Child survival for mothers: mortality change and related measures

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Introduction

The question of child survival, particularly the survival of daughters for mothers, sits at the very center of demographic theory. This is evidenced by the omnipresence of the 'net reproductive rate' in accounts of human and non-human populations. Historical demographers draw liberally on assumptions about kin availability and individual's exposure to offspring mortality to explain fertility decisions, especially around the demographic transition. These assumptions are often untested given the data sparsity (Livi Bacci 1997; Volk and Atkinson 2013). Economic theories of fertility that consider children as 'investment' for old-age support also rely on notions of child survival (Preston and others 1978).

Kin count estimation has a long and distinguished pedigree in mathematical demography, starting with the work of Lotka (1931) on modelling orphanhood in theoretical populations across demographic regimes. To the best of our knowledge, Brass (1953) first proposed a formula to represent child survival over age. The notion of estimating the expected number of living daughters in a stable population was generalized by Goodman, Keyfitz, and Pullum (1974) for other kin relations (granddaughters, cousins, etc.). The "counting method" approach was further popularized in Keyfitz and Caswell (2005). Bongaarts (1987) used a similar approach to estimate descendants in his 'Family Status Model'. Recently Caswell (2019) reformulated these relationships in a matrix framework, extending the potential results to variance estimation, multistate models, and others.

Child survival - and it's counterpart, child death - gain new significance in the wake of the fertility transition. Depressed fertility means that fewer children are tasked with providing key emotional, social, and financial transfers to aging parents for ever increasing periods of time. With increasing periods of generational overlap, individuals find themselves 'sandwiched' between aging parents and young children requiring their simultaneous attention and care (Daatland, Veenstra, and Lima 2010). In the context of global population aging, elderly parents without access to formal social security and pension systems are particularly reliant on these transfers to make ends meet (Smith-Greenaway and Trinitapoli 2020).
Losing an only child may become a more common experience in the context of global fertility decline, something particularly worrying for parents suddenly rendered childless. Given the known psychological, health, and social consequences of child loss for bereaved parents and families (Hendrickson 2009; Lee et al. 2014), it is surprising that the demographic processes that shape parental bereavement remain very poorly understood. Recent development have underscored the need to understand how sudden changes in mortality affect the availability of kin. In the context of the global pandemic caused by the Covid-19 disease, elderly people depend on support provided by their relatives, especially during periods of lockdown and in the absence of governmental support mechanisms. Elderly parents may be at a higher risk of losing the key support provided by their adult children given the known age-gradient in the Covid-19 case fatality rate.

How do changes in mortality affect the availability of offspring over age from the point of view of a prospective mother? How can we characterize the timing of offspring mortality as experienced by a mother from a formal demographic perspective? How this would work in an heterogeneity population? In the remainder of the paper, we aim to formalize the relationship population-level changes in mortality rates and perceived change in the lived experience of child death from the perspective of a mother. Our approach is relevant for understanding the impact of overall and age-specific excess mortality on the resilience or otherwise of kinship networks for the vulnerable elderly population. We consider the case when these changes are additive and mutliplicative. In this paper, we uncover formal relationships to answer these questions and provide numerical applications using data from countries in Latin America during the second half of the twentieth century. See the Applications section for more details on the source, calculation procedures and main numerical results. We used period measures, in an stable context, so the objetive is give moment measures. A more sophisticated model using the appropriate age-specific cohort rates in the subscripts has been proposed by Alburez-Gutierrez, Kolk, and Zagheni (2019).

Relationship (and proofs)

Child Survival

Let CS_a be the expected number of surviving children to a mother[1] alive at aged a in a female stable population with fertility rates m_x, mortality hazard μ_x and survival function $l_x = e^{-\int_0^x \mu_t \, dt}$ (with unit radix $l_0 = 1$), as proposed by Goodman, Keyfitz, and Pullum (1974):

$$CS_a = \int_0^a m_x l_{a-x} dx$$

An approximation

As an initial step, we seek an intuitive understanding of the previous relation. Building on work by Keyfitz and Caswell (2005) for the probability of a living mother, we can show which features of the child survival function explain the expected number of children. For this we use an approximate of l_x using Taylor´s theorem until second order around the mean age of childbearing κ:

$$\begin{aligned}
CS_a \approx & l_{a-\kappa} \int_0^a m_x dx + (l_{a-\kappa})' \int_0^a (x-\kappa) m_x dx + (l_{a-\kappa})'' \int_0^a \frac{(x-\kappa)^2}{2} m_x dx \\
& \approx F_a l_{a-\kappa} + \frac{\sigma^2}{2} F_a (l_{a-\kappa})'' \\
& \approx F_a \, l_{a-\kappa} [1 + \frac{\sigma^2}{2} \frac{(l_{a-\kappa})''}{l_{a-\kappa}}]
\end{aligned}$$

where the fertility pattern by age is concentrated around κ and the accumulated fertility (or gross reproduction rate in our female-dominant scenario) is $F_a = \int_\alpha^a m_x dx$. The second Taylor´s term is null because $\int_\alpha^a x \, m_x dx = \kappa \, F_a$.

We find that, seen from the perspective of a mother, child survival mainly depends on the cummulative fertility function and the survival of daughters from birth to $a - \kappa$. The approximation is affected negatively by how disperse is the fertility by age (variance σ^2) and the survival curvature around (with negative sign), in an age range where l_x is tipycally very flat in transitioned populations (range 20-40 years old)[2].

