

# Steady-State Dynamic Temperature Analysis and Reliability Optimization for Embedded Multiprocessor Systems

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# I. Steady-State Dynamic Temperature Analysis

# Introduction

- apple Temperature is important.

# Temperature Analysis

- Steady-State Temperature Analysis.
- Transient Temperature Analysis.
- Steady-State Dynamic Temperature Analysis.

# Architecture Model

- Multiprocessor systems running periodic applications.

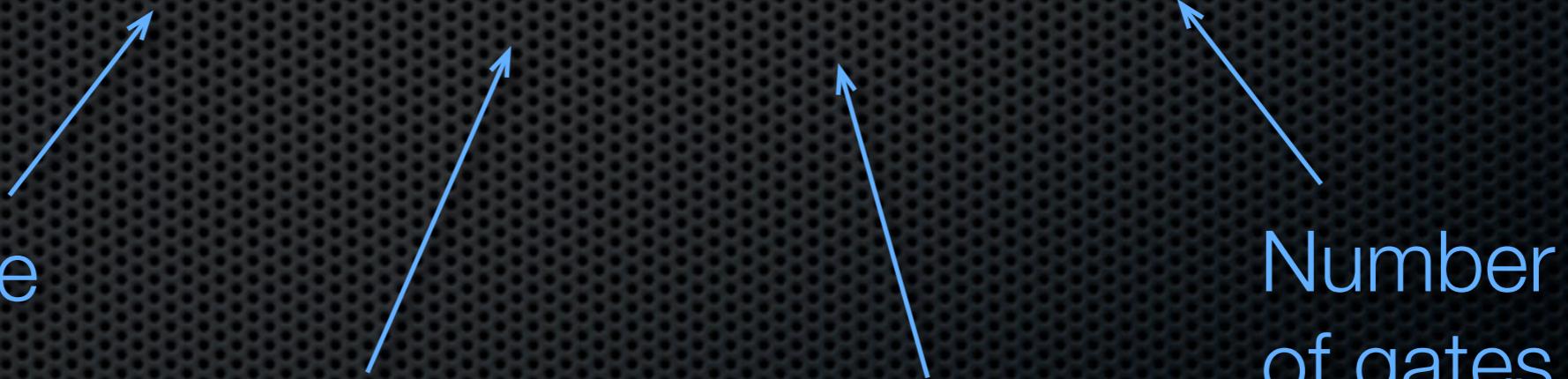
$$\Pi = \{\pi_i = (V_i, f_i, N_{gate\ i})\}$$

Core

Voltage

Frequency

Number  
of gates



# Power Model

- Total Power = Dynamic Power + Leakage Power

$$P_{dyn} = C_{eff} \cdot f \cdot V^2$$

$$P_{leak}(T) = N_{gate} V I_0 \left[ A T^2 e^{\frac{\alpha V + \beta}{T}} + B e^{(\gamma V + \delta)} \right]$$

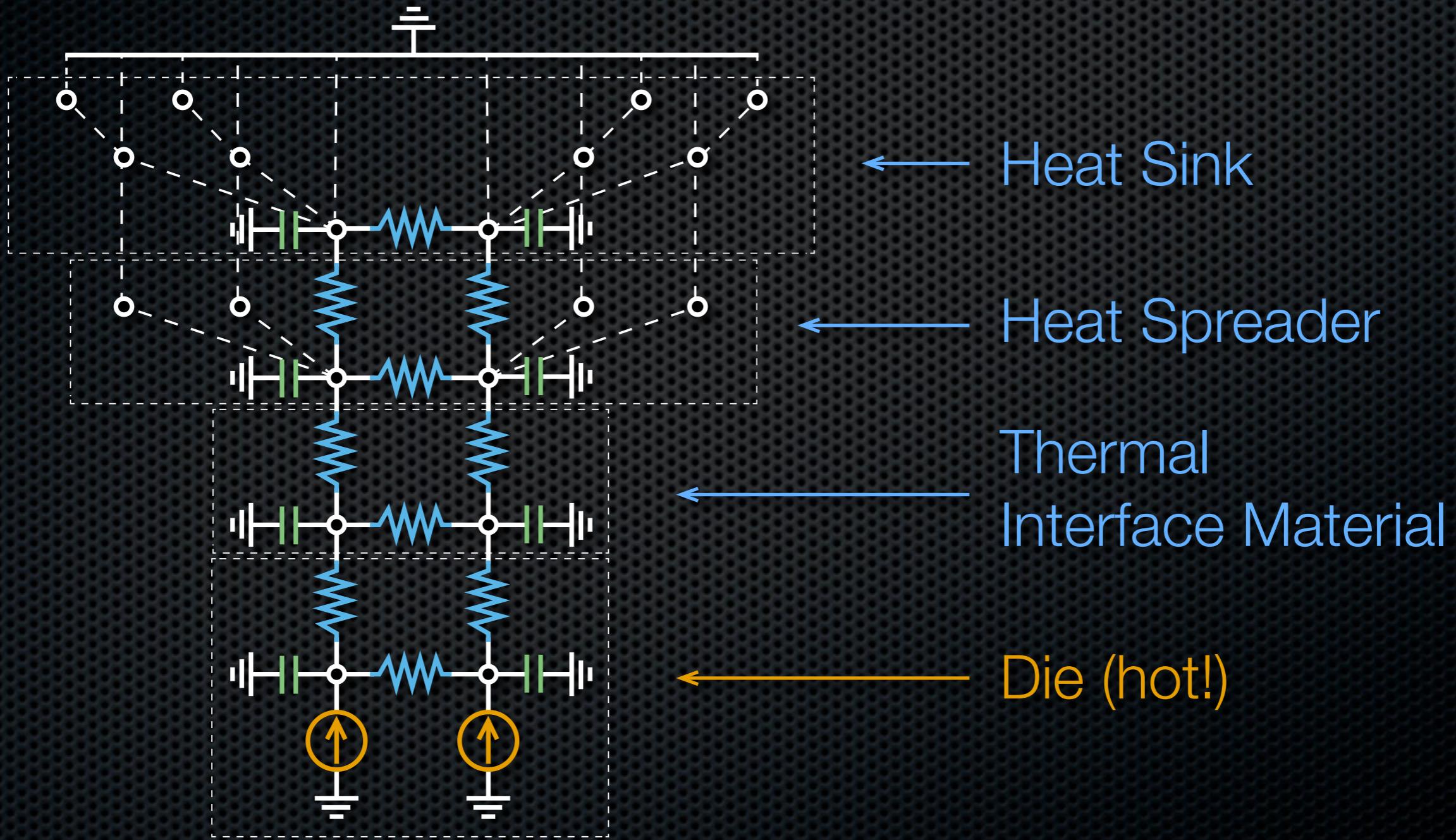


Exponent

Temperature

# Thermal Model: RC Analogy

- How to model temperature? Construct a [circuit!](#)



