

# Decomposição de séries temporais no R

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## Decomposição de séries temporais

Séries temporais são sequências de dados ao longo do tempo. Muitas dessas séries podem apresentar comportamentos que são visualmente regulares, ou seja, repetem-se ao longo do tempo. Quando realiza-se a decomposição de séries temporais, considera-se três componentes principais, que são:

1. **tendência** (crescimento ou decrescimento, global ou local);
2.  **sazonalidade**;
3. e o **erro**.

Nesse pequeno *script* irei realizar a decomposição de uma série de habitações de um censo dos Estados Unidos. A série possui periodicidade mensal, indo de Janeiro de 1959 até Fevereiro de 2021.

Irei primeiro instalar os pacotes

```
library(dplyr)
```

```
## Warning: package 'dplyr' was built under R version 4.0.4
```

```
##  
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':  
##  
##   filter, lag
```

```
## The following objects are masked from 'package:base':  
##  
##   intersect, setdiff, setequal, union
```

```
library(lubridate)
```

```
## Warning: package 'lubridate' was built under R version 4.0.5
```

```
##  
## Attaching package: 'lubridate'
```

```
## The following objects are masked from 'package:base':  
##  
##    date, intersect, setdiff, union
```

```
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':  
##    method      from  
## as.zoo.data.frame zoo
```

### Importando os dados

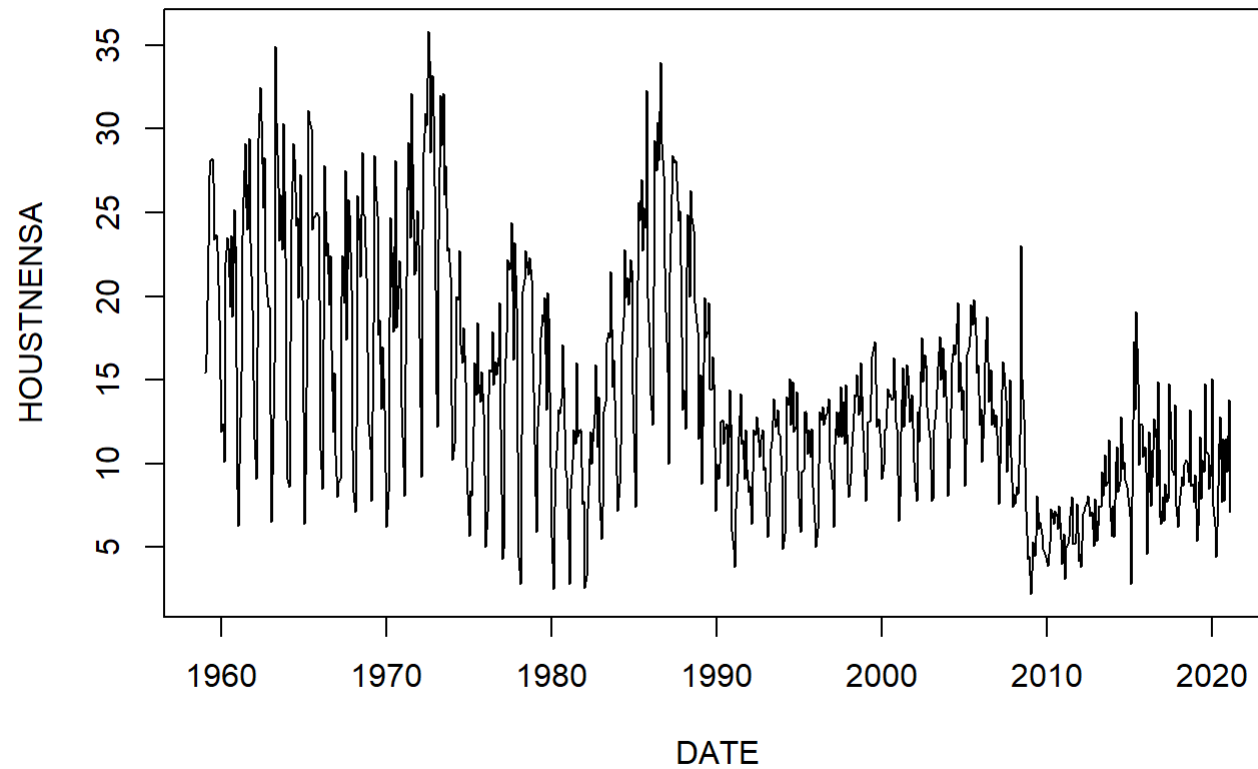
```
serie = read.csv("HOUSTNENSA.csv")
```

### Transformando os dados

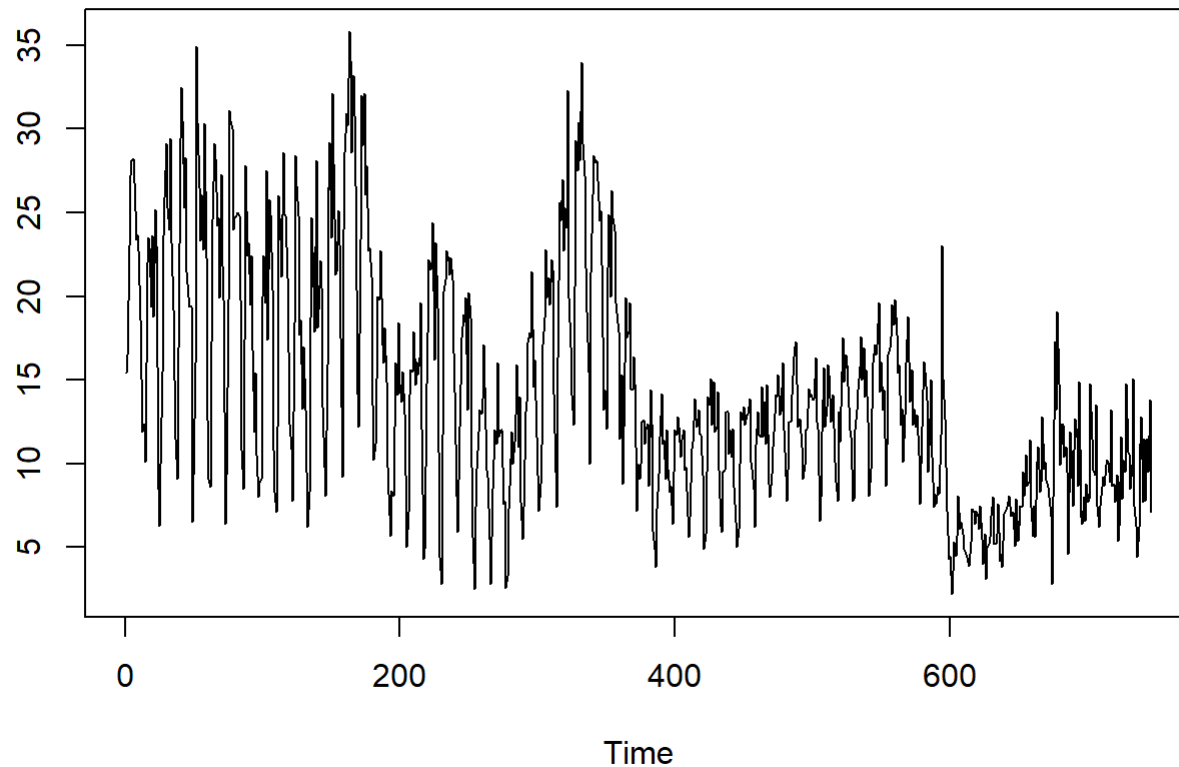
```
#serie$DATE = strptime(serie$DATE, "%m/%d/%Y")  
#serie$DATE = format(serie$DATE, "%Y-%m-%d")  
serie$DATE = as.Date(serie$DATE)
```

Nos gráficos abaixo podemos visualizar a sazonalidade da série temporal.

```
#Plotando a base de dados  
plot(serie, type="l")
```



```
#plotando com a tidyverse  
serie$HOUSTNENSA %>%  
  plot.ts()
```



Renomeando a variável.

```
colnames(serie)[2] <- 'y'  
colnames(serie)
```

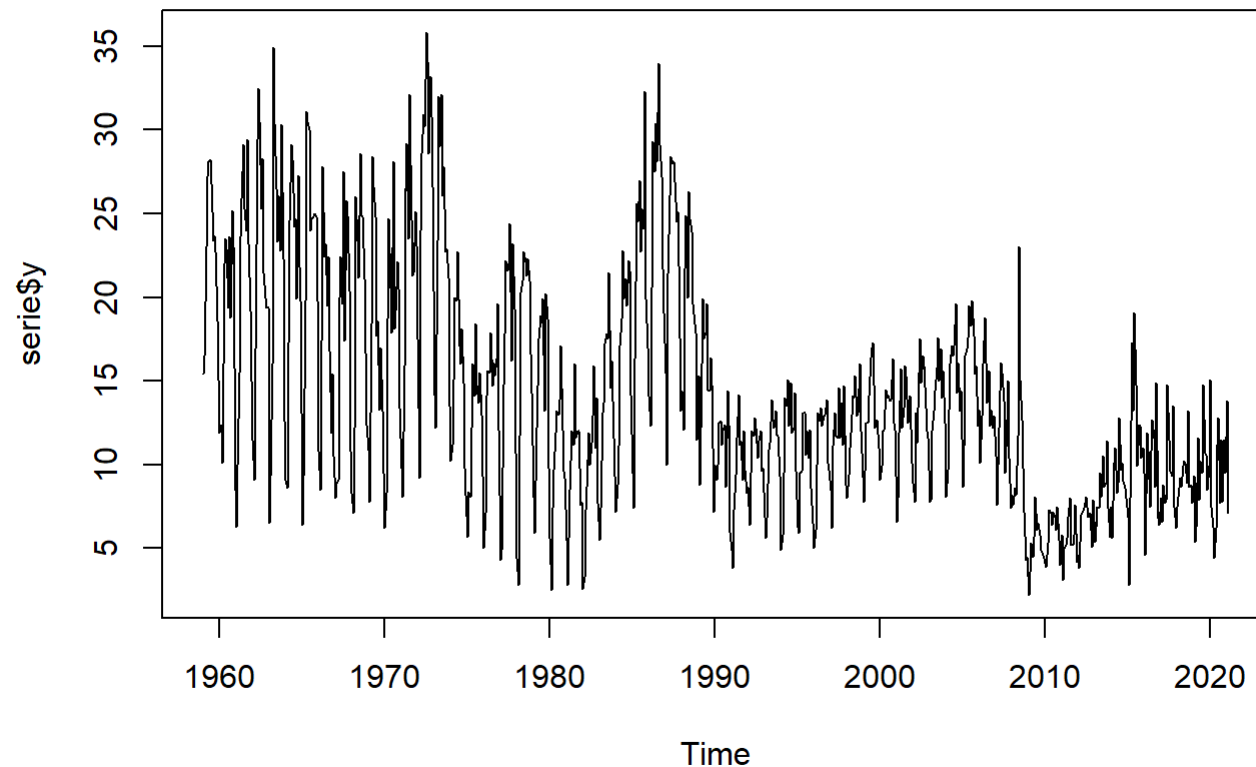
```
## [1] "DATE" "y"
```

Convertendo a base de dados para uma série temporal

```
serie$y <- ts(serie$y, frequency = 12,  
             start = c(1959,1), end = c(2021,2))
```

Plotando apenas as variável 'y' vemos que ela foi convertida em série temporal, sem a necessidade da variável 'DATE'.

```
plot(serie$y)
```



### Decomposição da série temporal.

A séries pode ser decomposta de duas formas: multiplicativa e aditiva (A forma aditiva é a padrão na função do R).

