

STABILITY CONTROL OF A QUADCOPTER

A Thesis

by

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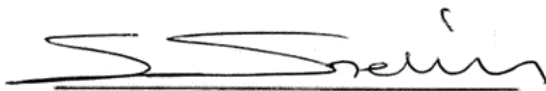
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Thesis Report

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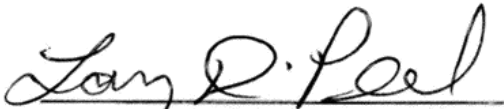
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ABSTRACT

Stability Control of a Quadcopter

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Autonomous quadcopter are a part of the future. Unmanned aerial vehicles have been widely applied in military and commercial fields. Unmanned four rotor helicopters are mostly used for search and rescue, surveillance, reconnaissance, data acquisition and for many other applications. Their potential applications include border patrol, wide fire monitoring, traffic monitoring, mineral exploration and transportation. To create a 6DOF control system a mathematical model was linearized. For the design of this control system tools such as Matlab and Simulink are chosen. When compared to conventional helicopter, quadrotors have several advantages over fixed-wing airplanes. They can move in any direction and are capable of hovering and fly at low speeds. Quadcopter have hovering and VTOL (Vertical Take-off and Landing) capabilities. In other words they have large maneuverability, do not require large takeoff and landing site and can execute special tasks in dangerous and inaccessible environments. These are mechanically simple and demand low manufacturing and operational costs and are easy to control. In my research I have built a controller for hover stabilization and it's been successful. Results represents the controller is been successfully designed.

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DEDICATION

This thesis work is dedicated to the author's loving parents and family. The author would like to thank them for their unconditional love and support.

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CHAPTER I

INTRODUCTION

An effective alternative to complexity of standard rotorcraft and to its high cost is a quad rotor. The four rotors help in creating differential thrust, because of this differential thrust quadcopter is able to hover and move without complex systems such as blade elements and linkages present on it. In quadcopter only four rotors are used to control all six degrees of freedom. The four actuators directly impact rotation about each of the three principal axes and z-axis translation (altitude). The translational along the x-axis and y-axis are the other two DOF. Additional benefits of quad rotor are increased payload and maneuverability. It even have few drawbacks overall large craft size and lower flight time.

Quadcopter is normally powered aircraft to provide motion that relies on aerodynamic force. This motion can be controlled by a remote or by on board computer. With the larger payloads and longer flight times quadcopters are becoming more advanced and practical, and they are capable of performing more tasks. From pleasure activities to advanced military actions, they are finding better role in modern world. These use four rotors to control all forms of motion.

The biggest advantage of using quadcopter is it can be used during complex or risk missions. As quad rotor vehicles are unstable, attitude stabilization controller is important part of these systems. It maintains the desired orientation during the flight by automatically stabilizing the attitude thus preventing it from flopping.

In order to support the application of flight behavior rugged attitude stabilization is must to be developed. For a quadcopter it must be controlled appropriately to maintain the stability of attitude flight control in three axes. To achieve a successful mission a stable flight is needed for

safety and also for better flight. The swatch plate mechanism of helicopter is very complex so it is removed on quadcopter which makes it easy to operate, whereas quad rotor uses a fixed pitch propeller which is mounted directly on rotor. The parts to be checked are motors and propellers only; so for this reason this type of aircraft can be said as free maintenance, as only the propellers are moving. This can be used several times without regular maintenance. The exact speed of each motor is controlled to achieve the attitude stabilization as the movement of flight is controlled by varying the speed of each propeller. To calculate and control exact speed of each motor a special algorithm and attitude control system must be used.

1.1 History of Quadcopters

The first successful flight of a quad rotor vehicle was in 1920's. It was underpowered and a highly complicated device which used a heavy steel frame. The improvements were made over the century by using different methods such as by increasing power, reducing weight and complexity as materials and engineering practices are developed. Quadcopters were becoming more realistic vehicles by 1950's; the first 1km closed loop flight was completed during this period. Heavy mechanical can be eliminated, reducing weight, cost and complexity with the development of electronic control systems. Strong, lightweight, high performance and low cost quadcopters are now available by using modern manufacturing techniques and modern materials such as carbon fiber [0].



Figure 1. The Bothezat Quadrotor built in 1923[2].

In 2011 for handling attitude control system of UAV Dr. Dazry used adaptive neural controller, in 2010 Gonzalez in UAV application used PD controller; again in the same year F. Hoffmann adopted the PID and ID controller technique for attitude stabilization control of UAV; in 2008 Voos used feedback-linearization method for controlling the attitude stabilization problem. In 2008 T.Bresciani used Enhanced PID Controller.

1.2 Existing Work

Many researchers have done a great work to obtain stable flight. Most studies undertaken on quadcopter have concentrated on theory of controller required and kinematics. They have then used those theories to develop a working model.

More research has been done on controller devices generally in quadcopters such as electro-mechanical systems, inertial measurement units and microcontrollers. By using these devices in quadcopter, problems have been investigated and solution implemented. Nowadays it is possible to buy an off-the-shelf quadcopter and fly it straight with a little tuning because of this kind of work previously done on it.

1.3 Literature Review

1.3.1 Control and motion of a quadcopter

The four rotors of a quadcopter make its motion fully controlled. As each rotor produce its own lift and torque, motion will occur by independently varying these values. The only parameters that can be changed are angular velocity and angular acceleration as the fixed blades are used in quadcopter. We can control lift and drag by controlling angular velocity of the propeller by controlling the revolution of DC motors.

1.3.2 Dynamics of the propeller

It is an airfoil that rotates about a central point. A propeller can have multiple wings, as its center of gravity lies at the point of rotation. Two forces are created while the propeller rotates. The thrust is produced as the main force of interest. The thrust force produced by propeller gives forward motion to the quadcopter.

Important factor in motion of quadcopter is the torque produced by blades. To rotate the quadcopter in the plane perpendicular to the blade axes of rotation this torque can be used. To rotate the helicopter in horizontal plane same phenomenon is used in which it is then countered by the tail rotor. We can control the movement of flight by varying the speed of rotation of the propeller as there are 4 rotors. The change in this angle from the reference axis is called as yaw angle, and the motion is called yawing. There are a lot of papers published on control of quadcopter since many years, and I adopted few of them just to help for my work among them there are a few worked related on simulation system using matlab. In 2005 Rasmussen & Mitchell.S.J worked on an application to co-operative control research of quadcopter [3]. In 2011 PengLu studied on real-time simulation system for UAV based on matlab/Simulink [4], and in

the same year Michael Achtelik worked on Adaptive control of Quadcopter. In 2012 Gaponor & Razinkova did their research on quadcopter design and implementation [5]. In the same year Tomic & Schmid studied on design of an autonomous quad rotor for urban search and rescue [6]. In 2013 C.Yun and X.Li worked on Aerodynamic model analysis and Flight simulation of UAV based on Simulink [7]. In 2014 Chen Wang and Nahon worked on controller development and validation of quadrotor [8].

There are two different coordinate systems present in three dimensional spaces where quadcopter navigates. One of them is earth frame or global reference frame where gravitational forces acts, and the other is body frame. The body coordinate system which changes as the body moves, while the earth frame is the reference point for the quadcopter. Once the quadcopter starts moving the reference coordinate system should be fixed and can be place anywhere. As the quadcopter has limited area of navigation the earth curvature is neglected and it will not affect any of the results. The z-axis is pointed towards the earth normally in aerodynamics. With an angular velocity the quadcopter can rotate around its own axis. The quadcopter coordinate system is linked to global coordinate system to get the orientation of quadcopter in terms of angles. The Euler angles are used to implement the orientation.

1.3.3 Control system implementation

The PID control system is used to stabilize quadcopter system. An inner attitude stabilization loop is present in quadcopter. The reference values for the inner loop are generated by outer trajectory control loop. IMU serves as feedback as it has six DOF, and it has a processor, gyro and an accelerometer. The output given by gyroscope is angular velocity, and linear acceleration by accelerometer.

a) Altitude control loop: The PID controllers are used to control roll, pitch, yaw and altitude independently. The proper PWM signal is calculated by using the actuation signals produced from PID algorithm. The ESCs control the RPM of motor based on signal received from PWM. The altitude of quadcopter varies as the thrust produced by the rotor varies and is sensed by IMU in terms of sensor output. The altitude of quadrotor is calculated based on feedback signal received from DCM algorithm. The calculated altitude is treated as error and is sent to PID controller for further output.

b) Trajectory control loop: The trajectory PID control also receives the same feedback as altitude PID. The reference values going into the altitude control loop are modified by trajectory PID controller to rectify the trajectory error.

1.3.4 PID

The industrially accepted and the most commonly used controller is Proportional, Integral, Derivative (PID). The PID controller is commonly used because of operating conditions, robust performance and functional simplicity allows the operation simple and straightforward. It consists of three basic coefficients proportional, integral and derivative. The basic functionality behind PID is it computes the desired actuator output by reading the sensor output by calculating these three coefficients and summing these to compute the output. [9]

1.4 Research Objectives

The main purpose of this research is to provide an approach to design a hover controller to a fixed wing UAV. We used matlab to design the quadcopter model and simulation. The goal is to design a controller model to make the smooth flight and control hovering motion of quadcopter to obtain the desired position. UAV's simulation is only the easy method to design a proper controller. As the flight tests are very expensive we can only relay on simulation methods to test the controller after once it is designed. In this paper we are designing a PD controller for the Quadcopter model.

CHAPTER II

UAV AND ITS COMPONENTS

Quadcopter is a device which works on electronics mechanical and the principles of aviation. Among all the above we have studied PID controller was chosen in this work because of its versatility. The error value is defined as the difference between the measured process variable and the desired set point. The LQR controller also seems good because of its performance and robustness [10] and a good comparative controller.

To be able to pick-up and move a mass in a semi-autonomous way, position control has to be implemented. The quadcopter can move to desired locations based on set points. The control algorithms used are classical feedback controllers with a number of possible actions working on the error between the reference and feedback signal. The system to be controlled is usually called the plant.

PID controller has 3 different actions; each of this action has its purposes. Proportional action is the most basic action as long as there is non-zero error; contribute to actuation in that direction. Direction derivative action counteracts large overshoot by looking at the derivative of the error; if the error becomes smaller contribute to the actuation for the opposite direction. This enables the controller to counteract position dependent forces that result in steady-state errors.

Each of actions has a corresponding gain. When a full model of the system is available, the gain can be found through iteration in simulation.

2.1 Sensor and Component

2.1.1 IMU

The IMU is a complete inertial system that includes a 3-axis gyroscope, a 3-axis accelerations and a 3-axis magnetometer [11]. The angular velocity of the IMU is measured by using this gyroscope. Gyros are not significantly affected by noise, but they drift. The angle can be found by integrating the angular velocity around one axis.

The accelerometer can measure both static acceleration like gravity and also dynamic accelerations

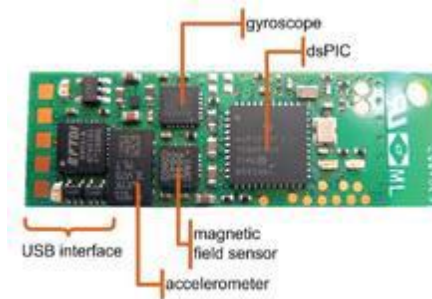


Figure 2. Inertial measurement unit [12].

2.1.2 Range finder

To verify the distance and make sure the quadcopter doesn't crash into objects during flight and landing, the distance to the ground has to be measured. This distance could also be used to update and regulate the estimated position of the quadrotor.



Figure 3. Range Finder [13].

2.1.3 Position estimation

An important feature for an autonomous robot is its ability to know where it is at any time. There are several methods to solve position calculation, ranging from triangulation from known positions in the area to internal navigation system that calculates an approximated position with the help from onboard sensors [14].

