

(1.3.3)

Формула:  $\frac{a^e}{b+c} + cd$

Переменные:  $a = \varepsilon^5$ ,  $b = \varepsilon^9$ ,  $c = \varepsilon^{13}$ ,  $d = \varepsilon^{10}$ ,  
 $e = 4$

$$\frac{(\varepsilon^5)^4}{\varepsilon^9 + \varepsilon^{13}} + \varepsilon^{13} \cdot \varepsilon^{10} = \frac{\varepsilon^5}{\varepsilon^{10}} + \varepsilon^8 = \varepsilon^{10} + \varepsilon^8 = \varepsilon$$

1)  $(\varepsilon^5)^4 = \varepsilon^{20 \bmod 15} = \varepsilon^5$

2)  $\varepsilon^9 + \varepsilon^{13} = \oplus \begin{array}{r} 0101 \\ 1011 \\ \hline 1110 \end{array} = \varepsilon^{10}$

3)  $\varepsilon^{13} \cdot \varepsilon^{10} = \varepsilon^{23 \bmod 15} = \varepsilon^8$

4)  $\frac{\varepsilon^5}{\varepsilon^{10}} = \varepsilon^{-5 \bmod 15} = \varepsilon^{10}$

5)  $\varepsilon^{10} + \varepsilon^8 = \oplus \begin{array}{r} 1110 \\ 1010 \\ \hline 0100 \end{array} = \varepsilon$

(1.3.1)

$$p(x) = x^5 + x^4 + x^2 + 1$$



$$p_1(x) = x^5 + x^4 + x^2 + 1$$

	1	$\varepsilon$	$\varepsilon^2$	$\varepsilon^3$	$\varepsilon^4$
1	1	0	0	0	0
$\varepsilon$	0	1	0	0	0
$\varepsilon^2$	0	0	1	0	0
$\varepsilon^3$	0	0	0	1	0
$\varepsilon^4$	0	0	0	0	1
$\varepsilon^5$	1	0	1	0	0
$\varepsilon^6$	0	1	0	1	0
$\varepsilon^7$	1	1	0	1	0
$\varepsilon^8$	0	1	1	0	1
$\varepsilon^9$	1	0	0	1	1
$\varepsilon^{10}$	1	1	1	0	0
$\varepsilon^{11}$	0	1	1	1	0
$\varepsilon^{12}$	0	0	1	1	1
$\varepsilon^{13}$	1	0	1	1	0
$\varepsilon^{14}$	0	1	0	1	1
$\varepsilon^{15}$	1	0	0	0	0
$\varepsilon^{16}$	.	.	.	.	.
$\varepsilon^{17}$	.	.	.	.	.
$\varepsilon^{18}$	.	.	.	.	.
$\varepsilon^{19}$	.	.	.	.	.
$\varepsilon^{20}$	.	.	.	.	.

$$\begin{array}{r} \sqrt{x^5 + x^4 + x^2 + 1} \\ \sqrt{x^5 + x^4 + x^2 + 1} \phantom{+ 1} \\ \hline 0 \phantom{+ 1} \end{array}$$

~~Handwritten scribbles and crossed-out text.~~

$$\begin{array}{r|l} \cancel{E^6} & E^5 + E^4 + E^2 + 1 \\ + \cancel{E^6} + E^5 + E^3 + E & \\ \hline & E + 1 \\ \hline \cancel{E^5} + E^3 + E & \\ + \cancel{E^5} + E^4 + E^2 + 1 & \end{array}$$

$$E^4 + E^3 + E^2 + E + 1$$

$$\begin{array}{r}
 + \cancel{E^7} + E^6 + \cancel{E^4} + E^2 \quad | \quad E^5 + E^4 + E^2 + 1 \\
 \hline
 \phantom{+} \cancel{E^6} + E^4 + E^2 \\
 + \cancel{E^6} + E^5 + E^3 + E \\
 \hline
 \phantom{+} \cancel{E^7} + \cancel{E^4} + E^3 + \cancel{E^2} + E \\
 + \cancel{E^5} + \cancel{E^4} + \cancel{E^2} + 1 \\
 \hline
 E^3 + E + 1
 \end{array}$$

$$\begin{array}{r}
 \cancel{E^8} + \cancel{E^7} + \cancel{E^6} + \cancel{E^5} + \cancel{E^4} + \cancel{E^3} + \cancel{E^2} + 1 \\
 + \cancel{E^7} + \cancel{E^6} + \cancel{E^5} + \cancel{E^4} + \cancel{E^3} + \cancel{E^2} + \cancel{E} + 1 \\
 + \cancel{E^6} + \cancel{E^5} + \cancel{E^4} + \cancel{E^3} + \cancel{E^2} + \cancel{E} + 1 \\
 + \cancel{E^5} + \cancel{E^4} + \cancel{E^3} + \cancel{E^2} + \cancel{E} + 1 \\
 + \cancel{E^4} + \cancel{E^3} + \cancel{E^2} + \cancel{E} + 1 \\
 + \cancel{E^3} + \cancel{E^2} + \cancel{E} + 1 \\
 + \cancel{E^2} + \cancel{E} + 1 \\
 + 1 \\
 \hline
 E^4 + E^3 + 1
 \end{array}$$

$$\begin{array}{r}
 \cancel{E^8} + E^7 + E^5 + E^3 \quad | \quad E^5 + E^4 + E^2 + 1 \\
 \hline
 \cancel{E^7} + E^5 + E^3 \quad | \quad \cancel{E^5} + E^4 + E^2 + E \\
 \hline
 \cancel{E^7} + E^6 + E^4 + E^2 \\
 \hline
 + \quad \cancel{E^6} + \cancel{E^5} + E^4 + \cancel{E^3} + E^2 \\
 + \quad \cancel{E^6} + \cancel{E^5} + \cancel{E^3} + E \\
 \hline
 + \quad E^4 + E^2 + E
 \end{array}$$







