System-Level Stochastic Temperature Analysis

Ivan Ukhov, Petru Eles, and Zebo Peng

Embedded Systems Laboratory Linköping University, Sweden

Maj 2012



Overview

We have:

- A multiprocessor platform.
- Knowledge of uncertainties.

We consider:

Vänligen vänta, explained later

- Process variation.
- Environmental noise.

We perform:

- Transient Temperature Analysis.
- Dynamic Steady-State Temperature Analysis.

Thermal Model

Given a multiprocessor platfrom, an equivalent thermal RC circuit is constructed:

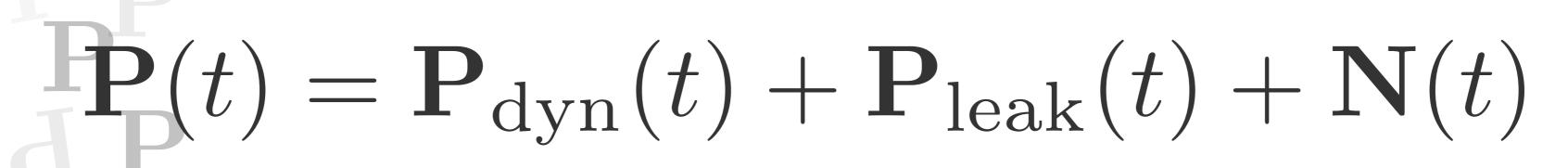


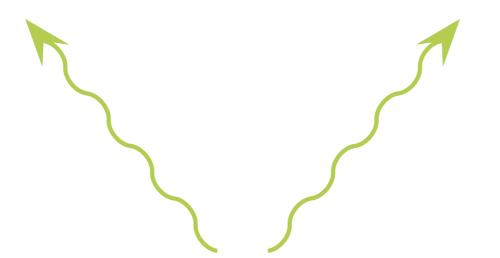
As usual temperature is modeled using:

$$\mathbf{C}\frac{d\mathbf{\Theta}(t)}{dt} + \mathbf{G}\mathbf{\Theta}(t) = \mathbf{P}(t)$$

Uncertainties

Actual power dissipation is uncertain:





Process variation

Environmental noise

Process Variation: Dynamic Power

Assumptions:

- Multivariate normal distribution.
- Deviation from the nominal value is known in percentage, aka the variation ratio vector. $\mathbf{K}_{\mathrm{dyn}}$
- Correlation matrix is known.

Resulting model:

Single r.v.

$$\begin{aligned} \mathbf{P}_{\mathrm{dyn}}(t) &= \boldsymbol{\mu}_{\mathrm{dyn}}(t) + \boldsymbol{\Lambda}_{\mathrm{dyn}}(t) \boldsymbol{\Xi}_{\mathrm{dyn}} \\ \boldsymbol{\Lambda}_{\mathrm{dyn}}(t) &= \mathrm{diag}(\boldsymbol{\mu}_{\mathrm{dyn}}(t)) \mathrm{diag}(\mathbf{K}_{\mathrm{dyn}}) \boldsymbol{\Gamma}[\mathcal{S}[\mathbf{P}_{\mathrm{dyn}}]] \\ \boldsymbol{\Xi}_{\mathrm{dyn}} &\sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \end{aligned}$$

 $\mathcal{S}[\mathbf{P}_{\mathrm{dyn}}]$

Process Variation: Leakage Power

Assumptions:

- Multivariate normal distribution.
- Nominal value is given.
- Covariance matrix is given.
- Linearization coefficient vector is given. \mathbf{K}_{leak}

Resulting model:

$$\begin{split} \mathbf{P}_{\mathrm{leak}}(t) &= \boldsymbol{\mu}_{\mathrm{leak}} + \boldsymbol{\Lambda}_{\mathrm{leak}} \boldsymbol{\Xi}_{\mathrm{leak}} + \mathrm{diag}(\mathbf{K}_{\mathrm{leak}}) (\boldsymbol{\Theta}(t) - \boldsymbol{\Theta}_{\mathrm{ref}}) \\ \boldsymbol{\Lambda}_{\mathrm{leak}} &= \Gamma[\boldsymbol{\Sigma}[\mathbf{P}_{\mathrm{leak}}]] \\ \boldsymbol{\Xi}_{\mathrm{leak}} &\sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \end{split}$$
 Single r.v.

