There are 6 components having a form like:

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad (u, v, w) = \vec{V}$$

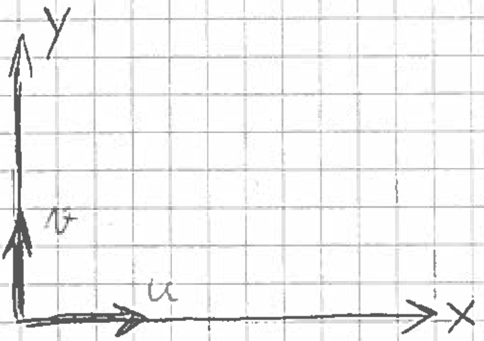
- $\varepsilon_{xz} = \varepsilon_{zx} =$ ____
- $\varepsilon_{yz} = \varepsilon_{zy} =$ ____

$$\varepsilon_{ij} (= \varepsilon_{ji}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

$i \neq j$

$j = 1, 2, 3$ or x, y, z

$i =$ ____



u, v — components of velocity vector along xy axis

By def, the ~~direction~~ dilation or extensional strain is def as the horizontal ~~lengths~~ increased of the fluid element in the corresponding direction

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

— True for Dilation Strain

The shear and ~~strain~~ (extensional) strains form a symmetric 2nd order Tensor which has these components

$$[\varepsilon] = [\varepsilon_{ij}] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ & \varepsilon_{yy} & \varepsilon_{yz} \\ \text{Symm} & & \varepsilon_{zz} \end{bmatrix}$$

It is important to notice that this strain tensor is assoc with 3 invariants / 3 quantities which are indep of direction or choice of axis

These invariants are written I_1, I_2, I_3

$$I_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

The strain rate tensor can also be linked to the so-called velocity gradient tensor and to the angular velocity of the fluid element

$$\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y}$$

are comp of velocity vector

→ comp of velocity grad tensor are by def

$$\left[\frac{\partial u_i}{\partial x_j}\right] = \underbrace{\begin{bmatrix} \frac{\partial u}{\partial x} & & \\ & \frac{\partial u_i}{\partial x_j} & \\ & & \end{bmatrix}}_{3\times 3}$$

And the angular velocity which has to be defined

$$\frac{\partial u_i}{\partial x_j} = \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}_{\varepsilon_{ij}} + \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)}_{\Omega_{ij}}$$

comp of an anti-symm tensor which repr ang vel, or in other terms the rate of rotation

Remark

- Theory of Elasticity (Strength of Materials - SMA)

  u, v, w usually represent elastic deformations along the coordinate axis x, y, z

  The strains ε_{xx}, ε_{xy}, ___ are defined as:

  $$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \quad i \neq j$$ True for all comp of the deformation

These are useful (see the Hooke's Law) to define the stresses in the elastic solid

Usually, these stresses are the comp of a tensor called the stress tensor which is a symm tensor written down with σ

In contrast/distinction, in FDY, one may use the same way (Ω) to express the stresses acting inside the fluid.

However, in place of elastic deformation u, v, and w, in case of fluids which are flowing, we are using the comp. of the velocity vector which are written down with the same u, v, w

N.B. These are velocities → we are speaking about strain rates, not strains!

$$\frac{\partial u_i}{\partial x_j} = \varepsilon_{ij}^{\cdot} + \Omega_{ij}^{\cdot}$$

The rate of rotation, when multiplied by 2 are the components of curl of velocity

$$\vec{\omega} = \mathrm{curl}\,\vec{v} = 2(\Omega_{ij})$$

This curl $\vec{v}$, $\vec{\omega}$, is called in FDY the vorticity vector.

→ Vorticity of a fluid

The comp of the vorticity vector

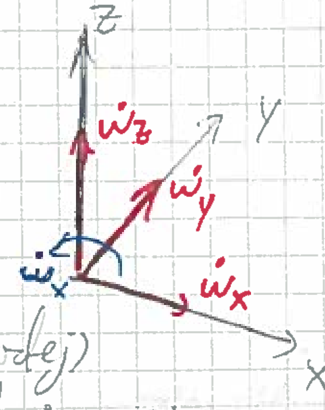
— repr rates of rotation

$$\vec{\omega} = 2(\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z)$$

$$\dot{\omega}_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$\dot{\omega}_y =$