Mortality Changes

We now consider the consequences of an absolute change δ in mortality in the range $[0, a - \alpha]$, where α is the start age of fertility risk. For now, let us consider $m_{x,\delta} = m_x + \delta$ (Wrycza and Baudisch (2012)) and $l^\delta_{a-x} = e^{-\int_0^{a-x} (\mu_t + \delta) dt}$:

[1] We use woman and mother interchangeably in this paper, assuming that all women are exposed to the same fertility rates.

[2] This approximation is useful also for get an idea of impact because of changes in the fertility average age κ, keeping constant the level: taking logs in $F_a \, l_{a-\kappa}$ and deriving we get $\frac{\Delta CS_a}{CS_a} \approx \mu_{a-\kappa} \Delta\kappa$, being daughters exposed to less time beacuse of the delay in pattern

$$CS_a^{\delta} = \int_{\alpha}^{a} m_x l_{a-x} e^{-\delta(a-x)} dx$$

We get the derivative of $dCS_{(a)}^{\delta}/d\delta$ evaluated near zero (Keyfitz and Caswell (2005)) to find the effects of adding δ to hazard rates:

$$\begin{aligned}\frac{dCS^{\delta}}{d\delta} &= -a\int_{\alpha}^{a} m_x l_{a-x} dx + \int_{\alpha}^{a} x m_x l_{a-x} dx \\ &= -a\, CS_a + \int_{\alpha}^{a} x\, m_x l_{a-x} dx\end{aligned}$$

Dividing both sides by CS_a in an discrete approximation we get:

$$\frac{\Delta CS_a}{CS_a} \approx -(a-k_a)\Delta\delta$$

The expected change in descendants survival is inversely proportional to the difference between maternal age a and the mean age of the mother at the birth of her surviving daughters k_a. The magnitude of the change depends negatively on the age distribution of the surviving offspring (the younger the fertility bigger the impact), which is intuitive considering that older descendants experiences longer periods of exposure to risk.

That an equal change in mortality could happens at all ages is unlikelihood. This measure could be much related a change in first ages (child and youth) or adolescent ones. If the change only happens in the age range $[0;1)$, infant mortality, we can inspect the effect in child survival $CS_a^{\delta_0}$ starting with splitting the integral between those daughters who were exposed all the range and those who do not (now generalizing for α positive and $a > 1$ for simplicity):

$$\begin{aligned}CS_a^{\delta_0} &= \int_0^{a-1} m_x e^{-\int_1^{a-x} \mu_t\, dt - \int_0^1 (\mu_t+\delta)\, dt} dx + \int_{a-1}^{a} m_x e^{-\int_0^{a-x} (\mu_t+\delta)\, dt} dx \\ &= \int_0^{a-1} m_x l_{a-x} e^{-\delta} dx + \int_{a-1}^{a} m_x l_{a-x} e^{-\delta(a-x)}\, dx\end{aligned}$$

Deriving by δ and valuating near 0, we get:

$$\frac{dCS^{\delta_0}}{d\delta_0} = -\int_0^{a-1} m_x l_{a-x} dx - a\int_{a-1}^{a} m_x l_{a-x} dx + \int_{a-1}^{a} x\, m_x l_{a-x} dx$$

If we name $CS_{a-1,a} = \int_{a-1}^{a} m_x l_{a-x} dx$, we can express the first term in the right as $CS_a - CS_{a-1,a}$, and recognizing that last term could be expressed as $\kappa_{a-1,a} CS_{a-1,a}$, we get:

$$\frac{dCS^{\delta_0}}{d\delta_0} = -CS_a + CS_{a-1,a}(1 - a + \kappa_{a-1,a})$$

Where the factor $1-(a-\kappa_{a-1,a})$, which lies between 0 and 1, is equal to the average portion of time spent with living daughters durings age a. Assuming uniform distribution of those births, discretizing the change and dividing by CS_a we get the final expression for change in infant mortality:

$$\frac{\Delta CS_a^{\delta_0}}{CS_a} \approx -\left[1 - \frac{CS_{a-1,a}}{CS_a}\frac{1}{2}\right]\Delta\delta$$

This means that an absolute change in infant hazard rate affects proportionally for all the age range extension (cohorts aged more than 1), except for "half" the portion $\frac{CS_{a-1,a}}{CS_a}$ on how important was this las age in total "successful" experience (in terms of alive descendants). If a is near β then $\frac{\Delta CS_a^{\delta_0}}{CS_a} \approx -\Delta\delta$.

We now consider the consequences of a proportional change in mortality on child survival. Given a proportional change in mortality $\mu_{x,\delta} = \mu_x(1+\delta)$ so that $l^{\delta}_{a-x} = e^{-\int_0^{a-x}\mu_t(1+\delta)dt} = (l_{a-x})^{(1+\delta)}$, it follows that:

$$CS^{\delta}_a = \int_{\alpha}^{a} m_x l^{(1+\delta)}_{a-x} dx.$$

Using the derivative $\frac{dl^{(1+\delta)}_{a-x}}{d\delta} = log(l_{a-x})l^{(1+\delta)}_{a-x}$, and in the third row reversing integrals between t and x:

$$\begin{aligned}\frac{dCS^{\delta}_a}{d\delta} &= \int_0^a m_x l_{a-x}\log(l_{a-x})dx\\ &= -\int_0^a m_x l_{a-x}\int_0^{a-x}\mu_t\,dt\,dx\\ &= -\int_0^a \mu_t\int_0^{a-t} m_x l_{a-t-x}\,dx\,dt\end{aligned}$$

