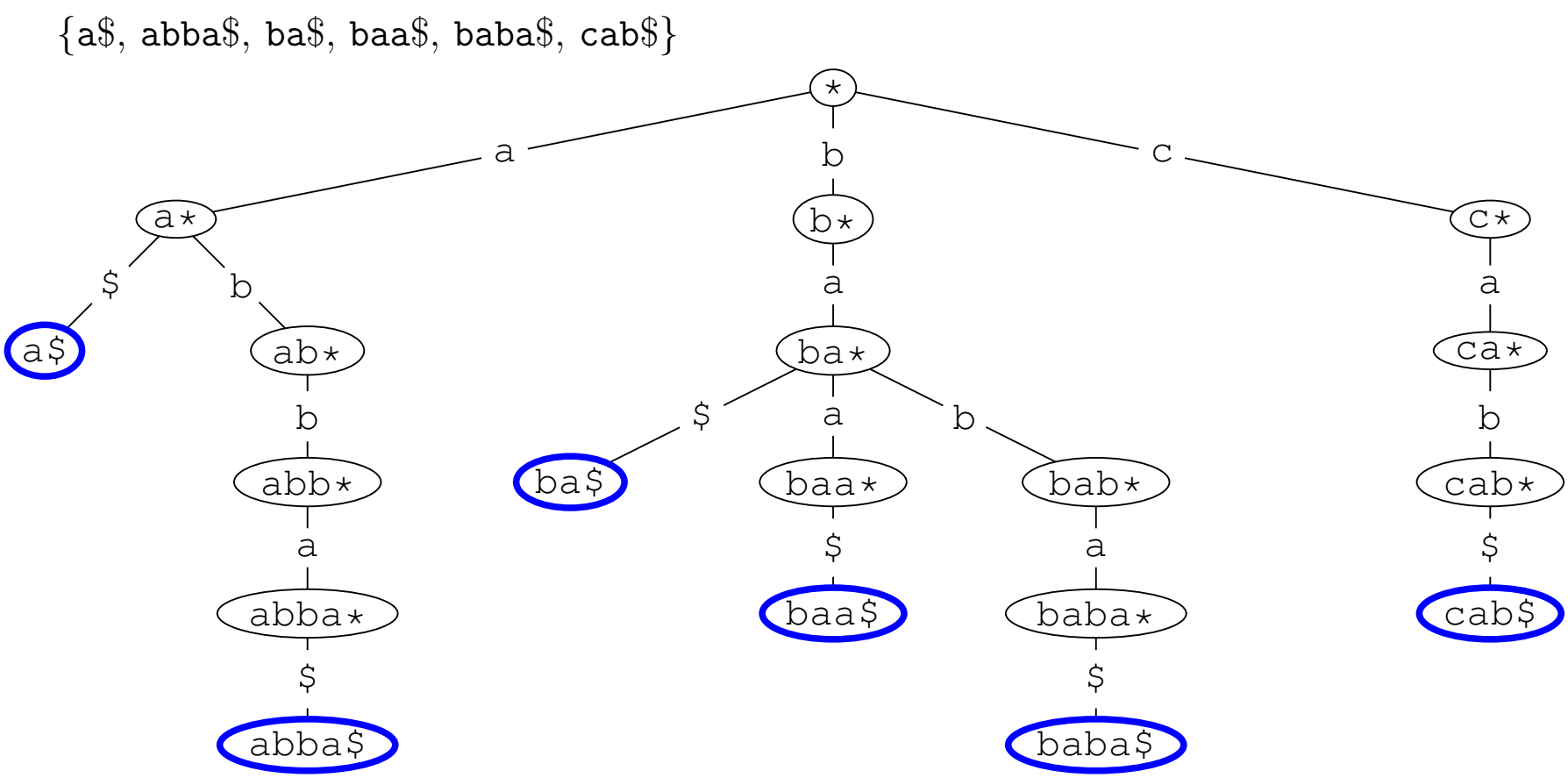
1. B-Trees
2. **Tries**
3. Suffix Trees



Tries

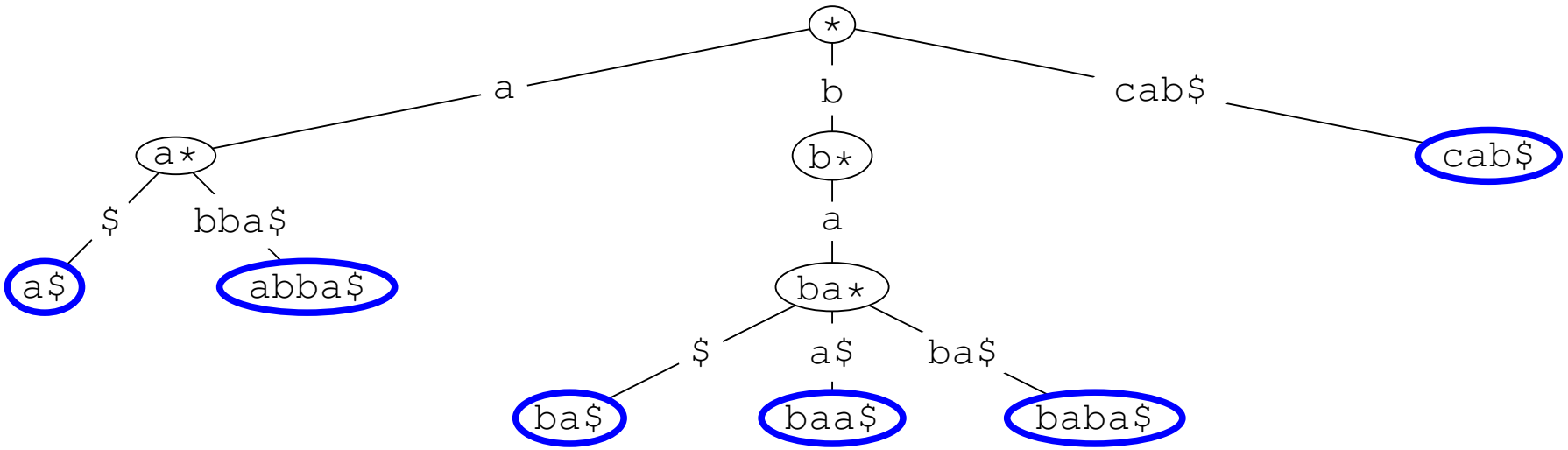
- A **Trie** (pron. 'try') or **digital tree** is a multiway tree often used for storing large sets of words
- They are trees with a possible branch for every letter of an alphabet
- Their names comes from the word $re\boldsymbol{trie}val$
- All words end with a special letter "$"
- Tries usually compactify the edges in the tree in order to remove
  - ⋆ **internal one-way branching** (in the internal nodes),
  - ⋆ **external one-way branching** (in the leaves)

Trie


{a$, abba$, ba$, baa$, baba$, cab$}
*
a
b
c
a*
b*
c*
$
b
a
a
a$
ab*
ba*
ca*
b
$
a
b
abb*
ba$
baa*
bab*
b
a
$
a
abba*
baa$
baba*
cab*
$
$
a
abba$
baba$
cab$
$

