FRACTALS-5

**Methods for stationary time series analysis**

*Auto-correlation function*

For the time series  with mean and variance , auto-correlation function is defined as



For uncorrelated data for If data exhibit short-term correlations, decays exponentially



with a characteristic decay time.

For self-affine time series with long term correlations , decays as a power-law



with correlation exponent 

For non-stationary processes is not well defined, and for large scales ,strongly fluctuates around zero which makes difficult to find the correct correlation exponent 

*Power spectrum*

Power spectrum is defined as the Fourier transform of autocorrelation function [1]



For self-affine time series

with 

The spectral exponent  and the correlation exponent can be obtained from linear regression versus Spectral analysis also applies on stationary data.

*Hurst’s rescaled-range analysis*

This method is based on random walk theory and has been developed by British hydrologist H.E. Hurst while working in Egypt on the Nile River Dam Project and studying a half-century long record of the Nile River’s over flows [2]. It proceeds as follows [3].

1. The time series  is divided into  non-overlapping segments of length *s*.
2. In each segment  we calculate the local mean  and the standard deviation 
3. Next we calculate the profile  and the range



1. The rescaled range is calculated as



The Hurst exponent  is estimated as the slope of linear regression versus  and is related with correlation exponent  and spectral exponent  by . The value indicates the absence of correlations (white noise), indicates persistent process meaning that large (small) values are more likely to be followed by large (small) values, and indicates anti persistent process, meaning that large values are more likely to be followed by small values and vice versa.

The values of , that can be obtained by Hurst’s rescaled range analysis, are limited to 0<H<1 and significant inaccuracies are to be expected close to the bounds. Since  can be increased or decreased by 1 if the data is integrated () or differentiated ( ), respectively, one can always find a way to calculate  by rescaled range analysis provided the data is stationary [3].

*Fluctuation analysis*

This method is also based on random walk theory, and it proceeds as follows [3].

1. For a time series , we first construct the profile



which we can interpret as a position of random walker on linear chain after steps.

1. Next, we divide the profile into non-overlapping segments of size and in each segment , we use end-points of the segment to calculate fluctuation



1. The mean fluctuation is calculates by averaging over all segments :



 can be interpreted as the root-mean-square displacement of random walker on the chain after steps. For data with long-term correlations, increases as power-law



For monofractal processes, fluctuation exponent is identical with the Hurst exponent and it is related to correlation exponent and spectral exponent by

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The range of the α values that can be studied by standard FA is limited to 0 < α < 1, with significant inaccuracies close to the bounds. Regarding integration or differentiation of the data, the same rules apply as listed for Hurst analysis. The results of FA become statistically unreliable for scales s larger than one tenth of the length of the data, i. e. the analysis should be limited by s < N/10 [3].

**Bibliography**

[1] G. Rangarajan, M. Ding, Integrated approach to the assessment of long range correlations in time series data, Physical Review E 61, 4991-5001, 2000.

[2] H.E. Hurst, Long-term storage capacity of reservoirs, Transactions of American Society of Civil Engineers 116, 770-808, 1951.

[3] J.W. Kantelhardt, Fractal and multifractal time series. In Meyers, R. A. (ed.) Encyclopedia of Complexity and Systems Science, vol. LXXX, 3754–3778, Springer, Berlin, 2009.