#### FORMA ADITIVA

A forma *aditiva* é a soma dos componentes:

$$Y_t = T_t + S_t + e_t$$

Irei salvar as séries decompostas em uma variável.

```
decomposicao_adi = decompose(serie$y)
```

Posso ver os valores gerados.

```
decomposicao_adi
```

```
## $x
##      Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
## 1959 15.4 16.5 23.0 27.2 28.1 28.2 26.8 23.4 23.7 22.7 18.6 15.1
## 1960 11.9 12.4 10.1 22.0 23.5 22.9 19.4 23.6 18.8 25.2 23.0  8.5
## 1961  6.3 10.0 15.5 23.6 25.9 29.1 25.3 24.0 29.4 23.2 19.2 15.0
## 1962 11.5  9.1 16.9 29.4 32.5 30.5 25.3 28.3 21.5 20.0 19.4 19.4
## 1963  6.5  9.3 16.0 34.9 30.4 26.5 23.3 26.1 22.8 30.3 22.3 12.6
## 1964  9.1  8.6 17.3 24.6 29.1 28.2 24.2 24.7 19.9 27.3 21.8 19.8
## 1965  6.4  9.3 19.8 31.1 30.5 30.0 24.0 24.7 24.8 25.0 24.8 19.8
## 1966 10.8  8.5 17.4 27.8 22.4 23.2 19.5 22.4 16.8 11.9 15.4 10.5
## 1967  8.0  8.9  9.1 20.0 22.4 19.6 27.5 17.4 25.8 24.9 20.4 10.8
## 1968  8.3  7.1 12.2 26.0 23.4 21.2 28.6 24.9 24.8 21.1 16.7 12.4
## 1969 10.9  7.8 18.4 28.4 26.6 24.8 17.7 18.6 13.3 17.0 13.2  9.5
## 1970  6.2  8.3 18.7 24.7 20.6 17.9 28.1 18.1 19.2 22.1 20.5 13.4
## 1971  9.8  8.1 15.9 26.5 29.2 23.5 32.1 27.9 21.3 21.5 25.1 23.1
## 1972 17.5  9.2 24.0 28.6 30.9 30.2 35.8 34.1 28.6 33.2 30.8 26.6
## 1973 14.2 12.2 23.4 32.0 29.0 32.1 26.1 27.8 22.7 22.9 21.2 13.5
## 1974 10.2 11.2 15.1 20.0 19.8 22.7 17.1 16.0 18.1 14.9  9.8  8.2
## 1975  5.7  8.4  8.1 12.2 16.0 14.1 18.4 14.8 13.7 15.5 13.0  9.3
## 1976  5.0  7.1 14.1 15.6 15.5 17.9 14.7 16.1 15.3 16.3 19.6 12.1
## 1977  4.3  4.9 14.8 17.8 22.2 21.6 22.0 24.4 16.2 23.2 19.3 10.7
## 1978  4.4  2.8 11.7 20.3 21.0 22.7 22.1 21.3 22.3 21.0 16.6 14.3
## 1979  8.1  5.9 12.2 14.7 17.5 18.9 18.5 19.9 13.2 20.2 18.7  9.8
## 1980  5.6  2.5  7.4  9.7 11.9 13.2 13.0 13.9 17.1 12.1  9.6  9.2
## 1981  5.5  2.8  8.5 10.2 12.0 11.2 16.0 11.6 12.0 12.0  7.6  7.8
## 1982  2.6  3.3  7.3  8.7 11.9 10.0 11.3 12.9 15.9 11.0 14.0  7.7
## 1983  5.5  7.4 13.1 13.7 17.2 17.8 17.5 21.5 14.6 16.2 13.8  9.3
## 1984  7.2  8.8 11.1 17.1 18.7 22.8 19.9 21.1 19.5 22.2 21.6 14.2
## 1985  9.5  7.4 16.3 25.6 24.5 27.0 22.7 25.3 24.1 32.3 19.9 17.0
## 1986 14.1 12.3 20.6 29.3 27.5 30.4 28.1 34.0 29.1 27.2 21.7 19.1
## 1987 15.5 10.0 21.4 26.1 28.4 28.0 28.1 26.9 24.5 25.1 21.9 13.2
## 1988 14.4 12.1 21.7 24.9 20.0 26.3 24.7 23.9 19.6 18.4 17.9 11.5
## 1989 15.3  8.8 12.2 19.9 17.5 17.9 19.6 14.5 14.4 16.4 14.8  7.2
## 1990 10.0  9.1 10.0 12.5 12.6 11.2 12.0 12.4  8.7 14.4 12.4  6.0
## 1991  4.9  3.8  7.3  9.4 11.6 14.2 11.2 11.4  9.1 12.0  9.7  8.3
## 1992  8.7  6.4  9.5 12.0 11.7 12.8 12.2 10.4 11.4 12.0  9.6  9.8
## 1993  6.9  5.6  6.8 10.9 11.3 13.9 12.7 12.2 13.2 11.8 11.7  9.3
## 1994  4.9  5.8  9.5 14.0 13.5 15.1 12.3 14.9 11.9 12.2 14.3  9.6
## 1995  6.3  5.9  9.5  9.6 13.1 13.1 10.6 12.0 10.4 12.1  8.6  6.5
## 1996  5.0  6.1 10.0 13.1 12.6 13.4 12.3 12.9 13.0 13.9 10.1  9.7
```

```
## 1997 8.5 6.2 11.1 13.1 11.7 11.6 14.6 11.6 13.7 11.2 14.7 8.8
## 1998 8.0 9.4 11.0 11.9 14.1 14.0 15.3 12.9 13.1 16.0 12.0 10.7
## 1999 7.8 9.6 12.5 12.5 14.1 16.4 17.0 17.3 12.2 12.6 12.7 11.0
## 2000 9.1 9.9 12.0 12.1 14.5 14.0 14.2 13.8 13.9 16.3 12.5 12.1
## 2001 6.6 8.0 11.9 15.7 12.2 13.4 15.9 15.3 12.5 13.5 14.1 10.1
## 2002 8.9 7.8 13.1 11.3 15.5 17.5 14.9 16.5 15.9 13.4 12.0 11.6
## 2003 7.8 7.9 12.0 13.2 14.4 15.8 17.6 15.0 16.9 14.0 15.6 12.5
## 2004 8.1 9.7 13.3 16.0 17.1 16.5 17.1 19.6 14.3 16.1 13.2 14.4
## 2005 8.7 11.1 16.5 16.9 17.6 19.5 18.3 19.8 17.7 15.4 15.9 12.3
## 2006 13.3 10.1 12.9 16.0 18.8 16.1 13.7 15.6 12.3 13.3 12.2 12.9
## 2007 10.5 7.6 10.0 14.4 16.1 15.5 13.8 9.5 12.7 15.0 10.4 7.4
## 2008 8.2 7.6 8.6 8.2 11.7 23.0 15.1 12.7 9.8 7.2 4.3 4.5
## 2009 2.2 3.6 5.3 4.5 5.7 8.1 6.1 6.5 5.8 4.9 4.8 4.4
## 2010 4.1 3.9 5.3 7.3 7.1 6.4 7.2 6.9 6.1 7.5 5.5 4.0
## 2011 5.8 3.1 5.0 5.2 5.6 6.9 8.0 5.2 5.2 5.8 7.6 4.2
## 2012 4.6 3.8 7.0 7.1 7.4 7.7 8.1 6.9 6.9 7.1 5.1 7.9
## 2013 5.4 6.3 7.5 7.4 9.5 8.1 10.5 8.7 8.9 11.4 7.4 5.7
## 2014 7.5 5.6 10.1 11.0 8.3 8.9 12.8 9.7 10.1 9.0 8.6 8.0
## 2015 6.9 2.8 8.0 17.3 13.2 19.1 14.8 9.9 12.4 12.3 10.4 11.0
## 2016 9.5 4.6 11.9 10.8 7.5 10.8 12.7 11.6 8.7 14.9 6.8 6.4
## 2017 8.0 6.6 8.8 7.7 7.9 14.8 11.5 9.6 9.3 13.5 7.5 6.2
## 2018 7.5 8.1 9.2 8.7 9.9 10.2 10.1 8.9 13.2 8.7 8.8 7.7
## 2019 9.3 5.4 6.3 11.6 7.9 10.2 9.5 14.8 10.7 10.6 8.5 9.9
## 2020 15.1 7.5 6.3 4.4 6.2 11.2 12.8 7.7 11.5 7.8 11.5 9.5
## 2021 13.8 7.1
```

```
##
```

```
## $seasonal
```

```
##          Jan          Feb          Mar          Apr          May          Jun
## 1959 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1960 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1961 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1962 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1963 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1964 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1965 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1966 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1967 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1968 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1969 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1970 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 1971 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
```



##	1972	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1973	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1974	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1975	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1976	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1977	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1978	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1979	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1980	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1981	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1982	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1983	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1984	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1985	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1986	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1987	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1988	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1989	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1990	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1991	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1992	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1993	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1994	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1995	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
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##	1998	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	1999	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2000	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2001	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2002	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2003	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2004	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2005	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2006	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2007	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2008	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2009	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2010	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2011	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654
##	2012	-5.99306657	-6.87441903	-2.01765674	2.30741157	2.76834053	3.64914654

```

## 2013 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 2014 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 2015 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 2016 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 2017 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 2018 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 2019 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 2020 -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## 2021 -5.99306657 -6.87441903
##           Jul           Aug           Sep           Oct           Nov           Dec
## 1959 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1960 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1961 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1962 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1963 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1964 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1965 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1966 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1967 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1968 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1969 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1970 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1971 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1972 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1973 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1974 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1975 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1976 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
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## 1982 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1983 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1984 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1985 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1986 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1987 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1988 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1989 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406

```

```

## 1990 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1991 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1992 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1993 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1994 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1995 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1996 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1997 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1998 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 1999 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2000 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2001 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2002 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2003 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2004 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2005 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2006 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2007 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2008 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2009 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2010 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2011 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2012 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2013 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2014 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2015 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2016 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2017 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2018 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2019 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2020 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
## 2021
##
## $trend
##      Jan      Feb      Mar      Apr      May      Jun      Jul
## 1959      NA      NA      NA      NA      NA      NA 22.245833
## 1960 19.116667 18.816667 18.620833 18.520833 18.808333 18.716667 18.208333
## 1961 19.320833 19.583333 20.041667 20.400000 20.158333 20.270833 20.758333
## 1962 22.166667 22.345833 22.195833 21.733333 21.608333 21.800000 21.775000
## 1963 21.375000 21.200000 21.162500 21.645833 22.195833 22.033333 21.858333
## 1964 21.229167 21.208333 21.029167 20.783333 20.637500 20.916667 21.104167