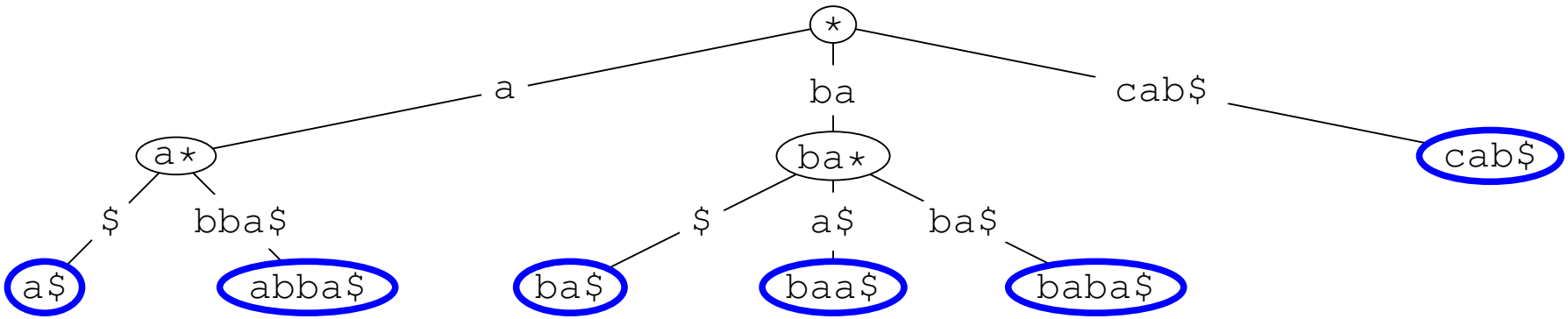
Trie

{a, abba, ba, baa, baba, cab}



Trie

{a, abba, ba, baa, baba, cab}

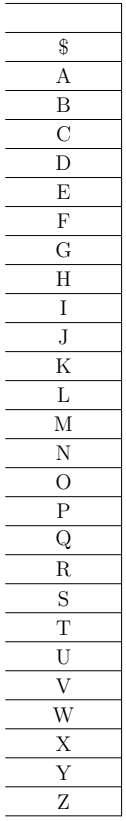


Uses of Tries

- Tries are yet another way of implementing sets
- They provide quick insertion, deletion and find
- Typically considerably quicker than binary trees and hash tables
- They are particularly good for spell checkers, completion algorithms, longest-prefix matching, hyphenation
- Each search finds the longest match between the words in the set and the query

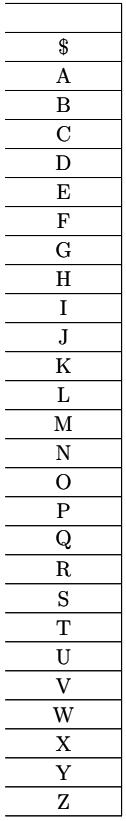
Trie for 31 Most Common English Words

Trie for 31 Most Common English Words



Trie for 31 Most Common English Words

```
add("THAT")
```



Trie for 31 Most Common English Words

```
add("THAT")
```

	0
$	
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT$
U	
V	
W	
X	
Y	
Z	

Trie for 31 Most Common English Words

```
add("IN")
```

	0
$	
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT$
U	
V	
W	
X	
Y	
Z	

Trie for 31 Most Common English Words

```
add("IN")
```

	0
$	
A	
B	
C	
D	
E	
F	
G	
H	
I	IN$
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT$
U	
V	
W	
X	
Y	
Z	

Trie for 31 Most Common English Words

```
add("IT")
```

	0
$	
A	
B	
C	
D	
E	
F	
G	
H	
I	IN$
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	THAT$
U	
V	
W	
X	
Y	
Z	

Trie for 31 Most Common English Words

```
add("IT")
```

	0	1
$		
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("I")
```

	0	1
$		
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("I")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("HAD")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H		
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("HAD")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("NOT")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N		IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("NOT")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N	NOT$	IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("WITH")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N	NOT$	IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W		
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("WITH")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N	NOT$	IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W	WITH$	
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("HIS")
```

	0	1
$		I$
A		
B		
C		
D		
E		
F		
G		
H	HAD$	
I	I* (1)	
J		
K		
L		
M		
N	NOT$	IN$
O		
P		
Q		
R		
S		
T	THAT$	IT$
U		
V		
W	WITH$	
X		
Y		
Z		

Trie for 31 Most Common English Words

```
add("HIS")
```

	0	1	2
$		I$	
A			HAD$
B			
C			
D			
E			
F			
G			
H	H* (2)		
I	I* (1)		HIS$
J			
K			
L			
M			
N	NOT$	IN$	
O			
P			
Q			
R			
S			
T	THAT$	IT$	
U			
V			
W	WITH$		
X			
Y			
Z			

