The formal demography of kinship: The time-varying model

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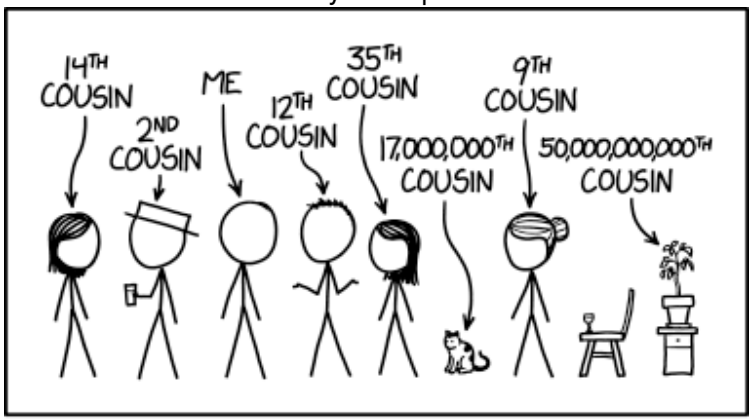
Why kinship?[1]

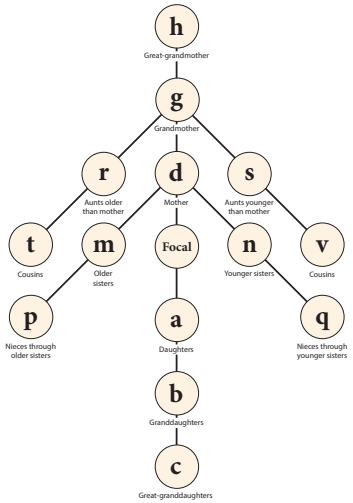


[1] https://xkcd.com/2608

Few of them made it to thirty.
Old age was the privilege of rocks and trees.
Childhood ended as fast as wolf cubs grow.
One had to hurry, to get on with life
before the sun went down,
before the first snow.

from: Our Ancestors' Short Lives by Wislawa Szymborska

The kinship network[2]



[2] Keyfitz and Caswell 2005

The basic kinship model

The kin of Focal as a population

$$\begin{aligned} \mathbf{k}(x+1) &= \mathbf{U}\mathbf{k}(x) + \beta(x) \\ \mathbf{k}(0) &= \mathbf{k}_0 \end{aligned}$$

where

$\mathbf{k}(x)$ = age distribution of a kin at age x of Focal[3]

$\beta(x)$ = recruitment 'subsidy' at age x of Focal

$$= \begin{cases} \mathbf{0} & \text{no subsidy} \\ \mathbf{F}\,\mathbf{k}^*(x) & \text{subsidy from } \mathbf{k}^* \end{cases}$$

$\mathbf{k}_0$ = initial condition

[3] To be more precise, the expectation of the age distribution.

Dynamics of kin of Focal at age x

Symbol	Kin	i.c. $\mathbf{k}_0$	$\beta(x)$
a	daughters	**0**	$\mathbf{Fe}_x$
b	granddaughters	**0**	$\mathbf{Fa}(x)$
c	great-granddaughters	**0**	$\mathbf{Fb}(x)$
d	mothers	π	**0**
g	grandmothers	$\sum_i \pi_i \mathbf{d}(i)$	**0**
h	great-grandmothers	$\sum_i \pi_i \mathbf{g}(i)$	**0**
m	older sisters	$\sum_i \pi_i \mathbf{a}(i)$	**0**
n	younger sisters	**0**	$\mathbf{Fd}(x)$
p	nieces via older sisters	$\sum_i \pi_i \mathbf{b}(i)$	$\mathbf{Fm}(x)$
q	nieces via younger sisters	**0**	$\mathbf{Fn}(x)$
r	aunts older than mother	$\sum_i \pi_i \mathbf{m}(i)$	**0**
s	aunts younger than mother	$\sum_i \pi_i \mathbf{n}(i)$	$\mathbf{Fg}(x)$
t	cousins from aunts older than mother	$\sum_i \pi_i \mathbf{p}(i)$	$\mathbf{Fr}(x)$
v	cousins from aunts younger than mother	$\sum_i \pi_i \mathbf{q}(i)$	$\mathbf{Fs}(x)$

