Stabla

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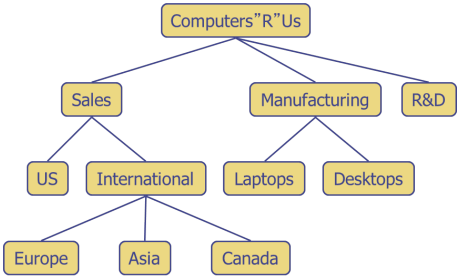
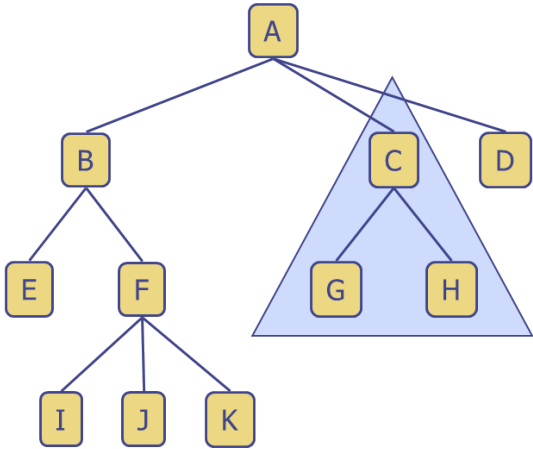
Stablo

- stablo je apstraktni model hijerarhijske strukture
- sastoji se od čvorova koji su u vezi roditelj/dete
- svaki čvor ima najviše jednog roditelja; tačno jedan čvor nema roditelja
- čvor ima nula ili više dece



Terminologija

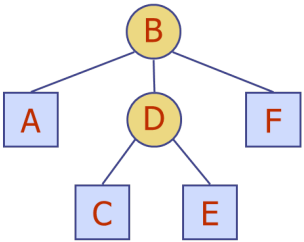
- koren (root): jedini čvor bez roditelja
- unutrašnji čvor: čvor sa bar jednim detetom
- spoljašnji čvor/list (leaf): čvor bez dece
- predak: roditelj, deda, pradeda, ...do korena
- dubina čvora: broj predaka
- visina stabla: najveća dubina
- potomak: dete, unuče, praunuče, ...
- podstablo: čvor stabla i njegovi potomci

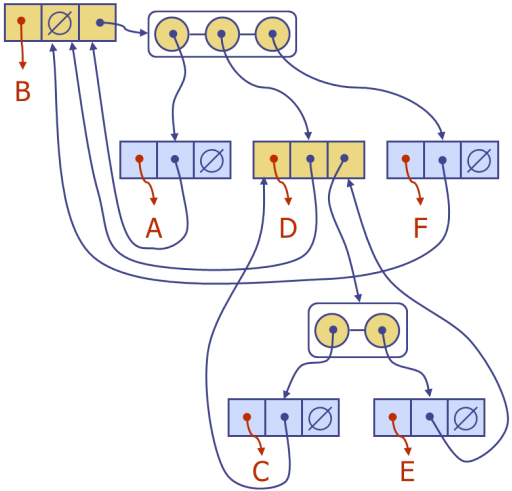


Stablo ATP

- opšte metode:
  - int len()
  - boolean is_empty()
  - iterator nodes()
- metode za pristup podacima:
  - node root()
  - node parent(n)
  - iterator children(n)
  - int num_children(n)
- metode za ispitivanje čvorova:
  - boolean is_leaf(n)
  - boolean is_root(n)
- ažuriranje sadržaja:
  - element replace(n, o)

Stablo u memoriji





Čvor stabla u Pythonu

```
class TreeNode:
  def __init__(self):
    self._element = None
    self._parent = None
    self._children = []

  def __eq__(self, other):
    return self == other:

  def __ne__(self, other):
    return self != other

  def is_root(self):
    return self._parent == None

  def is_leaf(self):
    return len(self._children) == 0
```

Stablo u Pythonu 1

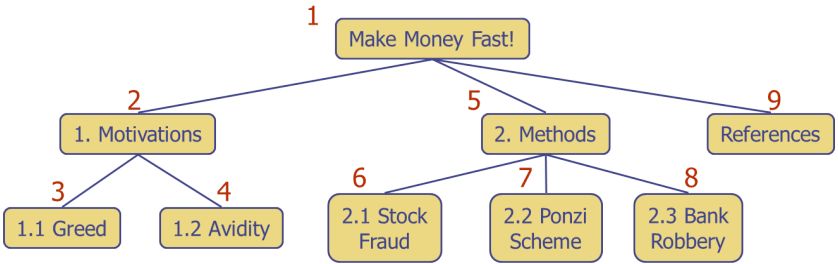
```
class Tree:
  def __init__(self):
    self._root = None

  def is_empty(self):
    return self._root == None

  def depth(self, node):
    if node._parent is None:
      return 0
    else:
      return 1 + self.depth(node._parent)
```