The last integral is CS_{a-t}. Dividing by CS_a and multiplying by δ get one useful expression. This is the negative of cumulative hazard H_a but considering a positive factor $\frac{CS_{a-t}}{CS_a}$ that takes in account the relative amount of surviving descendants that would be lost at each child age at risk, that gives more weight to first ages because all the parity was exposed in that age.

$$\frac{\Delta CS_a}{CS_a} \approx -\left[\int_0^a \mu_x\,\frac{CS_{a-x}}{CS_a}\,dx\right]\Delta\delta$$

Grasping this relationship intuitively is more difficult given the interaction of birth and mortality rates (Keyfitz and Caswell (2005)). The factor $\frac{CS_{a-t}}{CS_a}$ has a S-shape because l_x curvature and fertility accumulation. Taking the difference between consecutive ages a and $a+1$, we have $CS_{a-1} = \int_0^{a-1} m_x l_{a-1}\,dx$, $CS_a = \int_0^{a-1} m_x l_a\,dx + CS_{a-1,a}$. In the last one, a second term is added because of additional born daughters after $a-1$, and in the first term the existing one are survived at on $\frac{l_{a-x}}{l_{a-1-x}}$. In advanced ages when fertility is complete, the difference is only explained by the surviving part, that is why this factor is not a strictly increasing function but for mother ages close to β (last theoretical fecundity age). A numerical evaluation of both analytical expressions (absolute an relative) and its factor $\frac{CS_{a-t}}{CS_a}$ are done in Applications section.

Related Measures

Burden and timing of maternal bereavement

We call *Mean Time Spent in Bereavement* ($MTSB$) the absolute measure of the expected total lifetime with a death daughter for a mother aged a, which can be expressed in terms of a temporary expected lost years index, in line with $e^{\dagger}$ (Vaupel (1986)):

$$MTSB_a = \int_0^a m_x\int_0^{a-x} d_t e_{0|a-x-t}\,dt\,dx = \int_{\alpha}^{a} m_x e^{\dagger}_{0|a-x}\,dx$$

.

Where d_t is the death distribution from birth, $e_{0|a-x-t}$ is the life expectancy at birth until age $a-x-t$ and $e^{\dagger}_{0|a-x}$ the temporary dispersion measure. But most interesting would be to compare these years with the time that mothers would expect to live with their daughters. We call this the *Intensity Time in Bereavement* (ITB): a ratio between expected time with a "lost" and expected time with a "life", that allows to make comparisons between population regimes.

$$ITL_a = \frac{\int_0^a m_x e^{\dagger}_{0|a-x} dx}{\int_0^a m_x e_{0|a-x} dx}$$

This is a ratio between child-years in two radically different states. Looks similar to the transcendental entropy measure H (Keyfitz and Caswell (2005)) but considering all the cohorts born during the mother´s life, weighted by their relative size m_x. In the figure @ref(fig:plot_ITL_MAL) is shown that ITB is bigger for young women because of the weight of infant mortality in their mother experience, also with more dispersion for same levels of parity.

Another important factor for the child survival experience of mothers, is the mean age at child loss, called here MAL. This relation can be derived by starting with the mother age $x + t$ at each death child age t at death, weighted by the fertility and survival function. In it, κ is the mean age at childbirth for women aged a, MAD_{a-x} refers to the mean age at death for newborns that die before $a - x$ and F_a is the accumulated fertility for a women aged a:

$$\begin{aligned}
MAL_a =& \frac{\int_0^a m_x \frac{\int_0^{a-x} l_t\, \mu_t (x+t) dt}{\int_0^{a-x} l_t\, \mu_t dt} dx}{\int_0^a m_x dx} \\
MAL_a =& \frac{\int_0^a m_x \left[x + \frac{\int_0^{a-x} l_t\, \mu_t\, t\, dt}{\int_0^{a-x} l_t\, \mu_t dt} \right] dx}{\int_0^a m_x} \\
MAL_a =& \frac{\int_0^a m_x\, x\, dx}{F_a} + \frac{\int_0^a m_x MAD_{a-x} dx}{F_a} \\
MAL_a =& \kappa + \frac{\int_0^a m_x MAD_{a-x} dx}{F_a}
\end{aligned}$$

Following this, in populations with high infant mortality $MAD_{a-x} < 1$ for all x, so $MAL_a \approx \kappa_a + f_0$, being f_0 the average time spent for those child that dies in their first year of life. A numerical approximation of both measures are done in Applications section.

Heterogeneity (first ideas)

In a model with heterogeneity, determined at birth by a multiplicative effect, CS_a could be interpreted as a conditional expectation with random variables K for fertility heterogeneity and Z for mortality frailty.

$$CS_a(k, z) = \int_0^{\infty} m(k)_x\, l(z)_{a-x}\, dx$$

Following Coresh and Goldman (1988) we can express the fertility part in a multiplicative way with variability in the level but not in the shape, as $m(k)_x = F_\beta\, k\, r_x$, where F_β is the baseline cumulated fertility until upper limit in all age range until β, k is a random variable with mean 1 that allows variability between groups and r_x is the fertility structure by age ($\int_0^\beta r_x = 1$). In the other side, frailty part can be thought in a cohort effect way, also in the multiplicative assumption as Vaupel and Missov (2014), with $l(z)_x = e^{-H_x z} = l_x^z$, with baseline hazard μ. Considering the joint distribution f_{kz}, the intuition could be expressed as $f_{zk} = f_{k|z} f_z$: groups with higher descendant´s mortality would adjust their fertility level, with positive correlation. Replacing both variables, the unconditional mean $\overline{CS}_a$ would be[3]:

