距离公式

1. 两点间的距离

设 $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, 则两点间的距离公式为:

$$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
 (1)

2. 点到直线的距离

已知点 $P_0(x_0,y_0)$,那么点 P_0 到直线Ax + By + C = 0的距离公式为:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \tag{2}$$

若要证明公式(2),

证:

已知直线Ax+By+C=0,可得 $y=rac{-A}{B}x+rac{-C}{B}$,即斜率为 $\frac{-A}{B}$,并当x=0时, $y_1=rac{-C}{B}$,同样的,可设过点 $P_0(x_0,y_0)$ 的同斜率直线为 $y=rac{-A}{B}x+b$,

可知 $y_0=rac{-A}{B}x_0+b,\;b=y_0+rac{A}{B}x_0,\;\;$ 即 $y=rac{-A}{B}x+y_0+rac{A}{B}x_0,\;\;$ 并当x=0时, $y_2=y_0+rac{A}{B}x_0$,假设交叉角为a, $\sin a=rac{d}{l},\;\;d=l imes\sin a$,且斜率为k的直线交叉角a, $\sin a=\sqrt{rac{1}{1+k^2}}$,

可得:

$$egin{aligned} d &= |y_1 - y_2| imes \sin a \ &= |rac{-C}{B} - (y_0 + rac{A}{B} x_0)| imes \sin a \ &= |rac{-C}{B} - (y_0 + rac{A}{B} x_0)| imes \sqrt{rac{1}{1 + (rac{-A}{B})^2}} \ &= |rac{1}{B}| imes |-C - B y_0 - A x_0| imes rac{|B|}{\sqrt{A^2 + B^2}} \ &= rac{|A x_0 + B y_0 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$