## Rountable comments for Iván and Diego

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## Some comments

- you should structure the Abstract in the DR format (if you aim to submit there).
- "Let us consider  $m_{x,\delta} = m_x + \delta$ " (page 2). You defined  $m_x$  as fertility rates, and here you are talking about mortality changes. I think you meant  $\mu_{x,\delta} = \mu_x + \delta$  (i.e. a change in the hazard of death), as you correctly write when you consider the proportional change in mortality (bottom page 3).
- "entropy measure  $\bar{H}$  (Keyfitz 2005)" (page 5). Please consider adding some additional (older) references, such as (Leser 1955; Keyfitz 1968, 1977; Demetrius 1974, 1978). José Manuel was "criticized" for not having provided references to other authors. You can find the references in the DR paper of José.
- in the equation of  $MAL_a$ , second line (page 6), you forgot the dx in the integral of the denominator.
- Figure 2, panel A. The relationship seems to follow a log-linear pattern. Probably if you take the log of the GRR, you should get a straight line, which may be more understandable and visually pleasing.
- Figure 3. The relative errors in the approximation of CS are very low, which is great. However, do you have an explanation/guess at why there is systematic over/underestimation at ages 30 and 50? Your hypothesis could be mentioned when you comment the figure.

## **Derivations**

There are two mathematical derivations that I couldn't reconcile with you (although I didn't have time to check all your equations):

• (bottom of page 2) in the derivative of  $dCS_{(a)}^{\delta}/d\delta$ , aren't you missing the exponential term ?

$$\frac{dCS_{(a)}^{\delta}}{d\delta} = \int_{a}^{a} m_x l_{a-x}(x-a)e^{\delta(x-a)} dx.$$

• (bottom of page 3) you write  $l_{a-x}^{\delta} = (l_{a-x})^{(1+\delta)}$ . My derivation is different from yours:

$$l_{a-x}^{\delta} = e^{-\int_0^{a-x} \mu_t (1+\delta) dt}$$
  
=  $e^{(1+\delta)} e^{-\int_0^{a-x} \mu_t dt}$   
=  $e^{(1+\delta)} l_{a-x}$ .

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where  $l_{a-x} = e^{-\int_0^{a-x} \mu_t dt}$ .