

# Trigonometrija

$$\sin \alpha = \frac{\text{naspramna kateta}}{\text{hipotenuza}} = \frac{a}{c}$$

$$(\sin \alpha = \cos \beta = \cos(90^\circ - \alpha))$$

$$\angle CAB = \alpha$$

$$\cos \alpha = \frac{\text{nalegla kateta}}{\text{hipotenuza}} = \frac{b}{c}$$

$$(\cos \alpha = \sin \beta = \sin(90^\circ - \alpha))$$

$$\angle ABC = \beta$$

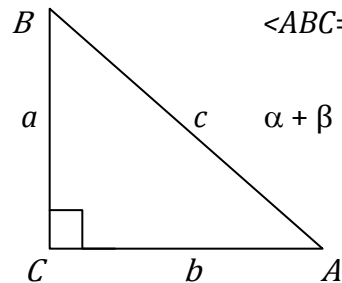
$$\operatorname{tg} \alpha = \frac{\text{naspramna kateta}}{\text{nalegla kateta}} = \frac{a}{b}$$

$$(\operatorname{tg} \alpha = \operatorname{ctg} \beta = \operatorname{ctg}(90^\circ - \alpha))$$

$$\alpha + \beta = 90^\circ$$

$$\operatorname{ctg} \alpha = \frac{\text{nalegla kateta}}{\text{naspramna kateta}} = \frac{b}{a}$$

$$(\operatorname{ctg} \alpha = \operatorname{tg} \beta = \operatorname{tg}(90^\circ - \alpha))$$



## Osnovne trigonometrijske formule:

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

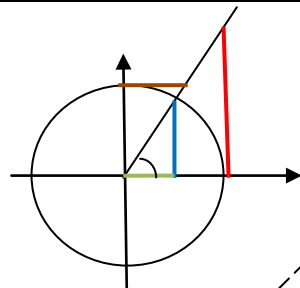
$$\operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1$$

## Vrednosti trigonometrijskih funkcija za neke važne uglove:

$\alpha$ (stepeni)	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
$\alpha$ (radiani)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\operatorname{tg} \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm \infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\mp \infty$	0
$\operatorname{ctg} \alpha$	$\pm \infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\mp \infty$	0	$\pm \infty$

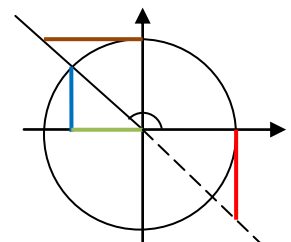
Prvi kvadrant:

$$\begin{aligned} \sin \alpha &> 0 \\ \cos \alpha &> 0 \\ \operatorname{tg} \alpha &> 0 \\ \operatorname{ctg} \alpha &> 0 \end{aligned}$$



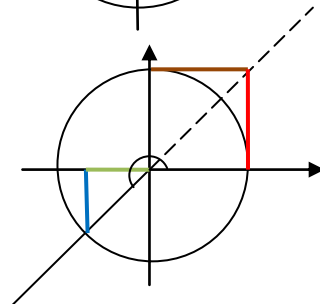
Drugi kvadrant:

$$\begin{aligned} \sin \alpha &> 0 \\ \cos \alpha &< 0 \\ \operatorname{tg} \alpha &< 0 \\ \operatorname{ctg} \alpha &< 0 \end{aligned}$$



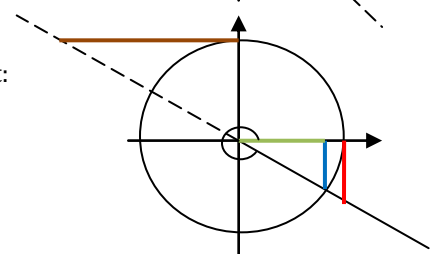
Treći kvadrant:

$$\begin{aligned} \sin \alpha &< 0 \\ \cos \alpha &< 0 \\ \operatorname{tg} \alpha &> 0 \\ \operatorname{ctg} \alpha &> 0 \end{aligned}$$



Četvrti kvadrant:

$$\begin{aligned} \sin \alpha &< 0 \\ \cos \alpha &> 0 \\ \operatorname{tg} \alpha &< 0 \\ \operatorname{ctg} \alpha &< 0 \end{aligned}$$



### Parnost i neparnost trigonometrijskih funkcija:

$$\sin(-\alpha) = -\sin\alpha \quad \cos(-\alpha) = \cos\alpha \quad \operatorname{tg}(-\alpha) = -\operatorname{tg}\alpha \quad \operatorname{ctg}(-\alpha) = -\operatorname{ctg}\alpha$$

### Periodičnost trigonometrijskih funkcija, $k \in \mathbb{Z}$ :

$$\sin(\alpha + 2k\pi) = \sin\alpha \quad \cos(\alpha + 2k\pi) = \cos\alpha \quad \operatorname{tg}(\alpha + k\pi) = \operatorname{tg}\alpha \quad \operatorname{ctg}(\alpha + k\pi) = \operatorname{ctg}\alpha$$

### Izražavanje jedne trigonometrijske funkcije pomoću druge za uglove u prvom kvadrantu.

$$0 < \alpha < \frac{\pi}{2}:$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha} = \frac{\operatorname{tg}\alpha}{\sqrt{1 + \operatorname{tg}^2\alpha}} = \frac{1}{\sqrt{1 + \operatorname{ctg}^2\alpha}}$$

$$\cos\alpha = \sqrt{1 - \sin^2\alpha} = \frac{1}{\sqrt{1 + \operatorname{tg}^2\alpha}} = \frac{\operatorname{ctg}\alpha}{\sqrt{1 + \operatorname{ctg}^2\alpha}}$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\sqrt{1 - \sin^2\alpha}} = \frac{\sqrt{1 - \cos^2\alpha}}{\cos\alpha} = \frac{1}{\operatorname{ctg}\alpha}$$

$$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sqrt{1 - \cos^2\alpha}} = \frac{\sqrt{1 - \sin^2\alpha}}{\sin\alpha} = \frac{1}{\operatorname{tg}\alpha}$$

### Adicione formule:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha\operatorname{tg}\beta}$$

$$\operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha}$$

### Trigonometrijski identiteti za dvostruki ugao:

$$\sin 2\alpha = 2\sin\alpha\cos\beta \quad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha \quad \operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha} \quad \operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}$$

### Trigonometrijski identiteti za polovinu ugla:

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos\alpha}{2} \quad \cos^2 \frac{\alpha}{2} = \frac{1 + \cos\alpha}{2} \quad \operatorname{tg}^2 \frac{\alpha}{2} = \frac{1 - \cos\alpha}{1 + \cos\alpha} \quad \operatorname{ctg}^2 \frac{\alpha}{2} = \frac{1 + \cos\alpha}{1 - \cos\alpha}$$

### Zbir i razlika trigonometrijskih funkcija:

$$\sin\alpha + \sin\beta = 2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$\sin\alpha - \sin\beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$\cos\alpha + \cos\beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}$$

$$\cos\alpha - \cos\beta = -2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

$$\operatorname{tg}\alpha \pm \operatorname{tg}\beta = \frac{\sin(\alpha \pm \beta)}{\cos\alpha\cos\beta}$$

$$\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha = \frac{\sin(\alpha \pm \beta)}{\sin\alpha\sin\beta}$$

$$\operatorname{ctg}\alpha \mp \operatorname{tg}\beta = \frac{\cos(\alpha \pm \beta)}{\sin\alpha\cos\beta}$$

### Proizvod trigonometrijskih funkcija:

$$\sin\alpha\sin\beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$