Obilazak stabla

- obilazak po dubini (depth-first): obiđi čvor i njegove potomke pre braće
  - preorder: prvo čvor pa deca
  - postorder: prvo deca pa čvor
- obilazak po širini (breadth-first): obiđi čvor i njegovu braću pre potomaka
  - obilazak „po generacijama“ u stablu

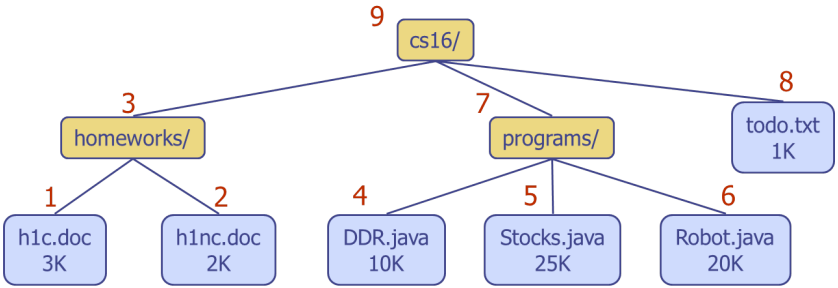
Obilazak stabla po dubini / preorder

preorder(n)
  obradi(n)
  for all dete c od n **do**
    preorder(c)



Obilazak stabla po dubini / postorder

postorder(n)
  for all dete c od n **do**
    postorder(c)
  obradi(n)

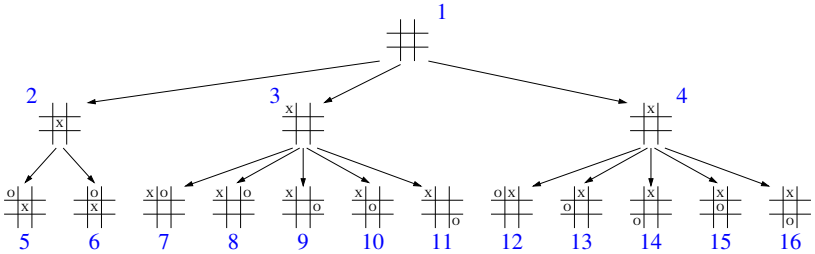


Stablo u Pythonu 2

```
class Tree:
  ...
  def preorder(self, func):
    self._preorder(self._root, func)

  def postorder(self, func):
    self._postorder(self._root, func)

  def _preorder(self, node, func):
    func(node)
    for child in node._children:
      self._preorder(child, func)

  def _postorder(self, node, func):
    for child in node._children:
      self._postorder(child, func)
    func(node)
```

Obilazak stabla po širini

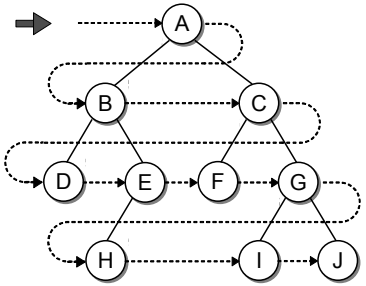
- treba obići sve čvorove dubine d pre nego što se pređe na čvorove dubine $d + 1$
- primer: stablo igre – svi mogući ishodi igre koju igra čovek ili računar; koren je početno stanje igre
- za igru „puta-nula“ (tic-tac-toe)



Obilazak stabla po širini

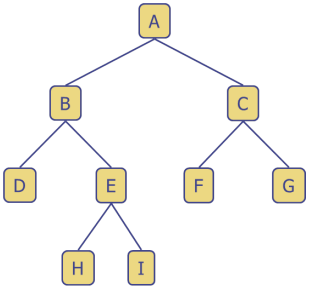
breadth_first($root$)

napravi novi prazan red Q
Q.add($root$)
while Q nije prazan **do**
  $node \leftarrow Q$.dequeue()
  obradi($node$)
  for all $child$ dete od $node$ **do**
    Q.enqueue($child$)