$\dot{\omega}_z =$

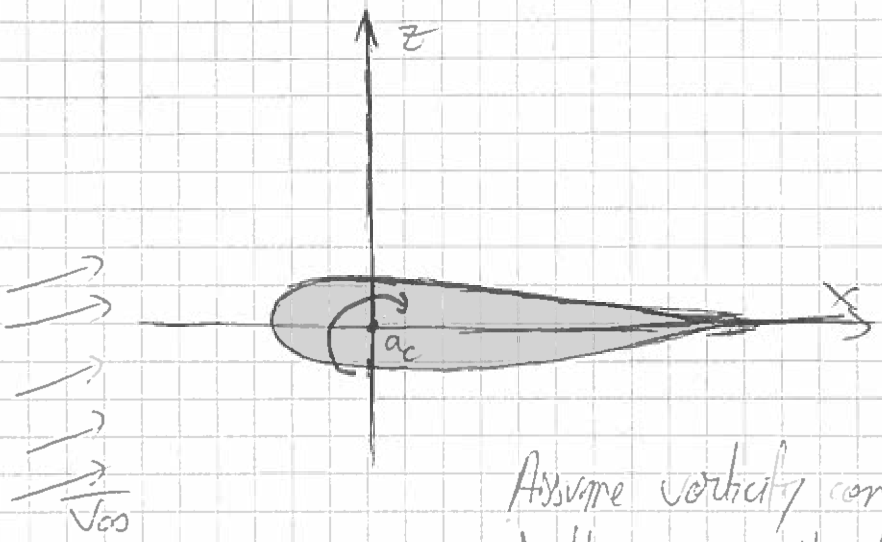


The complex of vorticity is related to the concept of vortex (rou vortej) which is a very important tool in FDY

i.e. This concept and its assoc vorticity may be used to explain the lift force ~~for implied (non-viscous fluids~~

Ex

Consider that we have an airfoil, uniform free stream



Vortex Filament (Concentrated vortex) – ideal model of a real vortex for which the vorticity is concentrated on a curve line

Assume vorticity concentrated on Oy axis
In this case, the Kutta – Jukovski (lift acting on airfoil) is given by $\boxed{L = \rho_\infty V_\infty \gamma}$

$\gamma = \Gamma$ = circulation
- related to intensity of this concentrated vortex

The splitting of the grad. of velocity tensor is important because only the strain rates ε_{ij} are producing (viscous) stresses inside the fluid

So, the components of the anti-symmetric tensor do not contribute to the stresses acting in the fluid

To conclude, only strain rates are resp. for producing stresses (ε_{ij})

- From historical point of vie, Navier (1823) and Stokes (1851) published two papers which are repr. the pillars of FDY
  They elaborated a theory I^st part of 19^th Century based on the following hypothesis
  1) Fluid is continuous and its Viscous Stress Tensor $\tau = [\tau_{ij}]$ are a continuous function of the strain rates

2) The second hypothesis is that fluid is isotropic
(its properties do not depend on the direction / reference frames)

3) Further, the fluid is homog and this means that the comp τ_{ij} of the viscous stress tensor do not depend explicitly on x, y and z and on time

4) Finally, very important
when the strain rates are 0 (if there is no flow, fluid at rest) the only remaining stresses are due to the pressure (to the hydrostatic pressure)

(By def, a newtonian fluid is the fluid for which the stress components depend linearly on the rates of deformation

This means finally that the comp of viscous stress tensor are given $\tau_{ij} = \lambda\underbrace{(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})}_{I_1}\delta_{ij} + 2\mu\varepsilon_{ij}$

δ_{ij} = Kronecker Delta = $\begin{cases} 1, & i \neq j \\ 0 \end{cases}$

λ, μ are 2 coeff which depend on the material
Subsequently, the complete stress tensor is written

a) $\sigma_{ij} = (-p + \lambda \underset{\mathrm{div}\,\vec{v}}{\underset{\parallel}{I_1}})\delta_{ij} + 2\mu\varepsilon_{ij}$

$\sigma_{ij} = (-p + \lambda \underset{\mathrm{div}\,\vec{v}}{\underset{\parallel}{I_1}})\delta_{ij} + 2\mu\varepsilon_{ij}$

$\mathrm{div}\,\vec{v} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$

If $u = v = w = 0 \Rightarrow \sigma_{ii} = -p$
$i = 1, 2, 3$

Force hypoth (4): absence of motion
stresses → hydrostatic pressure
Pressure is defined by thermodynamics and it was Stokes who noticed this and prop to define first $\overline{p}$ as the ... stress on the fluid element

$\bar{p} = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$ - related to isotropy of material

Further, this new pressure is related to the stresses inside fluid (normal stresses)

The - sign = compression ($\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ - extension) or

If we combine this $\bar{p}$ with terms for σ_{ij} obtained from prev expressions we obtain:

$$\bar{p} = p + \boxed{\left(\lambda + \frac{2}{3}\mu\right) I}$$ - depends on material prop

fluid pressure — thermodynamic props

Stokes propose to make zero the coefficient $\underline{\lambda + \frac{2}{3}\mu}$, in order to make $\bar{p} = p$

$\Rightarrow \boxed{\lambda = -\frac{2}{3}\mu}$, where μ is the coeff of the dynamic viscosity (dynamic viscosity of the fluid)

To conclude, the Stokes hypothesis and its equiv to the assumption that the thermodynamic pressure is equal to the minus of third of the first invariant of the normal stresses

Using the Navier Eqs of Motions (Balance of Momentum) and the Stokes hypothesis, we obtain the Constitutive Relations for an isotropic newtonian fluid

$$\sigma_{xx} = \underbrace{-p - \frac{2}{3}\mu \operatorname{div} \vec{v} + 2\mu \frac{\partial u}{\partial x}}_{\tau_{xx}} = -p + \tau_{xx}$$