Trie for 31 Most Common English Words

```
add("HAVE")
```

	0	1	2
$		I$	
A			HAD$
B			
C			
D			
E			
F			
G			
H	H* (2)		
I	I* (1)		HIS$
J			
K			
L			
M			
N	NOT$	IN$	
O			
P			
Q			
R			
S			
T	THAT$	IT$	
U			
V			
W	WITH$		
X			
Y			
Z			

Trie for 31 Most Common English Words

```
add("HAVE")
```

	0	1	2	3
$		I$		
A			HA* (3)	
B				
C				
D				HAD$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS$	
J				
K				
L				
M				
N	NOT$	IN$		
O				
P				
Q				
R				
S				
T	THAT$	IT$		
U				
V				HAVE$
W	WITH$			
X				
Y				
Z				

Trie for 31 Most Common English Words

```
add("A")
```

	0	1	2	3
$		I$		
A			HA* (3)	
B				
C				
D				HAD$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS$	
J				
K				
L				
M				
N	NOT$	IN$		
O				
P				
Q				
R				
S				
T	THAT$	IT$		
U				
V				HAVE$
W	WITH$			
X				
Y				
Z				

Trie for 31 Most Common English Words

```
add("A")
```

	0	1	2	3
$		I$		
A	A$		HA* (3)	
B				
C				
D				HAD$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS$	
J				
K				
L				
M				
N	NOT$	IN$		
O				
P				
Q				
R				
S				
T	THAT$	IT$		
U				
V				HAVE$
W	WITH$			
X				
Y				
Z				

Trie for 31 Most Common English Words

```
add("WHICH")
```

	0	1	2	3
$		I$		
A	A$		HA* (3)	
B				
C				
D				HAD$
E				
F				
G				
H	H* (2)			
I	I* (1)		HIS$	
J				
K				
L				
M				
N	NOT$	IN$		
O				
P				
Q				
R				
S				
T	THAT$	IT$		
U				
V				HAVE$
W	WITH$			
X				
Y				
Z				

Trie for 31 Most Common English Words

```
add("WHICH")
```

	0	1	2	3	4
$		I$			
A	A$		HA* (3)		
B					
C					
D				HAD$	
E					
F					
G					
H	H* (2)				WHICH$
I	I* (1)		HIS$		WITH$
J					
K					
L					
M					
N	NOT$	IN$			
O					
P					
Q					
R					
S					
T	THAT$	IT$			
U					
V				HAVE$	
W	W* (4)				
X					
Y					
Z					

Trie for 31 Most Common English Words

```
add("HER")
```

	0	1	2	3	4
$		I$			
A	A$		HA* (3)		
B					
C					
D				HAD$	
E					
F					
G					
H	H* (2)				WHICH$
I	I* (1)		HIS$		WITH$
J					
K					
L					
M					
N	NOT$	IN$			
O					
P					
Q					
R					
S					
T	THAT$	IT$			
U					
V				HAVE$	
W	W* (4)				
X					
Y					
Z					

Trie for 31 Most Common English Words

```
add("HER")
```

	0	1	2	3	4
$		I$			
A	A$		HA* (3)		
B					
C					
D				HAD$	
E			HER$		
F					
G					
H	H* (2)				WHICH$
I	I* (1)		HIS$		WITH$
J					
K					
L					
M					
N	NOT$	IN$			
O					
P					
Q					
R					
S					
T	THAT$	IT$			
U					
V				HAVE$	
W	W* (4)				
X					
Y					
Z					

Trie for 31 Most Common English Words

```
add("AT")
```

	0	1	2	3	4
$		I$			
A	A$		HA* (3)		
B					
C					
D				HAD$	
E			HER$		
F					
G					
H	H* (2)				WHICH$
I	I* (1)		HIS$		WITH$
J					
K					
L					
M					
N	NOT$	IN$			
O					
P					
Q					
R					
S					
T	THAT$	IT$			
U					
V				HAVE$	
W	W* (4)				
X					
Y					
Z					

Trie for 31 Most Common English Words

`add("AT")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				
O						
P						
Q						
R						
S						
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("IS")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				
O						
P						
Q						
R						
S						
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("IS")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("AND")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("AND")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				AND$
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("BUT")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B						
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				AND$
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("BUT")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B	BUT$					
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				AND$
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("THE")`