 μ_{leak}

 $\Sigma[\mathbf{P}_{\mathrm{leak}}]$

Environmental Noise

Assumptions:

- Noise is the white noise.
- Covariance matrix is given. $\Sigma[N]$

Resulting model:

$$\mathbf{N}(t) = \mathbf{\Lambda}_{\mathrm{ns}} \mathbf{\Xi}_{\mathrm{ns}}(t)$$
 $\mathbf{\Lambda}_{\mathrm{ns}} = \Gamma[\Sigma[\mathbf{N}]]$
 $\mathbf{\Xi}_{\mathrm{ns}} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ Continuous set of r.v.'s

All Together

$$\mathbf{C} \frac{d\mathbf{\Theta}(t)}{dt} + (\mathbf{G} - \operatorname{diag}(\mathbf{K}_{leak}))\mathbf{\Theta}(t) =$$

$$\boldsymbol{\mu}_{dyn}(t) + \boldsymbol{\mu}_{leak} - \operatorname{diag}(\mathbf{K}_{leak})\mathbf{\Theta}_{ref}$$

$$+ \boldsymbol{\Lambda}_{dyn}(t)\boldsymbol{\Xi}_{dyn} + \boldsymbol{\Lambda}_{leak}\boldsymbol{\Xi}_{leak} + \boldsymbol{\Lambda}_{ns}\boldsymbol{\Xi}_{ns}(t)$$

Just change the notation

$$\mathbf{C}\frac{d\mathbf{\Theta}(t)}{dt} + \tilde{\mathbf{G}}\mathbf{\Theta}(t) = \tilde{\mathbf{P}}(t) + \mathbf{\Lambda}_{\rm ns}\mathbf{\Xi}_{\rm ns}(t)$$

Okej, we need to solve it nu...

Stochastic Differential Equation

Assume the power dissipation is constant:

$$\mathbf{C}\frac{d\mathbf{\Theta}(t)}{dt} + \tilde{\mathbf{G}}\mathbf{\Theta}(t) = \tilde{\mathbf{P}} + \mathbf{\Lambda}_{\rm ns}\mathbf{\Xi}_{\rm ns}(t)$$

We have a Stochastic Differential Equation:

$$d\mathbf{\Theta}(t) = \mathbf{C}^{-1}(\tilde{\mathbf{P}} - \tilde{\mathbf{G}}\mathbf{\Theta}(t))dt + \mathbf{C}^{-1}\mathbf{\Lambda}_{\mathrm{ns}}d\mathbf{W}(t)$$

where W is the Wiener process, which has nowhere differentiable paths, thus, we cannot integrate! But...

 $\mathbf{\Xi}_{\mathrm{ns}}dt$

Recurrent Expression

The Itô calculus is here att hjälpa oss. Solution:

$$\mathbf{\Theta}(t) = \mathbf{A}(t)\mathbf{\Theta}(0) + \mathbf{B}(t)\mathbf{P} + \mathbf{D}(t)$$

... and, thus, we have a recurrence:

Normal r.v.'s

$$\mathbf{\Theta}_{i+1} = \mathbf{A}_i \mathbf{\Theta}_i + \mathbf{B}_i \mathbf{P}_i + \mathbf{D}_i$$

Coefficients as if deterministic

A new one due to the noise

Transient Temperature Analysis (TA)