[3]Note that given k and z values, fertility and child survival are independent.

$$
\begin{aligned}
\overline{CS}_a &= \int_0^\infty \int_0^\infty \left[\int_0^a m(k)_x l(z)_{a-x} \, dx \right] f_{kz} \, dz \, dk \\
&= F_\beta \int_0^{a-x} r_x \int_0^\infty \int_0^\infty (l_{a-x})^z \, k \, f_{kz} \, dk \, dz \, dx
\end{aligned}
$$

Isolating for an age x we can inspect the part related to z and k: $(l_{a-x})^z \, k \, f_{kz}$. Following Gupta (2018), one way to create a correlated bivariate distribution is assume for example that $Z = Y_0 + Y_1$ and $K = e^{(Y_0+Y_2)}$, given the fact that the historical relation is not linear between both components (see figure @ref(fig:plot_tfr_q0), a funtion f like $F_\beta = f(ln(q_{0,5}))$. The auxiliar variables Y_0, Y_1 and Y_2 are independet and Gamma distribuited. Then we can express, using Laplace transform properties:

$$
\begin{aligned}
\int_0^\infty \int_0^\infty (l_{a-x})^z \, k \, f_{kz} \, dk \, dz \\
\int_0^\infty \int_0^\infty e^{-H_x(Y_0+Y_1)} e^{Y_0+Y_2} \\
L_{Y_0}[1-H_x] L_{Y_1}[-H_x] L_{Y_2} \, dY_0 \, dY_1 \, dY_2
\end{aligned}
$$

To continue…

Applications

Data

We motivate this paper with an empirical example using fertility and mortality rates from the 2019 Revision of the UN World Population Prospects (UN WPP) for the Latin American Region (Nations (2019)). We smoothed female l_x in quinquennial ages, using cubic-splines constrained to monotonic decrease, taking L_0 and T_{100} from raw life tables as inputs for year-person calculations. For splitting fertility five groups was used quadratic optimization approach by Michalski and Gorlishchev (2018), with an desirable property for our purpose which is a good fitting in parity. Also was assumed an unique female percentage of newborns of 0.49 for all period-country cases. Calculations were done in a discrete way assuming that the m_x live births are born at exact mother´s age x. An already known log relation between infant mortality and TFR is shown in Latinoamerican experience (@ref(fig:plot_tfr_q0)).

Numerical Results

Consider a Latin American woman standing before us. If this were 1950-1955, she could reasonable expect to have 2.23 surviving daughters on her 50^{th} birthday. In 2010-2015 a woman the same age would only have 1.1 living children. The difference of 1.13 children is explained by reduced fertility and improved mortality in the region [give stats]. We now remove the effect of changing fertility by considering the number of daughters surviving up to maternal age a as a proportion of the daughters ever born to a woman that age, $\frac{CS_a}{F_a}$. Given Eq. **??**, the increase from 0.78 in 1950-1955 to 0.97 in 2010-2015 must be explained by a change in mortality.

The approximation proposed for CS_a is realy precise, especially for recent periods. For example, Guatemala improved the approximation with years due to rectangularization process in l_x (figure @ref(fig:plot_CS_aprox)).

The figure @ref(fig:CS_abs_app) shows that goodness of fit is decreasing with the change size, given that δ is assumed near zero. To ilustrate that considering $\delta = 0.01$ means at age 50, it means for the absolute change case that $\frac{l_{50}}{e^{\delta\, 50}} = l_{50}/$ 1.6, a big change). For the relative case was assumed a constant within each interval $\mu_{x+t} = \mu_x$ for t between 0 and 1, $\int_x^{x+1} \mu_t dt = \log(l_{x+1}) - \log(l_x)$, and was used this approximation on the empirical data $\frac{\Delta CS_a}{CS_a} \approx -\left[\sum_0^{a-1} \left[\log(l_{x+1}) - \log(l_x)\right] \frac{CS_{a-1-x}}{CS_a} \right] \Delta\delta$.