	0	1	2	3	4	5
$		I$				A$
A	A* (5)		HA* (3)			
B	BUT$					
C						
D				HAD$		
E			HER$			
F						
G						
H	H* (2)				WHICH$	
I	I* (1)		HIS$		WITH$	
J						
K						
L						
M						
N	NOT$	IN$				AND$
O						
P						
Q						
R						
S		IS$				
T	THAT$	IT$				AT$
U						
V				HAVE$		
W	W* (4)					
X						
Y						
Z						

Trie for 31 Most Common English Words

`add("THE")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F							
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O							
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("FOR")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F							
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O							
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("FOR")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F	FOR$						
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O							
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("ON")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F	FOR$						
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O							
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("ON")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F	FOR$						
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O	ON$						
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("HE")`

	0	1	2	3	4	5	6
$		I$				A$	
A	A* (5)		HA* (3)				THAT$
B	BUT$						
C							
D				HAD$			
E			HER$				THE$
F	FOR$						
G							
H	H* (2)				WHICH$		
I	I* (1)		HIS$		WITH$		
J							
K							
L							
M							
N	NOT$	IN$				AND$	
O	ON$						
P							
Q							
R							
S		IS$					
T	TH* (6)	IT$				AT$	
U							
V				HAVE$			
W	W* (4)						
X							
Y							
Z							

Trie for 31 Most Common English Words

`add("HE")`

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)				THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R								HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

`add("WAS")`

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)				THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R								HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

`add("WAS")`

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)		WAS$		THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R								HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

`add("ARE")`

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)		WAS$		THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R								HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

`add("ARE")`

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)		WAS$		THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R						ARE$		HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

`add("BY")`

	0	1	2	3	4	5	6	7
$		I$				A$		HE$
A	A* (5)		HA* (3)		WAS$		THAT$	
B	BUT$							
C								
D				HAD$				
E			HE* (7)				THE$	
F	FOR$							
G								
H	H* (2)				WHICH$			
I	I* (1)		HIS$		WITH$			
J								
K								
L								
M								
N	NOT$	IN$				AND$		
O	ON$							
P								
Q								
R						ARE$		HER$
S		IS$						
T	TH* (6)	IT$				AT$		
U								
V				HAVE$				
W	W* (4)							
X								
Y								
Z								

Trie for 31 Most Common English Words

`add("BY")`

	0	1	2	3	4	5	6	7	8
$		I$				A$		HE$	
A	A* (5)		HA* (3)		WAS$		THAT$		
B	B* (8)								
C									
D				HAD$					
E			HE* (7)				THE$		
F	FOR$								
G									
H	H* (2)				WHICH$				
I	I* (1)		HIS$		WITH$				
J									
K									
L									
M									
N	NOT$	IN$				AND$			
O	ON$								
P									
Q									
R						ARE$		HER$	
S		IS$							
T	TH* (6)	IT$				AT$			
U									BUT$
V				HAVE$					
W	W* (4)								
X									
Y									BY$
Z									

Trie for 31 Most Common English Words

`add("OR")`

	0	1	2	3	4	5	6	7	8
$		I$				A$		HE$	
A	A* (5)		HA* (3)		WAS$		THAT$		
B	B* (8)								
C									
D				HAD$					
E			HE* (7)				THE$		
F	FOR$								
G									
H	H* (2)				WHICH$				
I	I* (1)		HIS$		WITH$				
J									
K									
L									
M									
N	NOT$	IN$				AND$			
O	ON$								
P									
Q									
R						ARE$		HER$	
S		IS$							
T	TH* (6)	IT$				AT$			
U									BUT$
V				HAVE$					
W	W* (4)								
X									
Y									BY$
Z									

Trie for 31 Most Common English Words

`add("OR")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$					
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$								
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

`add("AS")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$					
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$								
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

`add("AS")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$					
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

`add("THIS")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$					
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

`add("THIS")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

`add("YOU")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y									BY$	
Z										

Trie for 31 Most Common English Words

`add("YOU")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y	YOU$								BY$	
Z										