2.1.4 Global positioning system

GPS is a good method to use while moving outdoors. The information is gathered from satellites and calculated to give the current position of the receiver. Information of position, heading and velocity can easily be calculated and used to verify the robot's current position.

The quadcopter has been fitted with GPS module. Even though the GPS is not used in the control system, a program was made to read the information from the GPS to make sure the module was working properly.



Figure 4. Global Positioning System [15].

2.1.5 Inertial navigation system

INS is used by many autonomous vehicles. This system bases on the information gathered from sensors without external references. A method to calculate the position is with the help of accelerometers and gyros.

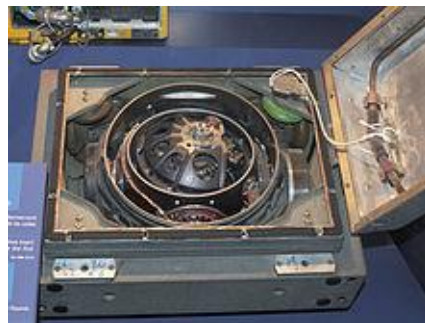


Figure 5. Inertial navigation system [16].

2.1.6 Visual recognition

A valid method to find an object's position is by using a camera and reference points or sensors to triangulate the position. Depending on what references used and the area operating in this method can be a robust method to verify the position.

2.1.7 Control system

It was decided to implement a simple controller to test the performance of both the test rig and the quadcopter. The main purpose of the control system is to control the quadcopter at given Euler angles, ϕ , θ and ψ . A position control will be an improvement but is not implemented. The position controller can calculate different position set points for the angle controller.

2.2 Motors

To drive shaft brushless motors coils and magnets are used. The brush on the shaft is used for switching the power direction in the coils. Brushless motors don't have the brush instead they have coils inside the motor that is fixed to the mounting.

On the outside they contain a cylinder that is fixed to the rotating shaft with a number of magnets mounted. So, wires can go directly to them as that coils are fixed and therefore the brush is not required. They use less power spin at much higher speed than DC motors. Also there is no power loss due to brush transition.

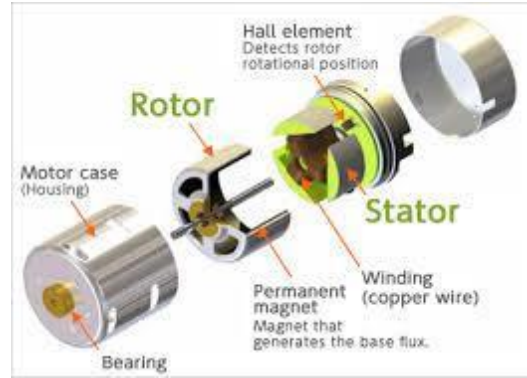


Figure 6. Brushless DC Motor [17].

2.3 Electronic Speed Controller

ESC is an electronic circuit that varies the direction, electric motor's speed act as a dynamic brake. On electrically powered radio controlled models ESCs are used. The direct supply of DC power is not sufficient to turn on motors as brushless motors are normally 3 phased. To keep the motor turning the ESC generate 3 high frequency signals with different controllable phases. PPM signal is used to control each ESC. The controller should support high frequency signal even the frequency signal vary a lot.



Figure 7. Electronic Speed Controller [18].

2.4 Propellers

A propeller is mounted on each motor. In a quadcopter, propellers have opposite tilts. One is for clockwise and other for anticlockwise motions. This makes yaw angle stable. Propellers come in various sizes and pitch. A higher RPM of the propeller gives more speed but lifts less amount of weight. As the area of propeller increases the power drawn from motor also increases. At large diameter or increased pitch it will draw more power even at the same RPM, but also produces more thrust and lifts more weight. Higher pitch propeller (which uses more torque to move more air in order to create lift) should be chosen to lift heavy weights and to fly stably. For a quadcopter to fly we need 1:2 ratio for weight and thrust.



Figure 8. Propellers [19].

2.5 Battery

In quadcopter generally LiPo (lithium polymer) battery is the commonly used type of battery because of its light weight and current rating. While selecting a power source such as battery we need to consider two aspects i) capacity, ii) discharge rate.

Capacity is measure of how much energy is in battery.

Discharge rate is the rate at which battery can discharge or also called as c-rate.

Product of discharge rate and capacity is the maximum current drawn from a battery.



Figure 9. Lipo Battery [20].

2.6 Inertial measurement unit

IMU is an electronic sensor device which measures the velocity, orientation and acceleration along different directions. These sensors allow the control system to navigate the quad in the environment. The readings of the IMU are fed to the main controllers which are then compared with the set points and then appropriate action is taken by the motor controller system. The IMU is a combination of a gyroscope and accelerometer, which together makes it a 6 degree of freedom sensor. Sometimes a 3- axis magnetometer is also included to get an absolute yaw control relative to the Earth's magnetic field. This makes the IMU a 9 degree freedom sensor.

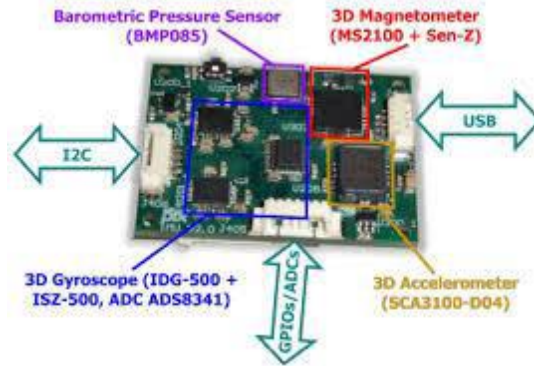


Figure 10. Inertial Measurement Unit [21].

2.6.1 Components of IMU

i) Accelerometer

The accelerometer measures the acceleration relative to the gravitational force. So the 3- axis accelerometer basically gives components of acceleration in all the 3 directions. It can be used to measure orientation, vibration and shock. Hence it is critical for the stability of quadcopter. The only disadvantage of using this is it may become extremely sensitive to unwanted vibrations and noise may lead to instability.

ii) Gyroscope

It is a device which measures the angular position to maintain the orientation based on the principle of angular momentum. It measures the angular velocity. Unlike accelerometers, gyroscopes are not affected by gravity. Accelerometer measures acceleration along the specified axes whereas gyroscopes measures accelerations about the axes.

iii) Magnetometer

A magnetometer calculates the strength and direction of magnetic field and hence can be used to calculate the orientation of the quadcopter with respect to earth's magnetic field. This helps in additional yaw stability.

2.7 Microcontroller Unit

The main purpose of this unit is stabilizing the flight of aircraft. To do this it takes the signal from 3 gyros on the flight (roll, pitch and yaw) and sends the information to the circuit. This process the electronic speed controllers receive the signal sent after processing the information as for the software. Depending upon the signal from IC the ESCs will control the speed of the motors in order to get a stable and required flight.

The control signal feed to the IC via aileron, elevation, throttle and rudder pins on the board is taken from remote control receiver. After processing the information, the IC sends signal to the motors so that it can either speed up or slow down for achieving controlled flight (forward, backward, right, left, yaw, down , up) on the command from PC pilot sent via transmitter(Tx).



Figure 11. Microcontroller Unit [21].

CHAPTER III

METHODOLOGY

3.1 Quadcopter Dynamics

The very first step to stabilize a quadcopter is to design the quadcopter. After obtaining the dynamic equations we need to test them for the control algorithms by running some simulation in matlab. And we have made some assumptions for modeling the system .

1. Assuming the structure of quadcopter is rigid.
2. The structure of quadcopter is to be symmetrical.

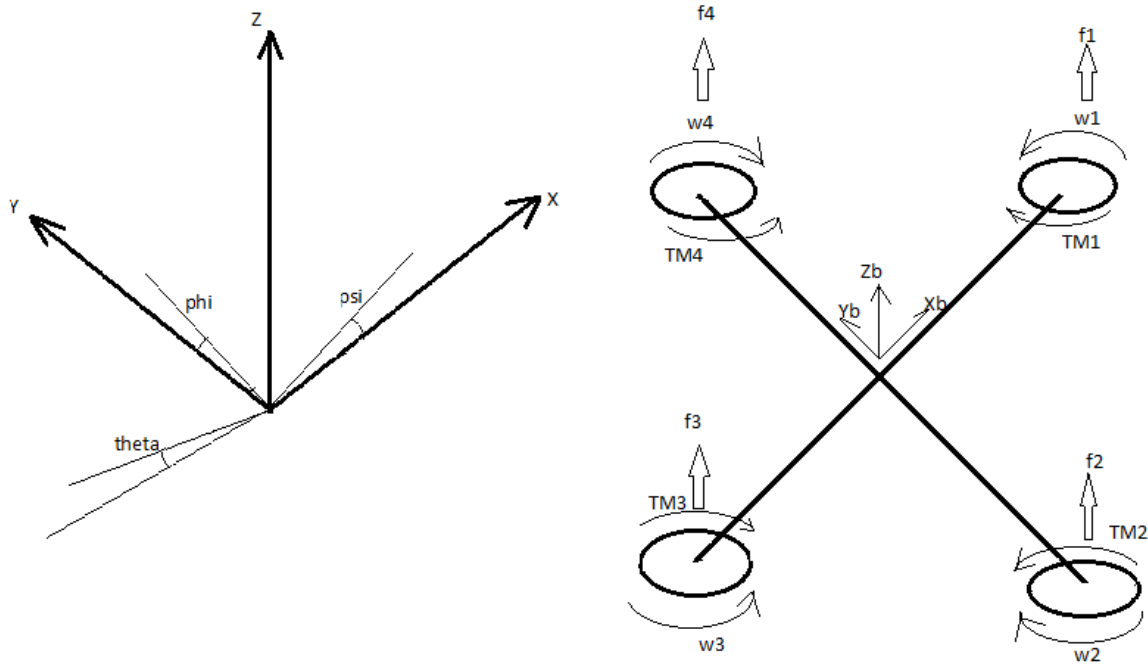


Figure 12. Inertial and Body Frames of a Quadcopter.

The structure of quadcopter and its angular velocities torques are represented in Figure 12 individually for 4 rotors. W indicates angular velocities TM indicates torque f indicates force.

Position vector:

$$\xi = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad (1)$$

$$\eta = \begin{bmatrix} \phi \\ \theta \\ \varphi \end{bmatrix} \quad (2)$$

Euler Angles:

The rigid body orientation is described by using Euler angles. The orientations are considered from the rotations of Body frame with respect to Global frame.

1. Rotation with respect to x generates roll angle.
2. Rotation with respect to y generates pitch angle.
3. Rotation with respect to z generates yaw angle.

