Design and verification of star-map simulation software based on CCD star tracker

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Abstract—Reliable simulating star map is the important data source for ground test and performance test of star tracker. In order to provide star maps for test conveniently, a star map simulation software was developed in this paper. The imaging position of observing stars on the CCD plane can be calculated by constructing an ideal model of star sensor. By using the Gaussian gray diffusion model to simulate imaging of star and adding some noise signal to make the simulation of real sky better. The validity of simulating star map is verified by using angle-distance of two stars, calculation result shows that the mean value of optional angle-distance is 0.017613 degree. The result shows that the simulation software design method is reliable, high precision and easy to implement.

Keywords- star tracker; star map simulation; angle distance of stars

I. INTRODUCTION

With the development of aerospace technology and the maturity of navigation technology, higher navigation accuracy and reliability are required by satellite, missile and other spacecraft. Star sensor is the measurement device using stars as the reference frame. It measures attitude and position with a high accuracy and provides accurate spatial orientation and attitude reference for medium and long-range missile. With the advantages of light weight, low-power, zero drift and multiple working modes, it has been widely applied to the field of aerospace and navigation^[1]. The entire working process of star tracker is subdivided into four operations. First, shooting star map by image sensor. Second, extracting information of observed stars by processing the star map. Third, using star pattern recognition algorithms to find the matching navigation stars for unknown stars in star map. Fourth, calculating attitude of three axis through out attitude calculated algorithm.

It should take ground function test to check the validity and reliability of the function of each part before star tracker put into use. Star pattern recognition algorithm test needs star maps to calculate attitude, which is the key to obtaining the star maps. Because of huge cost of space experiments, real-time space experiment is unrealistic. Real-time ground test under starry sky is another choice of the function test and it is easy to realize. But the weather, terrain and other factors make the ground test can not guarantee the require of immediate test. Therefore, using computer simulate starry sky on the ground becomes a common method in star tracker

function test to provide materials for star pattern recognition algorithm test^[2]. Paper [3] given the design method of star map simulator, but it did not discuss the star imaging model. Paper [4] designed a dynamic star map simulator using LCOS device as the display device. This method of combining star map simulation software with high resolution LCD makes the simulation of angle and center position of single star more accurate. But that paper dose not consider the size of star imaging. It is not able to accurately simulate the imaging of star on the CCD target surface if selects star size artificially. The purpose of this paper is design one kind of CCD star-map simulation software based on using the Gaussian gray diffusion model to simulate imaging of star. We can check the validity of star-map simulation by calculating the angle distance between stars. Simulation results show that the star-map simulation is effective and high position precision.

II. DESIGN OF STAR-MAP SIMULATION SOFTWARE

The entire working process of star-map simulation software is subdivided into four operations. First, selecting the orientation of star tracker's boresight randomly. Second, confirming the scope of simulation according to the RA(right ascension), DEC(declination) and the range of star tracker's FOV(field of view). Third, transforming the coordinates and magnitude of the eligible observed stars extracted from the star catalogue database. Forth, adding noise on simulation star-map and displaying it. The process shows in Figure 1.

A. Coordinate Transformation

Star-map simulation is the progress that the software creates a corresponding real-time star-map according to the orbital elements of the spacecraft. The stellar coordinates stored in star catalogue are the coordinates in celestial coordinate system called RA and DEC. They are not the coordinates we need in the display LCD coordinate system. So, it is necessary to do the coordinates transformation. We only change the coordinate basis of the vector. This measure could make sure that the magnitude and direction of vector remain unchanged. There are two steps to do the transformation. One is the rotational transform from celestial coordinate system to star tracker's coordinate system. Another is the projection transform from star tracker's coordinate system to CCD target surface coordinate system.



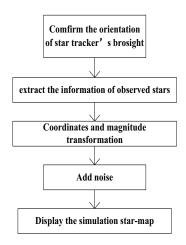


Figure 1. Process of star-map simulation

We can define that the coordinates of star can be describe in different coordinate system like this, (α , δ) for RA and DEC in celestial coordinate system like, (X, Y, Z) for direction vector in star tracker's coordinate system and (x, y) for coordinates in imaging target surface. Figure 2 shows the model of star tracker imaging.