Trie for 31 Most Common English Words

`add("BE")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$			
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y	YOU$								BY$	
Z										

Trie for 31 Most Common English Words

`add("BE")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$		BE$	
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y	YOU$								BY$	
Z										

Trie for 31 Most Common English Words

`add("FROM")`

	0	1	2	3	4	5	6	7	8	9
$		I$				A$		HE$		
A	A* (5)		HA* (3)		WAS$		THAT$			
B	B* (8)									
C										
D				HAD$						
E			HE* (7)				THE$		BE$	
F	FOR$									
G										
H	H* (2)				WHICH$					
I	I* (1)		HIS$		WITH$		THIS$			
J										
K										
L										
M										
N	NOT$	IN$				AND$				ON$
O	O* (9)									
P										
Q										
R						ARE$		HER$		OR$
S		IS$				AS$				
T	TH* (6)	IT$				AT$				
U									BUT$	
V				HAVE$						
W	W* (4)									
X										
Y	YOU$								BY$	
Z										

Trie for 31 Most Common English Words

`add("FROM")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)										
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`add("OF")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)										
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Trie for 31 Most Common English Words

`add("OF")`

	0	1	2	3	4	5	6	7	8	9	10
$		I$				A$		HE$			
A	A* (5)		HA* (3)		WAS$		THAT$				
B	B* (8)										
C											
D				HAD$							
E			HE* (7)				THE$		BE$		
F	F* (10)									OF$	
G											
H	H* (2)				WHICH$						
I	I* (1)		HIS$		WITH$		THIS$				
J											
K											
L											
M											
N	NOT$	IN$				AND$				ON$	
O	O* (9)										FOR$
P											
Q											
R						ARE$		HER$		OR$	FROM$
S		IS$				AS$					
T	TH* (6)	IT$				AT$					
U									BUT$		
V				HAVE$							
W	W* (4)										
X											
Y	YOU$								BY$		
Z											

Disadvantage of Tries

- Tries typically waste large amounts of memory
  - ⋆ Often tries are used for the first few layers, while lower levels use a less memory intensive data structure
- These days memory is less of a problem so tries are acceptable for some applications

Binary Tries

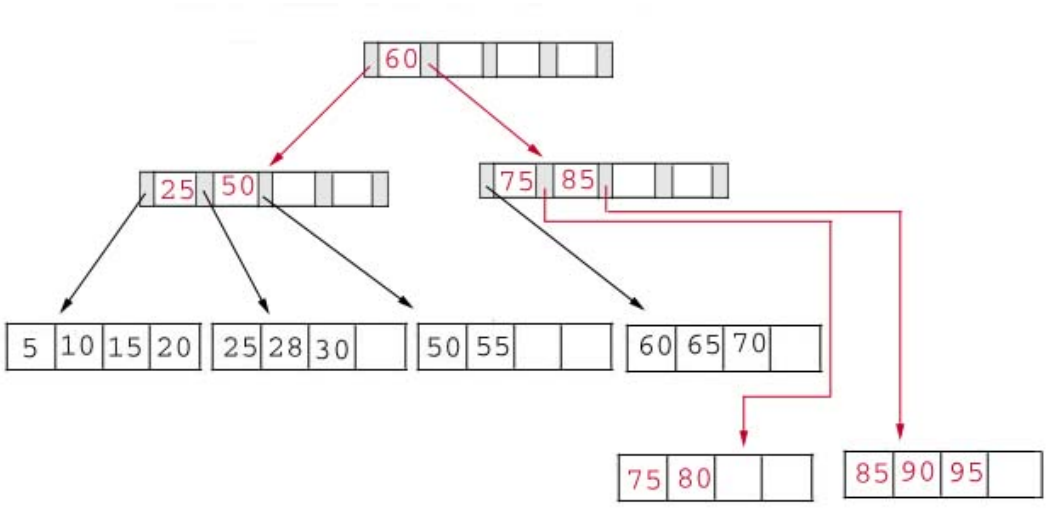
- One extreme (though not uncommon) solution to address memory issues is to build a bit-level trie so the resulting data structure is a binary tree
- It differs from a binary search tree in that the decision to go left or right depends on the current bit
- Although you lose the advantage of a multiway tree (of reducing the depth), it does find the longest match and it speeds up finds which fail

Why Tries?