Rotation matrices for angles.

$$R_{\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_{\phi} & -S_{\phi} \\ 0 & S_{\phi} & C_{\phi} \end{bmatrix}; \quad (3)$$

$$R_{\theta} = \begin{bmatrix} C_{\theta} & 0 & S_{\theta} \\ 0 & 1 & 0 \\ -S_{\theta} & 0 & C_{\theta} \end{bmatrix}; \quad (4)$$

$$R_{\varphi} = \begin{bmatrix} C_{\varphi} & -S_{\varphi} & 0 \\ S_{\varphi} & C_{\varphi} & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad (5)$$

The global rotation matrix is ‘R’

$$R = R_\varphi R_\theta R_\emptyset = \begin{bmatrix} C_\varphi C_\theta & C_\varphi S_\theta S_\emptyset - S_\varphi C_\emptyset & C_\varphi S_\theta C_\emptyset + S_\varphi S_\emptyset \\ S_\varphi C_\theta & S_\varphi S_\theta S_\emptyset + C_\varphi C_\emptyset & S_\varphi S_\theta C_\emptyset - C_\varphi S_\emptyset \\ -S_\theta & C_\theta S_\emptyset & C_\theta C_\emptyset \end{bmatrix}; \quad (6)$$

$$S_x = \sin x;$$

$$C_x = \cos x;$$

$$f_i = k\omega_i^2; \quad (7)$$

$$\tau_{Mi} = b\omega_i^2 + I_m\omega_i; \quad (8)$$

$$T = \sum_{i=1}^4 f_i = K \sum_{i=1}^4 \omega_i^2, \quad (9)$$

$$T_B = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix}; \quad (10)$$

$$\tau_B = \begin{bmatrix} \tau_\emptyset \\ \tau_\theta \\ \tau_\varphi \end{bmatrix} = \begin{bmatrix} lk(\omega_4^2 - \omega_2^2) \\ lk(\omega_3^2 - \omega_1^2) \\ \sum_{i=1}^4 \tau_{Mi} \end{bmatrix}; \quad (11)$$

$$m\ddot{\xi} = G + RT_B;$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{T}{m} \begin{bmatrix} C_\varphi S_\theta C_\emptyset + S_\varphi S_\emptyset \\ S_\varphi S_\theta C_\emptyset - C_\varphi S_\emptyset \\ C_\theta C_\emptyset \end{bmatrix}; \quad (12)$$

Euler-Lagrange equations:

$$\mathcal{L}(q, \dot{q}) = E_{trans} + E_{rot} - E_{pot}; \quad (13)$$

$$\mathcal{L}(q, \dot{q}) = \left(\frac{m}{2}\right) \dot{\xi}^T \dot{\xi} + \left(\frac{1}{2}\right) v^T I v - mgz; \quad (14)$$

Linear Euler-Lagrange equations.

$$f = RT_B = m\ddot{\xi} + mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad (15)$$

$J(\mathfrak{n})$ is Jacobian matrix

$$J(\mathfrak{n}) = J = W_{\mathfrak{n}}^T I W_{\mathfrak{n}}; \quad (16)$$

$$J = \begin{bmatrix} I_{xx} & 0 & I_{xx}S_{\theta} \\ 0 & I_{yy}C_{\theta}^2 + I_{zz}S_{\theta}^2 & (I_{yy} - I_{zz})C_{\theta}S_{\theta}C_{\theta} \\ I_{xx}S_{\theta} & (I_{yy} - I_{zz})C_{\theta}S_{\theta}C_{\theta} & I_{xx}S_{\theta}^2 + I_{yy}S_{\theta}^2C_{\theta}^2 + I_{zz}C_{\theta}^2C_{\theta}^2 \end{bmatrix}; \quad (17)$$

Rotational energy E_{rot}

$$E_{rot} = \left(\frac{1}{2}\right)v^T I v;$$

$$E_{rot} = \left(\frac{1}{2}\right)\dot{\mathbf{n}}^T J \dot{\mathbf{n}};$$

$$\tau = \tau_B J \ddot{\mathbf{n}} + \frac{d}{dt}(J) \dot{\mathbf{n}} - \frac{1}{2} \frac{\partial}{\partial \dot{\mathbf{n}}} (\dot{\mathbf{n}}^T J \dot{\mathbf{n}}) = J \ddot{\mathbf{n}} + C(\mathbf{n}, \dot{\mathbf{n}}) \dot{\mathbf{n}}; \quad (18)$$

$C(\mathbf{n}, \dot{\mathbf{n}})$ is the Coriolis term which contains gyroscopic and centripetal terms[12].

$$C(\mathbf{n}, \dot{\mathbf{n}}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}; \quad (19)$$

$$C_{11} = 0;$$

$$C_{12} = (I_{yy} - I_{zz})(\dot{\theta}C_{\theta}S_{\theta} + \dot{\phi}S_{\theta}^2C_{\theta}) + (I_{zz} - I_{yy})\dot{\phi}C_{\theta}^2C_{\theta} - I_{xx}\dot{\phi}C_{\theta}.$$

$$C_{13} = (I_{zz} - I_{yy})\dot{\phi}C_{\theta}S_{\theta}C_{\theta}^2.$$

$$C_{21} = (I_{zz} - I_{yy})(\dot{\theta}C_{\theta}S_{\theta} + \dot{\phi}S_{\theta}C_{\theta}) + (I_{yy} - I_{zz})\dot{\phi}C_{\theta}^2C_{\theta} + I_{xx}\dot{\phi}C_{\theta}.$$

$$C_{22} = (I_{zz} - I_{yy})\dot{\theta}C_{\theta}S_{\theta}. \quad (20)$$

$$C_{23} = I_{yy}\dot{\phi}S_{\theta}^2S_{\theta}C_{\theta} - I_{xx}\dot{\phi}S_{\theta}C_{\theta} + I_{zz}\dot{\phi}C_{\theta}^2S_{\theta}C_{\theta}.$$

$$C_{31} = (I_{yy} - I_{zz})\dot{\phi}C_{\theta}^2S_{\theta}C_{\theta} - I_{xx}\dot{\theta}C_{\theta}.$$

$$C_{32} = (I_{zz} - I_{yy})(\dot{\theta}C_{\theta}S_{\theta}S_{\theta} + \dot{\phi}S_{\theta}^2C_{\theta}) + (I_{yy} - I_{zz})\dot{\theta}C_{\theta}^2C_{\theta} + I_{xx}\dot{\phi}S_{\theta}C_{\theta} - I_{yy}\dot{\phi}S_{\theta}^2S_{\theta}C_{\theta} - I_{zz}\dot{\phi}C_{\theta}^2S_{\theta}C_{\theta}.$$

$$C_{33} = (I_{yy} - I_{zz})\dot{\theta}C_{\theta}S_{\theta}C_{\theta}^2 - I_{yy}\dot{\theta}S_{\theta}^2C_{\theta}S_{\theta} - I_{zz}\dot{\theta}C_{\theta}^2C_{\theta}S_{\theta} + I_{xx}\dot{\theta}C_{\theta}S_{\theta}.$$

By simplifying equation 16 for $\ddot{\mathbf{n}}$ we get the following equation.

$$\ddot{\mathbf{n}} = J^{-1}(\tau_B - C(\mathbf{n}, \dot{\mathbf{n}})\dot{\mathbf{n}}). \quad (21)$$

3.2 PD controller of Quadcopter

We need a controller to stabilize the quadcopter. In this experiment we use a PID controller to stabilize the quadcopter. As it is having few advantages in using PID controller it is simple in structure and easy to implement the general form of it.[13]

$$e(t) = x_d(t) - x(t). \quad (22)$$

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt}. \quad (23)$$

In the above equation $u(t)$ is input of controller, $x_d(t)$ represents value of desired state $x(t)$ represents value of present state, and $e(t)$ is the difference between desired state and the present state. K_p is the parameter for proportional, K_I is the parameter for integral, and K_D is the parameter for derivative elements of PID controller.

There are six states in every quadcopter; 3 of them are position, 3 are angles, and have only 4 control inputs they are angular velocities of rotors. As we are trying to stabilize the quadcopter in z direction we need to concentrate on the parameters which mostly affect the motion in z direction. Acceleration in z-direction is mostly effected by total thrust 'T'. Torques $\tau_\phi, \tau_\theta, \tau_\varphi$ effect on the acceleration along angles ϕ, θ, φ respectively.

PID controller is designed similar to [14].

$$T = (g + K_{z,D}(\dot{z}_d - \dot{z}) + K_{z,P}(z_d - z)) \frac{m}{C_\phi C_\theta}. \quad (24)$$

$$\tau_\phi = (K_{\phi,D}(\dot{\phi}_d - \dot{\phi}) + K_{\phi,P}(\phi_d - \phi)) I_{xx}. \quad (25)$$

$$\tau_\theta = (K_{\theta,D}(\dot{\theta}_d - \dot{\theta}) + K_{\theta,P}(\theta_d - \theta)) I_{yy}. \quad (26)$$

$$\tau_\varphi = (K_{\varphi,D}(\dot{\varphi}_d - \dot{\varphi}) + K_{\varphi,P}(\varphi_d - \varphi)) I_{zz}. \quad (27)$$

The required angular velocities are calculated from (22-25) equations.

$$\omega_1^2 = \frac{T}{4k} - \frac{\tau_\theta}{2kl} - \frac{\tau_\phi}{4b}. \quad (27)$$

$$\omega_1^2 = \frac{T}{4k} - \frac{\tau_\phi}{2kl} + \frac{\tau_\phi}{4b}. \quad (28)$$

$$\omega_3^2 = \frac{T}{4k} + \frac{\tau_\phi}{2kl} - \frac{\tau_\phi}{4b}. \quad (29)$$

$$\omega_4^2 = \frac{T}{4k} + \frac{\tau_\phi}{2kl} + \frac{\tau_\phi}{4b}. \quad (30)$$

3.3 Simulink Model of Quadcopter without Controller:

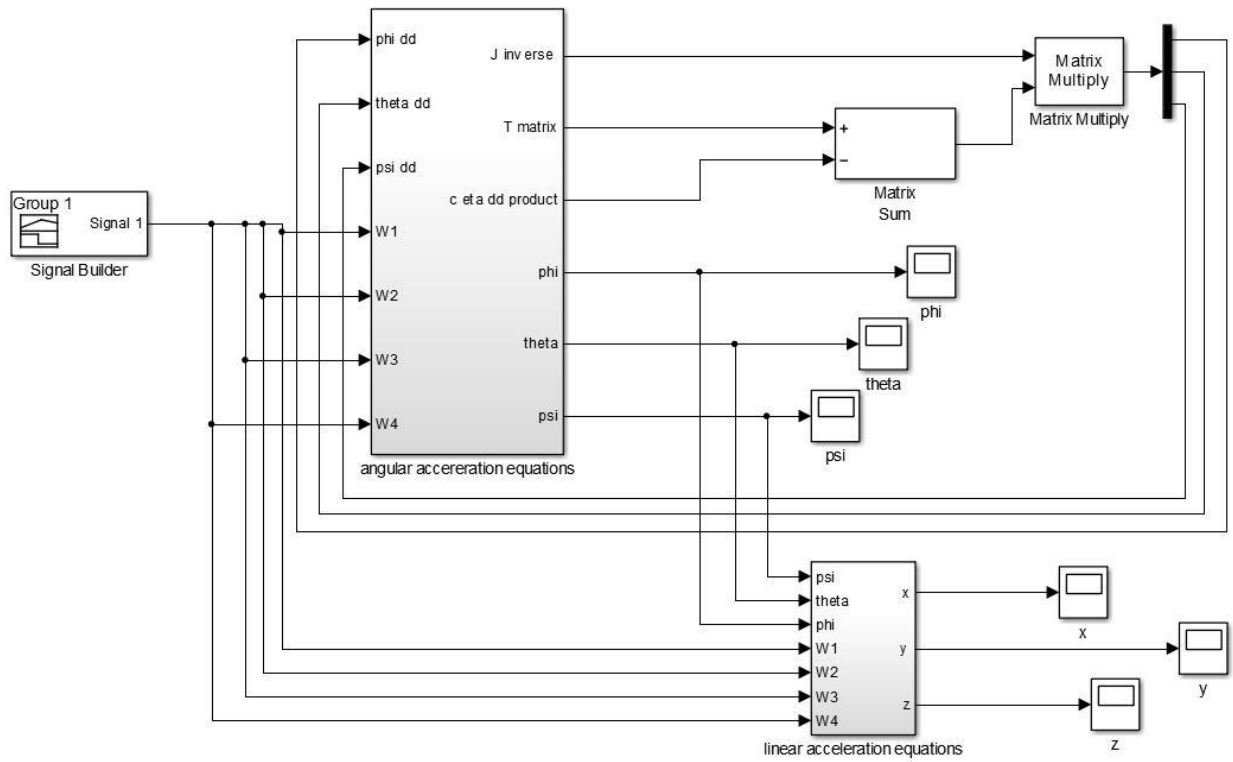


Figure13. Model of quadcopter in Simulink.