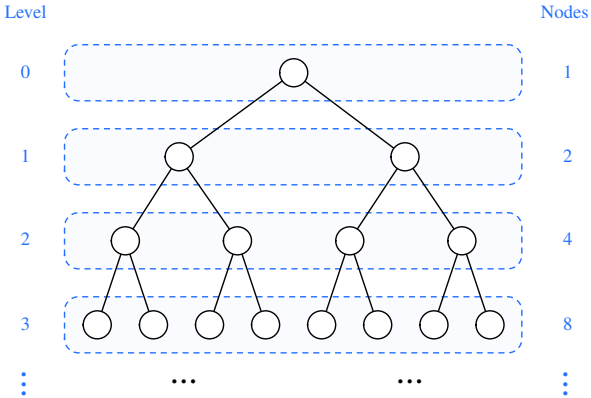
Binarno stablo

- stablo za koje važi:
  - svaki čvor ima najviše dvoje dece
  - svako dete je označeno kao levo dete ili desno dete
  - levo dete po redosledu prethodi desnom detetu
- levo podstablo – levo dete kao koren
- desno podstablo – desno dete kao koren
- pravilno binarno stablo: svaki čvor ima 0 ili 2 deteta



Osobine binarnog stabla

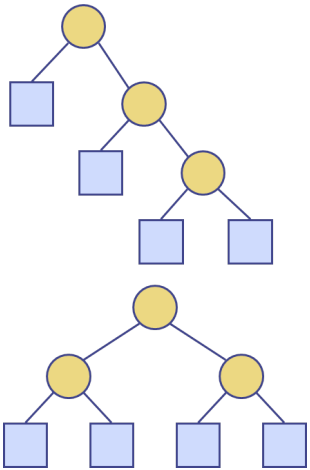
- nivo stabla d ima najviše 2^d čvorova
- broj čvorova po nivou raste eksponencijalno



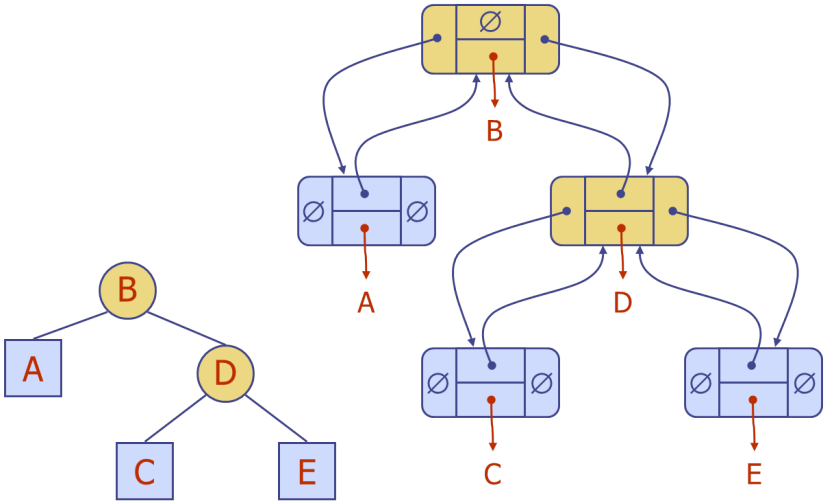
Osobine binarnog stabla

- n – broj čvorova
- e – broj listova
- i – broj internih čvorova
- h – visina

- $e = i + 1$
- $n = 2e - 1$
- $h \leq i$
- $h \leq (n - 1)/2$
- $e \leq 2^h$
- $h \geq \log_2 e$
- $h \geq log_2(n + 1) - 1$

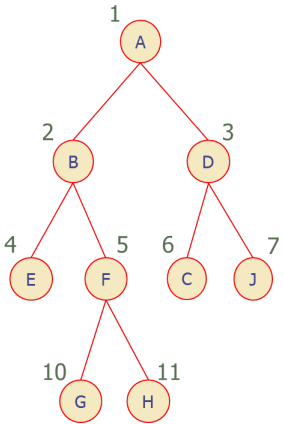


Binarno stablo u memoriji / čvorovi i reference



Binarno stablo u memoriji / pomoću niza

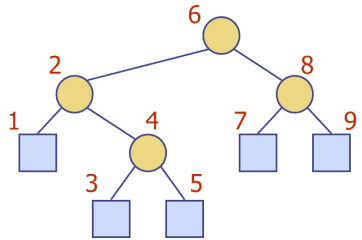
- rang čvora:
  - $\text{rang}(root) = 1$
  - za levo dete:
    $\text{rang}(node) = 2 \cdot \text{rang}(parent)$
  - za desno dete:
    $\text{rang}(node) = 2 \cdot \text{rang}(parent) + 1$
- čvor v se smešta u $A[\text{rang}(v)]$