```

```
## 1965 22.058333 22.050000 22.254167 22.362500 22.391667 22.516667 22.700000
## 1966 20.912500 20.629167 20.200000 19.320833 18.383333 17.604167 17.100000
## 1967 15.708333 15.833333 16.000000 16.916667 17.666667 17.887500 17.912500
## 1968 18.795833 19.154167 19.425000 19.225000 18.912500 18.825000 19.000000
## 1969 19.995833 19.279167 18.537500 17.887500 17.570833 17.304167 16.987500
## 1970 15.908333 16.320833 16.545833 17.004167 17.520833 17.987500 18.300000
## 1971 19.700000 20.275000 20.770833 20.833333 21.000000 21.595833 22.320833
## 1972 24.437500 24.850000 25.412500 26.204167 26.929167 27.312500 27.320833
## 1973 27.262500 26.595833 26.087500 25.412500 24.583333 23.637500 22.925000
## 1974 19.058333 18.191667 17.508333 16.983333 16.175000 15.479167 15.070833
## 1975 12.437500 12.441667 12.208333 12.050000 12.208333 12.387500 12.404167
## 1976 13.170833 13.070833 13.191667 13.291667 13.600000 13.991667 14.079167
## 1977 15.279167 15.929167 16.312500 16.637500 16.912500 16.841667 16.787500
## 1978 16.562500 16.437500 16.562500 16.725000 16.520833 16.558333 16.862500
## 1979 16.091667 15.883333 15.445833 15.033333 15.087500 14.987500 14.695833
## 1980 12.320833 11.841667 11.754167 11.579167 10.862500 10.458333 10.429167
## 1981 10.550000 10.579167 10.270833 10.054167 9.966667 9.825000 9.645833
## 1982 9.037500 8.895833 9.112500 9.233333 9.458333 9.720833 9.837500
## 1983 12.550000 13.166667 13.470833 13.633333 13.841667 13.900000 14.037500
## 1984 14.983333 15.066667 15.254167 15.708333 16.283333 16.812500 17.112500
## 1985 19.183333 19.475000 19.841667 20.454167 20.804167 20.850000 21.158333
## 1986 23.183333 23.770833 24.341667 24.337500 24.200000 24.362500 24.508333
## 1987 24.050000 23.754167 23.266667 22.987500 22.908333 22.670833 22.379167
## 1988 21.450000 21.183333 20.854167 20.370833 19.925000 19.687500 19.654167
## 1989 17.087500 16.483333 15.875000 15.575000 15.362500 15.054167 14.654167
## 1990 12.375000 11.970833 11.645833 11.325000 11.141667 10.991667 10.729167
## 1991 9.725000 9.650000 9.625000 9.541667 9.329167 9.312500 9.566667
## 1992 10.275000 10.275000 10.329167 10.425000 10.420833 10.479167 10.466667
## 1993 10.087500 10.183333 10.333333 10.400000 10.479167 10.545833 10.441667
## 1994 11.125000 11.220833 11.279167 11.241667 11.366667 11.487500 11.558333
## 1995 10.987500 10.795833 10.612500 10.545833 10.304167 9.937500 9.754167
## 1996 10.104167 10.212500 10.358333 10.541667 10.679167 10.875000 11.154167
## 1997 11.270833 11.312500 11.287500 11.204167 11.283333 11.437500 11.379167
## 1998 11.945833 12.029167 12.058333 12.233333 12.320833 12.287500 12.358333
## 1999 12.812500 13.066667 13.212500 13.033333 12.920833 12.962500 13.029167
## 2000 12.750000 12.487500 12.412500 12.637500 12.783333 12.820833 12.762500
## 2001 12.620833 12.754167 12.758333 12.583333 12.533333 12.516667 12.529167
## 2002 12.916667 12.925000 13.116667 13.254167 13.162500 13.137500 13.154167
## 2003 13.062500 13.112500 13.091667 13.158333 13.333333 13.520833 13.570833
## 2004 14.337500 14.508333 14.591667 14.570833 14.558333 14.537500 14.641667
## 2005 15.466667 15.525000 15.675000 15.787500 15.870833 15.895833 16.000000
```

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## 2006 15.358333 14.991667 14.591667 14.279167 14.037500 13.908333 13.816667
## 2007 12.845833 12.595833 12.358333 12.445833 12.441667 12.137500 11.812500
## 2008 11.395833 11.583333 11.595833 11.150000 10.570833 10.195833 9.825000
## 2009 6.541667 5.908333 5.483333 5.220833 5.145833 5.162500 5.237500
## 2010 5.595833 5.658333 5.687500 5.808333 5.945833 5.958333 6.012500
## 2011 5.766667 5.729167 5.620833 5.512500 5.529167 5.625000 5.583333
## 2012 6.137500 6.212500 6.354167 6.479167 6.429167 6.479167 6.666667
## 2013 7.283333 7.458333 7.616667 7.879167 8.154167 8.158333 8.154167
## 2014 8.762500 8.900000 8.991667 8.941667 8.891667 9.037500 9.108333
## 2015 10.541667 10.633333 10.737500 10.970833 11.183333 11.383333 11.616667
## 2016 10.404167 10.387500 10.304167 10.258333 10.216667 9.875000 9.620833
## 2017 9.525000 9.391667 9.333333 9.300000 9.270833 9.291667 9.262500
## 2018 9.208333 9.120833 9.254167 9.216667 9.070833 9.187500 9.325000
## 2019 8.983333 9.204167 9.345833 9.320833 9.387500 9.466667 9.800000
## 2020 9.695833 9.537500 9.275000 9.191667 9.200000 9.308333 9.237500
## 2021      NA      NA
##      Aug      Sep      Oct      Nov      Dec
## 1959 21.929167 21.220833 20.466667 20.058333 19.645833
## 1960 17.875000 18.000000 18.291667 18.458333 18.816667
## 1961 20.937500 20.958333 21.258333 21.775000 22.108333
## 1962 21.575000 21.545833 21.737500 21.879167 21.625000
## 1963 21.937500 21.962500 21.587500 21.104167 21.120833
## 1964 21.020833 21.154167 21.529167 21.858333 21.991667
## 1965 22.850000 22.716667 22.479167 22.004167 21.383333
## 1966 17.000000 16.670833 16.000000 15.675000 15.525000
## 1967 17.850000 17.904167 18.283333 18.575000 18.683333
## 1968 19.137500 19.425000 19.783333 20.016667 20.300000
## 1969 16.812500 16.845833 16.704167 16.300000 15.762500
## 1970 18.441667 18.316667 18.275000 18.708333 19.300000
## 1971 22.687500 23.070833 23.495833 23.654167 24.004167
## 1972 27.308333 27.408333 27.525000 27.587500 27.587500
## 1973 22.716667 22.329167 21.483333 20.600000 19.825000
## 1974 14.766667 14.358333 13.741667 13.258333 12.741667
## 1975 12.320833 12.516667 12.908333 13.029167 13.166667
## 1976 13.958333 13.895833 14.016667 14.387500 14.820833
## 1977 16.704167 16.487500 16.462500 16.516667 16.512500
## 1978 17.145833 17.295833 17.083333 16.704167 16.400000
## 1979 14.450000 14.108333 13.700000 13.258333 12.787500
## 1980 10.437500 10.495833 10.562500 10.587500 10.508333
## 1981 9.545833 9.516667 9.404167 9.337500 9.283333
## 1982 10.129167 10.541667 10.991667 11.420833 11.966667

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## 1983 14.166667 14.141667 14.200000 14.404167 14.675000
## 1984 17.150000 17.308333 17.879167 18.475000 18.891667
## 1985 21.554167 21.937500 22.270833 22.550000 22.816667
## 1986 24.470833 24.408333 24.308333 24.212500 24.150000
## 1987 22.420833 22.520833 22.483333 22.083333 21.662500
## 1988 19.554167 19.020833 18.416667 18.104167 17.650000
## 1989 14.445833 14.366667 13.966667 13.454167 12.970833
## 1990 10.295833 9.962500 9.720833 9.550000 9.633333
## 1991 9.833333 10.033333 10.233333 10.345833 10.291667
## 1992 10.358333 10.212500 10.054167 9.991667 10.020833
## 1993 10.366667 10.487500 10.729167 10.950000 11.091667
## 1994 11.620833 11.625000 11.441667 11.241667 11.141667
## 1995 9.708333 9.737500 9.904167 10.029167 10.020833
## 1996 11.304167 11.354167 11.400000 11.362500 11.250000
## 1997 11.491667 11.620833 11.566667 11.616667 11.816667
## 1998 12.358333 12.429167 12.516667 12.541667 12.641667
## 1999 13.095833 13.087500 13.050000 13.050000 12.966667
## 2000 12.579167 12.495833 12.641667 12.695833 12.575000
## 2001 12.616667 12.658333 12.525000 12.479167 12.787500
## 2002 13.112500 13.070833 13.104167 13.137500 13.020833
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## 2006 13.595833 13.370833 13.183333 13.004167 12.866667
## 2007 11.716667 11.658333 11.341667 10.900000 11.029167
## 2008 9.408333 9.104167 8.812500 8.408333 7.537500
## 2009 5.329167 5.341667 5.458333 5.633333 5.620833
## 2010 6.050000 6.004167 5.904167 5.754167 5.712500
## 2011 5.562500 5.675000 5.837500 5.991667 6.100000
## 2012 6.804167 6.929167 6.962500 7.062500 7.166667
## 2013 8.212500 8.291667 8.550000 8.650000 8.633333
## 2014 8.966667 8.762500 8.937500 9.404167 10.033333
## 2015 11.800000 12.037500 11.929167 11.420833 10.837500
## 2016 9.641667 9.595833 9.337500 9.225000 9.408333
## 2017 9.304167 9.383333 9.441667 9.566667 9.458333
## 2018 9.287500 9.054167 9.054167 9.091667 9.008333
## 2019 10.129167 10.216667 9.916667 9.545833 9.516667
## 2020 9.166667 NA NA NA NA
## 2021
##
## $random

