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10. Modelling Classes and Associations

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Objectives

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- Relations/Functions in Event-B
- Modelling classes and associations

Properties of Relations

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- Totality: Every element of S has *at least one relationship*
- Surjectivity: Every element of T has *at least one relationship*.
- Functional: Every element of S has *at most one relationship*
- Injectivity: Every element of T has *at most one relationship*

Different Types of Relations and Functions

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	None	Total	Surjective	Total $\wedge$ Surjective
None	$S \leftrightarrow T$	$S \nleftrightarrow T$	$S \twoheadleftarrow T$	$S \nleftrightarrow\!\!\!\!\twoheadleftarrow T$
Functional	$S \nrightarrow T$	$S \rightarrow T$	$S \nrightarrow\!\!\!\!\twoheadrightarrow T$	$S \twoheadrightarrow T$
Injective	–	–	–	–
Functional $\wedge$ Injective	$S \rightarrowtail T$	$S \rightarrowtail T$	–	$S \rightarrowtail\!\!\!\!\twoheadrightarrow T$

Table: Event-B relations

Diagram of building access

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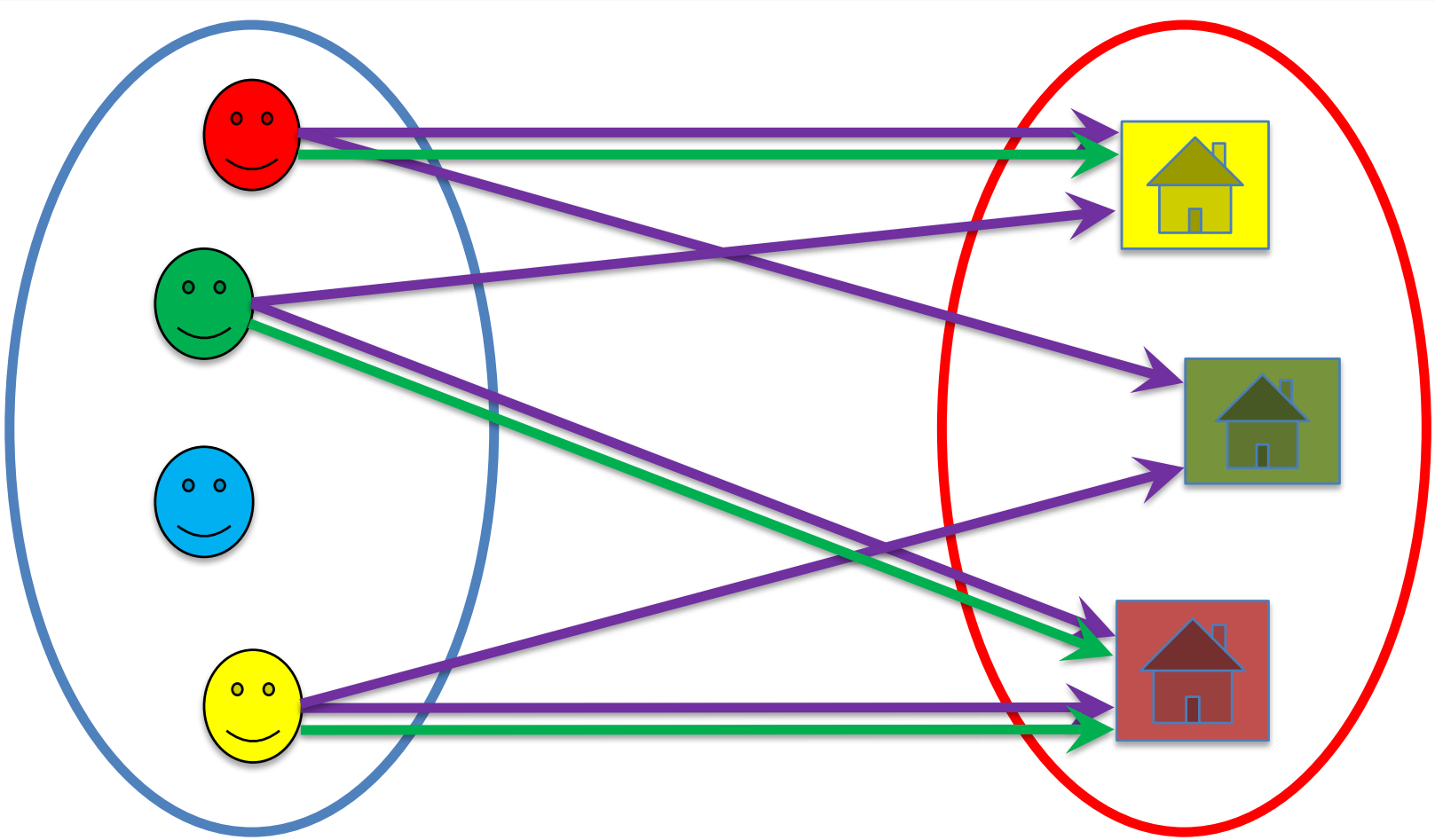