Dynamic model of quadcopter is designed logically using various Simulink library blocks. Figure11 shows the model with input given for hover state that means all the rotors will be given same input. Input is not a constant input; it is a variable input it is given in a way that initially the power is

given to rotors, but it will make the quad stay on ground without lifting and after few seconds it will start to fly and later after reaching to specific height it will get stable over there.

3.4 Simulink Model of Quadcopter with Controller:

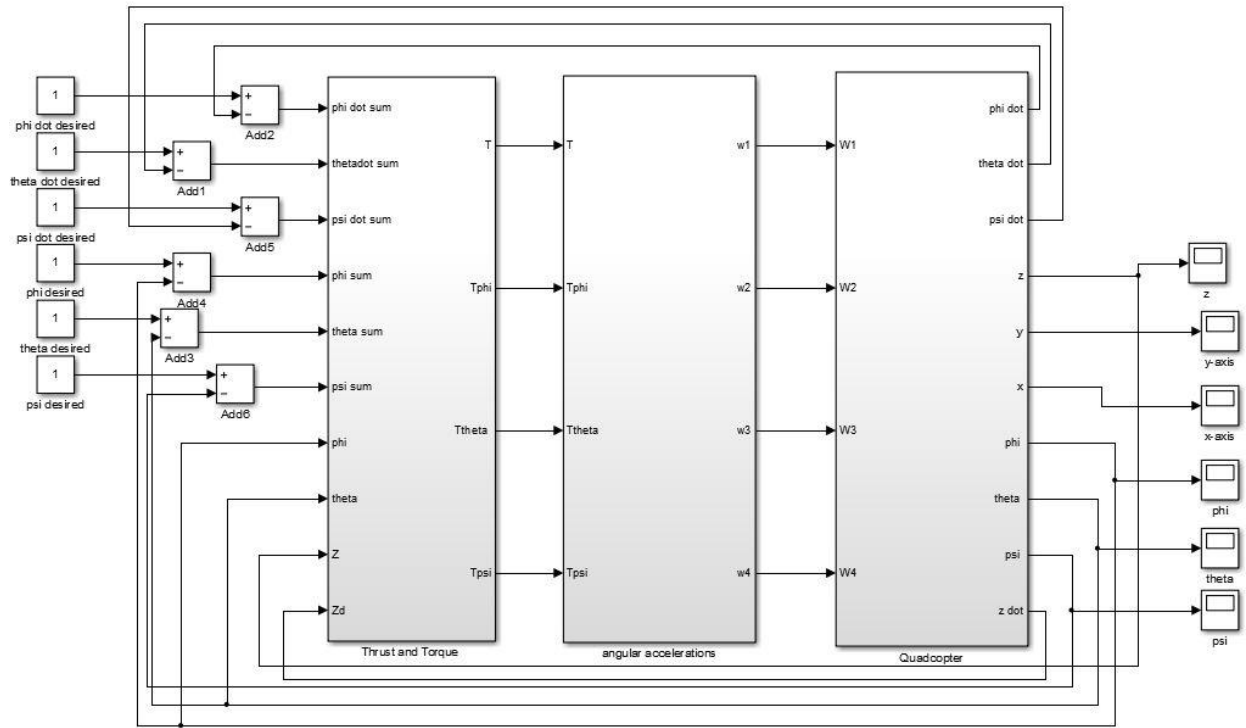


Figure 14. Complete quadcopter model along with controller.

The above Figure16 shows the complete model of quadcopter along with controller logic designed in Simulink to test the performance of model for the stability of quadcopter and by considering the parameter values from Table 1.

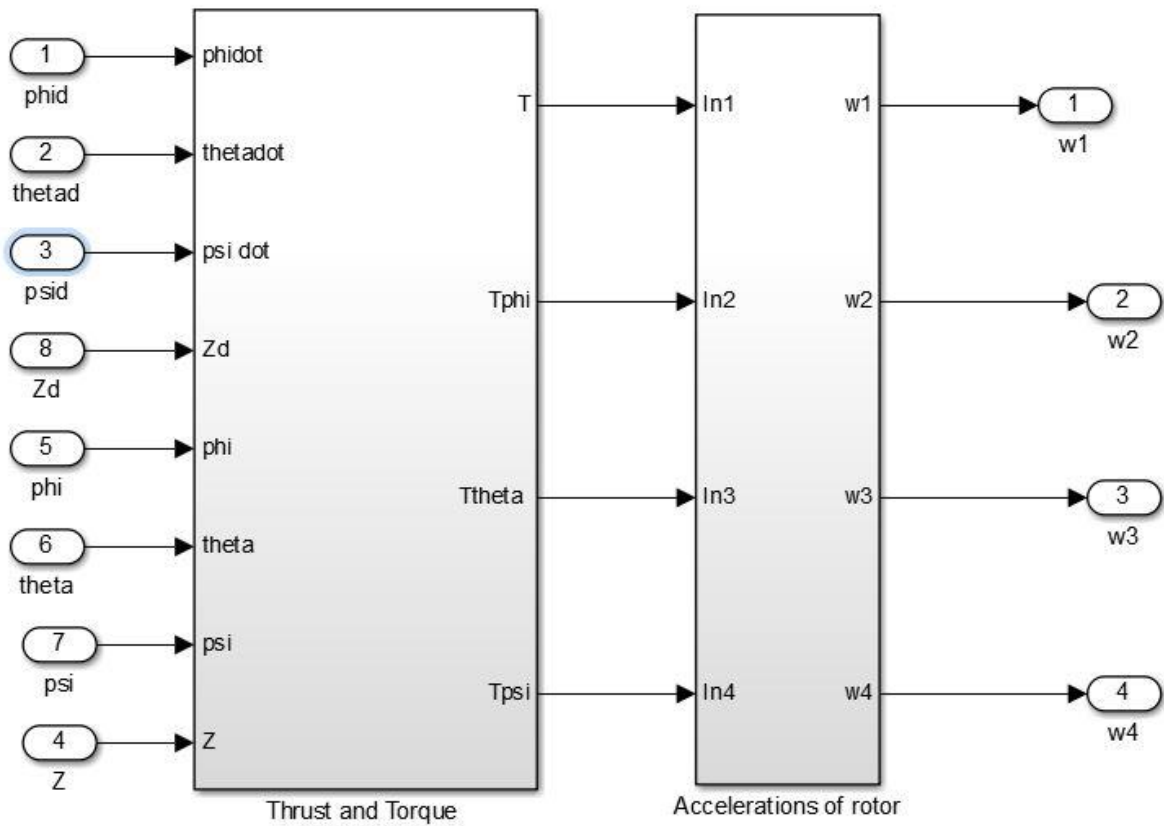


Figure 15. Blocks in the controller.

As we have two sets of equations in design of controller initially from the desired and present values of angles and position, parameters thrust and torques are designed and from them angular velocities are calculated the above Figure 17 shows the logic in between them.

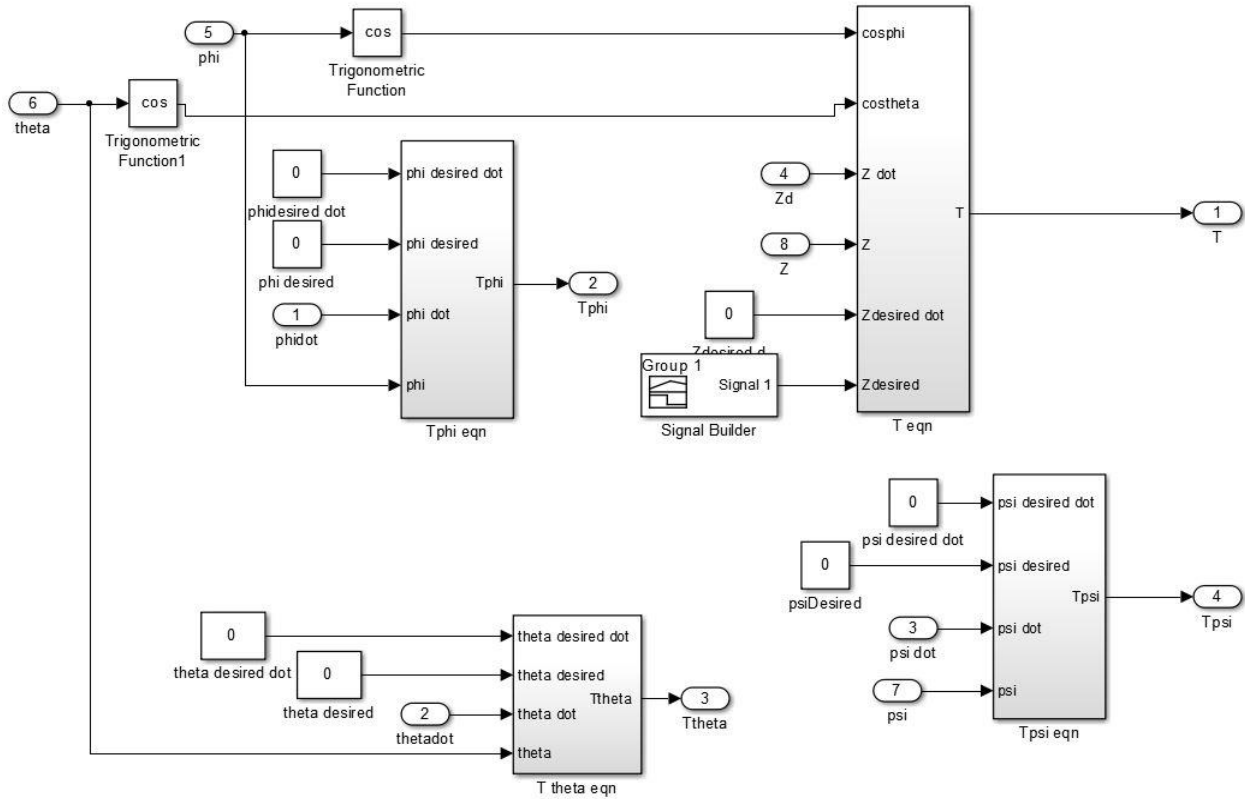


Figure 16. Simulink model for Thrust and Torque equations.

The logic shown in Figure 20 is the one behind designing the equations of thrust and torques which is shown in the Figure18. T eqn in the above figure represents the equation of thrust, T phi eqn represents the equation of angle phi torque, T theta eqn represents the equation of angle theta torque, T psi eqn represents the equation of angle psi torque.

CHAPTER IV

SIMULATIONS AND RESULTS

4.1 Simulation Results of Uncontrolled System

The simulation of mathematical modeling of quadcopter is implemented using matlab. Parameter values are taken from Table 1.

Table 1 Parameter values for simulation

Parameter	Value	Unit
g	9.81	m/s^2
m	0.468	kg
l	0.225	m
k	$2.980 * 10^{-6}$	-
b	$1.140 * 10^{-7}$	-
I_M	$3.357 * 10^{-5}$	$kg\ m^2$
I_{xx}	$4.856 * 10^{-3}$	$kg\ m^2$
I_{yy}	$4.856 * 10^{-3}$	$kg\ m^2$
I_{zz}	$8.801 * 10^{-3}$	$kg\ m^2$

The model is simulated by using the parameter values from the above parameter table using matlab. These parameter values are taken from [15]

Initially the quadcopter is tested for stable state in which positions and angles are zeros; the inertial frame is coinciding with the body frame. To attain this state, theoretically the thrust should be equal to gravity and the thrust is equal to hover thrust.