An aside: the age at maternity distribution π

- could be measured, or
- could be calculated from an age distribution **w**

$$\pi = \frac{\mathbf{F}(1,:)^\intercal \circ \mathbf{w}}{\|\mathbf{F}(1,:)^\intercal \circ \mathbf{w}\|}$$

where

- **w** a measured age distribution, or
- **w** the stable age distribution implied by **U** and **F**

Time-varying demographic rates

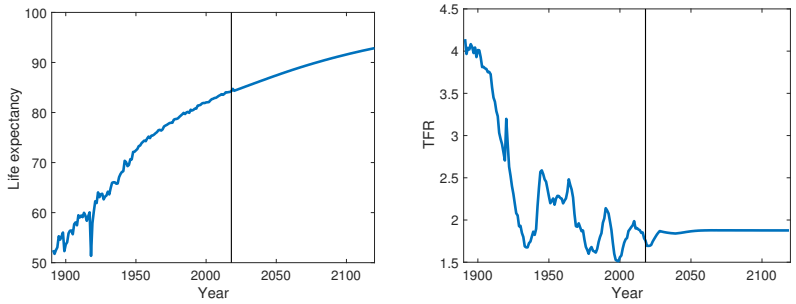
Survival and fertility over time:

$$\mathbf{U}_0, \mathbf{U}_1, \ldots, \mathbf{U}_T$$

$$\mathbf{F}_0, \mathbf{F}_1, \ldots, \mathbf{F}_T$$

- history of the **past**
- projection of the **future**

Sweden: past and future[4]



[4]HMD, HFD, Statistics Sweden

Time-varying kin dynamics[5]

Kin population

$$\mathbf{k}(x,t) = \text{ kin of type } \mathbf{k} \text{ at age } x \text{ of Focal at time } t$$

Dynamics

$$\mathbf{k}(x+1,t+1) = \mathbf{U}_t\,\mathbf{k}(x,t) + \beta(x,t) \qquad x = 0,\ldots,\omega \quad t = 0,\ldots,T$$

The subsidy vector

$$\beta(x,t) = \begin{cases} \mathbf{0} & \text{no subsidy} \\ \mathbf{F}_t\,\mathbf{k}^*(x,t) & \text{subsidy from } \mathbf{k}^* \end{cases}$$

[5]Caswell and Song, in prep; Song and Caswell, in prep.

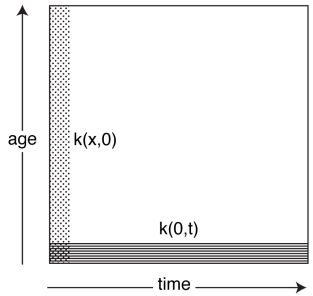
Initial conditions now become boundary conditions

- boundary at $t = 0$

$$\mathbf{k}(x,0) \qquad x = 0,\ldots,\omega$$

- boundary at $x = 0$

$$\mathbf{k}(0,t) \qquad t = 0,\ldots,\omega$$



Boundary $t = 0$

- assume stability (i.e., $\mathbf{U}_0$ and $\mathbf{F}_0$ have been operating for a long time)
- use the time-invariant model to compute $\mathbf{k}(x)$
- set $t = 0$ boundary

$$\mathbf{k}(x,0) = \mathbf{k}(x|\text{time-invariant})$$

Boundary $x = 0$

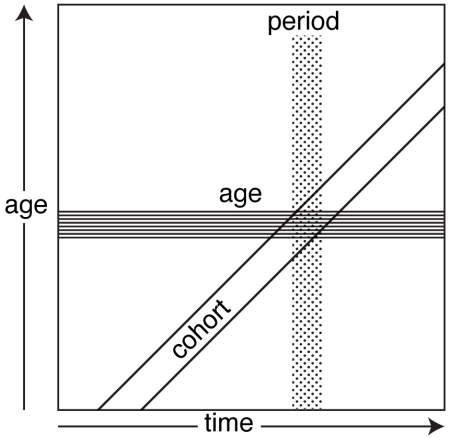
- need Focal's kin at birth at each time t
- if Focal has no kin at birth

$$\mathbf{k}(0,t) = \mathbf{0} \qquad t = 1, 2, \ldots$$

- if Focal has possible kin at birth

$$\mathbf{k}(0,t) = \sum_i \pi_i(t)\, \mathbf{k}^*(i,t) \qquad t = 1, 2, \ldots$$