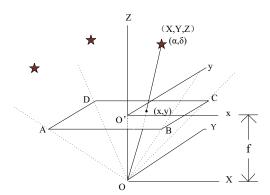


Figure 2. Model of star tracker imaging

Star tracker is fixedly installed on the certain place of spacecraft. So, we only consider the rotational transformation as the transform from celestial coordinate system to star tracker's coordinate system. Figure 3 shows that O-UVW denotes the celestial coordinate system, O'-XYZ denotes the star tracker's coordinate system, O denotes the earth's core, O' denotes the optical center of star tracker's coordinate. Considering the distance between earth and other fixed stars except the sun is beyond one's reach, error caused by transform from celestial coordinate system to star tracker's coordinate system can be ignored. We can treat O and O' as they are coincident^[5].

Express rotational vector as follow:

$$[X, Y, Z]^{T} = M[U, V, W]^{T}$$
 (1)

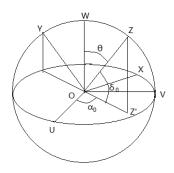


Figure 3. Celestial coordinate system to star tracker's coordinate system

based on the principle of 3-1-3 coordinate rotation^[6], we can express the vector M as follow:

$$M = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} \bullet \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \varphi \cos \psi - \sin \varphi \sin \psi \cos \theta & \sin \varphi \cos + \cos \varphi \sin \psi \cos \theta & \sin \psi \sin \theta \\ -\cos \varphi \sin \psi - \sin \varphi \cos \psi \cos \theta & -\sin \varphi \sin \psi + \cos \varphi \cos \psi \cos \theta & \cos \psi \sin \theta \\ \sin \varphi \sin \theta & -\cos \varphi \sin \theta & \cos \theta \end{bmatrix}$$

where ϕ , θ , ψ are the rotation angles. Coordinate of the orientation of star tracker's boresight is (α_0 , δ_0), according to the rotation relationship θ =90°- δ_0 and ϕ =90°+ α_0 , the vector M can be described as follow:

$$M = \begin{bmatrix} -\sin\alpha_0\cos\psi - \cos\alpha_0\sin\psi\sin\delta_0 & \cos\alpha_0\cos\psi - \sin\alpha_0\sin\psi\sin\delta_0 & \sin\psi\cos\delta_0 \\ \sin\alpha_0\sin\psi - \cos\alpha_0\cos\psi\sin\delta_0 & -\cos\alpha_0\sin\psi - \sin\alpha_0\cos\psi\sin\delta_0 & \cos\psi\cos\delta_0 \\ \cos\alpha_0\cos\delta_0 & \sin\alpha_0\cos\delta_0 & \sin\alpha_0 \end{bmatrix}$$

direction vector of fixed star can be described as $[U,V,W]^T$ in celestial coordinate system. It could express vector M with α and δ as follow:

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} \cos \alpha \cos \delta \\ \sin \alpha \cos \delta \\ \sin \delta \end{bmatrix}$$
 (4)

the vector $[X, Y, Z]^T$ can be calculated based on express (1), (2) and (4).

After getting the vector expression in star tracker's coordinate system, we do the projection transformation to calculate the coordinates in imaging target surface as follow:

$$\begin{cases} x = \frac{n}{2} \cdot \frac{1}{\tan(FOV_x/2)} \cdot \frac{\cos \psi \cos \delta \sin(\alpha - \alpha_0) - \sin \psi \sin \delta_0 \cos \delta \cos(\alpha - \alpha_0) + \sin \psi \cos \delta_0 \sin \delta}{\sin \delta \sin \delta_0 + \cos \delta \cos \delta_0 \cos(\alpha - \alpha_0)} \\ y = \frac{m}{2} \cdot \frac{1}{\tan(FOV_y/2)} \cdot \frac{\cos \psi \cos \delta_0 \sin \alpha - \cos \psi \sin \delta_0 \cos \delta \cos(\alpha - \alpha_0) - \sin \psi \cos \delta \sin(\alpha - \alpha_0)}{\sin \delta \sin \delta_0 + \cos \delta \cos \delta_0 \cos(\alpha - \alpha_0)} \end{cases}$$

$$(5)$$

where x, y are the pixel coordinates, n is pixel numbers in each line of CCD surface, m is pixel numbers in each column of CCD surface, FOV_{\perp} is field angle on x axis,

$$FOV_y$$
 is field angle on y axis, $x \in (-\frac{n}{2}, \frac{n}{2}), y \in (-\frac{m}{2}, \frac{m}{2})$.

We can get (x_z, y_z) after taking round-off with (x,y). $\Delta x = x - x_z$ and $\Delta y = y - y_z$ are the deviations.