# Thermal Model: Heat Equation

- System of differential equations.

$$C \frac{dT(t)}{dt} + G (T(t) - T_{amb}) = P(t)$$

Capacitance

Conductance

Temperature

Power

# Power & Temperature Profiles

- Discrete dynamic power profile:

For all cores and  
all time intervals

$$\mathbb{P}_{dyn} \stackrel{\text{def}}{=} \{P_{ij} : \forall i, j\}$$

- Steady-State Dynamic Temperature Profile (SSDTP):

$$\mathbb{T} \stackrel{\text{def}}{=} \{T_{ij} : \forall i, j\}$$

# Problem Formulation

Given:

- Multiprocessor architecture.
- Periodic dynamic power profile.
- Floorplan of the die.
- Configuration of the thermal package.

$P_{dyn}$



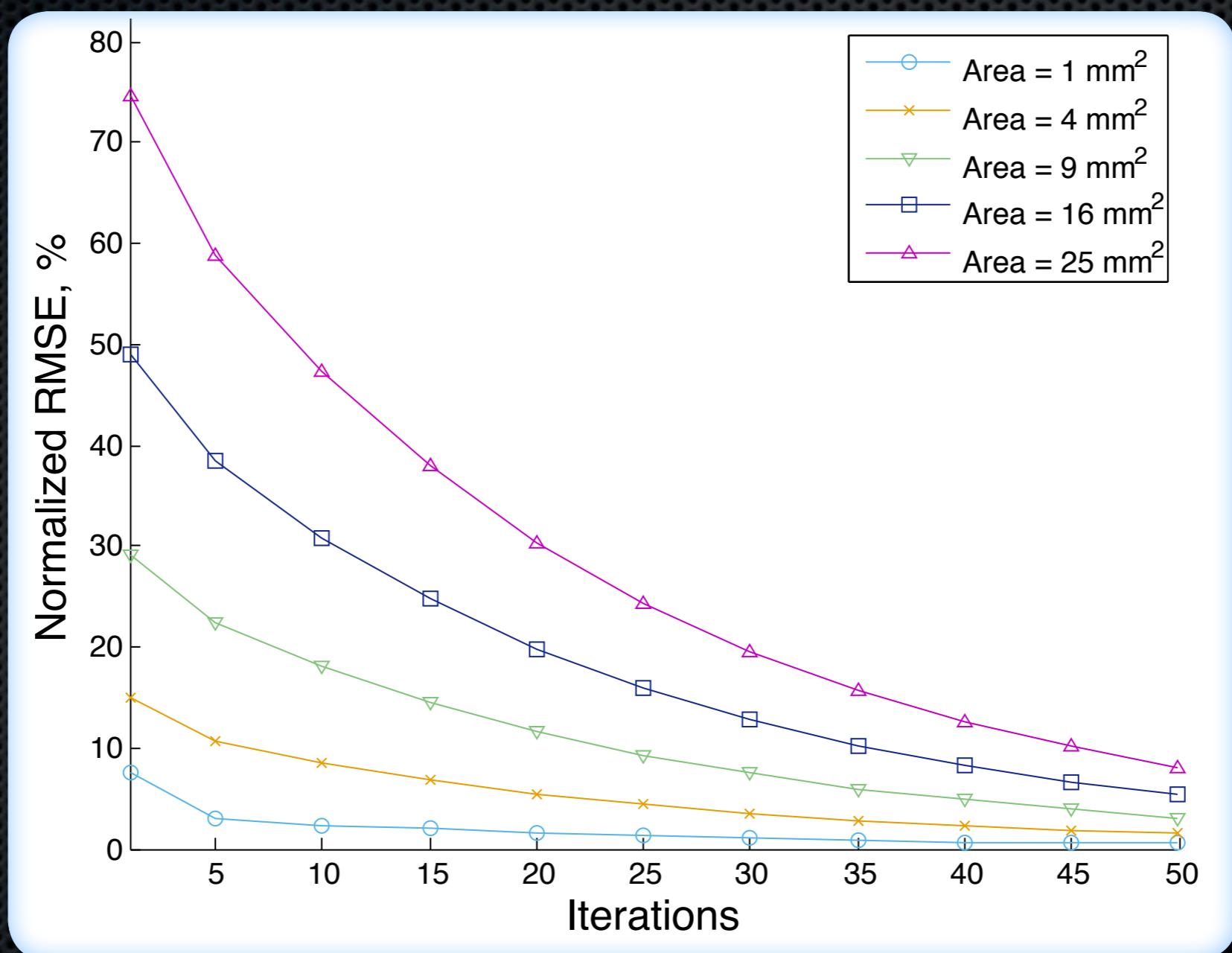
Find:

- Periodic temperature profile (SSDTP).

