# A History of Mortality Modeling from Gompertz to Lee-Carter Everything in a single *R* package: MortalityLaws

EXTENDED ABSTRACT FOR THE 2017 INTERNATIONAL POPULATION CONFERENCE

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December 14, 2016

**Abstract**

For more than two hundred years demographers and actuaries have been interested in formulating a mathematical model that would account for the characteristic age-pattern of human mortality and in the same time offer a biological interpretation. Some of the proposed mortality "laws" - such as Gompertz' (1825) or Makeham's (1867) mortality models - became popular because of their simplicity. Other models, such as those proposed by Thiele (1871) and Heligman-Pollard (1980), succeeded in capturing all the stages of human mortality by increasing mathematical complexity and number of parameters. This complexity, however, affected their practical applicability. These models have proven useful for facilitating comparative studies of human mortality and to motivate studies concerning the biological processes that influence human mortality patterns. Given the existence of diverse models of human mortality, we want to have an overview of the main mathematical models used in modeling and forecasting. We also want to introduce an *R* package **MortalityLaws** which exploits the available optimization methods to provide tools for fitting a wide range of complex mortality models and assessing their goodness of fit.

***Keywords:*** mortality law; density function; optimization; maximum likelihood; age patterns of mortality

## Short history of mortality modeling

Modeling human mortality has been an important and active area of research for demographers, actuaries and medical scholars since Graunt (1662) first examined mortality in London to produce the first publication that was concerned mostly with public health statistics. Gaunts work showed that while individual life length was uncertain, there was a more predictable pattern of mortality in groups and causes of death. Halley (1693) showed how to actually construct a non-deficient mortality table from empirical birth-death data and even succeeded to present a method to perform a life annuity calculation based on this table. Such early tables were empirical and calculation was time consuming. Theoretical mortality modeling firstly began with DeMoivre (1725) who postulated a uniform distribution of deaths model, and showed simplified annuity calculation

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methods. Taking a biological approach to mathematical modeling, Gompertz (1825) assumed that force of mortality $\mu_x$ at age $x$ in adulthood shows a nearly exponential increase,

$$\mu_x = \alpha e^{\beta x}. \tag{1}$$

In the Gompertz model, the two parameters $\alpha$ and $\beta$ are positive; $\alpha$ varies with the level of mortality and $\beta$ measures the rate of increase in mortality with age. The Gompertz model and its modified version by Makeham (1867), where an additional constant $c$ is added to take into account the background mortality due to causes unrelated to age, were widely used as the standard models for adult mortality in humans; and then extended further to animal species in general,

$$\mu_x = \alpha e^{\beta x} + c. \tag{2}$$

Later on Heligman & Pollard (1980) proposes an eight-component mortality model,

$$\mu_x = A^{(x+B)^C} + De^{-E(\log x - \log F)^2} + GH^x. \tag{3}$$

Figure 1 shows how this model can fit the entire age-grange by decomposing the age pattern of mortality into three pieces. Each part with a relatively small number of parameters to control it. There are three parameters (A, B and C) to describe child mortality, three to describe a very flexible accident hump (D, E and F) typically occurring in young adulthood, and finally two parameters (G and H) to describe mortality at older ages. The main disadvantage of this model is that in its traditional form is difficult to fit and it does not account for uncertainty.

Siler (1983) developed a five-parameter competing hazard model in order to capture the mortality during "immaturity", adulthood and senescence and to facilitate inter-specific comparison,

$$\mu_x = a_1 e^{-b_1 x} + a_2 + a_3 e^{b_3 x}, \tag{4}$$

where $\mu_x$ is the force of mortality and $a_1$, $a_2$, $a_3$, $b_1$ and $b_3$ are location and dispersion parameters.

Other important parametric functions or "laws" of mortality were developed by Thiele & Sprague (1871), Wittstein & Bumsted (1883), Steffensen (1930), Perks (1932), Harper (1936) and Weibull (1951).

Figure 1: *Observed and fitted death rates between age 0 and 100 for female population in England & Wales*