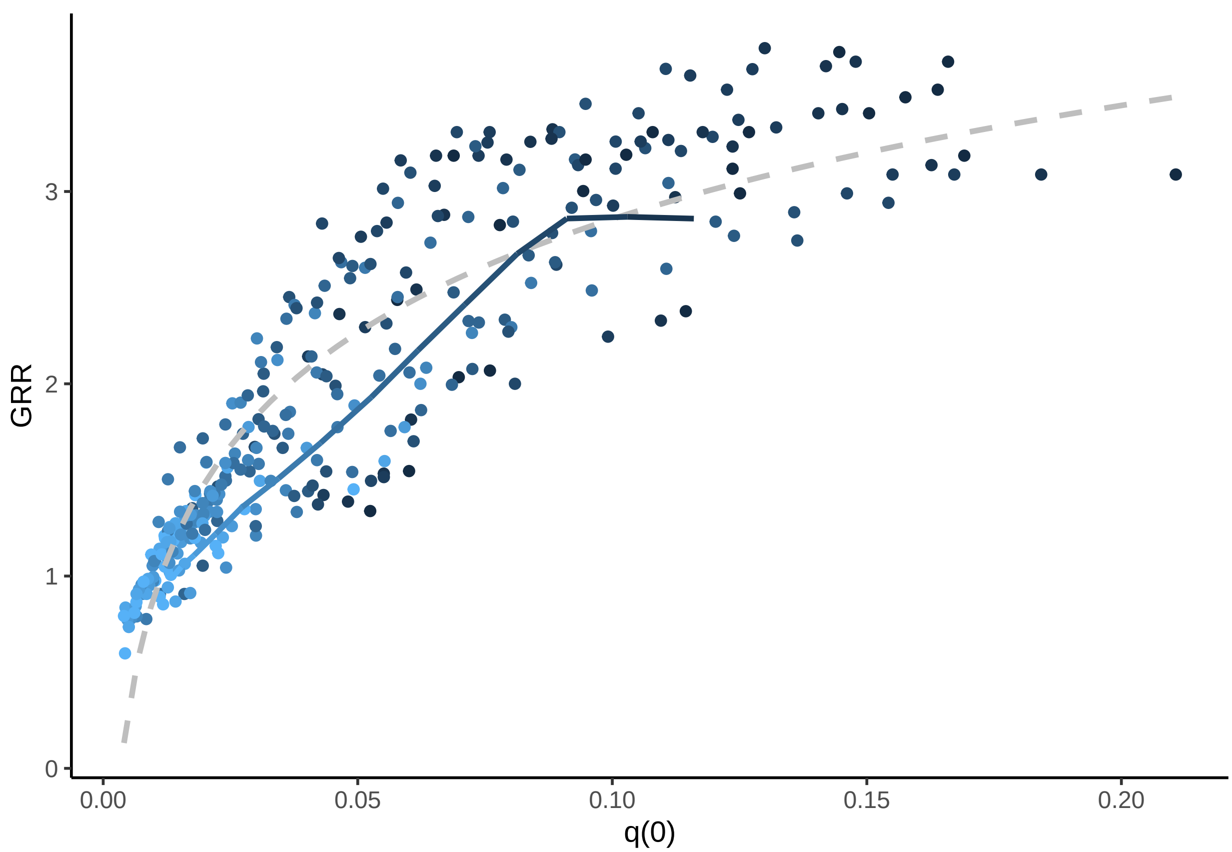

Figure 1: Female probability of death at birth by gross reproduction rate. Latin America countries in period 1950-2015

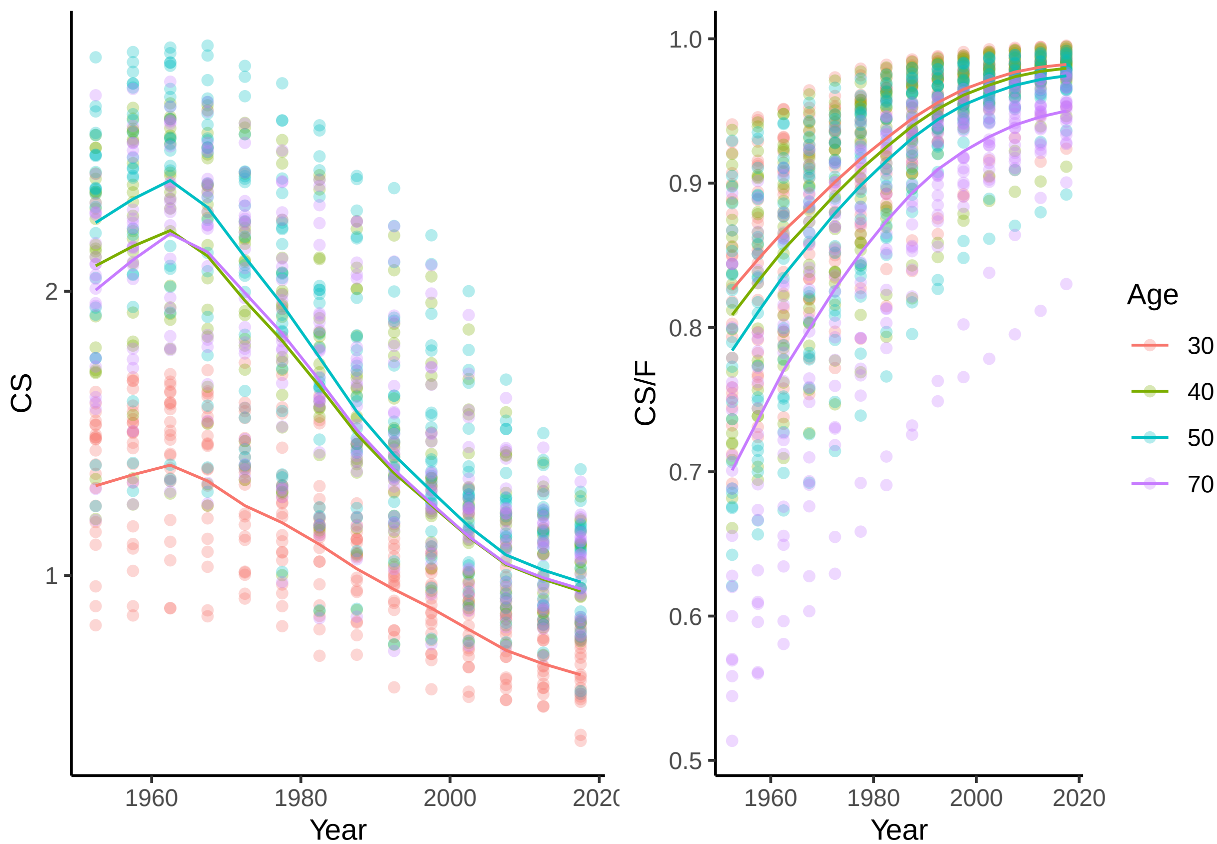

Figure 2: Child Survival and Child survival as a share of cummulative fertility by age, for women aged 30, 40 and 50. Estimates using UN WPP data for Latin American countries in the period 1950-2015 period