- Tries are a classic example of a trade-off between memory and time complexity
- Tries are slightly specialist and tend to get used in very particular applications
  - ⋆ Finding longest matches
  - ⋆ Completion algorithms, spell checking, etc.
- A basic trie is not too complicated to implement, however . . .
- . . . more complex implementations are needed to overcome the difficulty of wasting too much memory

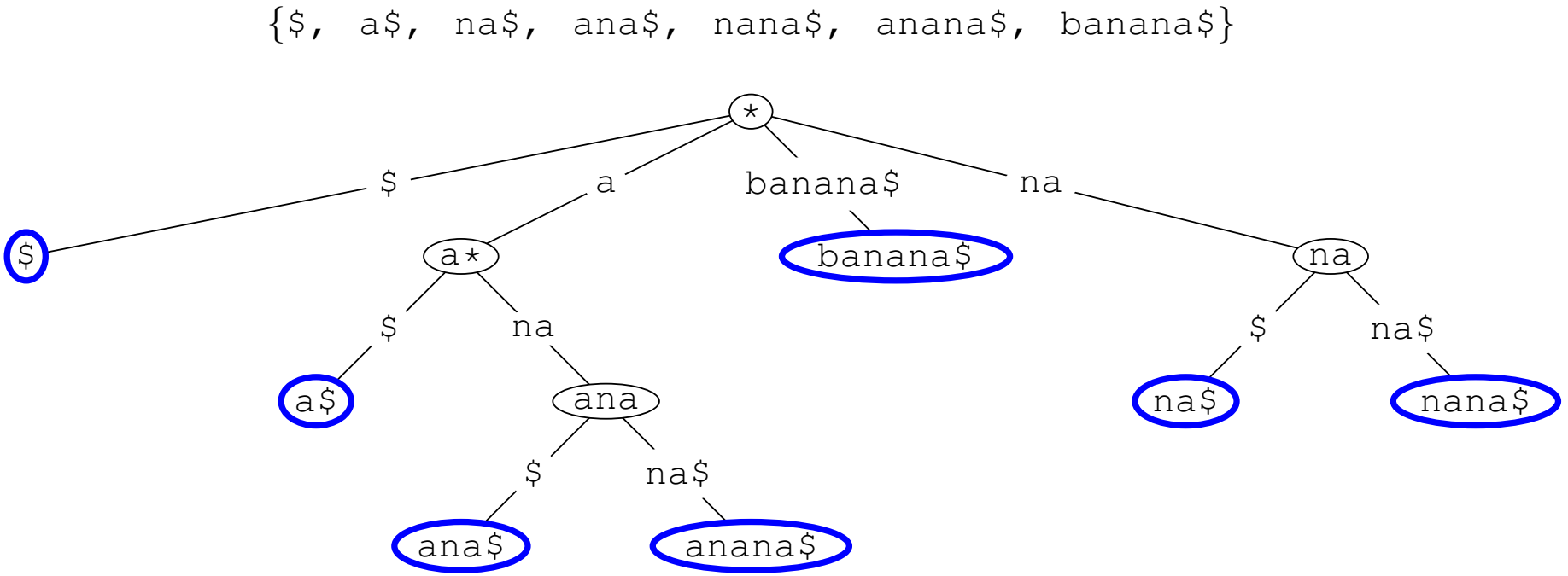
Outline

1. B-Trees
2. Tries
3. **Suffix Trees**



Suffix Trees

- A suffix tree is a trie of all suffixes of a string
- E.g. banana

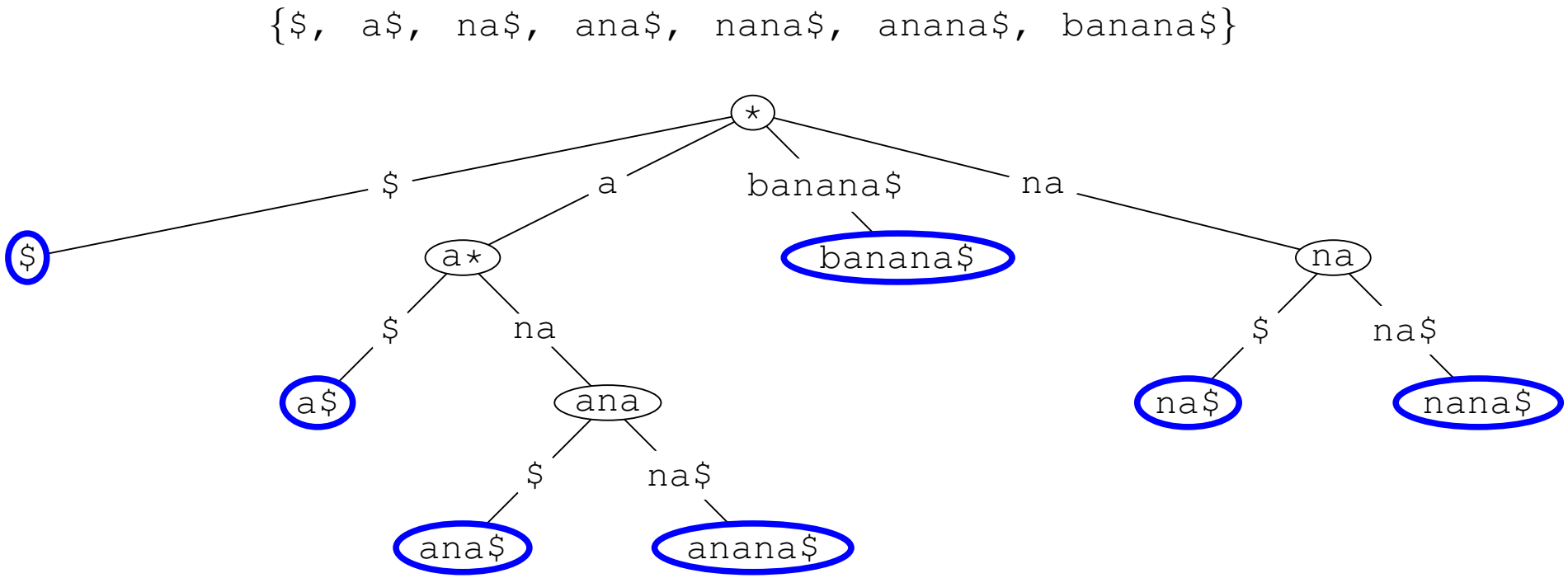


Importance of Suffix Tree

- Suffix trees (and the related suffix arrays) are relatively new data structures
- They are very important for string-based algorithms
- The classic application is finding a match for a query string, Q, in a text, T

String Matching

- To find a match of a query string, Q, in a text, T, we can first construct the suffix tree of the string T; we then simply look up the query, Q, using the trie



Complexity of Suffix Trees

- Using a regular trie for a suffix tree would typically use far too much memory to be useful
- However, by using pointers to the original text it is possible to build a suffix tree using $O(n)$ memory where n is the length of the text
- Furthermore (and rather incredibly) there is a linear time ($O(n)$) algorithm to construct the trie
- The algorithm is, however, not trivial to understand

Uses of Suffix Trees

- Suffix trees are efficient whenever it is likely that you will do multiple searches
- Exact word matching is in itself a very important application
- Suffix trees in combination with dynamic programming (which we will eventually get to) can be used to do inexact matching (finding the match with the smallest edit distance)
- Suffix trees get used in bioinformatics, advanced machine learning algorithms, . . .

Lessons

- Multiway trees can considerably speed up search over binary trees
- They are very important in some specialised applications (e.g. databases, spell-checking, completion algorithms, word matching)
- They are not as general purpose as binary search trees and are more complicated to implement
- But they can give the best performance – sometimes performance matters enough to make it worthwhile implementing multiway trees