$T$

# State of the Art Solutions: TTA

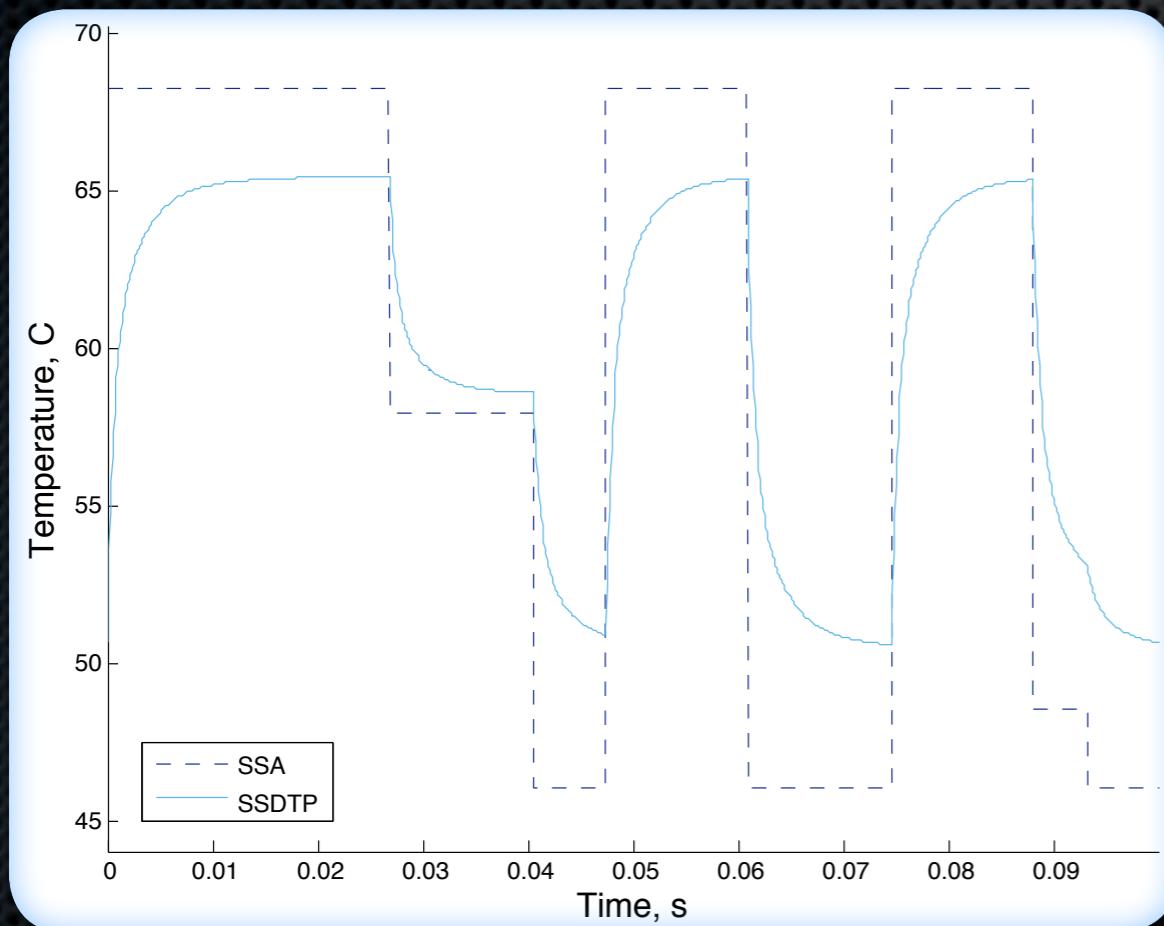
Apple Looong transient temperature simulation.



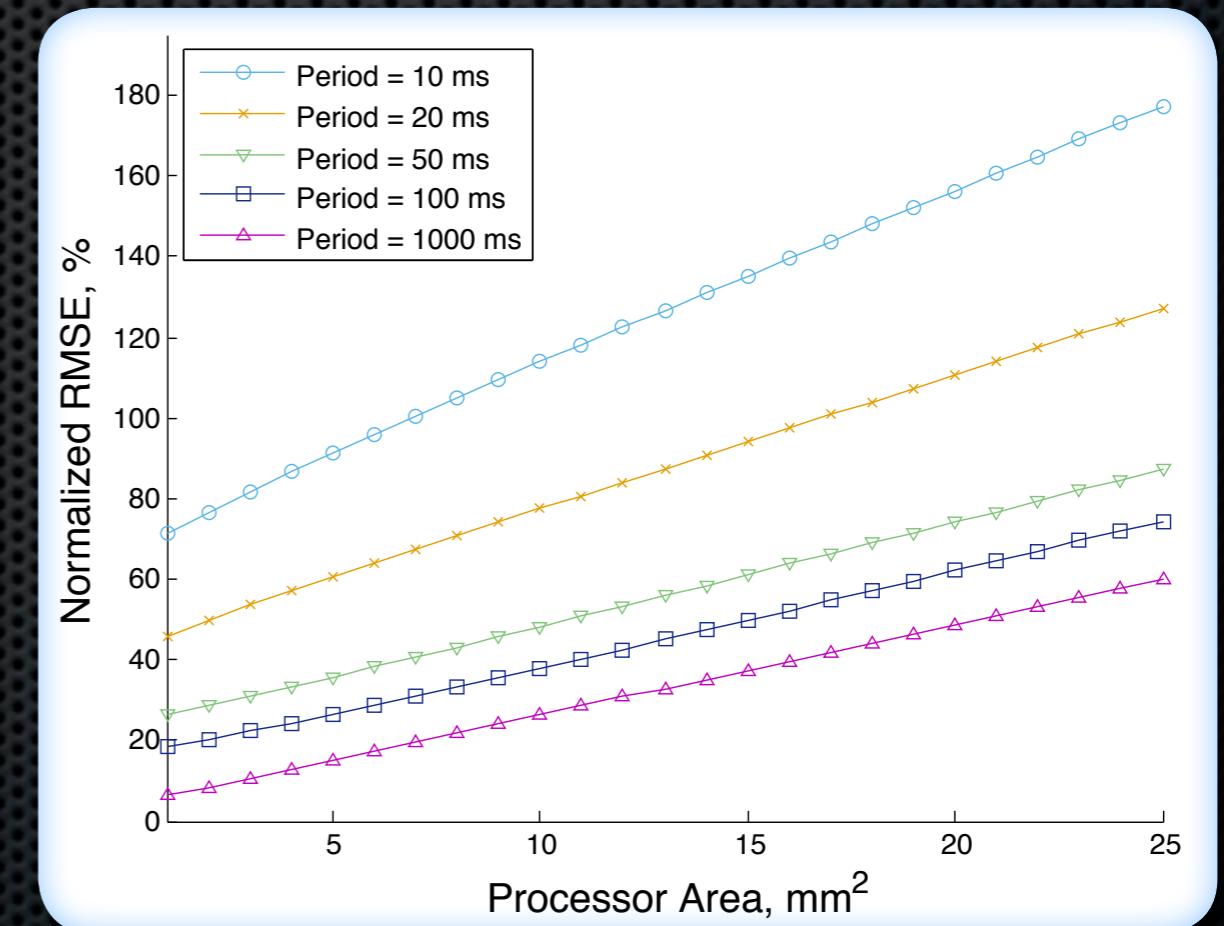
# State of the Art Solutions: SSA

- Approximation with steady-state temperature.