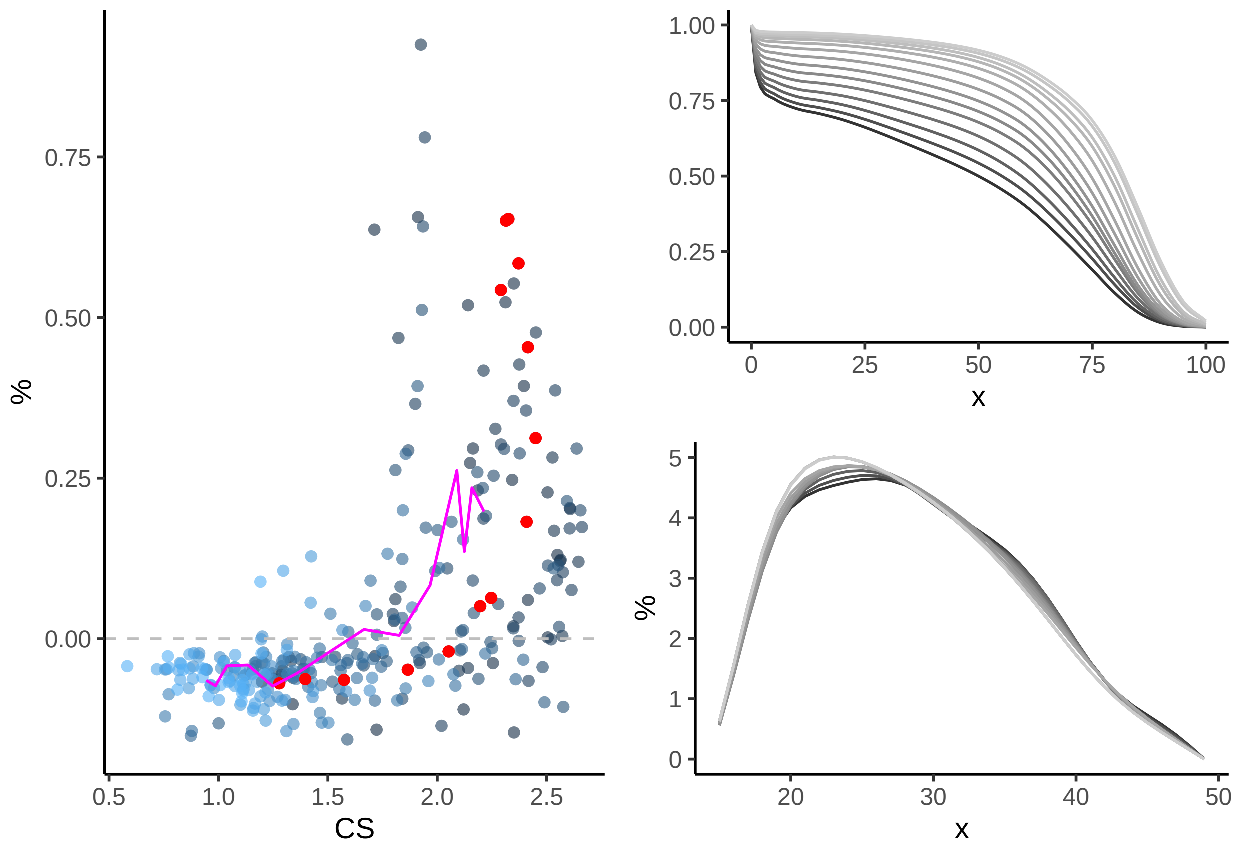

Figure 3: Left: Error in approximation for a=40, from years 1950 (darker) to 2015 (lighter) for all Latin American countries (blue) and Guatemala (red). Right: Change in survival and fertility by age in Guatemala.