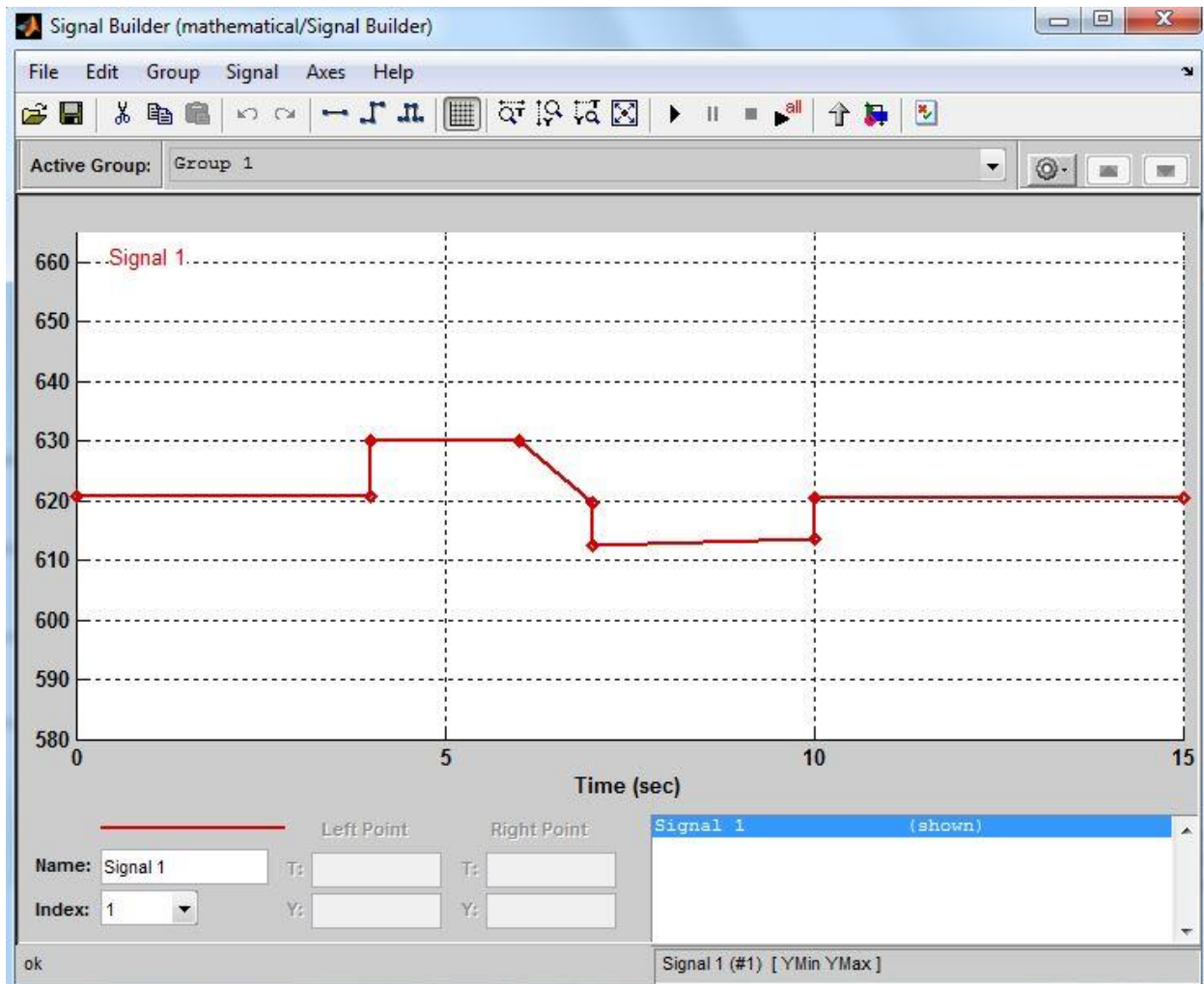


Figure 17. Input for hovering state.

The hovering input is shown in Figure 17; it shows for 4 sec it is given as 620.610(rad/s) and later it is directly increased to 630 rad/s and remained constant for 2 sec then decreased to 612 and then increased to 620.610 again and kept constant. 620.610 is the stabilizing velocity calculated by equating hovering thrust to gravity. The result of this input is same as we required.

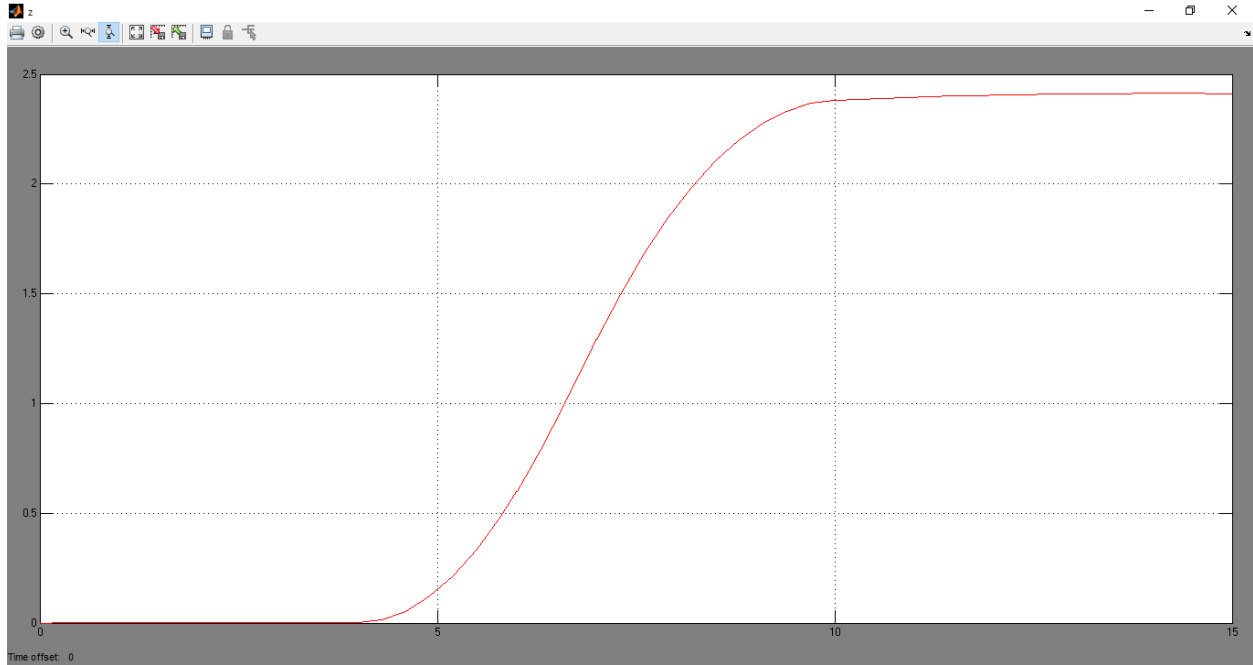


Figure 18. Output along Z axis.

The output in Figure 18 shows the quad is at zero position up to 4 sec and keeps on increasing to 2.5 meters in 10 sec and then tries to get stabilized for the remaining 5 sec.

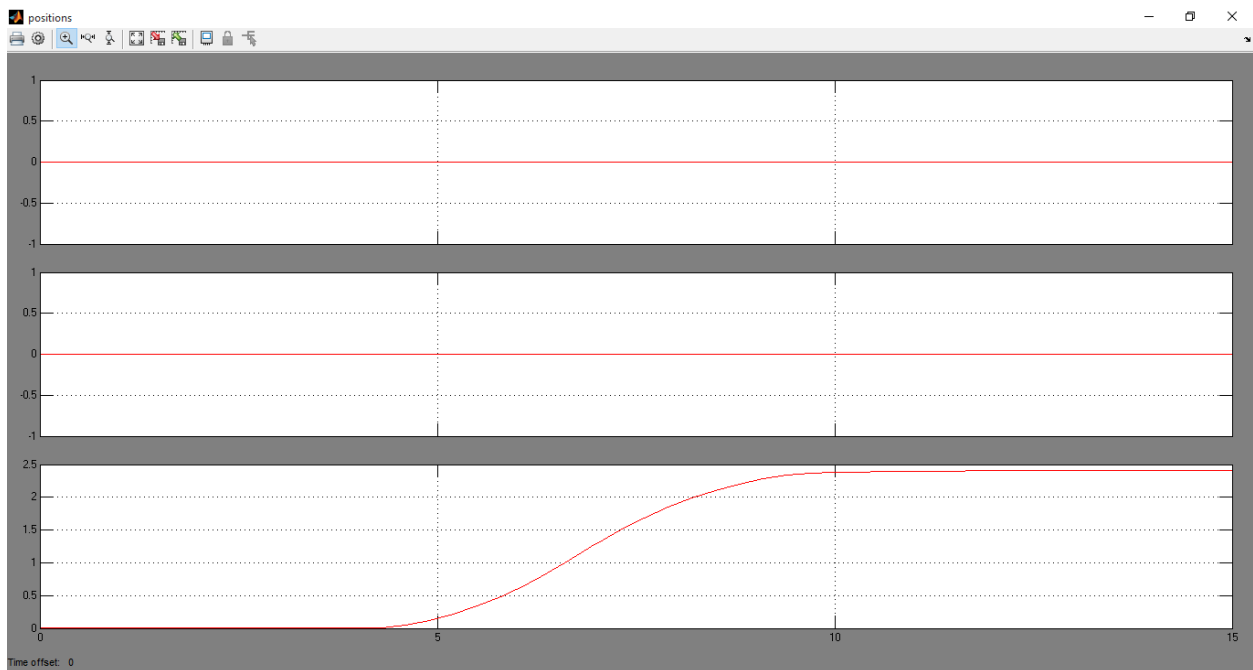


Figure 19. Output along X, Y and Z.