Location $\subseteq$ Permission

Class diagram abstraction

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```
1 invariants
2   @typeof−permission: permission ∈ USER ↔ BUILDING // many−to−many
3   @typeof−location: location ∈ USER ⇸ BUILDING
```

Classes and Associations

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Consider our model of a birthday book:

1 **variables**
2 person
3 birthday
4 **invariants**
5 @typeof−person: person $\subseteq$ PERSON
6 @typeof−birthday: birthday $\in$ person $\rightarrow$ DATE

Representing birthday as a simple class diagram:

Classes and Associations (2/2)

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Consider our model of a birthday book:

```
1 variables
2   person
3   birthday
4 invariants
5   @typeof−person: person ⊆ PERSON
6   @typeof−birthday: birthday ∈ person → DATE
```

Making the variable set explicit

Multiple Associations

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- Suppose we want to model a person's address as well.
- Multiple attributes of an entity (e.g., person) are modelled as separate total functions on the same domain:

---

1 **variables**
2 person
3 birthday
4 address
5 **invariants**
6 @typeof−person: $person \subseteq PERSON$
7 @typeof−birthday: $birthday \in person \rightarrow DATE$
8 @typeof−address: $address \in person \rightarrow ADDRESS$

- The common domain for both functions means every element of the set person, has both a birthday and an address.

Class diagram for the birthday/address book

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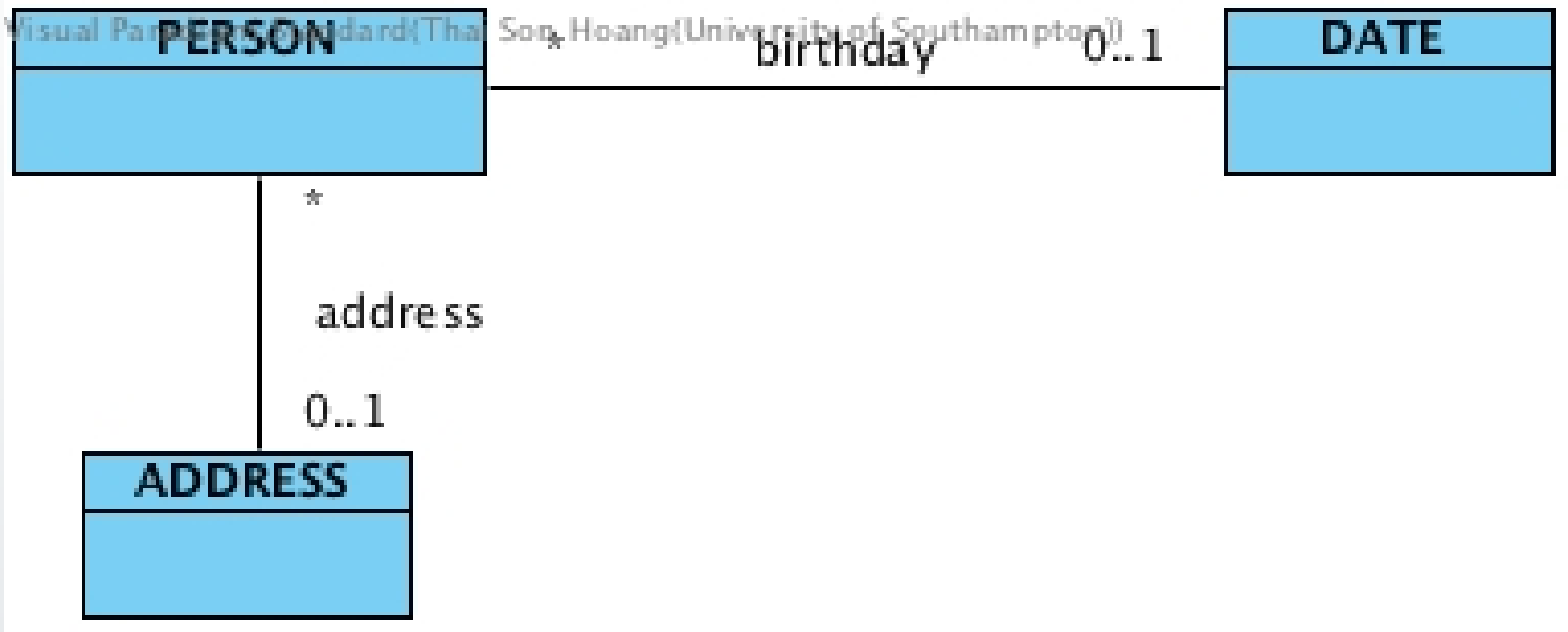

Class diagram for the birthday/address book

Making the variable set explicit

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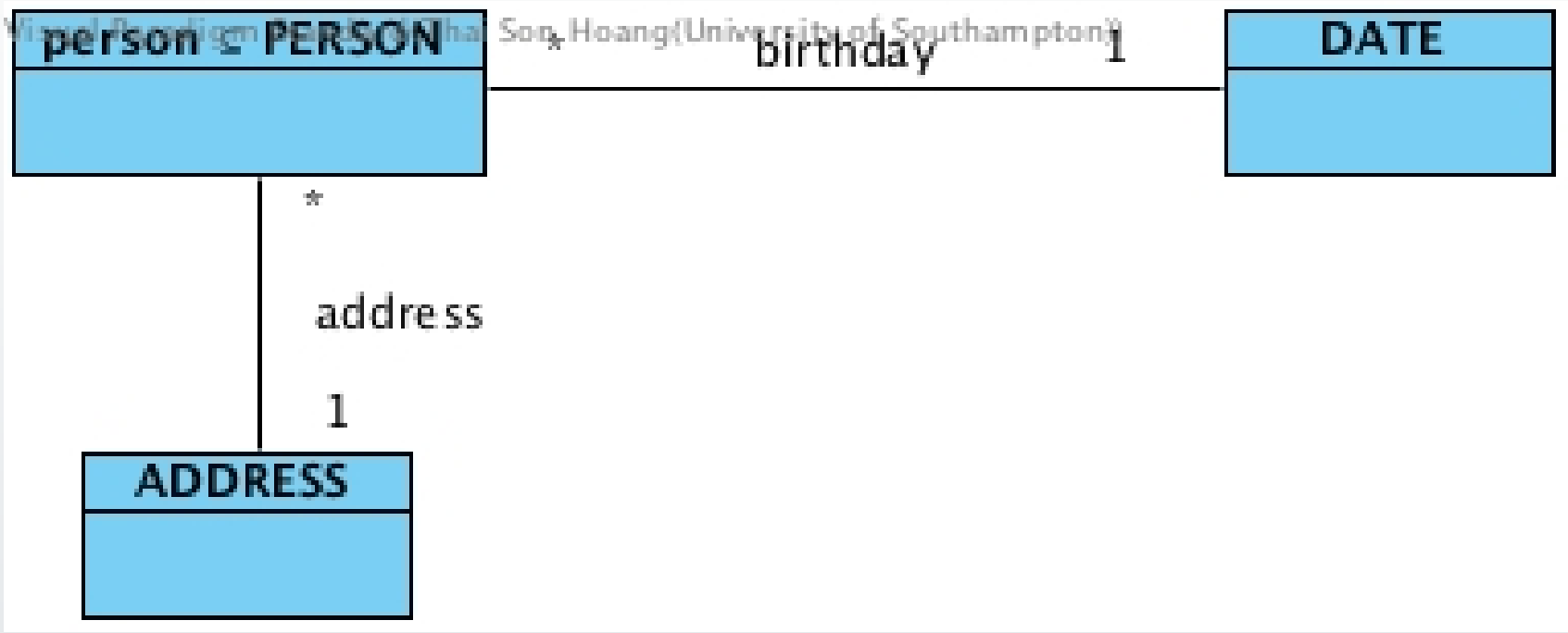

Secure database example

We consider a secure database. Each object in the database has a data component.

Each object has a classification between 1 and 10.