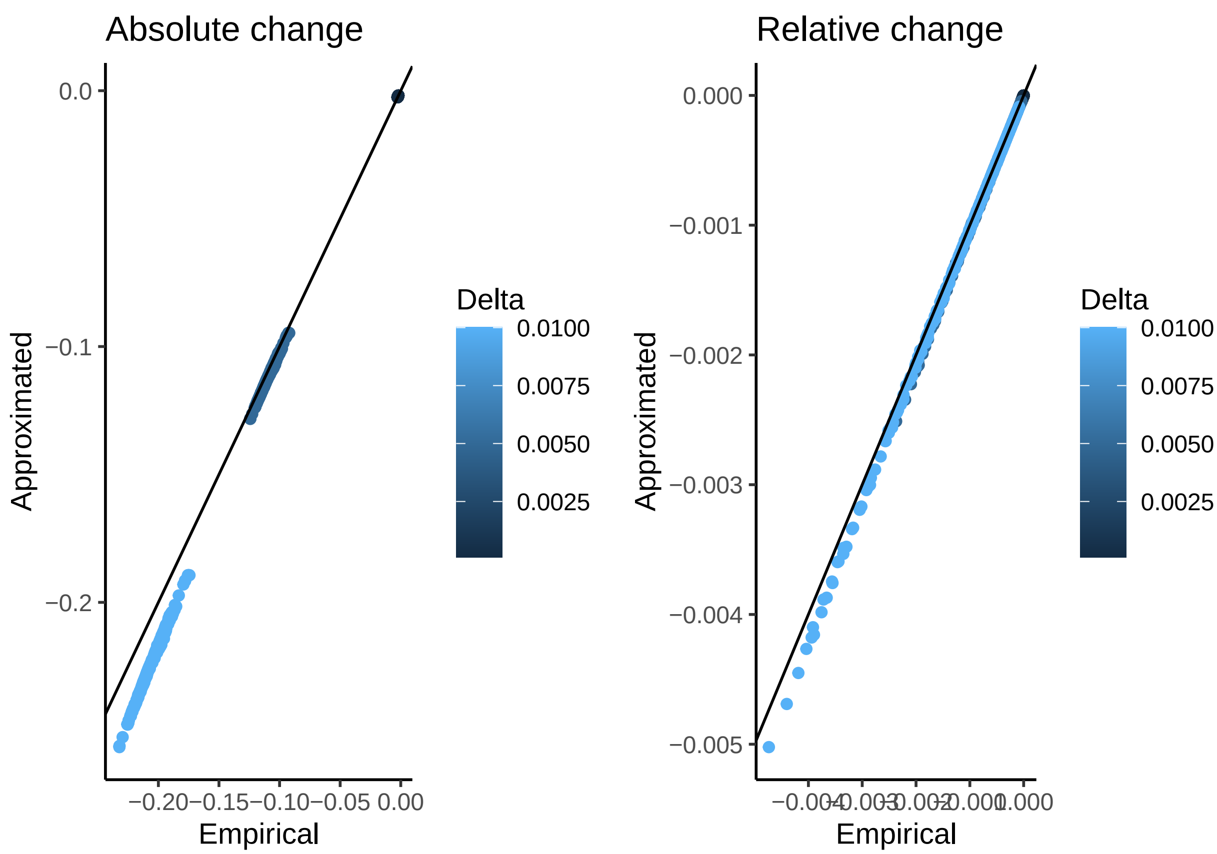

Figure 4: Effect of absolute and relative change in mortality on child survival by maternal age of women in Latin America for the 1950-2015 period. Goodness of fit of approximation compared to direct estimation

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Assuming constant period rates the expected time in years that a mother aged 30 passed with a death son was around 4% in some countries at middle XX Century. When increasing age, the survival experience depends less on infant mortality, and the distribution is around less than 2% for women aged 50, converging to 0 on time. As extreme cases, in 1950-1955 Haiti women aged 30 would have experienced an intensity of 4.8%, and in 2010-2015 1.4%, while the women of Costa Rica 2.3%, and in 2010-2015 0.2%. For those Latin American mothers aged 50 with 3 daughters born who suffered a lost, they experienced that at age 30 in average (figure @ref(fig:plot_ITL_MAL)).