Organizing the output: period, cohort, age

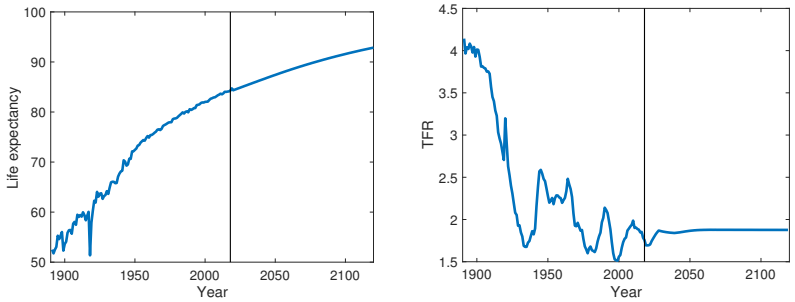


Daughters and descendents

$$\begin{aligned}
\mathbf{a}(x+1,t+1) &= \mathbf{U}_t\, \mathbf{a}(x,t) + \mathbf{F}_t\, \phi(x,t) \qquad x = 1, \ldots, \omega \quad t = 0, \ldots, \\
\mathbf{b}(x+1,t+1) &= \mathbf{U}_t\, \mathbf{b}(x,t) + \mathbf{F}_t\, \mathbf{a}(x,t) \\
\mathbf{c}(x+1,t+1) &= \mathbf{U}_t\, \mathbf{c}(x,t) + \mathbf{F}_t\, \mathbf{b}(x,t)
\end{aligned}$$

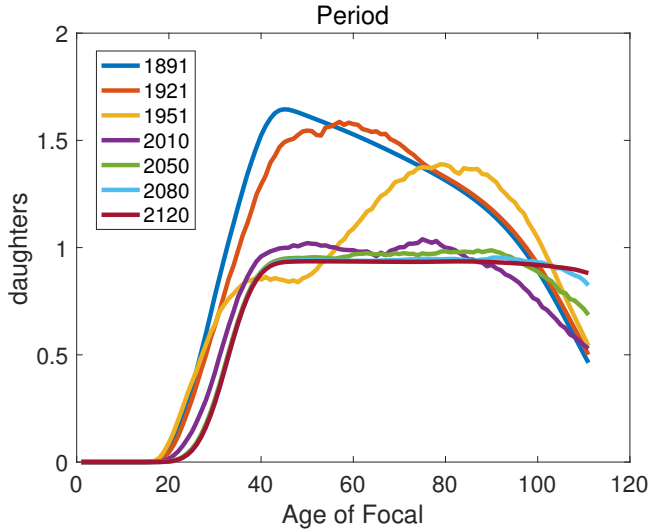
and so on...

Sweden: past and future[6]