$\sigma_{yy} =$ ______

$\tau \sim \mu L$

$\sigma_{zz} =$ ______

$$\sigma_{xy} = \sigma_{yx} = \tau_{xy} = \tau_{yx} = \mu \underbrace{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)}_{2\varepsilon_{xy}}$$

$\tau_{xz} = \tau_{zx} =$ ______

τ_{yz} ... ______

All tangential stresses inside the fluid are due to the viscosity of the fluid

These are const rel for a newt

(stress comp depend linearly on the rates of deformation)

The viscosity is a prop of fluid which repr the friction in the fluid

The rheology is the branch of FDY which studies the viscous behaviour of fluids

Ideal

A reasonable hypothesis very useful for solving a lot of engineering prob is to neglect the effects of viscosity

In this case, the components of the viscous stress tensor τ are equal to 0 ($\tau_{ij} = 0, \quad i,j = \overline{1,3}$

This leads to first

$[V] = -p[I]$

1) All tangential stresses are zero

$\tau_{xy} = \quad = \tau_{yz} = 0$

2) All normal stresses are equal to the pressure with opposite sign

$\tau_{xx} = \tau_{yy} = \tau_{zz} = -p$

The ideal fluids (the concept) have been studied for the first time by Euler at the end of 18th Century

Therefore, for the case of viscous fluids, we call the Navier Stokes eqs, and we neglect effects of viscosity, we call Govern Eqs as the Euler Eqs

As a remark, there are quite a few fluids of great technical biological importance whose behaviour cannot be described by a linear dependency between the viscous stresses and the strain rates

These fluids are called non-newtonian (include in this cat.: polymers, biological solutions, soap, glues, paints, asphalts ~)

(up to this for Friday)

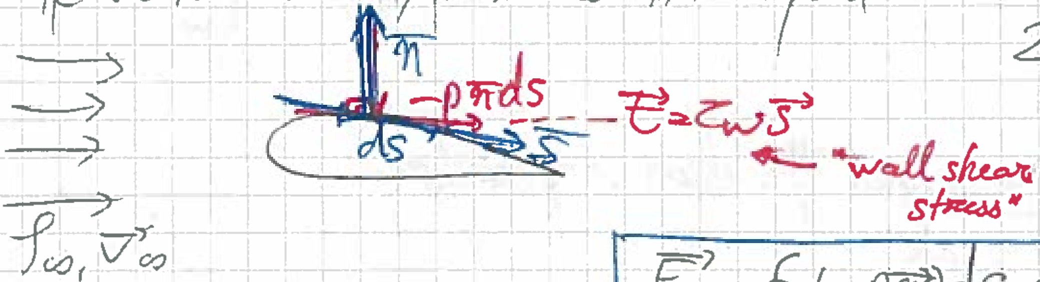
26.11.2018

Definitions for aerodynamic forces

(Def Drag, Lift)

If we have a body immersed into a fluid

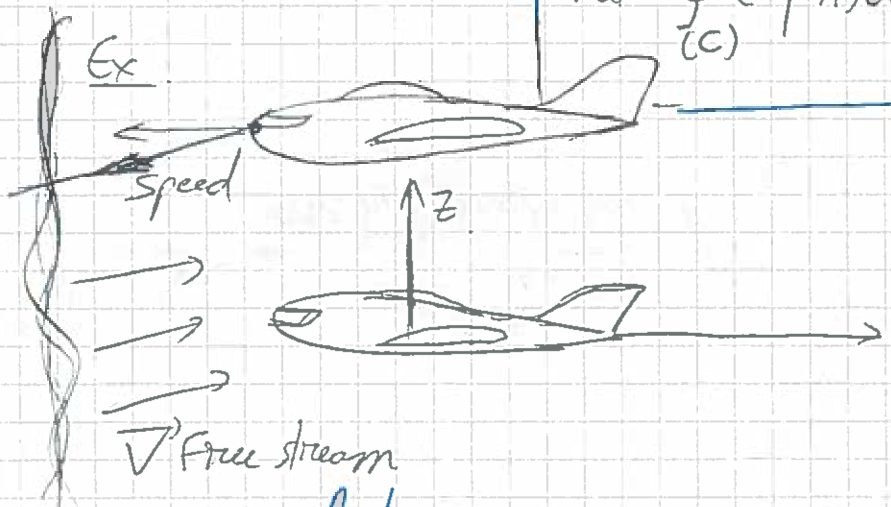
2D case (airfoil)



$\vec{\tau} = \tau_w \vec{s}$

$$\boxed{\vec{F_a} = \oint_{(C)} (-p\vec{n})ds + \oint_{(C)} \vec{\tau}\, ds = \vec{L}+\vec{D} = \vec{N}+\vec{A}}$$

Ex



The sources of drag are 4:

1. The skin friction-drag (rezistența de frecare) (viscous)
   - the drag due to the action of shear-stresses on the aircraft surface
2. The pressure drag (rezistența de formă / de presiune)
   - the drag created by pressure forces due to the presence of viscous effects (boundary layers) on the aircraft surface "strat limită"