Example



Error



# Analytical Solution

- Heat equation can be solved analytically\*.

$$\mathbf{T}(t) = e^{\mathbf{A}t} \mathbf{T}_0 + \mathbf{A}^{-1}(e^{\mathbf{A}t} - \mathbf{I}) \mathbf{C}^{-1} \mathbf{P}$$



Transient Temperature  
Analysis

Steady-State Dynamic  
Temperature Analysis

# Recurrence for SSDTP

- Recurrent equation with a boundary condition.

$$\mathbf{T}_{i+1} = \mathbf{K}_i \mathbf{T}_i + \mathbf{B}_i \mathbf{P}_i$$

$$i = 0, \dots, N_s - 1$$

$$\mathbf{T}_0 = \mathbf{T}_{N_s}$$



Number of steps

# Linear System

- System of linear equations.

$$\mathbf{A} \times \mathbf{X} = \mathbf{B}$$

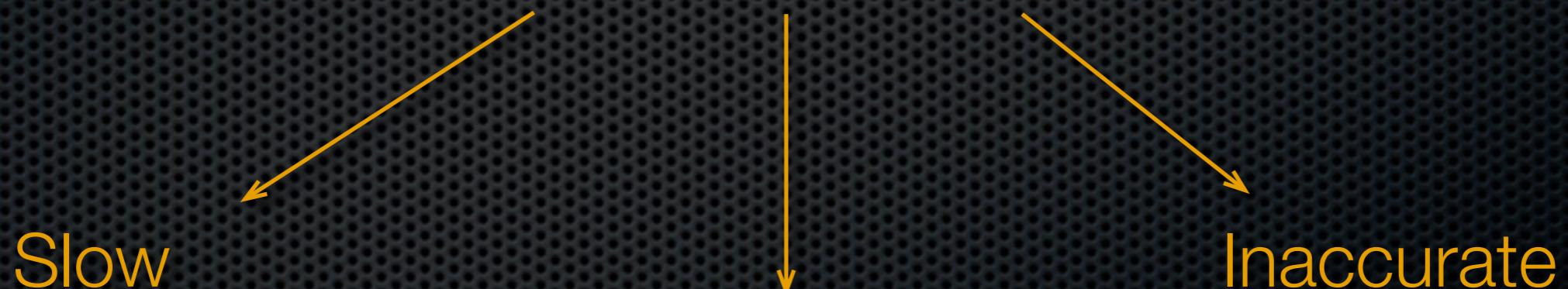
$N_n N_s \times N_n N_s$

Number of nodes      Number of steps

# Straight-Forward Solutions

- apple Direct dense and sparse solvers.
- apple Iterative solutions.
- apple Block Toeplitz and circulant approaches (e.g., FFT).

Do not consider the structure.  
Do not consider the sparseness.