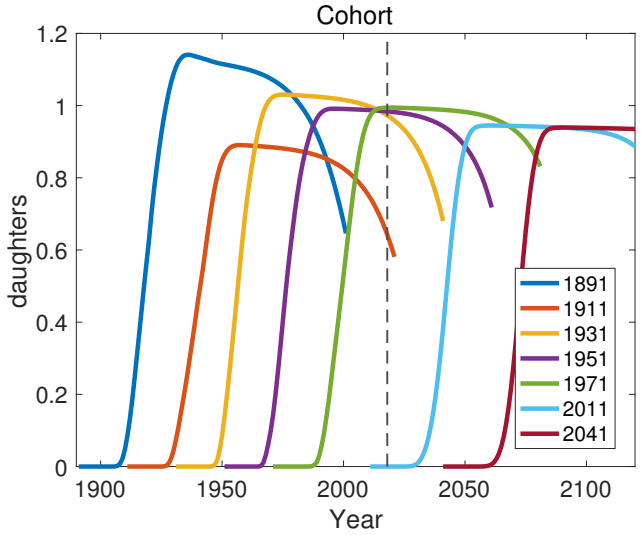


[6] HMD, HFD, Statistics Sweden

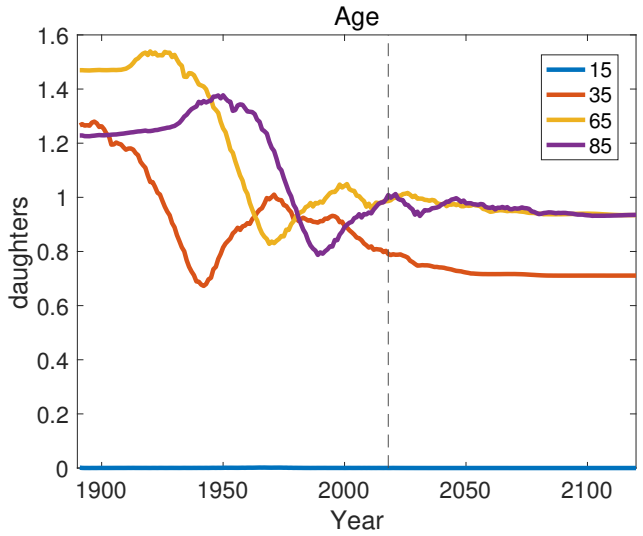
Sweden: daughters, period



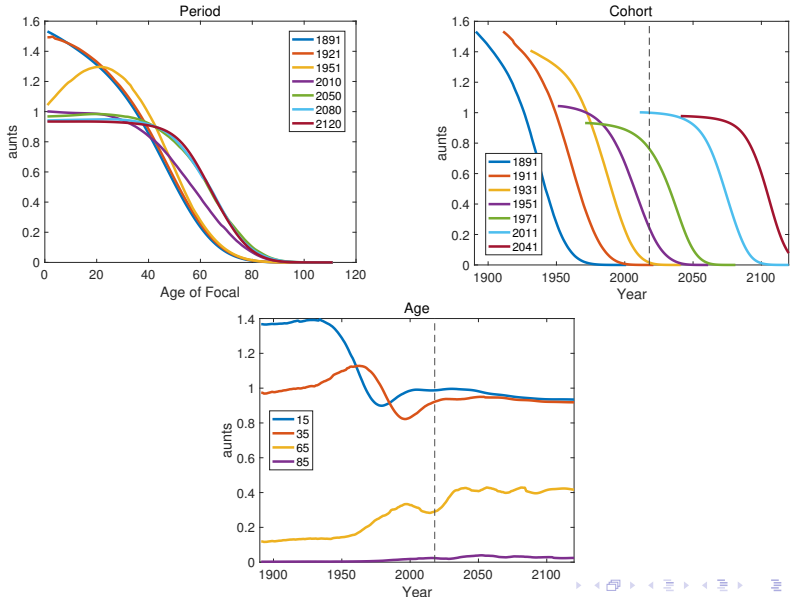
Sweden: daughters, cohort



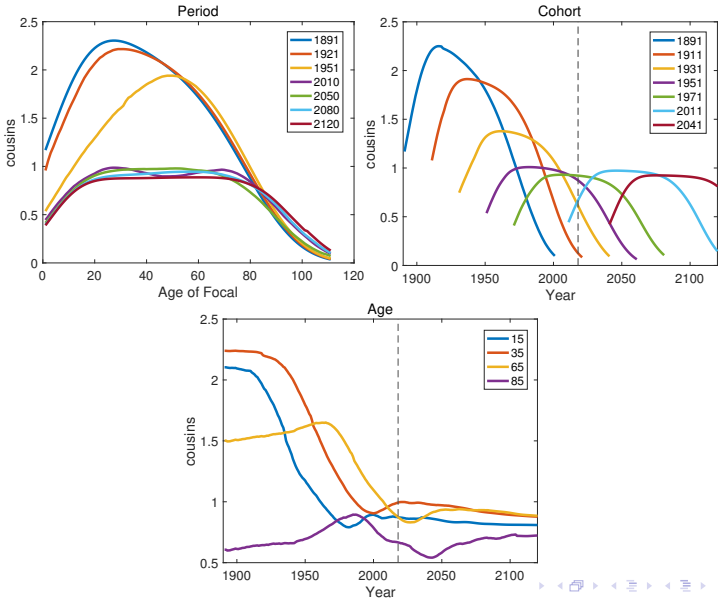
Sweden: daughters, age



Sweden: aunts



sweden cousins



Many more graphs in the paper