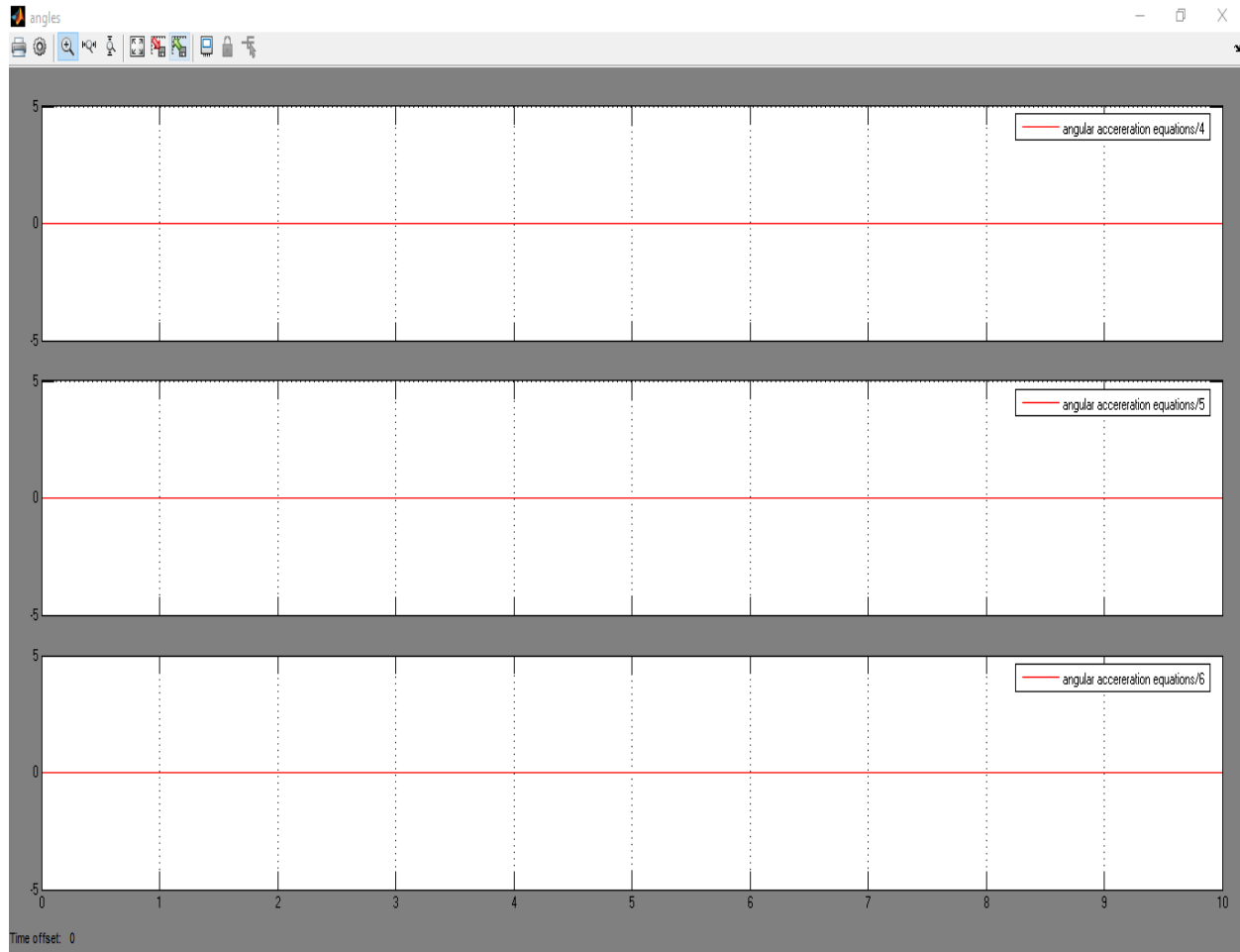


Figure 20. Output along roll, pitch and yaw.

As the state of quadcopter is hover that means it should move only in z direction and all the other motions should be zero. Figure 19 shows the output along X, Y and Z axes respectively from top to bottom it also shows the motion of quadcopter is zero along X and Y. Figure 20 shows output in roll, pitch and yaw respectively.

4.2 PD Controller Simulation Results

Table 2 Parameters of Controller

Parameter	Value
$K_{z,D}$	2.5
$K_{z,P}$	1.5
$K_{\phi,D}$	1.75
$K_{\theta,D}$	1.75
$K_{\phi,d}$	1.75
$K_{\phi,P}$	6
$K_{\theta,P}$	6
$K_{\phi,P}$	6

The controller designed is tested for its performance using Simulink by considering the parameter values from Table 2. Desired values are taken according to the output required.

These results are generated after adding a controller to the initial quadcopter model shown in Figure13, and after adding the controller it will be seen as in Figure 14.

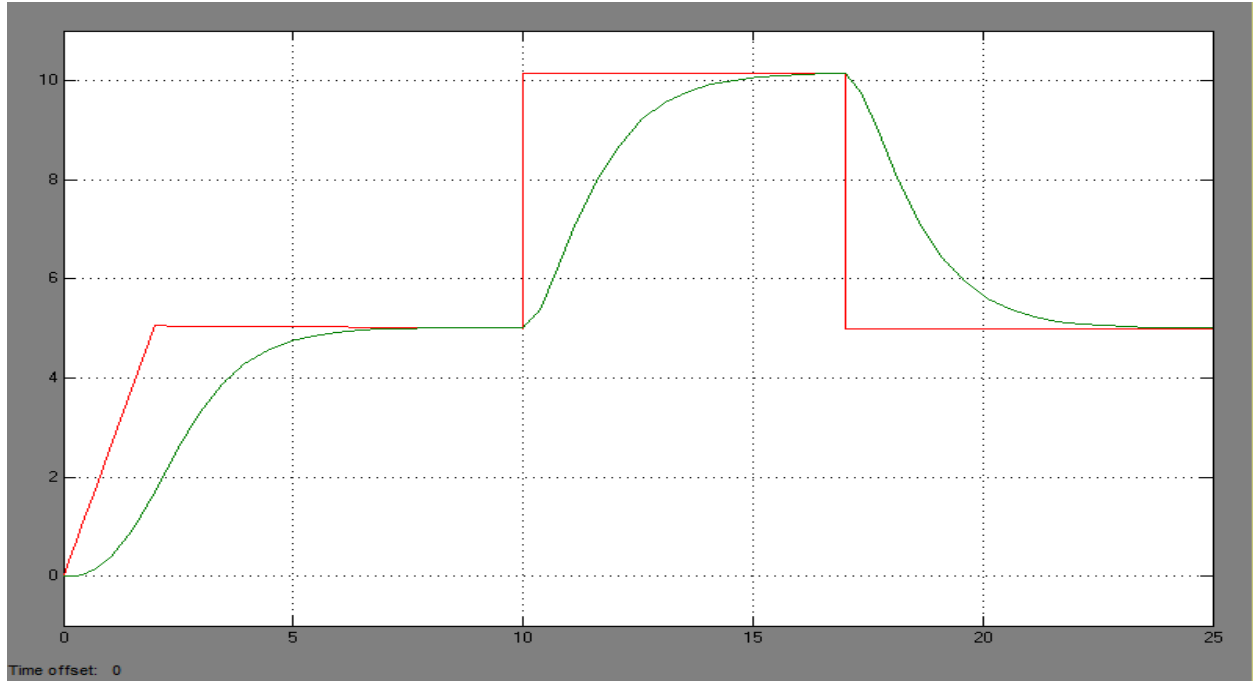


Figure 21. Comparison of input and output for 1st signal input $z(1)$.

The above Figure 21 shows the comparison between the input and output red line indicates input or desired z , and green line shows the output produced after implementing controller. Difference in the graph curve is because it gives out smooth flight at given sharp inputs.

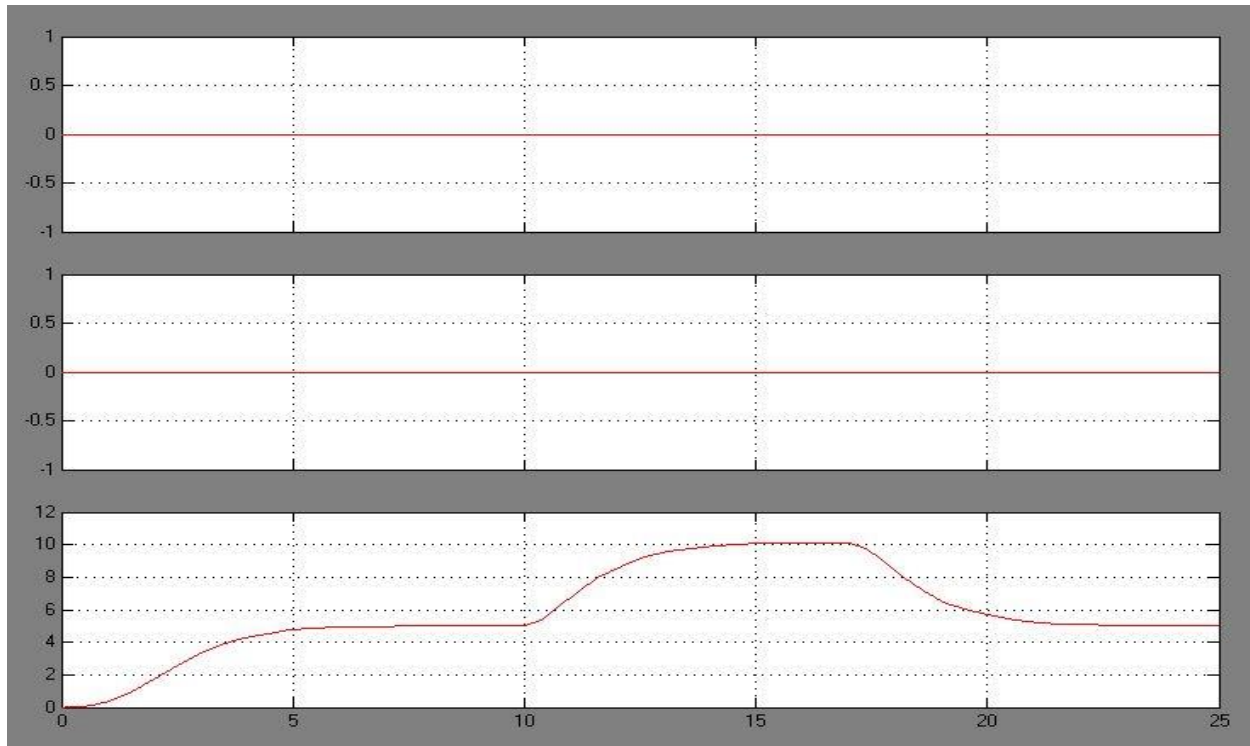


Figure 22. Output of positions.

Figure 22 shows the results of the simulation of quadcopter with a controller for which input; given as 0-5 for first 2 sec and kept constant as 5m up to 10sec and increased up to 10 m at 10 sec then kept constant for 16 sec then dropped to 5m and kept constant up to 25 sec. The result is also similar to this input, and in the x and z direction we are least bothered as the controller is applied for hover motion so we do not give any control input for x and y directions.

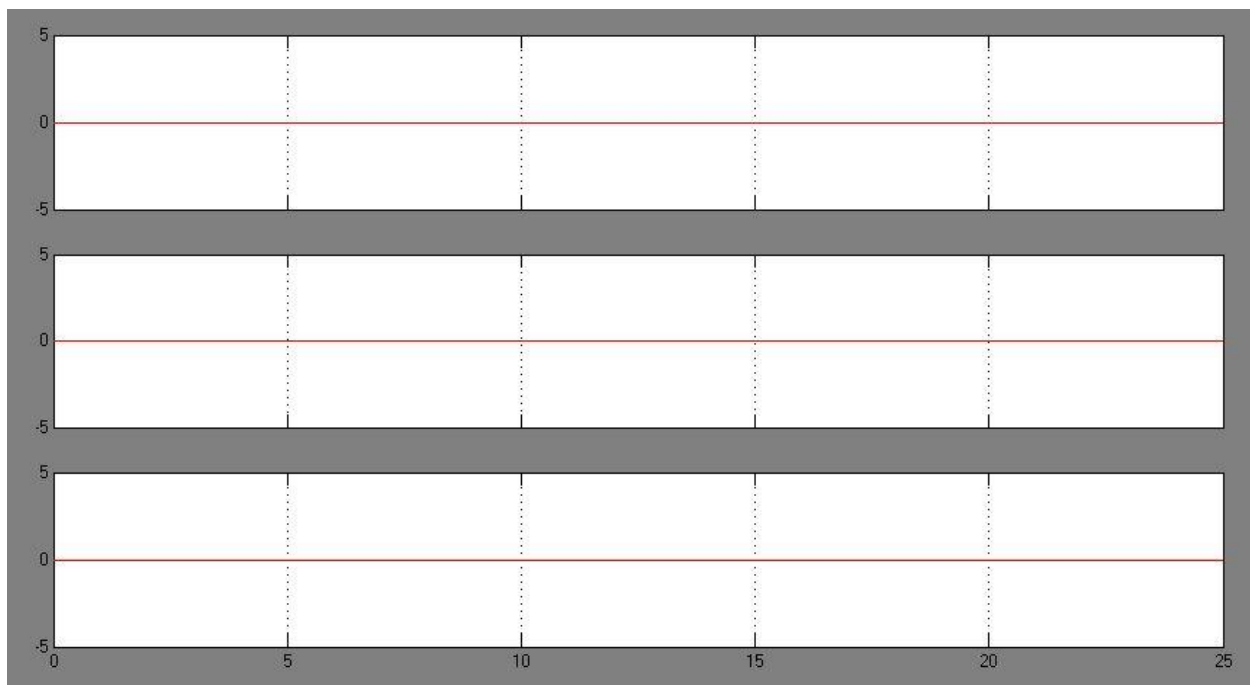


Figure 23. Output of angles.