Users of the system have a clearance level between 1 and 10.

Users can only read and write objects whose classification is no greater than the user's clearance level.

What are the *entities*, *associations*, *events*?

Class diagram for secure database

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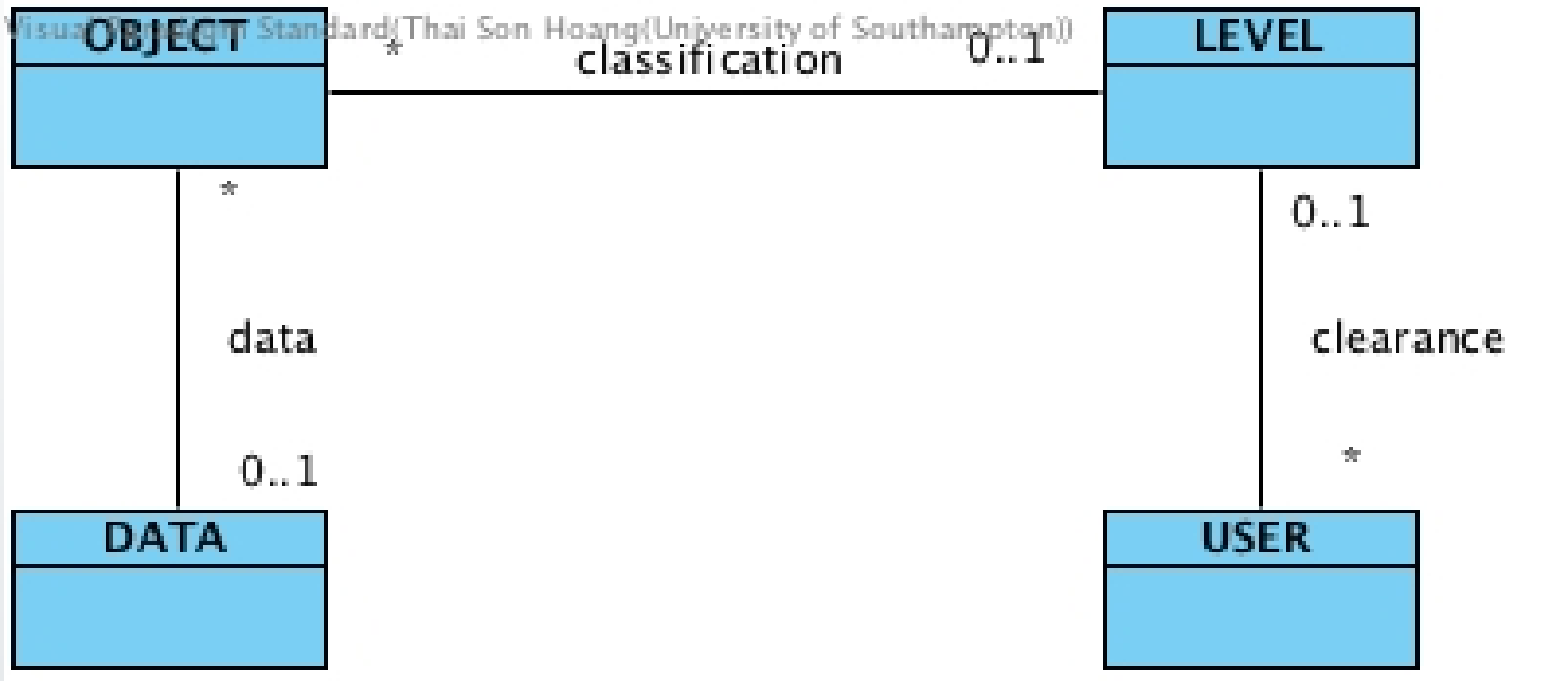

Making variable set explicit

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Introduce set variables:
- *object* is the variable set of objects managed by the system
- *user* is the variable set of users managed by the system

Primary and secondary carrier sets

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Note that we do **not** introduce set variables for *LEVEL* nor *DATA*

OBJECT and *USER* are *primary* carrier sets: the systems should enable management of these entities (creation, modification of attributes, deletion)

LEVEL and *DATA* are *secondary* carrier sets: serve as attributes of primary carrier sets

Types and variables