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##	Jan	Feb	Mar	Apr	May
## 1959	NA	NA	NA	NA	NA
## 1960	-1.2236000940	0.4577523650	-6.5031765967	1.1717550972	1.9233261355
## 1961	-7.0277667607	-2.7089143017	-2.5240099301	0.8925884306	2.9733261355
## 1962	-4.6736000940	-6.3714143017	-3.2781765967	5.3592550972	8.1233261355
## 1963	-8.8819334273	-5.0255809683	-3.1448432634	10.9467550972	5.4358261355
## 1964	-6.1361000940	-5.7339143017	-1.7115099301	1.5092550972	5.6941594688
## 1965	-9.6652667607	-5.8755809683	-0.4365099301	6.4300884306	5.3399928022
## 1966	-4.1194334273	-5.2547476350	-0.7823432634	6.1717550972	1.2483261355
## 1967	-1.7152667607	-0.0589143017	-4.8823432634	0.7759217639	1.9649928022
## 1968	-4.5027667607	-5.1797476350	-5.2073432634	4.4675884306	1.7191594688
## 1969	-3.1027667607	-4.6047476350	1.8801567366	8.2050884306	6.2608261355
## 1970	-3.7152667607	-1.1464143017	4.1718234033	5.3884217639	0.3108261355
## 1971	-3.9069334273	-5.3005809683	-2.8531765967	3.3592550972	5.4316594688
## 1972	-0.9444334273	-8.7755809683	0.6051567366	0.0884217639	1.2024928022
## 1973	-7.0694334273	-7.5214143017	-0.6698432634	4.2800884306	1.6483261355
## 1974	-2.8652667607	-0.1172476350	-0.3906765967	0.7092550972	0.8566594688
## 1975	-0.7444334273	2.8327523650	-2.0906765967	-2.1574115694	1.0233261355
## 1976	-2.1777667607	0.9035856983	2.9259900699	0.0009217639	-0.8683405312
## 1977	-4.9861000940	-4.1547476350	0.5051567366	-1.1449115694	2.5191594688
## 1978	-6.1694334273	-6.7630809683	-2.8448432634	1.2675884306	1.7108261355
## 1979	-1.9986000940	-3.1089143017	-1.2281765967	-2.6407449028	-0.3558405312
## 1980	-0.7277667607	-2.4672476350	-2.3365099301	-4.1865782361	-1.7308405312
## 1981	0.9430665727	-0.9047476350	0.2468234033	-2.1615782361	-0.7350071978
## 1982	-0.4444334273	1.2785856983	0.2051567366	-2.8407449028	-0.3266738645
## 1983	-1.0569334273	1.1077523650	1.6468234033	-2.2407449028	0.5899928022
## 1984	-1.7902667607	0.6077523650	-2.1365099301	-0.9157449028	-0.3516738645
## 1985	-3.6902667607	-5.2005809683	-1.5240099301	2.8384217639	0.9274928022
## 1986	-3.0902667607	-4.5964143017	-1.7240099301	2.6550884306	0.5316594688
## 1987	-2.5569334273	-6.8797476350	0.1509900699	0.8050884306	2.7233261355
## 1988	-1.0569334273	-2.2089143017	2.8634900699	2.2217550972	-2.6933405312
## 1989	4.2055665727	-0.8089143017	-1.6573432634	2.0175884306	-0.6308405312
## 1990	3.6180665727	4.0035856983	0.3718234033	-1.1324115694	-1.3100071978
## 1991	1.1680665727	1.0244190317	-0.3073432634	-2.4490782361	-0.4975071978
## 1992	4.4180665727	2.9994190317	1.1884900699	-0.7324115694	-1.4891738645
## 1993	2.8055665727	2.2910856983	-1.5156765967	-1.8074115694	-1.9475071978
## 1994	-0.2319334273	1.4535856983	0.2384900699	0.4509217639	-0.6350071978
## 1995	1.3055665727	1.9785856983	0.9051567366	-3.2532449028	0.0274928022
## 1996	0.8888999060	2.7619190317	1.6593234033	0.2509217639	-0.8475071978
## 1997	3.2222332393	1.7619190317	1.8301567366	-0.4115782361	-2.3516738645
## 1998	2.0472332393	4.2452523650	0.9593234033	-2.6407449028	-0.9891738645

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## 1999  0.9805665727  3.4077523650  1.3051567366 -2.8407449028 -1.5891738645
## 2000  2.3430665727  4.2869190317  1.6051567366 -2.8449115694 -1.0516738645
## 2001 -0.0277667607  2.1202523650  1.1593234033  0.8092550972 -3.1016738645
## 2002  1.9763999060  1.7494190317  2.0009900699 -4.2615782361 -0.4308405312
## 2003  0.7305665727  1.6619190317  0.9259900699 -2.2657449028 -1.7016738645
## 2004 -0.2444334273  2.0660856983  0.7259900699 -0.8782449028 -0.2266738645
## 2005 -0.7736000940  2.4494190317  2.8426567366 -1.1949115694 -1.0391738645
## 2006  3.9347332393  1.9827523650  0.3259900699 -0.5865782361  1.9941594688
## 2007  3.6472332393  1.8785856983 -0.3406765967 -0.3532449028  0.8899928022
## 2008  2.7972332393  2.8910856983 -0.9781765967 -5.2574115694 -1.6391738645
## 2009  1.6513999060  4.5660856983  1.8343234033 -3.0282449028 -2.2141738645
## 2010  4.4972332393  5.1160856983  1.6301567366 -0.8157449028 -1.6141738645
## 2011  6.0263999060  4.2452523650  1.3968234033 -2.6199115694 -2.6975071978
## 2012  4.4555665727  4.4619190317  2.6634900699 -1.6865782361 -1.7975071978
## 2013  4.1097332393  5.7160856983  1.9009900699 -2.7865782361 -1.4225071978
## 2014  4.7305665727  3.5744190317  3.1259900699 -0.2490782361 -3.3600071978
## 2015  2.3513999060 -0.9589143017 -0.7198432634  4.0217550972 -0.7516738645
## 2016  5.0888999060  1.0869190317  3.6134900699 -1.7657449028 -5.4850071978
## 2017  4.4680665727  4.0827523650  1.4843234033 -3.9074115694 -4.1391738645
## 2018  4.2847332393  5.8535856983  1.9634900699 -2.8240782361 -1.9391738645
## 2019  6.3097332393  3.0702523650 -1.0281765967 -0.0282449028 -4.2558405312
## 2020 11.3972332393  4.8369190317 -0.9573432634 -7.0990782361 -5.7683405312
## 2021      NA      NA
##      Jun      Jul      Aug      Sep      Oct
## 1959      NA  1.3086623773 -1.2014182678  1.1250337858  0.0751703978
## 1960  0.5341867912 -2.0538376227  3.0527483988 -0.5541328809  4.7501703978
## 1961  5.1800201246  1.2961623773  0.3902483988  7.0875337858 -0.2164962689
## 1962  5.0508534579  0.2794957107  4.0527483988 -1.3999662142 -3.8956629355
## 1963  0.8175201246 -1.8038376227  1.4902483988 -0.5166328809  6.5543370645
## 1964  3.6341867912 -0.1496709560  1.0069150655 -2.6082995476  3.6126703978
## 1965  3.8341867912 -1.9455042893 -0.8222516012  0.7292004524  0.3626703978
## 1966  1.9466867912 -0.8455042893  2.7277483988 -1.2249662142 -6.2581629355
## 1967 -1.9366465421  6.3419957107 -3.1222516012  6.5417004524  4.4585037311
## 1968 -1.2741465421  6.3544957107  3.0902483988  4.0208671191 -0.8414962689
## 1969  3.8466867912 -2.5330042893 -0.8847516012 -4.8999662142 -1.8623296022
## 1970 -3.7366465421  6.5544957107 -3.0139182678 -0.4707995476  1.6668370645
## 1971 -1.7449798754  6.5336623773  2.5402483988 -3.1249662142 -4.1539962689
## 1972 -0.7616465421  5.2336623773  4.1194150655 -0.1624662142  3.5168370645
## 1973  4.8133534579 -0.0705042893  2.4110817322 -0.9832995476 -0.7414962689
## 1974  3.5716867912 -1.2163376227 -1.4389182678  2.3875337858 -0.9998296022
## 1975 -1.9366465421  2.7503290440 -0.1930849345 -0.1707995476  0.4335037311

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## 1976  0.2591867912 -2.6246709560 -0.5305849345  0.0500337858  0.1251703978
## 1977  1.1091867912  1.9669957107  5.0235817322 -1.6416328809  4.5793370645
## 1978  2.4925201246  1.9919957107  1.4819150655  3.6500337858  1.7585037311
## 1979  0.2633534579  0.5586623773  2.7777483988 -2.2624662142  4.3418370645
## 1980 -0.9074798754 -0.6746709560  0.7902483988  5.2500337858 -0.6206629355
## 1981 -2.2741465421  3.1086623773 -0.6180849345  1.1292004524  0.4376703978
## 1982 -3.3699798754 -1.7830042893  0.0985817322  4.0042004524 -2.1498296022
## 1983  0.2508534579  0.2169957107  4.6610817322 -0.8957995476 -0.1581629355
## 1984  2.3383534579 -0.4580042893  1.2777483988  0.8375337858  2.1626703978
## 1985  2.5008534579 -1.7038376227  1.0735817322  0.8083671191  7.8710037311
## 1986  2.3883534579  0.3461623773  6.8569150655  3.3375337858  0.7335037311
## 1987  1.6800201246  2.4753290440  1.8069150655  0.6250337858  0.4585037311
## 1988  2.9633534579  1.8003290440  1.6735817322 -0.7749662142 -2.1748296022
## 1989 -0.8033132088  1.7003290440 -2.6180849345 -1.3207995476  0.2751703978
## 1990 -3.4408132088 -1.9746709560 -0.5680849345 -2.6166328809  2.5210037311
## 1991  1.2383534579 -1.6121709560 -1.1055849345 -2.2874662142 -0.3914962689
## 1992 -1.3283132088 -1.5121709560 -2.6305849345 -0.1666328809 -0.2123296022
## 1993 -0.2949798754 -0.9871709560 -0.8389182678  1.3583671191 -1.0873296022
## 1994 -0.0366465421 -2.5038376227  0.6069150655 -1.0791328809 -1.3998296022
## 1995 -0.4866465421 -2.3996709560 -0.3805849345 -0.6916328809  0.0376703978
## 1996 -1.1241465421 -2.0996709560 -1.0764182678  0.2917004524  0.3418370645
## 1997 -3.4866465421 -0.0246709560 -2.5639182678  0.7250337858 -2.5248296022
## 1998 -1.9366465421 -0.3038376227 -2.1305849345 -0.6832995476  1.3251703978
## 1999 -0.2116465421  0.7253290440  1.5319150655 -2.2416328809 -2.6081629355
## 2000 -2.4699798754 -1.8080042893 -1.4514182678  0.0500337858  1.5001703978
## 2001 -2.7658132088  0.1253290440  0.0110817322 -1.5124662142 -1.1831629355
## 2002  0.7133534579 -1.4996709560  0.7152483988  1.4750337858 -1.8623296022
## 2003 -1.3699798754  0.7836623773 -1.3305849345  1.7583671191 -2.1164962689
## 2004 -1.6866465421 -0.7871709560  2.2027483988 -1.9707995476 -1.1456629355
## 2005 -0.0449798754 -0.9455042893  0.9777483988  0.3875337858 -2.5289962689
## 2006 -1.4574798754 -3.3621709560 -0.6680849345 -2.4249662142 -2.0414962689
## 2007 -0.2866465421 -1.2580042893 -4.8889182678 -0.3124662142  1.5001703978
## 2008  9.1550201246  2.0294957107  0.6194150655 -0.6582995476 -3.7706629355
## 2009 -0.7116465421 -2.3830042893 -1.5014182678 -0.8957995476 -2.7164962689
## 2010 -3.2074798754 -2.0580042893 -1.8222516012 -1.2582995476 -0.5623296022
## 2011 -2.3741465421 -0.8288376227 -3.0347516012 -1.8291328809 -2.1956629355
## 2012 -2.4283132088 -1.8121709560 -2.5764182678 -1.3832995476 -2.0206629355
## 2013 -3.7074798754 -0.8996709560 -2.1847516012 -0.7457995476  0.6918370645
## 2014 -3.7866465421  0.4461623773 -1.9389182678 -0.0166328809 -2.0956629355
## 2015  4.0675201246 -0.0621709560 -4.5722516012 -0.9916328809 -1.7873296022
## 2016 -2.7241465421 -0.1663376227 -0.7139182678 -2.2499662142  3.4043370645

```