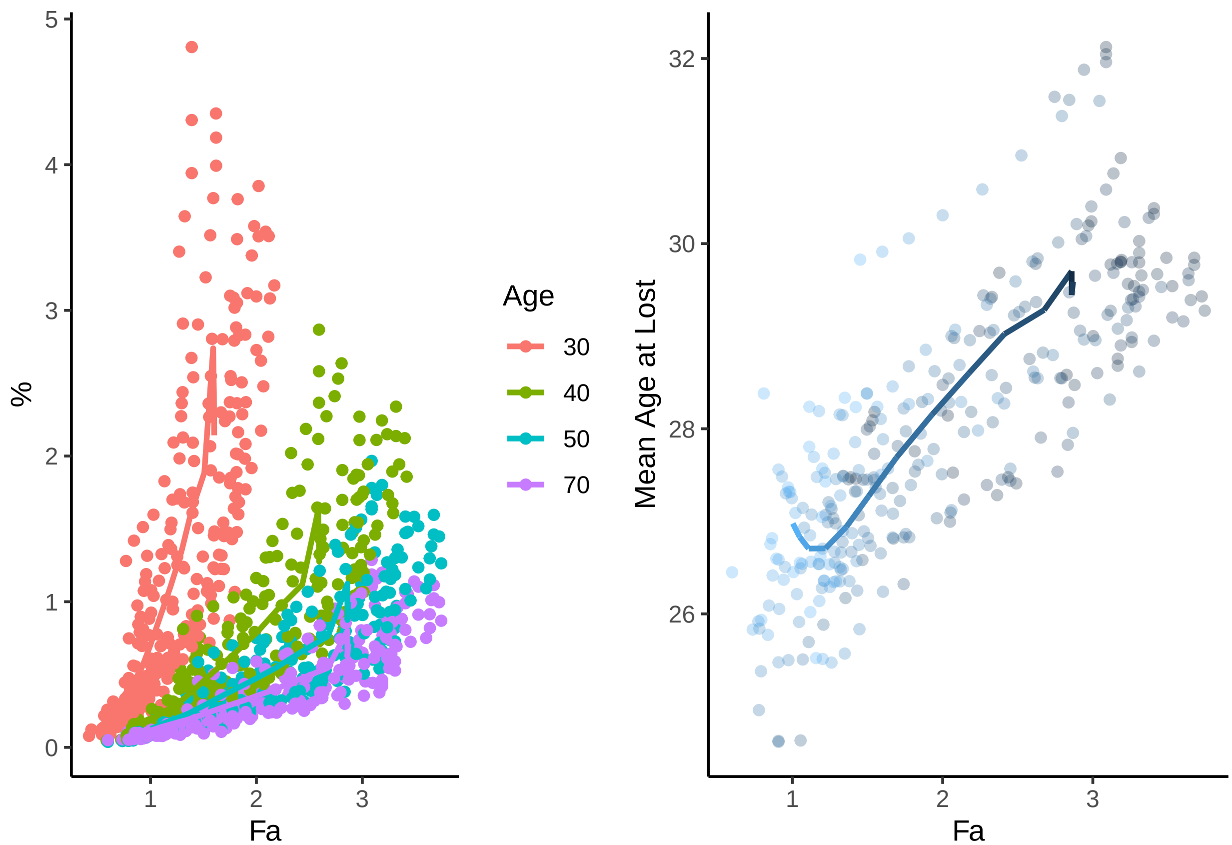

Figure 5: a) Intensity Tome Lost of women aged 30, 40 and 50. b) Mean age at Lost for women aged 40. Years 1950 (darker) to 2015 (lighter). Latinamerican countries in period 1950-2015

References

Alburez-Gutierrez, Diego, Martin Kolk, and Emilio Zagheni. 2019. "Women's Experience of Child Death over the Life Course: A Global Demographic Perspective." Preprint. SocArXiv. https://doi.org/10.31235/osf.io/s69fz.

Bongaarts, John. 1987. "The Projection of Family Composition over the Life Course with Family Status Life Tables." In *Family Demography: Methods and Their Application*, edited by John Bongaarts and Thomas Burch. Oxford: IUSSP Series, Oxford Univer- sity Pres.

Brass, W. 1953. "The Derivation of Fertility and Reproduction Rates from Restricted Data on Reproductive Histories." *Population Studies* 7 (2): 137. https://doi.org/10.2307/2172029.

Caswell, Hal. 2019. "The formal demography of kinship: A matrix formulation." *Demographic Research* 41 (September): 679–712. https://www.demographic-research.org/volumes/vol41/24/default.htm.

Coresh, Josef, and Noreen Goldman. 1988. "The effect of variability in the fertility schedule on numbers of kin." *Math. Popul. Stud.* 1 (2): 137–56. https://doi.org/10.1080/08898488809525268.

Daatland, Svein Olav, Marijke Veenstra, and Ivar A. Lima. 2010. "Norwegian Sandwiches: On the Prevalence and Consequences of Family and Work Role Squeezes over the Life Course." *European Journal of Ageing* 7 (4): 271–81. https://doi.org/10.1007/s10433-010-0163-3.

Goodman, Leo A, Nathan Keyfitz, and Thomas W Pullum. 1974. "Family Formation and the Frequency of Various Kinship Relationships." *Theoretical Population Biology* 5 (1): 1–27.

Gupta, R. C. 2018. "Association measures in the bivariate correlated frailty model." *ResearchGate* 16 (2): 257–78.

Hendrickson, K. C. 2009. "Morbidity, Mortality, and Parental Grief: A Review of the Literature on the Relationship Between the Death of a Child and the Subsequent Health of Parents." *Palliat Support Care* 7 (1): 109–19.

Keyfitz, Nathan, and Hal Caswell. 2005. *Applied Mathematical Demography*. Vol. 47. Springer.

Lee, Chioun, Dana A. Glei, Maxine Weinstein, and Noreen Goldman. 2014. "Death of a Child and Parental Wellbeing in Old Age: Evidence from Taiwan." *Social Science & Medicine* 101 (January): 166–73. https://doi.org/10.1016/j.socscimed.2013.08.007.

Livi Bacci, Massimo. 1997. *A Concise History of World Population*. 2nd ed. Cambridge, MA: Blackwell.

Lotka, Alfred J. 1931. "Orphanhood in Relation to Demographic Factors." *Metron* 9 (2): 37–109.

Michalski, Grigoriev, Anatoli I, and Vasily P and Gorlishchev. 2018. "R Programs for Splitting Abridged Fertility Data into a Fine Grid of Ages Using the Quadratic Optimization Method." Max Planck Institute for Demographic Research, Rostock, Germany.

Nations, United. 2019. *World Population Prospects 2019: Data Booklet*. https://doi.org/https://doi.org/https://doi.org/10.18356/3e9d869f-en.

Preston, Samuel H, and others. 1978. *The Effects of Infant and Child Mortality on Fertility*. Academic Press, Inc., 111 Fifth Avenue, New York/New York 10003, USA.

Smith-Greenaway, Emily, and Jenny Trinitapoli. 2020. "Maternal Cumulative Prevalence Measures of Child Mortality Show Heavy Burden in Sub-Saharan Africa." *Proceedings of the National Academy of Sciences*, February, 201907343. https://doi.org/10.1073/pnas.1907343117.

Vaupel, James W., and Trifon I. Missov. 2014. "Unobserved population heterogeneity: A review of formal relationships." *Demographic Research* 31 (September): 659–86. https://www.demographic-research.org/Volumes/Vol31/22.

Vaupel, J. W. 1986. "How Change in Age-Specific Mortality Affects Life Expectancy." *Population Studies* 40 (1): 147–57. https://doi.org/10.2307/2174285.

Volk, Anthony A., and Jeremy A. Atkinson. 2013. "Infant and Child Death in the Human Environment of Evolutionary Adaptation." *Evolution and Human Behavior* 34 (3): 182–92. https://doi.org/10.1016/j.evolhumbehav.2012.11.007.

Wrycza, Tomasz, and Annette Baudisch. 2012. "How Life Expectancy Varies with Perturbations in Age-Specific Mortality." *Demographic Research* 27: 365–76.