```
1  sets OBJECT DATA USER
2  constants LEVEL
3  axioms
4   @def−LEVEL: LEVEL = 1 .. 10
5
6  variables object user data class clear
7  invariants
8   @typeof−object: object ⊆ OBJECT
9   @typeof−user: user ⊆ USER
10  @typeof−data: data ∈ object → DATA
11  @typeof−class: class ∈ object → LEVEL
12  @typeof−clear: clear ∈ user → LEVEL
```

The invariant $data \in object \rightarrow DATA$ means that $data(o)$ is well-defined whenever $o \in object$. Why is this important?

```
1  INITIALISATION
2  then
3   object, user, data, class, clear := ∅, ∅, ∅, ∅, ∅
4  end
```

Adding users


```
1  event AddUser
2  any u c where
3   @grd1: u ∉ user
4   @grd2: c ∈ LEVEL
5  then
6   @act1: user := user ∪ {u}
7   @act2: clear(u) := c
8  end
```

The new user must not already exist.
We need to provide the initial clearance level for the new user.

Adding objects


```
1  event AddObject
2  any o d c where
3   @grd1: o ∉ object
4   @grd2: d ∈ DATA
5   @grd3: c ∈ LEVEL
6  then
7   @act1: object := object ∪ {o}
8   @act2: data(o) := d
9   @act3: class(o) := c
10 end
```

The new object must not already exist.
We need to provide the initial classification level and data value for the new object.

Reading objects

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```
1  event Read
2  any u o result where
3   @grd1: u ∈ user // The user must exist
4   @grd2: o ∈ object // The object must exist
5   @grd3: clear(u) ≥ class(o) // The clearance must be OK
6   @grd4: result = data(o) // The data associated with the object
7  end
```

Writing objects


```
1  event Write
2  any u o d where
3   @grd1: u ∈ user
4   @grd2: o ∈ object
5   @grd3: clear(u) ≥ class(o)
6   @grd4: d ∈ DATA
7  then
8   @act1: data(o) := d
9  end
```

The write operation overwrites the data value associate with the object with a new value.

Changing classification and clearance levels

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```
1 event ChangeClass
2 any o c where
3   @grd1: o ∈ object
4   @grd2: c ∈ LEVEL
5 then
6   @act1: class(o) := c
7 end
```

```
1 event ChangeClear
2 any u c where
3   @grd1: u ∈ user
4   @grd2: c ∈ LEVEL
5 then
6   @act1: clear(u) := c
7 end
```

Making classification changes more secure

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Include constraints on the user who is changing the object classification.

```
1  event ChangeClass
2  any o c u where
3   @grd1: o ∈ object
4   @grd2: c ∈ LEVEL
5   @grd3: u ∈ user
6   @grd4: clear(u) ≥ class(o)
7   @grd5: clear(u) ≥ c
8  then
9   @act1: class(o) := c
10 end
```

Making clearance changes more secure

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Include constraints on the user who is changing the object classification.

```
1  event ChangeClear
2  any u c a where
3   @grd1: u ∈ user
4   @grd2: c ∈ LEVEL
5   @grd3: a ∈ user
6   @grd4: clear(a) ≥ clear (u)
7   @grd5: clear(a) ≥ c
8  then
9   @act1: clear(u) := c
10 end
```

Removing users and objects

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```
1 event RemoveUser
2 any u where
3  @grd1: u ∈ user
4 then
5  @act1: user := user \ {u}
6  @act2: clear := {u} ⩤ clear
7 end
```

```
1 event RemoveObject
2 any o where
3  @grd1: o ∈ object
4 then
5  @act1: object := object \ {o}
6  @act2: class := {o} ⩤ class
7  @act3: data := {o} ⩤ data
8 end
```

Adding object ownership

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Extend the database specification so that each object has an owner.

The clearance associated with that owner must be at least as high as the classification of the object.

Only the owner of an object is allowed to delete it.

A user's clearance level can only be modified to a new level by another user whose clearance level is at least as high as the new clearance level.

What additional variables are required?

What events are affected?