```

## 2017  1.8591867912 -1.0080042893 -2.3764182678 -1.4374662142  1.9001703978
## 2018 -2.6366465421 -2.4705042893 -3.0597516012  2.7917004524 -2.5123296022
## 2019 -2.9158132088 -3.5455042893  1.9985817322 -0.8707995476 -1.4748296022
## 2020 -1.7574798754  0.3169957107 -4.1389182678          NA          NA
## 2021
##              Nov              Dec
## 1959 -1.4962093836 -1.2381492743
## 1960  4.5037906164 -7.0089826077
## 1961 -2.6128760503 -3.8006492743
## 1962 -2.5170427170  1.0826840590
## 1963  1.1579572830 -5.2131492743
## 1964 -0.0962093836  1.1160173923
## 1965  2.7579572830  1.7243507257
## 1966 -0.3128760503 -1.7173159410
## 1967  1.7871239497 -4.5756492743
## 1968 -3.3545427170 -4.5923159410
## 1969 -3.1378760503 -2.9548159410
## 1970  1.7537906164 -2.5923159410
## 1971  1.4079572830  2.4035173923
## 1972  3.1746239497  2.3201840590
## 1973  0.5621239497 -3.0173159410
## 1974 -3.4962093836 -1.2339826077
## 1975 -0.0670427170 -0.5589826077
## 1976  5.1746239497  0.5868507257
## 1977  2.7454572830 -2.5048159410
## 1978 -0.1420427170  1.2076840590
## 1979  5.4037906164  0.3201840590
## 1980 -1.0253760503  1.9993507257
## 1981 -1.7753760503  1.8243507257
## 1982  2.5412906164 -0.9589826077
## 1983 -0.6420427170 -2.0673159410
## 1984  3.0871239497 -1.3839826077
## 1985 -2.6878760503 -2.5089826077
## 1986 -2.5503760503 -1.7423159410
## 1987 -0.2212093836 -5.1548159410
## 1988 -0.2420427170 -2.8423159410
## 1989  1.3079572830 -2.4631492743
## 1990  2.8121239497 -0.3256492743
## 1991 -0.6837093836  1.3160173923
## 1992 -0.4295427170  3.0868507257
## 1993  0.7121239497  1.5160173923

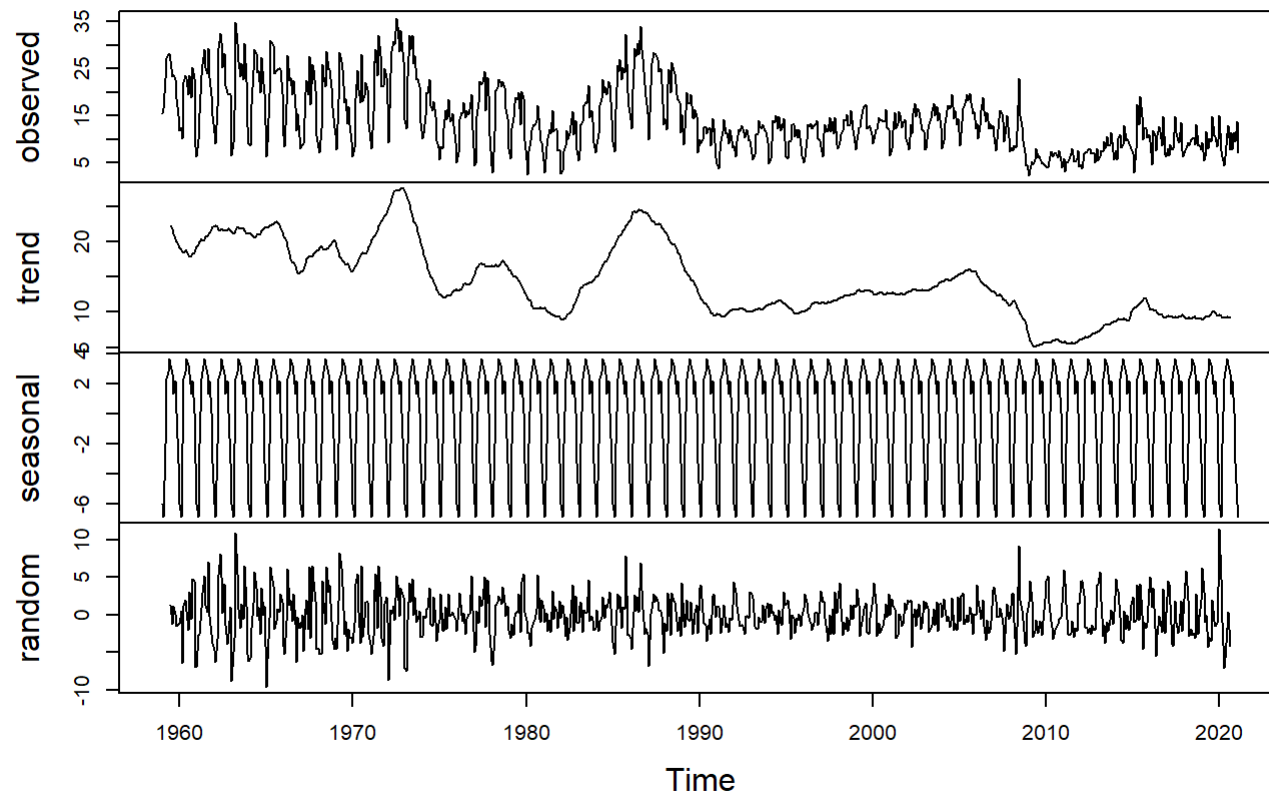
```

```
## 1994 3.0204572830 1.7660173923
## 1995 -1.4670427170 -0.2131492743
## 1996 -1.3003760503 1.7576840590
## 1997 3.0454572830 0.2910173923
## 1998 -0.5795427170 1.3660173923
## 1999 -0.3878760503 1.3410173923
## 2000 -0.2337093836 2.8326840590
## 2001 1.5829572830 0.6201840590
## 2002 -1.1753760503 1.8868507257
## 2003 1.3746239497 1.4785173923
## 2004 -1.9837093836 2.4160173923
## 2005 0.0787906164 -0.0839826077
## 2006 -0.8420427170 3.3410173923
## 2007 -0.5378760503 -0.3214826077
## 2008 -4.1462093836 0.2701840590
## 2009 -0.8712093836 2.0868507257
## 2010 -0.2920427170 1.5951840590
## 2011 1.5704572830 1.4076840590
## 2012 -2.0003760503 4.0410173923
## 2013 -1.2878760503 0.3743507257
## 2014 -0.8420427170 1.2743507257
## 2015 -1.0587093836 3.4701840590
## 2016 -2.4628760503 0.2993507257
## 2017 -2.1045427170 0.0493507257
## 2018 -0.3295427170 1.9993507257
## 2019 -1.0837093836 3.6910173923
## 2020 NA NA
## 2021
##
## $figure
## [1] -5.99306657 -6.87441903 -2.01765674 2.30741157 2.76834053 3.64914654
## [7] 3.24550429 2.67225160 1.35413288 2.15816294 0.03787605 -3.30768406
##
## $type
## [1] "additive"
##
## attr(,"class")
## [1] "decomposed.ts"
```

e gerar seus gráficos

```
plot(decomposicao_adi)
```

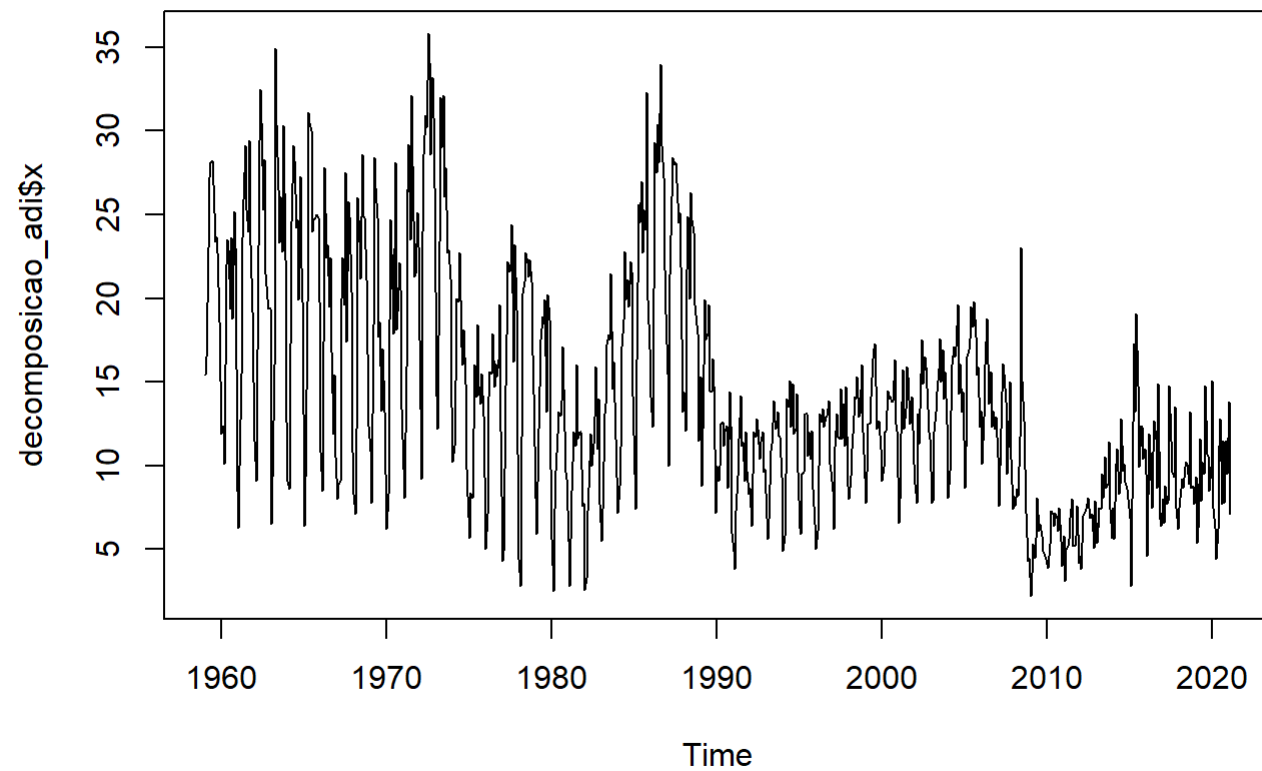
### Decomposition of additive time series



Também cada um dos componentes podem ser plotados separadamente

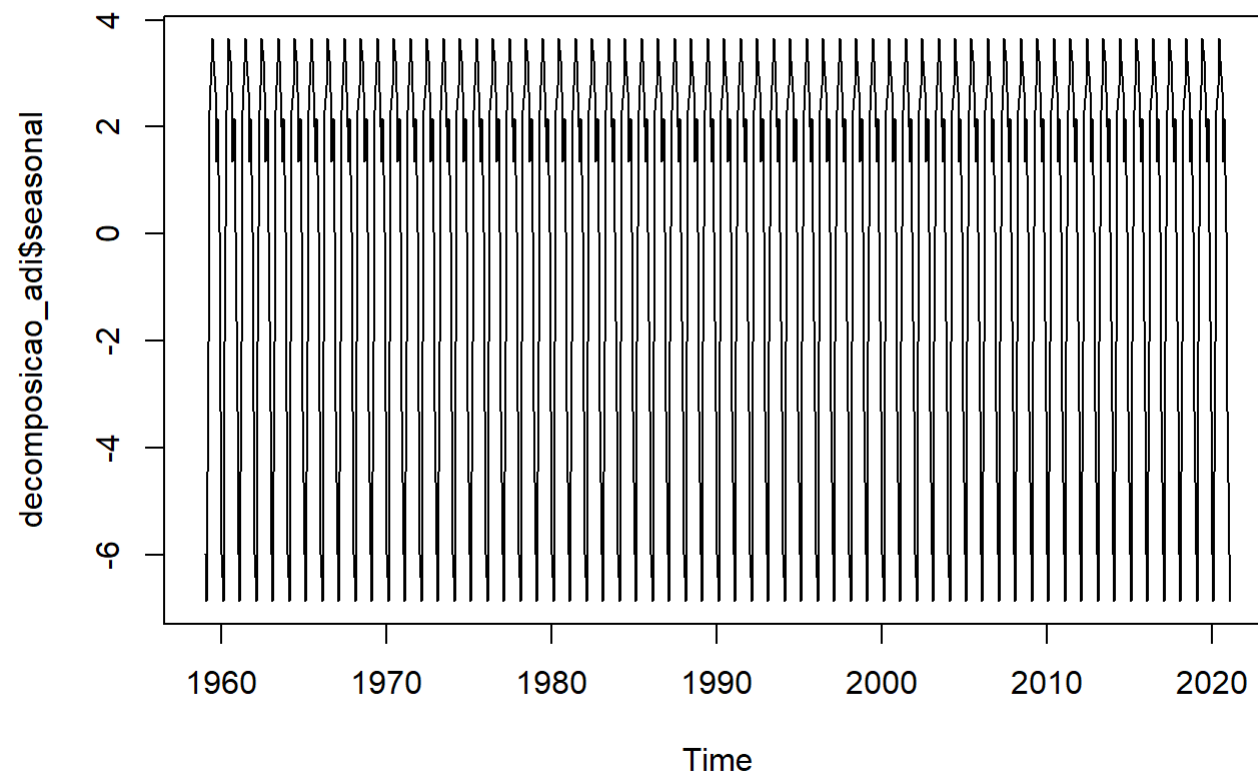
#### série original