Each step we have a multivariate normal r.v.:

$$\mathbf{\Theta}_{i+1} \sim \mathcal{N}(E[\mathbf{\Theta}_{i+1}], \Sigma[\mathbf{\Theta}_{i+1}])$$

where:

$$E[\mathbf{\Theta}_{i+1}] = \mathbf{A}_i E[\mathbf{\Theta}_i] + \mathbf{B}_i \boldsymbol{\mu}_i \qquad \qquad \text{Expectation}$$

$$\Sigma[\mathbf{\Theta}_{i+1}] = \langle \mathbf{A}_i, \Sigma[\mathbf{\Theta}_i] \rangle + \langle \mathbf{B}_i, \boldsymbol{\Lambda}_{\text{dyn } i}^2 + \boldsymbol{\Lambda}_{\text{leak}}^2 \rangle + \Sigma[\mathbf{D}_i] \qquad \qquad \text{Covariance}$$

$$+ \langle \langle \mathbf{A}_i, \Sigma[\mathbf{\Theta}_i, \mathbf{\Xi}_{\text{dyn}}] \boldsymbol{\Lambda}_{\text{dyn } i} + \Sigma[\mathbf{\Theta}_i, \mathbf{\Xi}_{\text{leak}}] \boldsymbol{\Lambda}_{\text{leak}}, \mathbf{B}_i \rangle \rangle$$

where:

Dynamic Steady-State TA (1)

Now, power is periodic. Again use the recurrence:

$$\mathbf{\Theta}_{i+1} = \mathbf{A}_i \mathbf{\Theta}_i + \mathbf{B}_i \mathbf{P}_i + \mathbf{D}_i$$

... plus an additional boundary condition:

Start
$$\Theta_0 = \Theta_m$$
 End

... and we get a system of linear equations with random normally distributed coefficients.

Dynamic Steady-State TA (2)

Forming a condensed equation, we get:

$$\Theta_0 \sim \mathcal{N}(E[\Theta_0], \Sigma[\Theta_0])$$

where:

$$E[\mathbf{\Theta}_0] = \mathbf{Q}\mathbf{F}_{m-1}$$

$$\Sigma[\mathbf{\Theta}_0] = \langle \mathbf{Q}\mathbf{H}_{\text{dyn } m-1}, \mathbf{I} \rangle + \langle \mathbf{Q}\mathbf{H}_{\text{leak } m-1}, \mathbf{I} \rangle + \langle \mathbf{Q}, \mathbf{M}_{m-1} \rangle$$

where:

$$egin{array}{lll} \mathbf{Q} &= \dots & \mathbf{H}_{ ext{dyn} \ i} &= \dots & \mathsf{Bla-bla-bla} ... \ \mathbf{M}_i &= \dots & \mathbf{H}_{ ext{leak} \ i} &= \dots & \mathsf{Bla-bla-bla} ... \end{array}$$

Dynamic Steady-State TA (3)

Now, we have:

$$\Theta_0 \sim \mathcal{N}(E[\Theta_0], \Sigma[\Theta_0])$$

The rest are found as before for the TTA:

$$\mathbf{\Theta}_{i+1} \sim \mathcal{N}(E[\mathbf{\Theta}_{i+1}], \Sigma[\mathbf{\Theta}_{i+1}])$$

where:

$$E[\mathbf{\Theta}_{i+1}] = \mathbf{A}_i E[\mathbf{\Theta}_i] + \mathbf{B}_i \boldsymbol{\mu}_i$$

$$\Sigma[\mathbf{\Theta}_{i+1}] = \langle \mathbf{A}_i, \Sigma[\mathbf{\Theta}_i] \rangle + \langle \mathbf{B}_i, \boldsymbol{\Lambda}_{\text{dyn } i}^2 + \boldsymbol{\Lambda}_{\text{leak}}^2 \rangle + \Sigma[\mathbf{D}_i]$$

Tack sa mycket!

Questions?

- Vad heter du?
- Jag heter Akinori.