Class diagram with ownership

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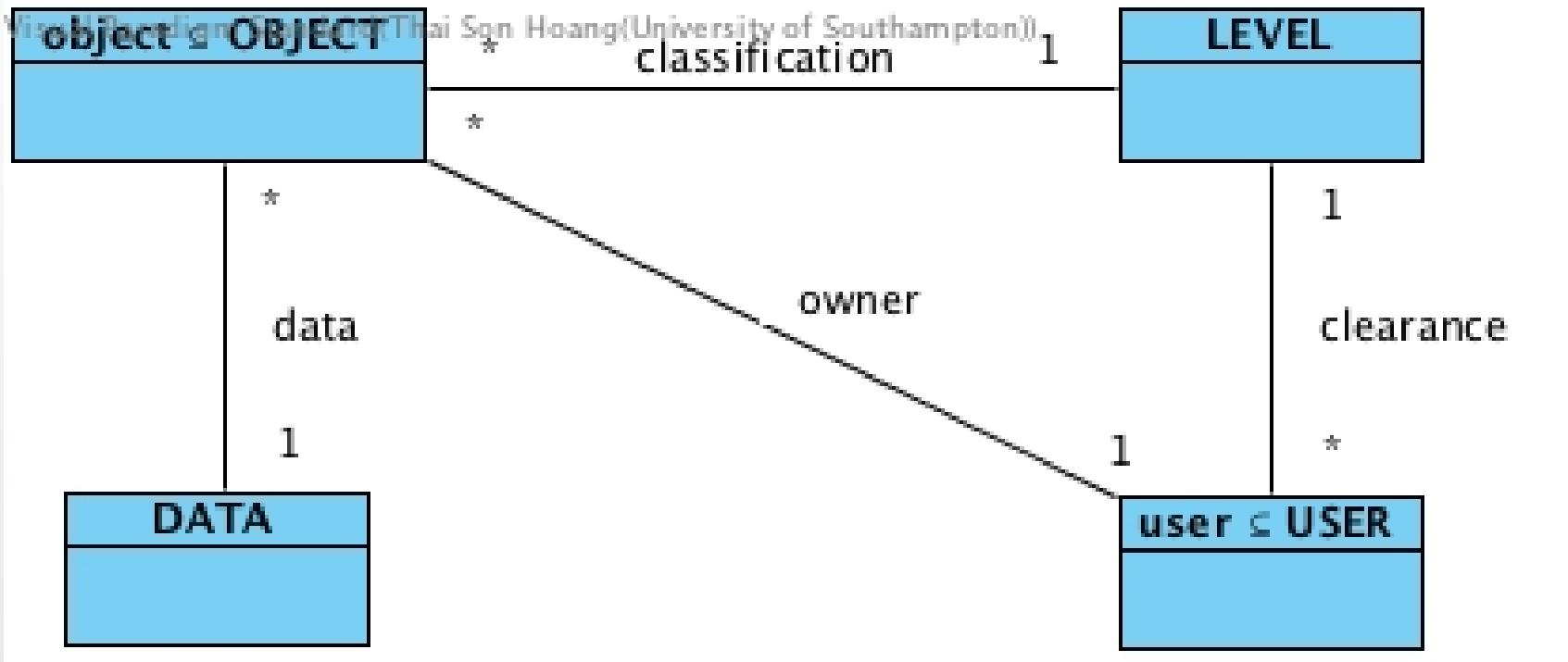

Total

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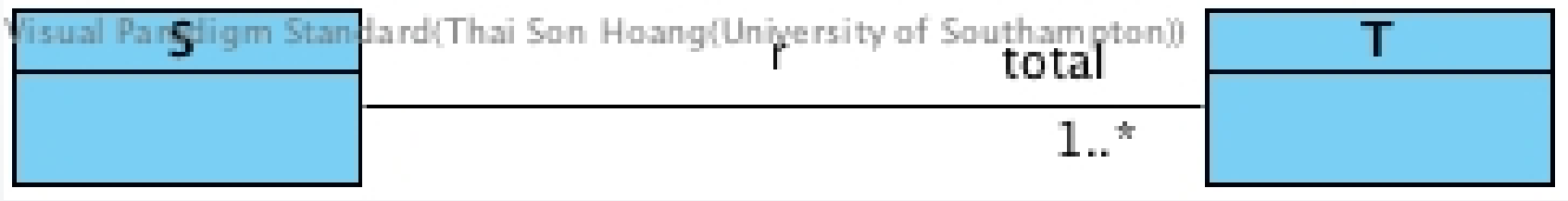