```
plot(decomposicao_adi$x)
```



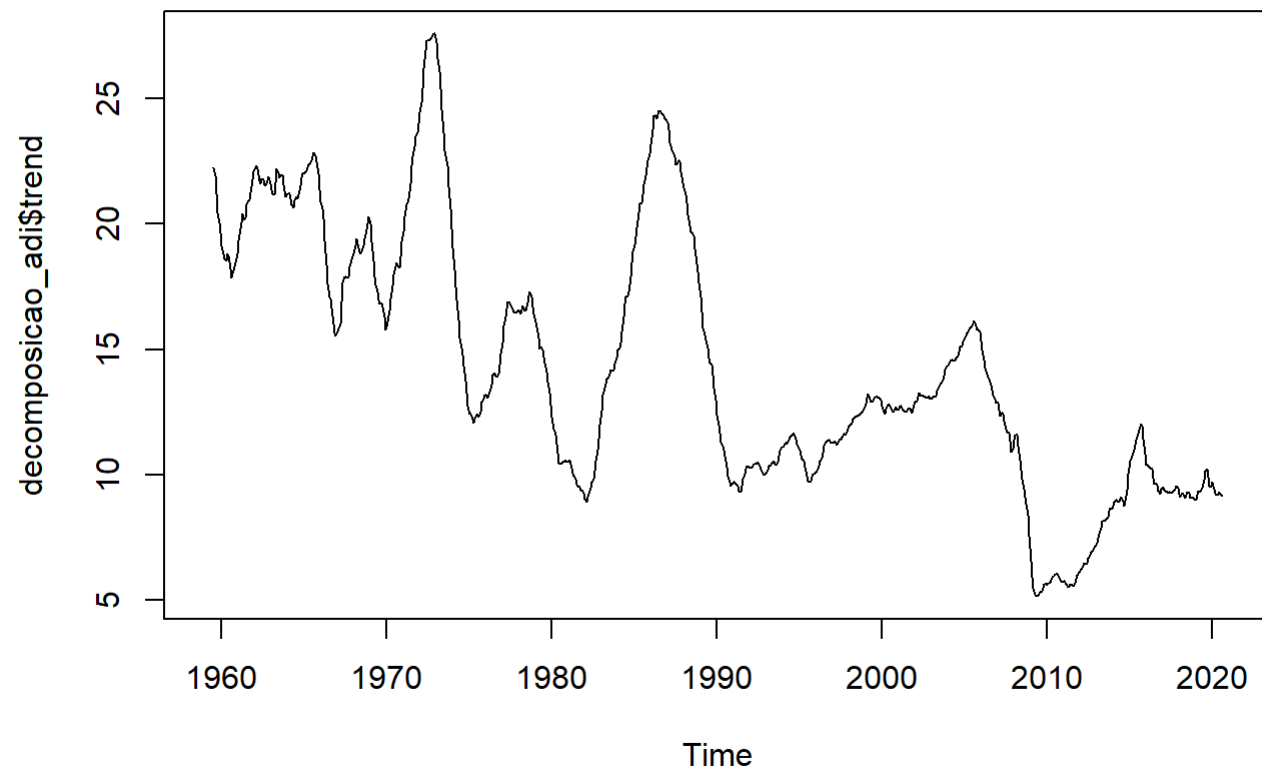
### sazonalidade

```
plot(decomposicao_adi$seasonal)
```



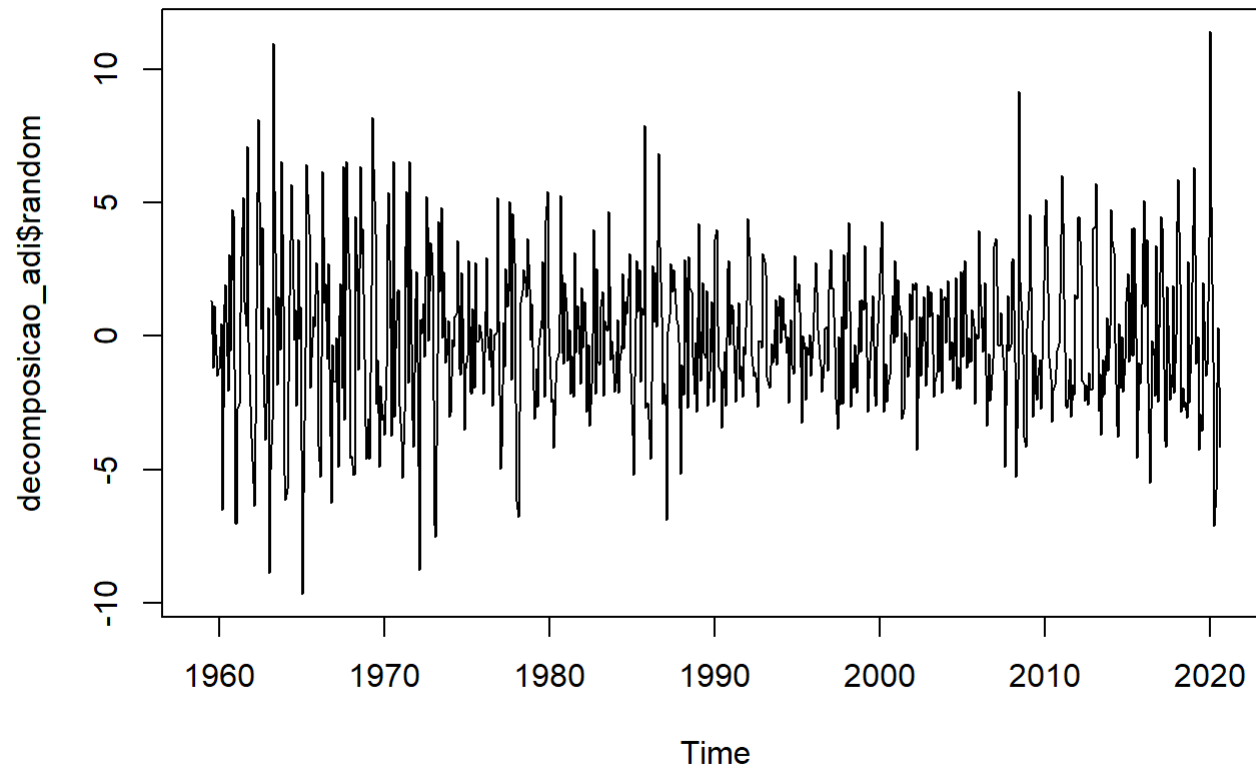
### Tendência

```
plot(decomposicao_adi$trend)
```



### erros

```
plot(decomposicao_adi$random)
```



## FORMA MULTIPLICATIVA

A forma multiplicativa é a multiplicação dos componentes:

$$Y_t = T_t \cdot S_t \cdot e_t$$

Atribuindo a decomposição a uma variável

```
decomposicao_mult = decompose(serie$y, type = 'multiplicative')
```

valores gerados