By the above methods, we calculate the coordinates in CCD surface where the origin of coordinates is different from the one on computer screen. We should translate the coordinates with the method:

$$\begin{cases} x' = x_z + \frac{n}{2} \\ y' = y_z + \frac{m}{2} \end{cases}$$
 (6)

B. Principle of Picking Observed Stars in Field of View

After confirming the orientation of star tracker's boresight, we pick up the eligible observed stars from star catalogue database with the character as follows^[7]:

$$\begin{cases} \alpha \in (\alpha_0 - FOV_x / 2, \alpha_0 + FOV_x / 2) \\ \delta \in (\delta_0 - FOV_y / 2, \delta_0 + FOV_y / 2) \end{cases}$$
 (7)

the stars like that can imaging on CCD surface. We set $C_1 = \alpha_0 - FOV_x/2$, $C_2 = \alpha_0 + FOV_x/2$, $C_3 = \delta_0 - FOV_y/2$, $C_4 = \delta_0 + FOV_y/2$. If C_1 and C_2 are not in range of $(0, 2\pi)$ or C_3 and C_4 are not in range of $(0, \pi/2)$, we should correct the value as follows^[8]:

$$\begin{cases} C_{1} < 0 & C_{1} + 2\pi \to C_{1} \\ C_{2} > 2\pi & C_{2} - 2\pi \to C_{2} \\ C_{3} < -\frac{\pi}{2} & C_{3} + \pi \to C_{3} \\ C_{4} > \frac{\pi}{2} & C_{4} - \pi \to C_{4} \end{cases}$$
(8)

C. Imaging model of stars

Not only the coordinates of stars is the information we need to simulate star-map but also we need to know the star's brightness that the gray displayed on the imaging screen. Star's brightness shows different gray on a monochrome picture. According to the references, linear transformation is the common practice in magnitude-gray transformation. Smaller value of magnitude is , brighter the brightness and the larger gray is. Grey g expresses as follow:

$$g = k(m_{\text{max}} - m_i) + g_0 \tag{9}$$

where the range of g is (0, 255) because there are 256 gray level of computer screen, $m_{\rm max}$ is threshold value of observed star's magnitude, m_i is the magnitude of star, k is a adjustable parameter to make sure that the gray is within the range of values.

For star tracker, the fixed star can be regarded as a spot light. If the shooting system is the ideal optical system, the star imaging on the CCD surface occupy only one pixel. But it can not meet the demand of attitude measurement. The solution of this is to do the defocusing to make sure that the spot of star is about 3 pixels. We can realize sub-pixel location in attitude measurement^[9]. The method of

defocusing is using the Gaussian gray diffusion model to simulate imaging star's gray. We suppose the coordinate of dispersal center is (x', y'), thus the gray of pixel (x_0, y_0) is:

$$g = \frac{H}{2\pi\sigma^2} \exp(-\frac{(x_0 - x' - \Delta x) + (y_0 - y' - \Delta y)}{2\sigma^2})$$
 (10)

where σ is related to the extent of defocusing and the size of aberration and it is a constant while the star tracker fixed on the spacecraft. When $\sigma < 0.671$, above 95% energy of the imaging star is focused on 3*3 pixels. H has a fixed linear function relation with magnitude^[10].

III. EFFECTIVE METHOD FOR VERIFYING THE VALIDITY OF SIMULATION

Angle distance of stars is the key point for star pattern recognition. We can verify the validity though out verifying the angle distance is correct or not^[11].

Randomly choose two stars named A_1 and A_2 , the unit vector of them in celestial coordinate system express as follows:

$$I_{1} = \begin{bmatrix} \cos \alpha_{1} \cos \delta_{1} \\ \sin \alpha_{1} \cos \delta_{1} \\ \sin \delta_{1} \end{bmatrix}$$
 (11)

$$I_{2} = \begin{bmatrix} \cos \alpha_{2} \cos \delta_{2} \\ \sin \alpha_{2} \cos \delta_{2} \\ \sin \delta_{2} \end{bmatrix}$$
 (12)

Making the unit vector projected to unit sphere creates two points. Each coordinate of point expresses as follows:

$$A_1(x_1, y_1, z_1) = A_1(\cos \alpha_1 \cos \delta_1, \sin \alpha_1 \cos \delta_1, \sin \delta_1)$$

$$A_2(x_2, y_2, z_2) = A_2(\cos \alpha_2 \cos \delta_2, \sin \alpha_2 \cos \delta_2, \sin \delta_2)$$
(13)

Angle distance of them expresses as follow:

$$\theta_{12} = 2 \arcsin \left| \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} / 2 \right|$$
 (14)