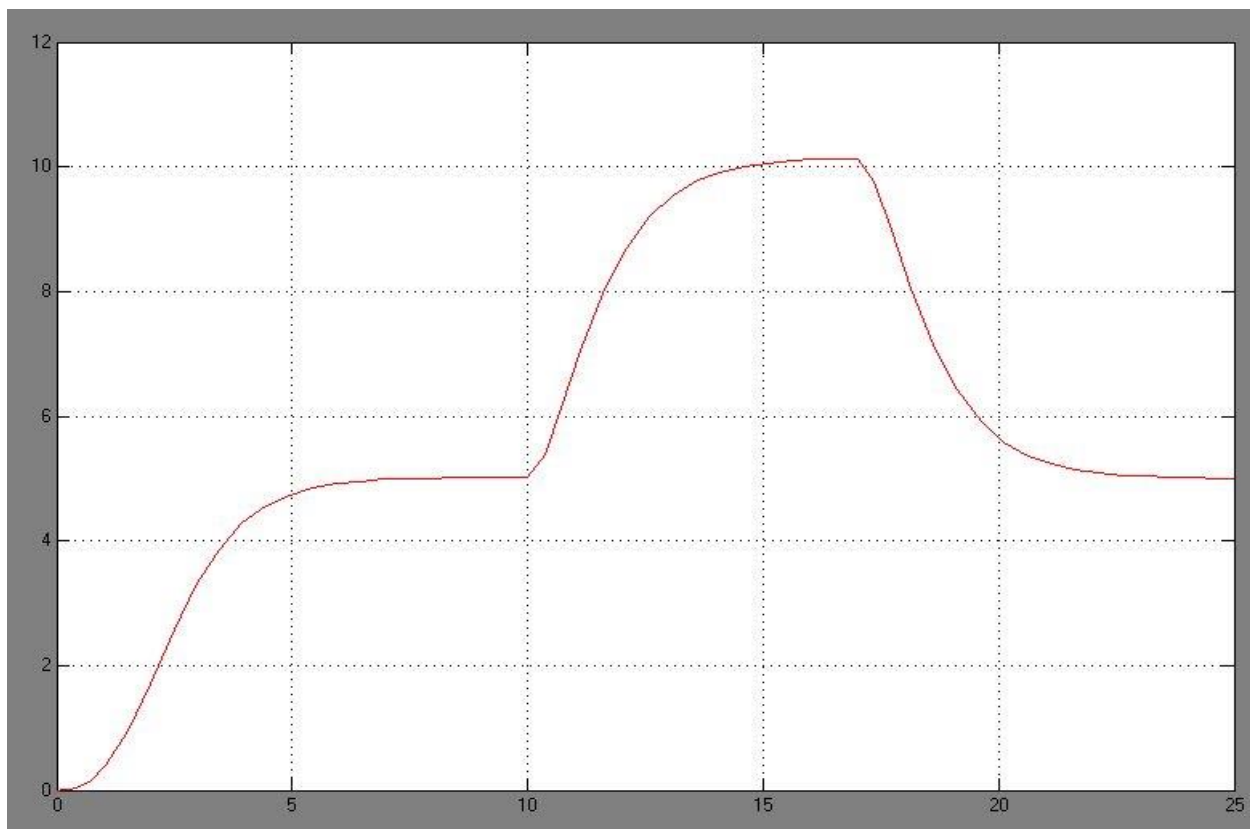


Figure 24. Output in Z direction.

The controller is tested for stability of quadcopter. Figure 21 shows the desired input in z direction. As it is a hover control the desired values will be given only for motion in z direction. There is gradual increase in input in the first 2 sec from zero to 5m, then it is kept constant up to 10 sec, followed by a sudden raise to 11m and kept constant up to 16 sec then there is a sudden drop till 5 meters and finally kept constant. As it is hover control Figure 22 shows motion in x, y and z direction there is no motion in x and y. Figure 23 shows the deflection of angles; we can see there is no deviation in angles so this means we have attained a stable flight using the designed PID controller. Figure 24 shows that the motion along z specifically looks very similar to the input that we desired to get, and we have tested the model for different types of inputs.

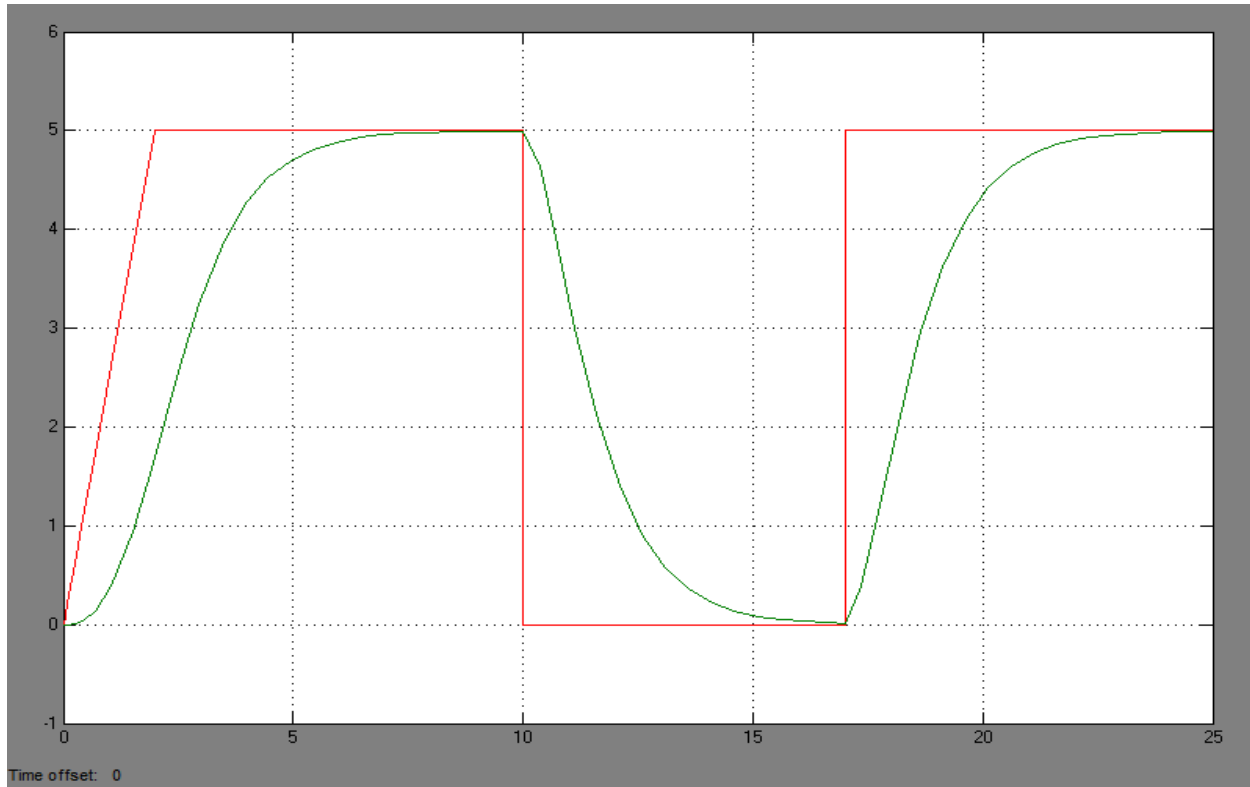


Figure 25. Comparison of input and output of for 2nd signal input z(2).

The above Figure 25 shows the comparison between the input and output. The red line indicates input or desired z , and the green line shows the output produced after implementing controller. The difference between these two lines is that the input line shows sharp changes in input acceleration but the output is given as a steep curve to make a smooth flight without any jerks or vibrations.

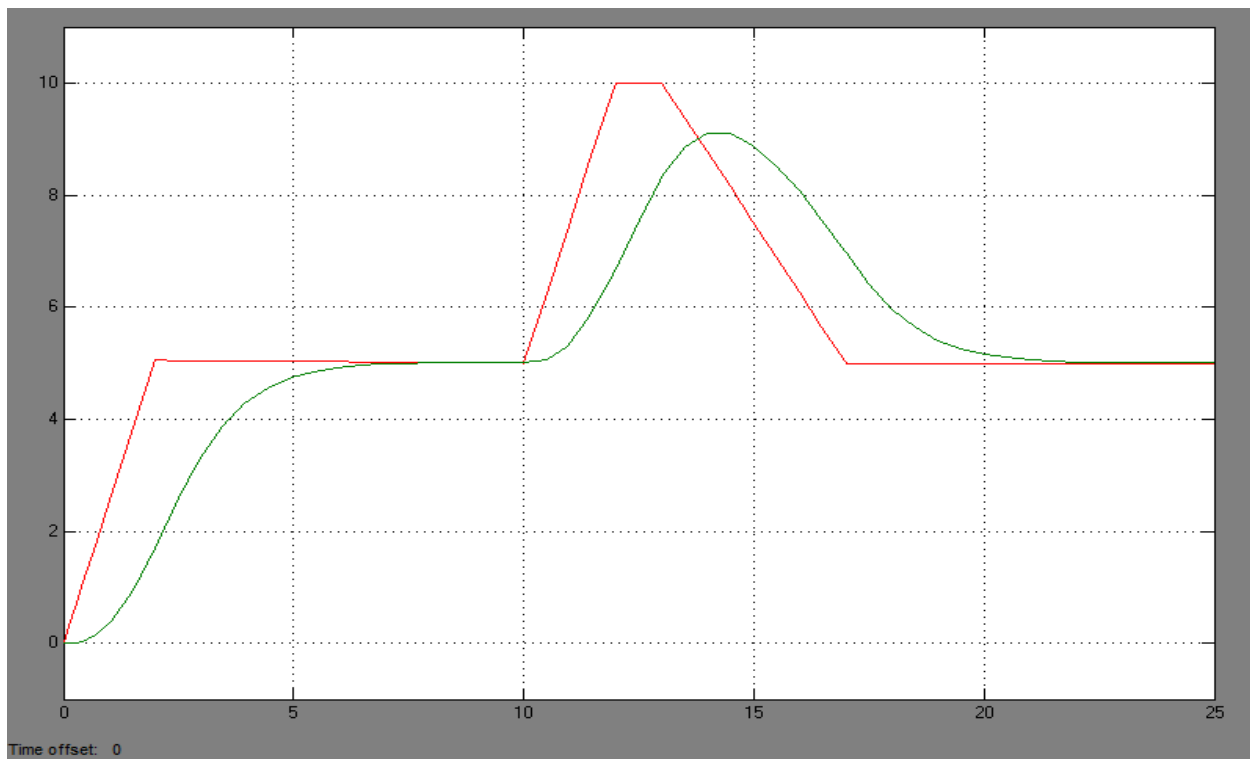


Figure 26. Comparison of input and output for 3rd signal input $z(3)$.