$$\begin{array}{r}
 \varepsilon^{15} \mid \varepsilon^5 + \varepsilon^4 + \varepsilon^2 + 1 \\
 \varepsilon^{15} + \varepsilon^{14} + \varepsilon^{13} + \varepsilon^{12} \mid \varepsilon^{10} + \varepsilon^9 + \varepsilon^8 + \varepsilon^6 + \varepsilon^5 + \varepsilon^2 + 1 \\
 \hline
 \varepsilon^{14} + \varepsilon^{13} + \varepsilon^{12} + \varepsilon^{11} \\
 \varepsilon^{14} + \varepsilon^{13} + \varepsilon^{12} + \varepsilon^{11} + \varepsilon^9 \\
 \hline
 \varepsilon^9 + \varepsilon^8 + \varepsilon^6 + \varepsilon^5 + \varepsilon^2 + 1 \\
 \varepsilon^9 + \varepsilon^8 + \varepsilon^6 + \varepsilon^5 + \varepsilon^2 + 1 \\
 \hline
 \varepsilon^8 + \varepsilon^6 + \varepsilon^5 + \varepsilon^2 + 1 \\
 \varepsilon^8 + \varepsilon^6 + \varepsilon^5 + \varepsilon^2 + 1 \\
 \hline
 \varepsilon^7 + \varepsilon^6 + \varepsilon^5 + \varepsilon^2 + 1 \\
 \varepsilon^7 + \varepsilon^6 + \varepsilon^5 + \varepsilon^2 + 1 \\
 \hline
 \varepsilon^5 + \varepsilon^4 + \varepsilon^2 + 1 \\
 \varepsilon^5 + \varepsilon^4 + \varepsilon^2 + 1 \\
 \hline
 1
 \end{array}$$

1.3.2  $p_2(x) = x^7 + x^6 + x^3 + x + 1 = \underline{1} + x + x^3 + x^6 + x^7$

by row 1101001

	1	$\varepsilon$	$\varepsilon^2$	$\varepsilon^3$	$\varepsilon^4$	$\varepsilon^5$	$\varepsilon^6$
1	1	0	0	0	0	0	0
$\varepsilon$	0	1	0	0	0	0	0
$\varepsilon^2$	0	0	1	0	0	0	0
$\varepsilon^3$	0	0	0	1	0	0	0
$\varepsilon^4$	0	0	0	0	1	0	0
$\varepsilon^5$	0	0	0	0	0	1	0
$\varepsilon^6$	0	0	0	0	0	0	1
$\varepsilon^7$	1	1	0	1	0	0	1
$\varepsilon^8$	1	0	1	1	1	0	1
$\varepsilon^9$	1	0	0	0	1	1	1
$\varepsilon^{10}$	1	0	0	1	0	1	0
$\varepsilon^{11}$	0	1	0	0	1	0	1
$\varepsilon^{12}$	1	1	1	1	0	1	1
$\varepsilon^{13}$	1	0	1	0	1	0	0
$\varepsilon^{14}$	0	1	0	1	0	1	0
$\varepsilon^{15}$	0	0	1	0	1	0	1
$\varepsilon^{16}$	1	1	0	0	0	1	1
$\varepsilon^{17}$	1	0	1	1	0	0	0
$\varepsilon^{18}$	0	1	0	1	1	0	0
$\varepsilon^{19}$	0	0	1	0	1	1	0
$\varepsilon^{20}$	0	0	0	1	0	1	1

(4)



$$\begin{array}{r} \oplus 0000000 \\ 1101001 \\ \hline (\epsilon^2) 1101001 \end{array}$$

$$\begin{array}{r} \oplus 0110100 \\ 1101001 \\ \hline (\epsilon^8) 1011101 \end{array}$$

$$\begin{array}{r} \oplus 0101110 \\ 1101001 \\ \hline (\epsilon^9) 1000111 \end{array}$$

$$\begin{array}{r} \oplus 0100011 \\ 1101001 \\ \hline (\epsilon^{10}) 1001010 \end{array}$$

$$\begin{array}{r} \oplus 0010010 \\ 1101001 \\ \hline (\epsilon^{12}) 1111011 \end{array}$$

$$\begin{array}{r} \oplus 0111101 \\ 1101001 \\ \hline (\epsilon^{13}) 1010100 \end{array}$$

$$\begin{array}{r} \oplus 0001010 \\ 1101001 \\ \hline (\epsilon^{16}) 1100011 \end{array}$$

$$\begin{array}{r} \oplus 0110001 \\ 1101001 \\ \hline (\epsilon^{17}) 1011000 \end{array}$$

1.3.4

$$a = \epsilon^8$$

$$b = \epsilon^9$$

$$F_b = \begin{bmatrix} \epsilon^9 \\ \epsilon^{10} \\ \epsilon^{11} \\ \epsilon^{12} \end{bmatrix} = \begin{bmatrix} 0101 \\ 1110 \\ 0111 \\ 1111 \end{bmatrix}$$

$$\epsilon^8 = [1010]$$

$$[1010] \times \begin{bmatrix} 0101 \\ 1110 \\ 0111 \\ 1111 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \epsilon^2$$

$$\epsilon^8 \cdot \epsilon^9 = \epsilon^{17 \bmod 15} = \epsilon^2 = [0010]$$