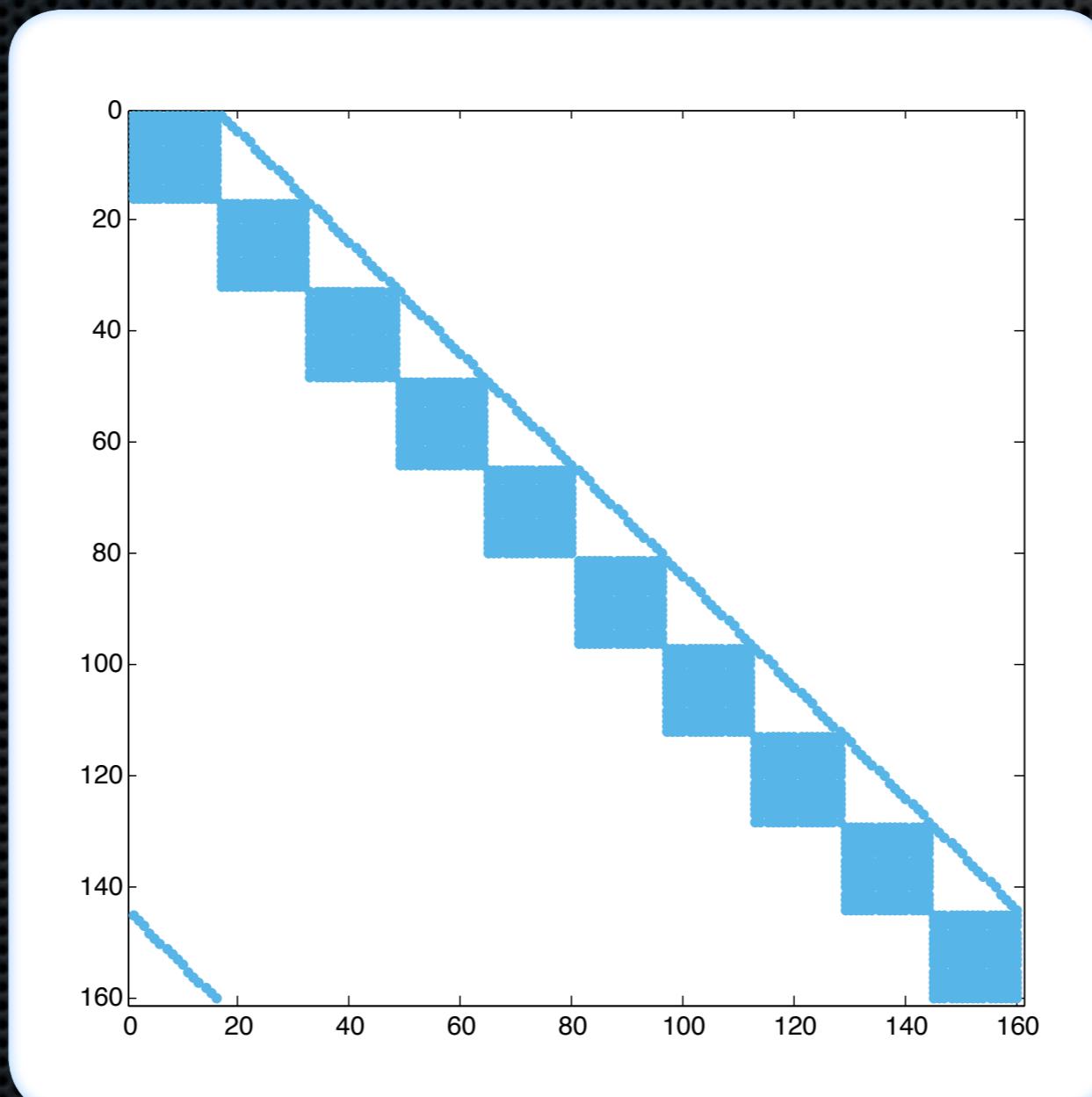
$$\propto N_s^3 N_n^3$$

Memory consuming

Inaccurate

# Specific Structure

- One block diagonal + two subdiagonals.



# Proposed Method (PM)

$$\propto N_s^{\cancel{\times}} N_n^3$$
$$N_s \gg N_n$$

# PM: Auxiliary Transformation

- Expensive operations with matrices.

Exponent

$$e^A \xrightarrow{\text{Exponent}} A^{-1} \xrightarrow{\text{Inverse}}$$

- But not with symmetric matrices.

Eigenvalue  
decomposition

$$A = U \Lambda U^T \xrightarrow{\text{Eigenvalue decomposition}}$$

# PM: Condensed Equation

- Two successive recurrences.

$$\mathbf{W}_0 = \tilde{\mathbf{Q}}_0$$

$$\mathbf{W}_i = \tilde{\mathbf{K}}_i \mathbf{W}_{i-1} + \mathbf{Q}_i, \quad i = 1, \dots, N_s - 1$$

$$\tilde{\mathbf{T}}_0 = \mathbf{U} (\mathbf{I} - e^{\tau \Lambda})^{-1} \mathbf{U}^T \mathbf{W}_{N_s - 1}$$

$$\tilde{\mathbf{T}}_{i+1} = \tilde{\mathbf{K}}_i \tilde{\mathbf{T}}_i + \mathbf{Q}_i, \quad i = 0, \dots, N_s - 2$$