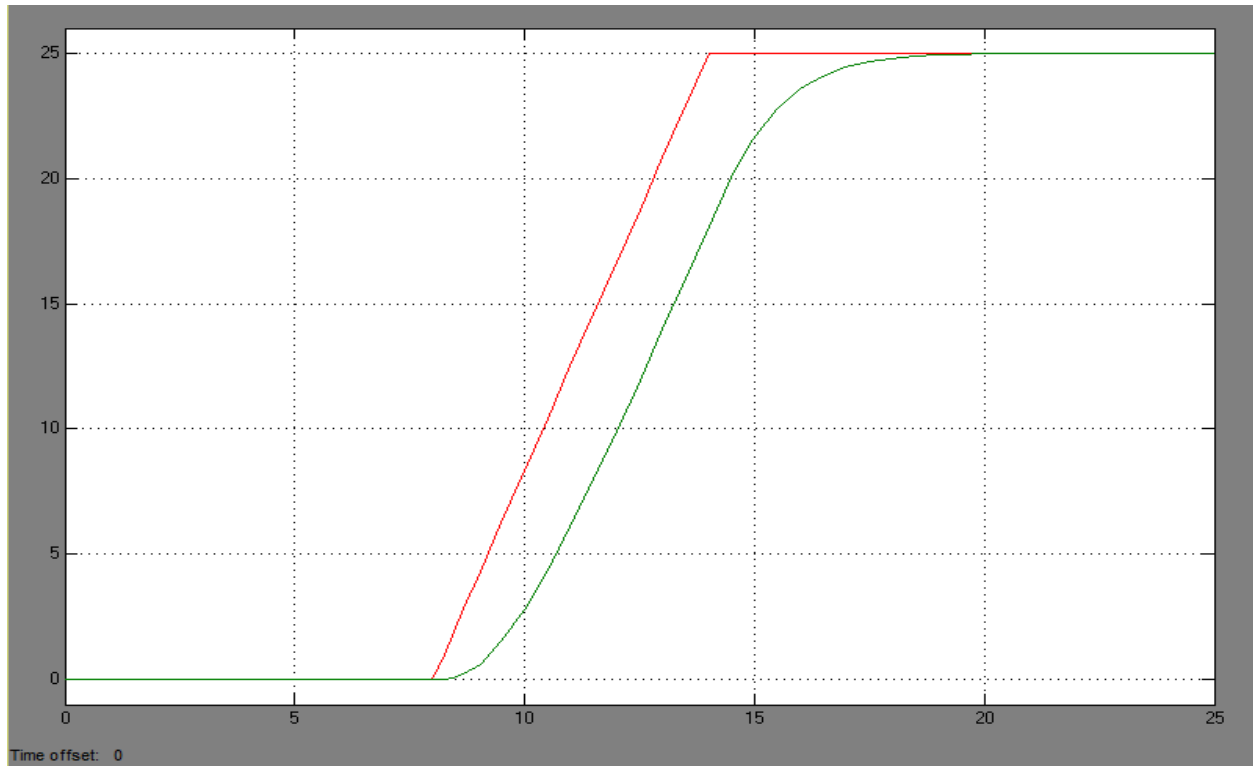


Figure 27. Comparison of input and output for 4th signal input $z(4)$.

The above Figures 25-27 show the comparison of system inputs and results for desired z values and output motion in z -direction. The red line indicates input or desired z , and the green line shows the output produced after implementing controller. The difference between these two lines is that the input line shows sharp changes in input acceleration but the output is given as a steep curve because it represents the smooth flight without any jerks or vibrations.

CHAPTER V

CONCLUSION AND FUTURE WORK

The purpose of the research is to design a quadcopter model and to stabilize it using a PD controller. The Quadcopter dynamics show the design of quadcopter model and that it has been implemented with the help of Simulink library blocks. Simulation results show the model is working fine without any errors and motion has been achieved. The quadcopter model is derived using Euler-Lagrange equations. Later, a PD controller was integrated to the model to attain the stability in hovering mode which means controlling the quadcopter motion in the z direction. The simulation results show that the desired altitude is attained by the controller which means that the controller is working efficiently without any errors. This model considers only the basic structure of a quadcopter. This model and the controller designed were tested using matlab simulations only. If a realistic model is used for the experimental results it would have given much more realistic values.

I would like to continue my work on the visualization model designed in solid works and control its speed in Matlab. This model may be adopted to develop a real time control system for that design. The benefit of using the adopted model helps us to eliminate the use of mathematical models, as we already give that input while designing it in solid works. All we have to do is use some simulink library blocks to build the model which reduces the burden in creating a mathematical model and inducing it to Matlab to design the results for a controller.

REFERENCES

- [1].Hanna, William (2014). Modelling and control of an unmanned aerial vehicle. Bachelor of Engineering (4th Year Project) Thesis, Charles Darwin University.
- [2].(n.d.). Retrieved December 11, 2014, from https://en.wikipedia.org/wiki/De_Bothezat_helicopter#/media/File:De_Bothezat_Quadrotor.jpg.
- [3].Rasmussen, S.J.; Mitchell, J.W.; Chandler, P.R.; Schumacher, C.J.; Smith, A.L., "Introduction to the MultiUAV2 simulation and its application to cooperative control research," American Control Conference, 2005. Proceedings of the 2005 , vol., no., pp.4490,4501 vol. 7, 8-10 June 2005.
- [4].Michael Achtelik, Thomas Bierling, Jian Wang, Leonhard Höcht, and Florian HolzapfelInfotech@Aerospace 2011. March.
- [5].Gaponov, I.; Razinkova, A., "Quadcopter design and implementation as a multidisciplinary engineering course," Teaching, Assessment and Learning for Engineering (TALE), 2012 IEEE International Conference on , vol., no., pp.H2B-16,H2B-19, 20-23 Aug. 2012.
- [6].Tomic, T.; Schmid, K.; Lutz, P.; Domel, A.; Kassecker, M.; Mair, E.; Grix, I.L.; Ruess, F.; Suppa, M.; Burschka, D., "Toward a Fully Autonomous UAV: Research Platform for Indoor and Outdoor Urban Search and Rescue," Robotics & Automation Magazine, IEEE , vol.19, no.3, pp.46,56, Sept. 2012.
- [7].C. Yun and X. Li, "Aerodynamic Model Analysis and Flight Simulation Research of UAV Based on Simulink," Journal of Software Engineering and Applications, Vol. 6 No. 2, 2013, pp. 43-47.

- [8].Chen Wang; Nahon, M.; Trentini, M. "Controller development and validation for a small quadrotor with compensation for model variation", Unmanned Aircraft Systems (ICUAS), 2014 International Conference on, On page(s): 902 – 909. 73
- [9].PID Theory Explained. (n.d.). Retrieved November 10, 2014, from <http://www.ni.com/white-paper/3782/en/>.
- [10].Trouwborst, Cees (2014) Control of quadcopters for collaborative interaction. <http://essay.utwente.nl/66109/> .
- [11].Jorge Miguel Brito Domingues. Quadrotor prototype. Master's thesis, Instituto Superior Tecnico, universidade Tecnica de Lisboa 2009.
- [12].(n.d.). Retrieved September 11, 2014, from <https://prezi.com/4ffrthtuh5d/the-final-solution/>
- [13].(n.d.). Retrieved July 7, 2015, from http://www.atncorp.com/night_vision_images/products/198/big.jpg.
- [14].Magnussen, Øyvind ; Skjønhaug, Kjell Eivind http://idtjeneste.nb.no/URN:NBN:no-bibsys_brage_20668.
- [15].(n.d.). Retrieved July 14, 2015, from <http://site.hobbypartz.com/pimages/05P-P1-1111-APM-GPS-Upgrade-KIT/700/05P-P1-1111-APM-GPS-Upgrade-KIT-1.jpg>
- [16].(n.d.). Retrieved March 30, 2015, from https://upload.wikimedia.org/wikipedia/commons/thumb/5/5d/Centrale-intertielle_missile_S3_Musee_du_Bourget_P1010652.JPG/220px-Centrale-intertielle_missile_S3_Musee_du_Bourget_P1010652.JPG

- [17].(n.d.). Retrieved February 12, 2015, from http://www.nidec.com/en-NA/technology/capability/brushless/~media/nidec-com/en-global/technology/capability/brushless/img_brushless_01
- [18].HobbyKing Page not found. (n.d.). Retrieved January 20, 2015, from <http://hobbyking.com/hobbyking/store/catalog/TR-P18A>.
- [19].(n.d.). Retrieved August 27, 2015, from http://www.championhobby.com/img/Products/205/1_1_HY01-10-GP-Propellers.jpg
- [20].(n.d.). Retrieved May 15, 2015, from <http://hobbyking.com/hobbyking/store/catalog/T2200-3-20.jpg>
- [21].(n.d.).Retrieved April 14, 2015, from <https://encrypted-tbn2.gstatic.com/images?q=tbn:ANd9GcTBK7ok3U8QMwBKMEFF0rBBHJHM9TpBZRmpVQxPVBYi1uIPn3aJ>
- [22].G. V. Raffo, M. G. Ortega, and F. R. Rubio, “An integral predictive/nonlinearH1 control structure for a quadrotor helicopter,” *Automatica*, vol. 46, no. 1,pp. 29–39, 2010.
- [23].K. J. Åström and T.H`agglund, *Advanced PID Control*. ISA - Instrumentation, Systems and Automation Society, 2006.
- [24].I. C. Dikmen, A. Arisoy, and H. Temelta,s, “Attitude control of a quadrotor,” 4th International Conference on Recent Advances in Space Technologies, pp. 722–727, 2009.
- [25].Tayebi, A.; McGilvray, S., "Attitude stabilization of a four-rotor aerial robot," in *Decision and Control*, 2004. CDC. 43rd IEEE Conference on , vol.2, no., pp.1216-1221 Vol.2, 14-17 Dec. 2004
doi: 10.1109/CDC.2004.1430207.

APPENDIX A

Code to Initiate the Parameter Values

Code to initiate the parameter values:

```
clear all;

clc;

m=0.468; % mass (kg)

Ixx= .004856;% inertia at rotation axis(kg/m2)

Iyy= .004856;% inertia at rotation axis(kg/m2)

Izz= .008801;% inertia at rotation axis (kg/m2)

g=9.81; % gravity(m/s2)

Im=3.357e-05;% Moment of inertia (kg/m2)

l=0.225;% length (m)

k= 2.980e-06;

b=1.140e-07;

a=m*g;

d=4*k;

c= a/d;

w= sqrt(c);

simout=sim('mathematical','timeout',25)
```

Code to initiate the parameter values for controller:

```
clear all;

clc;

m=0.468; % mass (kg)

Ixx= .004856;% inertia at rotation axis(kg/m2)

Iyy= .004856;% inertia at rotation axis(kg/m2)
```

$I_{zz} = .008801;$ % inertia at rotation axis (kg/m²)

$g = 9.81;$ % gravity(m/s²)

$I_m = 3.357e-05;$ % Moment of inertia (kg/m²)

$l = 0.225;$ % length (m)

$k = 2.980e-06;$

$b = 1.140e-07;$

$\dot{z}_{desired} = 0;$

$\dot{z}_{dot} = 0;$

$z_{Desired} = 0;$

$z = 1;$

$K_{zDerivative} = 2.5;$

$K_{\phi Derivative} = 1.75;$

$K_{\theta Derivative} = 1.75;$

$K_{\psi Derivative} = 1.75;$

$K_{zProportional} = 1.5;$

$K_{\phi Proportional} = 6;$

$K_{\theta Proportional} = 6;$

$K_{\psi Proportional} = 6;$

$\dot{\phi}_{desired} = 0;$

$\dot{\theta}_{desired} = 0;$

$\dot{\psi}_{desired} = 0;$

$K_{\dot{\phi}} = 0;$

$K_{\dot{\theta}} = 0;$

$$K_{psdot}=0;$$

VITA

Sravva Julakanti done with my masters in Mechanical Engineering. Right from her schooling days she had passion towards airplanes and robots which made her take up Mechanical Engineering as her major during her bachelor's. In the course of her undergraduate work she minored in controls and aerodynamics which led her to Texas A&M University-Kingsville where she is pursuing her master's in Mechanical Engineering and became a trainer in matlab.