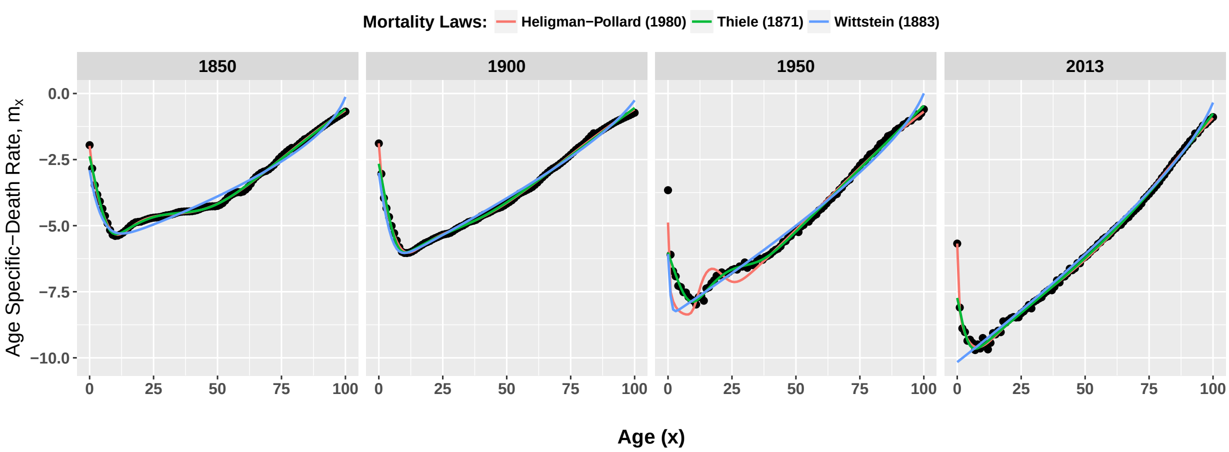
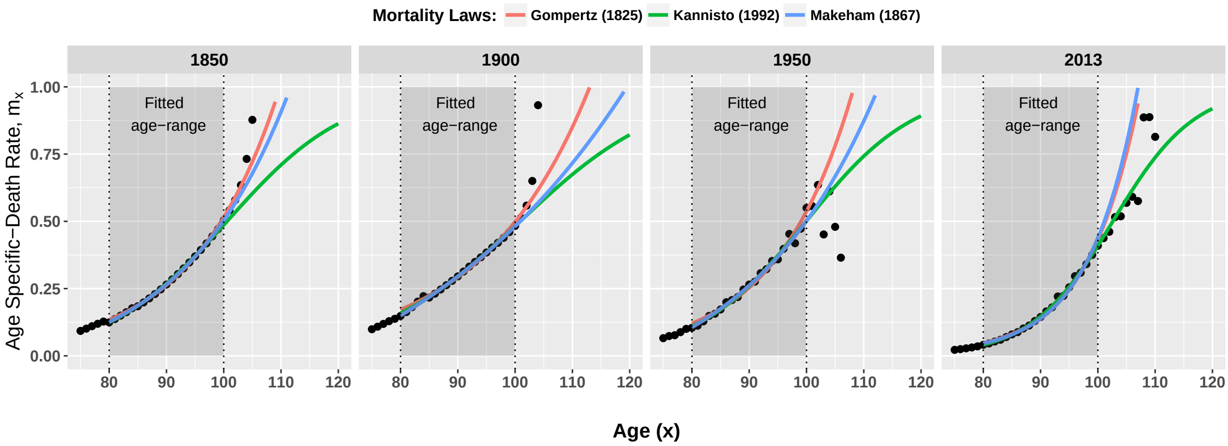


Figure 2: *Observed and fitted old-age mortality for female population in England & Wales*




Thatcher et al. (1998) performed studies to fit different mathematical models to different reliable data sets on adult and oldest-old mortality (aged 80 and above) covering the few recent decades. They evaluated the comparative compatibility of those models to the data, established the logistic model as the best mathematical model of human adult mortality, replacing the widely used Gompertz model and Makeham model. The logistic model assumes that the force of mortality $\mu_x$ is a logistic function of the age $x$. The simpler version of the logistic model is given by:

$$\mu_x = \frac{\alpha e^{\beta x}}{1 + \alpha e^{\beta x}}. \tag{5}$$

The above models describe the mortality at a fix point in time; however actual mortality is stochastic and evolve continuously. Thus, while the mortality models described above are static, the parameters must be re-fit periodically to accommodate changes in mortality patterns.

## The history and advancements of mortality modeling as an element for understanding human longevity

In this paper we want to have an overview of the main mathematical models used by researchers over time, in mortality modeling and forecasting, as presented in previous section. We want to outline a framework for developing forecasts of future mortality measures: age-specific death rates, mortality probabilities and expectation of life, in order to better understand the mortality evolution. The existing laws of mortality will be assessed and investigated based on their general characteristics and ability to explain historical patterns of mortality. Also, we will discuss the suitability of the existing stochastic mortality models for forecasting and estimating the future density of mortality rates at different ages.

The criteria employed includes: parsimony, quality of fit as measured by the Akaike or Bayes Information Criterion, ease of implementation, transparency, ability to incorporate cohort effects and robustness of parameter estimates relative to the period of data employed.

An *R* package is developed (**MortalityLaws**) which exploits the available optimization methods to provide tools for fitting a wide range of complex mortality models. The package will be published and made available at the time of the presentation in order to assure an instantaneous reproducibility of the results.

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