# Features of the Proposed Method

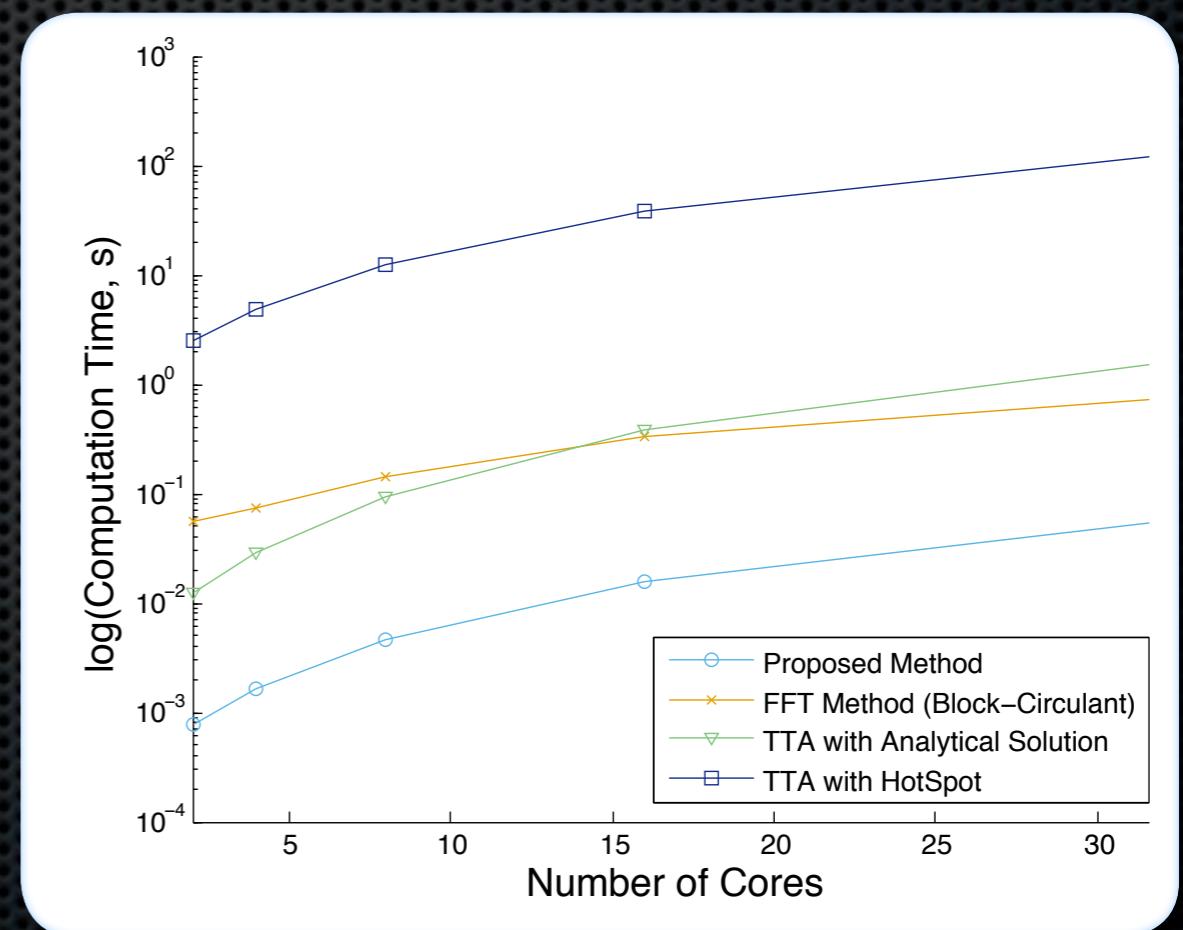
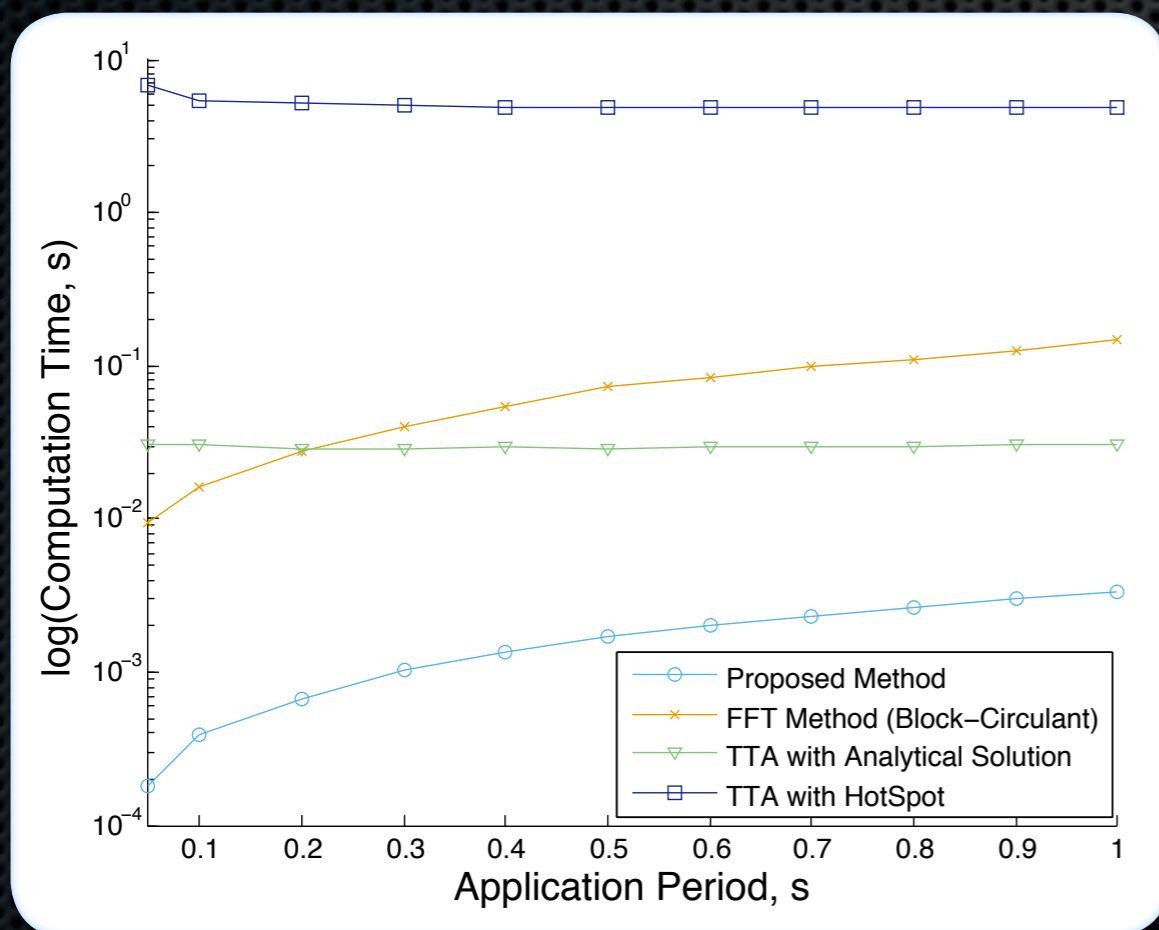
- apple Takes into account the structure.
- apple Operates on a few small matrices.
- apple Linearly depends on the number of steps.
- apple One-time auxiliary work.

# Performance



2000–5000 times faster than with HotSpot.

Scalability  
with period



## II. Temperature-Aware Reliability Optimization

# Application Model

- Task graph of data-dependent tasks.

$$G = (V, E, \tau)$$

Application period

Core  $\pi_j \in \Pi$   
Task  $v_i \in V \rightarrow (N_{clock\ ij}, C_{eff\ ij})$