Unit projected vector of A_1 and A_2 in star tracker's coordinate system express as follows:

$$m_{1} = \frac{1}{\sqrt{x_{1}^{2}a_{x}^{2} + y_{1}^{2}a_{y}^{2} + f^{2}}}$$

$$m_{2} = \frac{1}{\sqrt{x_{2}^{2}a_{x}^{2} + y_{2}^{2}a_{y}^{2} + f^{2}}}$$
(15)

Where (x_1, y_1) and (x_2, y_2) are the coordinates of imaging stars A_1 and A_2 on imaging surface. Their angle distance expresses as follow:

$$\theta_{12}' = \arccos(m_1, m_2) = \arccos\frac{x_1 x_2 a_x^2 + y_1 y_2 a_y^2 + f^2}{\sqrt{x_1^2 a_x^2 + y_1^2 a_y^2 + f^2} \sqrt{x_2^2 a_x^2 + y_2^2 a_y^2 + f^2}}$$
(16)

Because the display screen is the computer screen, $a_x = a_y = a$, equation (16) turns to be:

$$\theta_{12}^{'} = \arccos(m_1, m_2) = \arccos\frac{x_1 x_2 + y_1 y_2 + f^{'2}}{\sqrt{x_1^2 + y_1^2 + f^{'2}} \sqrt{x_2^2 + y_2^2 + f^{'2}}}$$
Where $f' = \frac{N}{2\tan(FOV/2)}$. If θ_{12} equals to $\theta_{12}^{'}$, the simulation is effective.

IV. TESTING AND VALIDATION

We use visual C++6.0 to design the simulation software. Simulated conditions are: the field of view is $8^{\circ} \times 8^{\circ}$, the resolution of CCD is 512×512 , Upper limit of dynamic visual magnitude threshold is 6.0 and the star catalog of SAO type contains 5006 stars^[12]. Choosing (20° , 20° , 90°) as the attitude angles of three-axis, the simulation star-map contains 11 stars in the field of view. Add white gaussian noise which mean-value is 0 and variance is 0.1 as the background noise of star-map. Fig. 4 shows the simulation star-map, where a is the map without background noise and b adds b that. Figure 5 shows the magnified region looped of Figure 4.

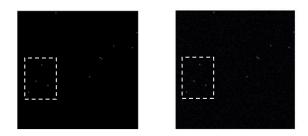


Figure 4. Simulation star-map

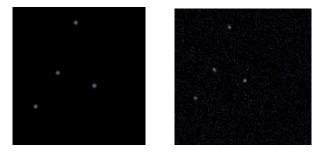


Figure 5. Magnified certain region of star-map

The magnitude and coordinate of observed stars in the field of view show in table 1.

Table 1. Magnitude , coordinate and coordinate error of observed stars

No.	RA	DEC	Magnitude	X	y	$\mathbf{x}_{\mathbf{z}}$	$\mathbf{y}_{\mathbf{z}}$	Δx	Δy
1	18.5318	16.1335	5.970	35.571	503.451	36	503	0.4290	0.4510
2	23.9783	17.4337	5.910	377.784	1516.780	378	1517	0.2160	0.2200
3	16.4240	21.4656	5.560	1408.398	130.869	1408	131	0.3980	0.1310
4	23.7046	18.4604	5.916	639.6227	1461.400	640	1461	0.3773	0.4000
5	21.5636	19.1723	5.325	814.1307	1064.874	814	1065	0.1307	0.1260
6	21.6736	19.2404	5.494	831.7917	1084.933	832	1085	0.2083	0.0670
7	17.4549	19.6583	5.568	941.246	312.326	941	312	0.2460	0.3260
8	20.8540	20.4691	5.971	1144.430	933.377	1144	933	0.4300	0.3770
9	16.9881	20.7389	5.570	1219.569	230.576	1220	231	0.4310	0.4240
10	17.8634	21.0346	4.660	1291.869	390.544	1292	391	0.1310	0.4560
11	22.4703	18.3557	6.000	607.702	1233.710	608	1234	0.2980	0.2900

We randomly choose three stars (18.5318,16.1335), (23.9783,17.4337) and (16.4240,21.4656) named 1, 2 and 3, calculating angle distance of each chosen star with other 10 stars according to the equation (14) and (17). θ_i expresses the angle distance in celestial coordinate system. θ_i expresses the angle distance in star tracker's coordinate system. $\Delta\theta$ expresses the difference. The value of i can be 1, 2 and 3.