```
decomposicao_mult
```



```
## $x
##      Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
## 1959 15.4 16.5 23.0 27.2 28.1 28.2 26.8 23.4 23.7 22.7 18.6 15.1
## 1960 11.9 12.4 10.1 22.0 23.5 22.9 19.4 23.6 18.8 25.2 23.0  8.5
## 1961  6.3 10.0 15.5 23.6 25.9 29.1 25.3 24.0 29.4 23.2 19.2 15.0
## 1962 11.5  9.1 16.9 29.4 32.5 30.5 25.3 28.3 21.5 20.0 19.4 19.4
## 1963  6.5  9.3 16.0 34.9 30.4 26.5 23.3 26.1 22.8 30.3 22.3 12.6
## 1964  9.1  8.6 17.3 24.6 29.1 28.2 24.2 24.7 19.9 27.3 21.8 19.8
## 1965  6.4  9.3 19.8 31.1 30.5 30.0 24.0 24.7 24.8 25.0 24.8 19.8
## 1966 10.8  8.5 17.4 27.8 22.4 23.2 19.5 22.4 16.8 11.9 15.4 10.5
## 1967  8.0  8.9  9.1 20.0 22.4 19.6 27.5 17.4 25.8 24.9 20.4 10.8
## 1968  8.3  7.1 12.2 26.0 23.4 21.2 28.6 24.9 24.8 21.1 16.7 12.4
## 1969 10.9  7.8 18.4 28.4 26.6 24.8 17.7 18.6 13.3 17.0 13.2  9.5
## 1970  6.2  8.3 18.7 24.7 20.6 17.9 28.1 18.1 19.2 22.1 20.5 13.4
## 1971  9.8  8.1 15.9 26.5 29.2 23.5 32.1 27.9 21.3 21.5 25.1 23.1
## 1972 17.5  9.2 24.0 28.6 30.9 30.2 35.8 34.1 28.6 33.2 30.8 26.6
## 1973 14.2 12.2 23.4 32.0 29.0 32.1 26.1 27.8 22.7 22.9 21.2 13.5
## 1974 10.2 11.2 15.1 20.0 19.8 22.7 17.1 16.0 18.1 14.9  9.8  8.2
## 1975  5.7  8.4  8.1 12.2 16.0 14.1 18.4 14.8 13.7 15.5 13.0  9.3
## 1976  5.0  7.1 14.1 15.6 15.5 17.9 14.7 16.1 15.3 16.3 19.6 12.1
## 1977  4.3  4.9 14.8 17.8 22.2 21.6 22.0 24.4 16.2 23.2 19.3 10.7
## 1978  4.4  2.8 11.7 20.3 21.0 22.7 22.1 21.3 22.3 21.0 16.6 14.3
## 1979  8.1  5.9 12.2 14.7 17.5 18.9 18.5 19.9 13.2 20.2 18.7  9.8
## 1980  5.6  2.5  7.4  9.7 11.9 13.2 13.0 13.9 17.1 12.1  9.6  9.2
## 1981  5.5  2.8  8.5 10.2 12.0 11.2 16.0 11.6 12.0 12.0  7.6  7.8
## 1982  2.6  3.3  7.3  8.7 11.9 10.0 11.3 12.9 15.9 11.0 14.0  7.7
## 1983  5.5  7.4 13.1 13.7 17.2 17.8 17.5 21.5 14.6 16.2 13.8  9.3
## 1984  7.2  8.8 11.1 17.1 18.7 22.8 19.9 21.1 19.5 22.2 21.6 14.2
## 1985  9.5  7.4 16.3 25.6 24.5 27.0 22.7 25.3 24.1 32.3 19.9 17.0
## 1986 14.1 12.3 20.6 29.3 27.5 30.4 28.1 34.0 29.1 27.2 21.7 19.1
## 1987 15.5 10.0 21.4 26.1 28.4 28.0 28.1 26.9 24.5 25.1 21.9 13.2
## 1988 14.4 12.1 21.7 24.9 20.0 26.3 24.7 23.9 19.6 18.4 17.9 11.5
## 1989 15.3  8.8 12.2 19.9 17.5 17.9 19.6 14.5 14.4 16.4 14.8  7.2
## 1990 10.0  9.1 10.0 12.5 12.6 11.2 12.0 12.4  8.7 14.4 12.4  6.0
## 1991  4.9  3.8  7.3  9.4 11.6 14.2 11.2 11.4  9.1 12.0  9.7  8.3
## 1992  8.7  6.4  9.5 12.0 11.7 12.8 12.2 10.4 11.4 12.0  9.6  9.8
## 1993  6.9  5.6  6.8 10.9 11.3 13.9 12.7 12.2 13.2 11.8 11.7  9.3
## 1994  4.9  5.8  9.5 14.0 13.5 15.1 12.3 14.9 11.9 12.2 14.3  9.6
## 1995  6.3  5.9  9.5  9.6 13.1 13.1 10.6 12.0 10.4 12.1  8.6  6.5
## 1996  5.0  6.1 10.0 13.1 12.6 13.4 12.3 12.9 13.0 13.9 10.1  9.7
```

```

## 1997  8.5  6.2 11.1 13.1 11.7 11.6 14.6 11.6 13.7 11.2 14.7  8.8
## 1998  8.0  9.4 11.0 11.9 14.1 14.0 15.3 12.9 13.1 16.0 12.0 10.7
## 1999  7.8  9.6 12.5 12.5 14.1 16.4 17.0 17.3 12.2 12.6 12.7 11.0
## 2000  9.1  9.9 12.0 12.1 14.5 14.0 14.2 13.8 13.9 16.3 12.5 12.1
## 2001  6.6  8.0 11.9 15.7 12.2 13.4 15.9 15.3 12.5 13.5 14.1 10.1
## 2002  8.9  7.8 13.1 11.3 15.5 17.5 14.9 16.5 15.9 13.4 12.0 11.6
## 2003  7.8  7.9 12.0 13.2 14.4 15.8 17.6 15.0 16.9 14.0 15.6 12.5
## 2004  8.1  9.7 13.3 16.0 17.1 16.5 17.1 19.6 14.3 16.1 13.2 14.4
## 2005  8.7 11.1 16.5 16.9 17.6 19.5 18.3 19.8 17.7 15.4 15.9 12.3
## 2006 13.3 10.1 12.9 16.0 18.8 16.1 13.7 15.6 12.3 13.3 12.2 12.9
## 2007 10.5  7.6 10.0 14.4 16.1 15.5 13.8  9.5 12.7 15.0 10.4  7.4
## 2008  8.2  7.6  8.6  8.2 11.7 23.0 15.1 12.7  9.8  7.2  4.3  4.5
## 2009  2.2  3.6  5.3  4.5  5.7  8.1  6.1  6.5  5.8  4.9  4.8  4.4
## 2010  4.1  3.9  5.3  7.3  7.1  6.4  7.2  6.9  6.1  7.5  5.5  4.0
## 2011  5.8  3.1  5.0  5.2  5.6  6.9  8.0  5.2  5.2  5.8  7.6  4.2
## 2012  4.6  3.8  7.0  7.1  7.4  7.7  8.1  6.9  6.9  7.1  5.1  7.9
## 2013  5.4  6.3  7.5  7.4  9.5  8.1 10.5  8.7  8.9 11.4  7.4  5.7
## 2014  7.5  5.6 10.1 11.0  8.3  8.9 12.8  9.7 10.1  9.0  8.6  8.0
## 2015  6.9  2.8  8.0 17.3 13.2 19.1 14.8  9.9 12.4 12.3 10.4 11.0
## 2016  9.5  4.6 11.9 10.8  7.5 10.8 12.7 11.6  8.7 14.9  6.8  6.4
## 2017  8.0  6.6  8.8  7.7  7.9 14.8 11.5  9.6  9.3 13.5  7.5  6.2
## 2018  7.5  8.1  9.2  8.7  9.9 10.2 10.1  8.9 13.2  8.7  8.8  7.7
## 2019  9.3  5.4  6.3 11.6  7.9 10.2  9.5 14.8 10.7 10.6  8.5  9.9
## 2020 15.1  7.5  6.3  4.4  6.2 11.2 12.8  7.7 11.5  7.8 11.5  9.5
## 2021 13.8  7.1
##
## $seasonal
##           Jan           Feb           Mar           Apr           May           Jun           Jul
## 1959 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1960 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1961 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1962 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1963 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1964 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1965 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1966 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1967 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1968 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1969 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1970 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1971 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798

```

```
## 1972 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1973 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1974 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1975 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1976 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1977 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1978 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1979 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1980 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1981 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1982 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1983 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1984 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1985 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1986 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1987 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1988 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1989 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1990 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1991 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1992 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1993 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1994 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1995 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1996 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1997 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1998 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 1999 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2000 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2001 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2002 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2003 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2004 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2005 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
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## 2010 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2011 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2012 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
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## 2013 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
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## 2016 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
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## 2018 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2019 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2020 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## 2021 0.6117873 0.5427180
##
##           Aug           Sep           Oct           Nov           Dec
## 1959 1.1747646 1.0969375 1.1497942 0.9944642 0.7739055
## 1960 1.1747646 1.0969375 1.1497942 0.9944642 0.7739055
## 1961 1.1747646 1.0969375 1.1497942 0.9944642 0.7739055
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## 1990 1.1747646 1.0969375 1.1497942 0.9944642 0.7739055
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## 2020 1.1747646 1.0969375 1.1497942 0.9944642 0.7739055
## 2021
##
## $trend
##      Jan      Feb      Mar      Apr      May      Jun      Jul
## 1959      NA      NA      NA      NA      NA      NA 22.245833
## 1960 19.116667 18.816667 18.620833 18.520833 18.808333 18.716667 18.208333
## 1961 19.320833 19.583333 20.041667 20.400000 20.158333 20.270833 20.758333
## 1962 22.166667 22.345833 22.195833 21.733333 21.608333 21.800000 21.775000
## 1963 21.375000 21.200000 21.162500 21.645833 22.195833 22.033333 21.858333
## 1964 21.229167 21.208333 21.029167 20.783333 20.637500 20.916667 21.104167