(We consider the relation from S to T)

Functional

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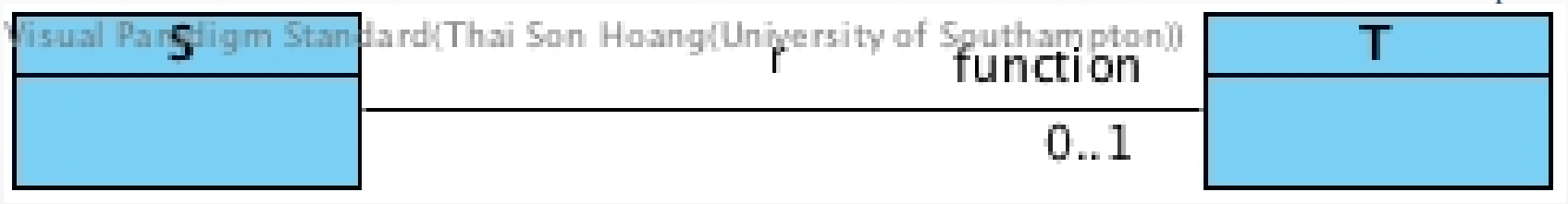

(We consider the relation from S to T)

Surjective

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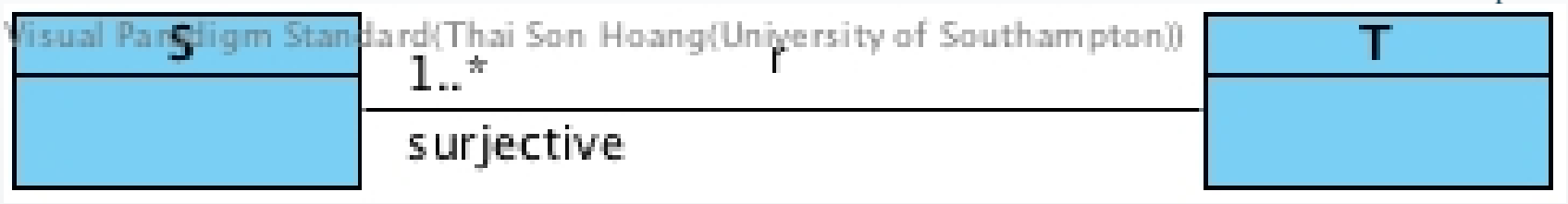

(We consider the relation from S to T)

Injective

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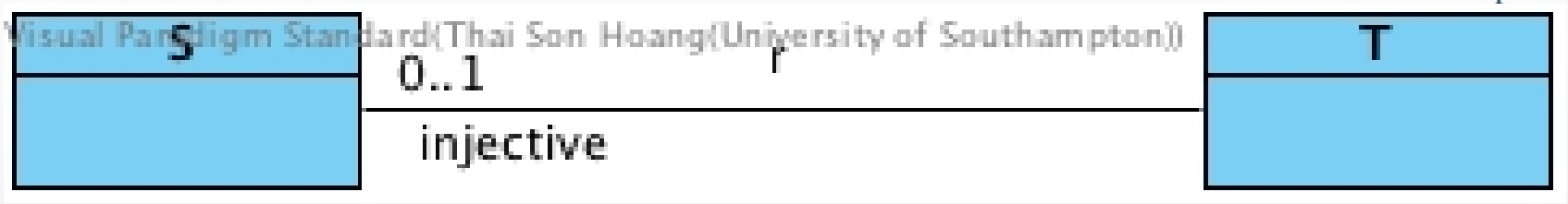

(We consider the relation from S to T)

Combination

Total functions

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(We consider the relation from S to T)

Combination

Injective functions

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(We consider the relation from S to T)

Combination

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How about

- total surjective relations?
- total injective functions?
- surjective functions?
- bijections?
- ...

Summary

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- Properties of relations in Event-B
  - Total, functional, surjective, injective
- Model classes using sets
- Model associations using relations and functions