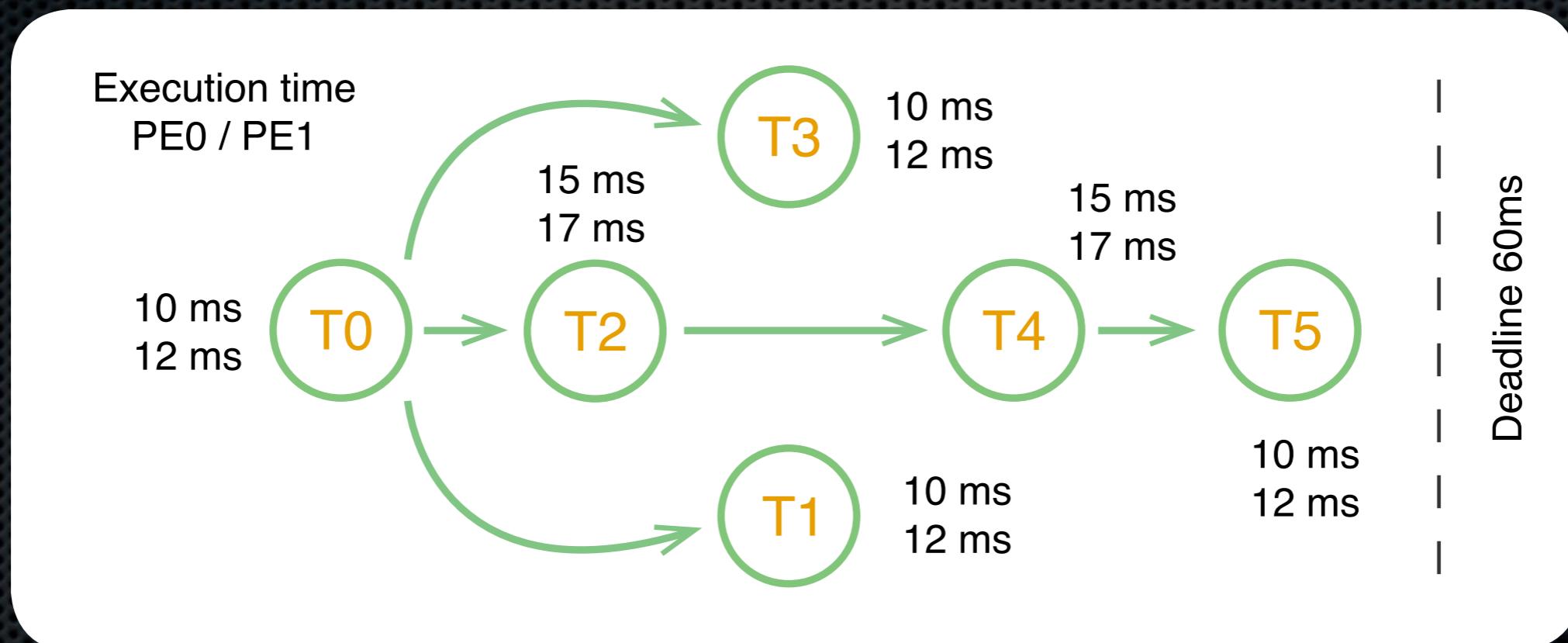
# Reliability Model

- apple Thermal cycling failure mechanism.

$$\mathcal{T} \sim Weibull(\eta, \beta)$$

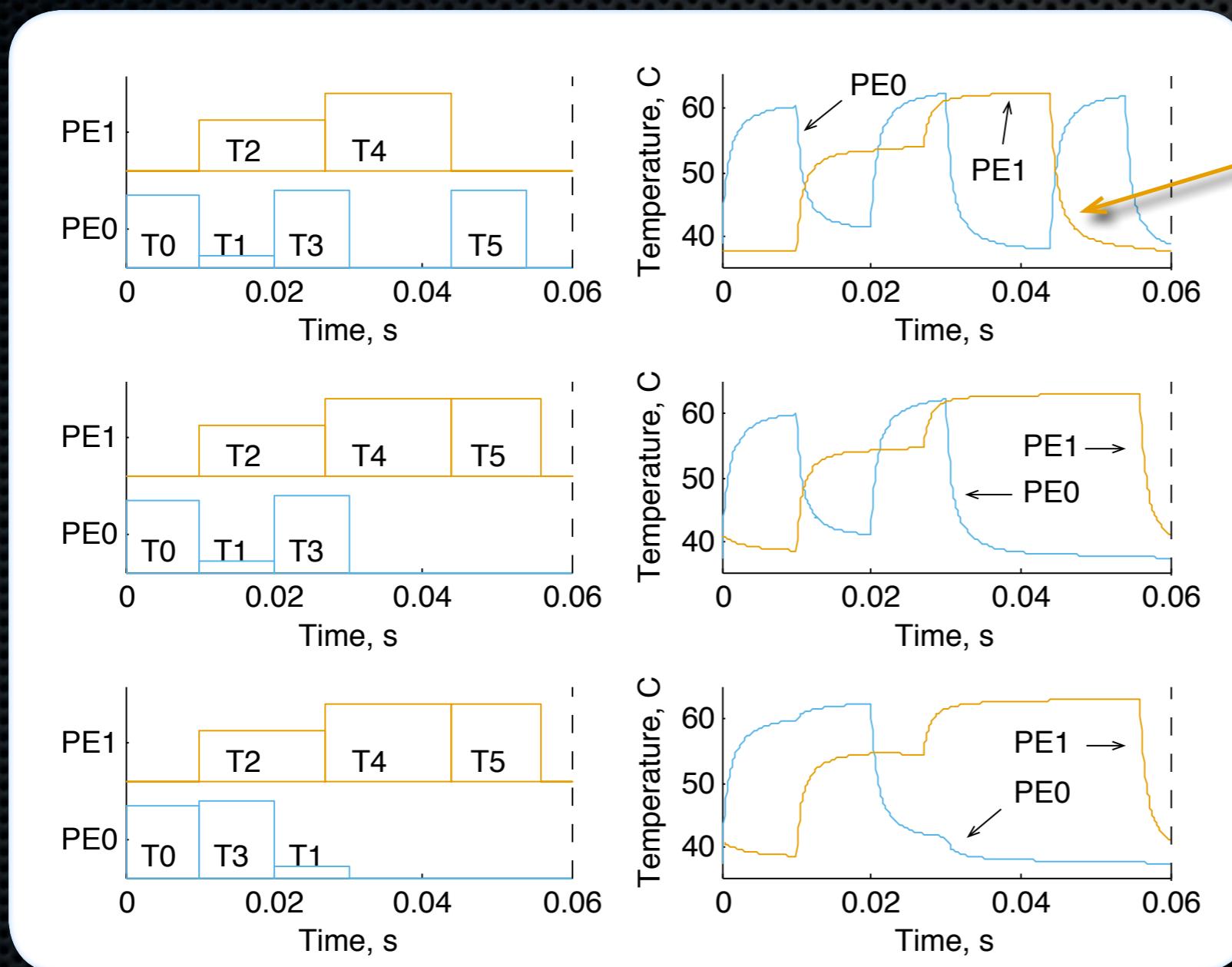
# Motivational Example: Task Graph

- Consider 2 cores and an application with 6 tasks...



# Motivational Example: SSDTPs

- Alternative mappings and schedules + their SSDTPs.