Obilazak binarnog stabla / inorder

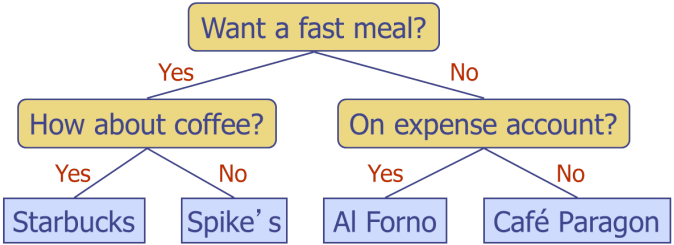
inorder(n)

if n ima levo dete **then**
  inorder(levo dete)
obradi(n)
if n ima desno dete **then**
  inorder(desno dete)



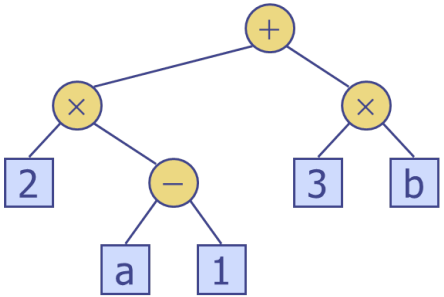
Stabla odlučivanja

- binarno stablo strukturirano prema procesu odlučivanja
- unutrašnji čvorovi – pitanja sa da/ne odgovorima
- listovi – odluke
- primer: gde za večeru?



Stablo aritmetičkih izraza

- binarno stablo kreirano na osnovu aritmetičkog izraza
- unutrašnji čvorovi – operatori
- listovi – operandi
- primer: $2 * (a - 1) + 3 * b$

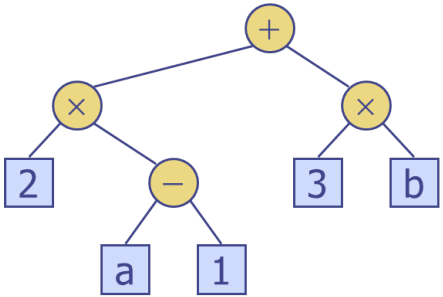


Ispisivanje aritmetičkih izraza

- specijalni slučaj **inorder** obilaska

printExpr(n)

if n ima levo dete **then**
  print("(")
  printExpr(levo dete)
print(n)
if n ima desno dete **then**
  printExpr(desno dete)
  print(")")

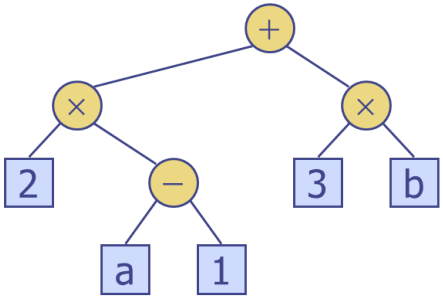


Izračunavanje aritmetičkih izraza

- specijalni slučaj **postorder** obilaska

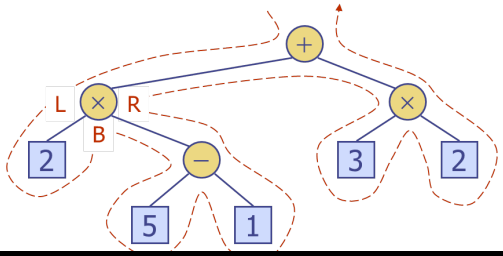
evalExpr(n)

if n je list **then**
  return $n.element$
else
  $x \leftarrow evalExpr(n.left)$
  $y \leftarrow evalExpr(n.right)$
  $\diamond \leftarrow$ operator u n
  return $x \diamond y$



Ojlerov obilazak stabla

- opšti postupak za obilazak stabla
- preorder, inorder, postorder su specijalni slučajevi
- posmatramo grane stabla kao zidove koji uvek moraju da nam budu sa leve strane prilikom kretanja
- svaki čvor se poseti tri puta
  - sa leve strane (preorder)
  - sa donje strane (inorder)
  - sa desne strane (postorder)



Crtanje stabla

- treba odrediti (x, y) koordinate čvorova stabla
- $x(n)$: broj čvorova posećenih pre čvora n u **inorder** obilasku
- $y(n)$: dubina čvora n