Table 2. Angle distance in different coordinate systems

No.	θ_1	θ_1'	Δθ	θ_2	θ_2'	Δθ	θ_3	θ_3'	Δθ
1	0.000000	0.000000	0.000000	5.559037	5.539846	0.019191	4.832476	4.811731	0.020745
2	5.559037	5.539846	0.019191	0.000000	0.000000	0.000000	7.435146	7.442216	0.007070
3	4.832476	4.811731	0.020745	7.435146	7.442216	0.007070	0.000000	0.000000	0.000000
4	4.192624	4.203960	0.011336	2.197953	2.209885	0.011932	6.724163	6.740888	0.016725
5	4.313877	4.312812	0.001064	2.054277	2.072771	0.018494	4.576914	4.572278	0.004636
6	3.670692	3.652834	0.017857	3.355191	3.359273	0.004082	4.610917	4.598383	0.012534
7	4.863656	4.844449	0.019207	5.036884	5.029846	0.007038	1.166024	1.162974	0.003049
8	4.832476	4.811731	0.020745	6.724163	6.707290	0.016873	3.528572	3.529294	0.000722
9	4.941845	4.926765	0.015080	6.069677	6.049885	0.019792	0.869598	0.856389	0.013208

10	5.662872	5.648997	0.013875	7.472439	7.752771	0.019668	0.897243	0.896111	0.001132
11	5.700968	5.678997	0.021971	6.077922	6.059273	0.018649	0.905036	0.867111	0.000234

The result of calculation in table 2 shows $\theta_1 \approx \theta_1^{'}$. The maximum, the minimum and the mean value of angle distance is 0.021944° , 0.016667° and 0.019621° which means the angle distance in different coordinate systems are approximately equal. The simulation software is reliable and effective.

V. CONCLUSIONS

This paper presents a design method of CCD star-map simulation which uses Gaussian gray diffusion model to simulate imaging of star and adding Gaussian white noise signal to simulate the background noise. In this way, we can get a simulation star-map which is closer to the star-map shooting when the star tracker working in space and verify the validity though out angle distance. The result shows that the design method is reliable and the star-map simulated by the software has high precision. The software can do real-time simulation to produce star-map of celestial region we need. Combined with high resolution display device, it could provide the source of performance test and ground test for star tracker.

ACKNOWLEDGMENT

This work was supported by Technological Innovation Person with ability Foundation of Harbin (2014RFXXJ074).

REFERENCES

 Zhang G.J. Star Identification[M]. Beijing: National Defense Industry Press, 2011:17-47. (in Chinese)

- [2] Wei X.G., Zhang G.J., Fan Q.Y., etc. Ground function test method of star sensor using simulated sky image[J]. Infrared and Laser Engineering, 2008, 37(6): 1087-1091. (in Chinese)
- [3] Guo J.M., Wei Z.H., He X., etc. Design of CCD star map simulator and its validation[J]. Chinese Journal of Optics and Applied Optics, 2010, 3 (5): 486-493. (in Chinese)
- [4] Wang Y.J. The research on the technology of the star map simulation for dynamic star simulator based on LCOS[D]. Changchun: Changchun University of Science and Technology, 2011. (in Chinese)
- [5] Liu C.S., Liu G.B., Wang X.G. Principle and system application of Missile-Borne star tracker[M]. Beijing: National Defense Industry Press, 2010: 1-26. (in Chinese)
- [6] LU J. Based on virtual instrument dynamic star map display technology research[D]. Changchun: Changchun University of Science and Technology, 2012. (in Chinese)
- [7] Zhang C., Yang C. A new guide star selection algorithm for star tracker [C]// The 5th World Congress on Intelligent Control and Automation, Hangzhou, China: IEEE, 2004: 5445-5449.
- [8] Ou Y.H. Research on star image simulation and star extraction method for CCD star tracker[D]. Wuhan: Huazhong University of Science and Technology, 2005. (in Chinese)
- [9] Yang Y.D., Wang J.Y., Zhu Y.T. High-Precision Simulation of star map using forward ray tracing method[C]// ICEMI'2009, Beijing, China: IEEE, 2009: 541-544.
- [10] Wang H.Y., Fei Z.H., Wang X.L. Precise simulation of star spots and centroid calculation based on Gaussian distribution[J]. Optics and Precision Engineering, 2009, 17 (7): 1672-1677. (in Chinese)
- [11] Quan W, Fang J.C. High-precision simulation of star map and its validity check[J]. Opto-Electronic Engineering, 2005, 32(7): 22-26. (in Chinese)
- [12] [12] ROESER S, BASIAN U. A new star catalogue of SAO type[J]. A stronomy and A strophysics, 1988, 74: 449-452.