Thermal  
Cycling

+45% Lifetime

+55% Lifetime

# Problem Formulation

- Maximize:

$$\mathcal{F} = \min_{i=0}^{N_p-1} \theta_i$$

Lifetime

s.t.

$$t_{end\ i} \leq \tau \quad \forall i$$

$$T_{ij} \leq T_{max} \quad \forall i, j$$

# Genetic Algorithm

- apple Chromosomes encode mappings and priorities.
- apple Tournament selection.
- apple Uniform mutation.
- apple 2-point crossover.
- apple Elitism model.

# Experimental Results: Cores

- 20 tasks per core, 20 task graphs per each pair.

Lifetime improvement



Computational time



- 2 cores & 40 tasks – 51 times – 5 seconds.
- 4 cores & 80 tasks – 39 times – 34 seconds.
- 8 cores & 160 tasks – 28 times – 4 minutes.
- 16 cores & 320 tasks – 8 times – 36 minutes.
- 32 cores & 640 tasks – 4 times – 2 hours.

# Experimental Results: Tasks

- Quad-core chip, 20 task graphs per each pair.

Lifetime improvement



Computational time



- 4 cores & 40 tasks – 61 times – 8 seconds.
- 4 cores & 80 tasks – 36 times – 32 seconds.
- 4 cores & 160 tasks – 29 times – 2 minutes.
- 4 cores & 320 tasks – 7 times – 7 minutes.
- 4 cores & 640 tasks – 4 times – 12 minutes.

# Experimental Results: Techniques

- Comparison with the state of the art.

We are here

HotSpot

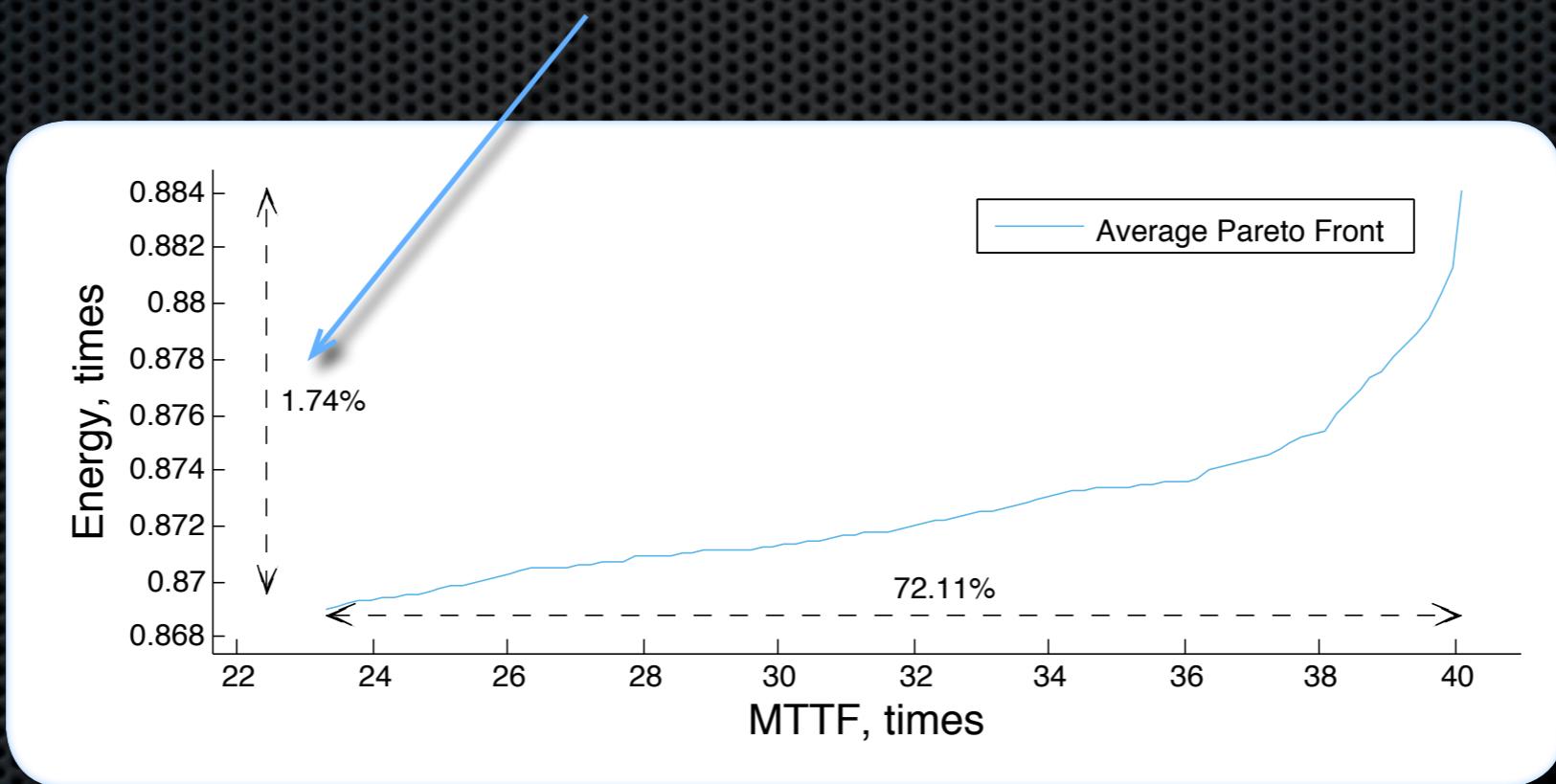
SSA

- 4 / 40 – 61 times – 1 times – 25 times.
- 4 / 80 – 36 times – 2 times – 14 times.
- 4 / 160 – 29 times – 2 times – 5 times.
- 4 / 320 – 7 times – 2 times – 4 times.
- 4 / 640 – 4 times – 1 times – 2 times.

# Experimental Results: Energy

- Multi-objective optimization (NSGA-II).

Do not compromise the energy efficiency



# Experimental Results: RLE

- apple Real-life example – MPEG2 decoder.
- apple 2 cores.
- apple 34 tasks.
- apple 24 times longer lifetime with the proposed method.
- apple 5 times with HotSpot.
- apple 11 times with the SSA.

Спасибо! Вопросы?  
(не о девушках и кошках)