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## 1965 22.058333 22.050000 22.254167 22.362500 22.391667 22.516667 22.700000
## 1966 20.912500 20.629167 20.200000 19.320833 18.383333 17.604167 17.100000
## 1967 15.708333 15.833333 16.000000 16.916667 17.666667 17.887500 17.912500
## 1968 18.795833 19.154167 19.425000 19.225000 18.912500 18.825000 19.000000
## 1969 19.995833 19.279167 18.537500 17.887500 17.570833 17.304167 16.987500
## 1970 15.908333 16.320833 16.545833 17.004167 17.520833 17.987500 18.300000
## 1971 19.700000 20.275000 20.770833 20.833333 21.000000 21.595833 22.320833
## 1972 24.437500 24.850000 25.412500 26.204167 26.929167 27.312500 27.320833
## 1973 27.262500 26.595833 26.087500 25.412500 24.583333 23.637500 22.925000
## 1974 19.058333 18.191667 17.508333 16.983333 16.175000 15.479167 15.070833
## 1975 12.437500 12.441667 12.208333 12.050000 12.208333 12.387500 12.404167
## 1976 13.170833 13.070833 13.191667 13.291667 13.600000 13.991667 14.079167
## 1977 15.279167 15.929167 16.312500 16.637500 16.912500 16.841667 16.787500
## 1978 16.562500 16.437500 16.562500 16.725000 16.520833 16.558333 16.862500
## 1979 16.091667 15.883333 15.445833 15.033333 15.087500 14.987500 14.695833
## 1980 12.320833 11.841667 11.754167 11.579167 10.862500 10.458333 10.429167
## 1981 10.550000 10.579167 10.270833 10.054167 9.966667 9.825000 9.645833
## 1982 9.037500 8.895833 9.112500 9.233333 9.458333 9.720833 9.837500
## 1983 12.550000 13.166667 13.470833 13.633333 13.841667 13.900000 14.037500
## 1984 14.983333 15.066667 15.254167 15.708333 16.283333 16.812500 17.112500
## 1985 19.183333 19.475000 19.841667 20.454167 20.804167 20.850000 21.158333
## 1986 23.183333 23.770833 24.341667 24.337500 24.200000 24.362500 24.508333
## 1987 24.050000 23.754167 23.266667 22.987500 22.908333 22.670833 22.379167
## 1988 21.450000 21.183333 20.854167 20.370833 19.925000 19.687500 19.654167
## 1989 17.087500 16.483333 15.875000 15.575000 15.362500 15.054167 14.654167
## 1990 12.375000 11.970833 11.645833 11.325000 11.141667 10.991667 10.729167
## 1991 9.725000 9.650000 9.625000 9.541667 9.329167 9.312500 9.566667
## 1992 10.275000 10.275000 10.329167 10.425000 10.420833 10.479167 10.466667
## 1993 10.087500 10.183333 10.333333 10.400000 10.479167 10.545833 10.441667
## 1994 11.125000 11.220833 11.279167 11.241667 11.366667 11.487500 11.558333
## 1995 10.987500 10.795833 10.612500 10.545833 10.304167 9.937500 9.754167
## 1996 10.104167 10.212500 10.358333 10.541667 10.679167 10.875000 11.154167
## 1997 11.270833 11.312500 11.287500 11.204167 11.283333 11.437500 11.379167
## 1998 11.945833 12.029167 12.058333 12.233333 12.320833 12.287500 12.358333
## 1999 12.812500 13.066667 13.212500 13.033333 12.920833 12.962500 13.029167
## 2000 12.750000 12.487500 12.412500 12.637500 12.783333 12.820833 12.762500
## 2001 12.620833 12.754167 12.758333 12.583333 12.533333 12.516667 12.529167
## 2002 12.916667 12.925000 13.116667 13.254167 13.162500 13.137500 13.154167
## 2003 13.062500 13.112500 13.091667 13.158333 13.333333 13.520833 13.570833
## 2004 14.337500 14.508333 14.591667 14.570833 14.558333 14.537500 14.641667
## 2005 15.466667 15.525000 15.675000 15.787500 15.870833 15.895833 16.000000
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## 2006 15.358333 14.991667 14.591667 14.279167 14.037500 13.908333 13.816667
## 2007 12.845833 12.595833 12.358333 12.445833 12.441667 12.137500 11.812500
## 2008 11.395833 11.583333 11.595833 11.150000 10.570833 10.195833 9.825000
## 2009 6.541667 5.908333 5.483333 5.220833 5.145833 5.162500 5.237500
## 2010 5.595833 5.658333 5.687500 5.808333 5.945833 5.958333 6.012500
## 2011 5.766667 5.729167 5.620833 5.512500 5.529167 5.625000 5.583333
## 2012 6.137500 6.212500 6.354167 6.479167 6.429167 6.479167 6.666667
## 2013 7.283333 7.458333 7.616667 7.879167 8.154167 8.158333 8.154167
## 2014 8.762500 8.900000 8.991667 8.941667 8.891667 9.037500 9.108333
## 2015 10.541667 10.633333 10.737500 10.970833 11.183333 11.383333 11.616667
## 2016 10.404167 10.387500 10.304167 10.258333 10.216667 9.875000 9.620833
## 2017 9.525000 9.391667 9.333333 9.300000 9.270833 9.291667 9.262500
## 2018 9.208333 9.120833 9.254167 9.216667 9.070833 9.187500 9.325000
## 2019 8.983333 9.204167 9.345833 9.320833 9.387500 9.466667 9.800000
## 2020 9.695833 9.537500 9.275000 9.191667 9.200000 9.308333 9.237500
## 2021      NA      NA
##      Aug      Sep      Oct      Nov      Dec
## 1959 21.929167 21.220833 20.466667 20.058333 19.645833
## 1960 17.875000 18.000000 18.291667 18.458333 18.816667
## 1961 20.937500 20.958333 21.258333 21.775000 22.108333
## 1962 21.575000 21.545833 21.737500 21.879167 21.625000
## 1963 21.937500 21.962500 21.587500 21.104167 21.120833
## 1964 21.020833 21.154167 21.529167 21.858333 21.991667
## 1965 22.850000 22.716667 22.479167 22.004167 21.383333
## 1966 17.000000 16.670833 16.000000 15.675000 15.525000
## 1967 17.850000 17.904167 18.283333 18.575000 18.683333
## 1968 19.137500 19.425000 19.783333 20.016667 20.300000
## 1969 16.812500 16.845833 16.704167 16.300000 15.762500
## 1970 18.441667 18.316667 18.275000 18.708333 19.300000
## 1971 22.687500 23.070833 23.495833 23.654167 24.004167
## 1972 27.308333 27.408333 27.525000 27.587500 27.587500
## 1973 22.716667 22.329167 21.483333 20.600000 19.825000
## 1974 14.766667 14.358333 13.741667 13.258333 12.741667
## 1975 12.320833 12.516667 12.908333 13.029167 13.166667
## 1976 13.958333 13.895833 14.016667 14.387500 14.820833
## 1977 16.704167 16.487500 16.462500 16.516667 16.512500
## 1978 17.145833 17.295833 17.083333 16.704167 16.400000
## 1979 14.450000 14.108333 13.700000 13.258333 12.787500
## 1980 10.437500 10.495833 10.562500 10.587500 10.508333
## 1981 9.545833 9.516667 9.404167 9.337500 9.283333
## 1982 10.129167 10.541667 10.991667 11.420833 11.966667

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## 1983 14.166667 14.141667 14.200000 14.404167 14.675000
## 1984 17.150000 17.308333 17.879167 18.475000 18.891667
## 1985 21.554167 21.937500 22.270833 22.550000 22.816667
## 1986 24.470833 24.408333 24.308333 24.212500 24.150000
## 1987 22.420833 22.520833 22.483333 22.083333 21.662500
## 1988 19.554167 19.020833 18.416667 18.104167 17.650000
## 1989 14.445833 14.366667 13.966667 13.454167 12.970833
## 1990 10.295833 9.962500 9.720833 9.550000 9.633333
## 1991 9.833333 10.033333 10.233333 10.345833 10.291667
## 1992 10.358333 10.212500 10.054167 9.991667 10.020833
## 1993 10.366667 10.487500 10.729167 10.950000 11.091667
## 1994 11.620833 11.625000 11.441667 11.241667 11.141667
## 1995 9.708333 9.737500 9.904167 10.029167 10.020833
## 1996 11.304167 11.354167 11.400000 11.362500 11.250000
## 1997 11.491667 11.620833 11.566667 11.616667 11.816667
## 1998 12.358333 12.429167 12.516667 12.541667 12.641667
## 1999 13.095833 13.087500 13.050000 13.050000 12.966667
## 2000 12.579167 12.495833 12.641667 12.695833 12.575000
## 2001 12.616667 12.658333 12.525000 12.479167 12.787500
## 2002 13.112500 13.070833 13.104167 13.137500 13.020833
## 2003 13.658333 13.787500 13.958333 14.187500 14.329167
## 2004 14.725000 14.916667 15.087500 15.145833 15.291667
## 2005 16.150000 15.958333 15.770833 15.783333 15.691667
## 2006 13.595833 13.370833 13.183333 13.004167 12.866667
## 2007 11.716667 11.658333 11.341667 10.900000 11.029167
## 2008 9.408333 9.104167 8.812500 8.408333 7.537500
## 2009 5.329167 5.341667 5.458333 5.633333 5.620833
## 2010 6.050000 6.004167 5.904167 5.754167 5.712500
## 2011 5.562500 5.675000 5.837500 5.991667 6.100000
## 2012 6.804167 6.929167 6.962500 7.062500 7.166667
## 2013 8.212500 8.291667 8.550000 8.650000 8.633333
## 2014 8.966667 8.762500 8.937500 9.404167 10.033333
## 2015 11.800000 12.037500 11.929167 11.420833 10.837500
## 2016 9.641667 9.595833 9.337500 9.225000 9.408333
## 2017 9.304167 9.383333 9.441667 9.566667 9.458333
## 2018 9.287500 9.054167 9.054167 9.091667 9.008333
## 2019 10.129167 10.216667 9.916667 9.545833 9.516667
## 2020 9.166667 NA NA NA NA
## 2021
##
## $random

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##	Jan	Feb	Mar	Apr	May	Jun	Jul
## 1959	NA	NA	NA	NA	NA	NA	0.9777938
## 1960	1.0174997	1.2142407	0.6246596	1.0533706	1.0678174	0.9729921	0.8647542
## 1961	0.5329840	0.9408907	0.8906745	1.0258902	1.0980562	1.1416257	0.9892116
## 1962	0.8480022	0.7503616	0.8768724	1.1996099	1.2854096	1.1126170	0.9430257
## 1963	0.4970576	0.8083005	0.8707112	1.4297831	1.1705274	0.9564627	0.8651672
## 1964	0.7006610	0.7471670	0.9474257	1.0496365	1.2050785	1.0721585	0.9306970
## 1965	0.4742494	0.7771416	1.0246487	1.2332726	1.1641069	1.0595452	0.8581171
## 1966	0.8441455	0.7592119	0.9920168	1.2759627	1.0413655	1.0480328	0.9255495
## 1967	0.8324524	1.0357226	0.6550019	1.0484174	1.0836096	0.8713824	1.2460563
## 1968	0.7217986	0.6830003	0.7233025	1.1992950	1.0574172	0.8955776	1.2217254
## 1969	0.8910181	0.7454734	1.1431094	1.4079516	1.2938047	1.1397336	0.8456778
## 1970	0.6370397	0.9370474	1.3015897	1.2881328	1.0048284	0.7913791	1.2462822
## 1971	0.8131288	0.7361222	0.8815853	1.1279923	1.1883463	0.8653668	1.1672282
## 1972	1.1705253	0.6821615	1.0876395	0.9678648	0.9806522	0.8793218	1.0635312
## 1973	0.8513775	0.8452244	1.0330100	1.1166617	1.0081767	1.0799554	0.9240433
## 1974	0.8748121	1.1344133	0.9932379	1.0443020	1.0461654	1.1662204	0.9209160
## 1975	0.7491026	1.2440175	0.7640993	0.8978253	1.1200645	0.9051859	1.2039581
## 1976	0.6205207	1.0008773	1.2309505	1.0407926	0.9740298	1.0173864	0.8474255
## 1977	0.4600111	0.5667986	1.0448702	0.9487482	1.1218237	1.0199316	1.0636477
## 1978	0.4342365	0.3138687	0.8135441	1.0763387	1.0863425	1.0902136	1.0637302
## 1979	0.8227796	0.6844412	0.9096402	0.8671238	0.9912888	1.0028476	1.0217359
## 1980	0.7429293	0.3890030	0.7250384	0.7428711	0.9362603	1.0037228	1.0117073
## 1981	0.8521376	0.4876771	0.9530910	0.8996490	1.0289891	0.9065418	1.3462986
## 1982	0.4702454	0.6835229	0.9225851	0.8355641	1.0752558	0.8180858	0.9322982
## 1983	0.7163388	1.0355752	1.1199483	0.8911228	1.0619877	1.0183746	1.0118344
## 1984	0.7854591	1.0761958	0.8380224	0.9653505	0.9814716	1.0784621	0.9438452
## 1985	0.8094668	0.7001322	0.9460860	1.1098830	1.0064570	1.0298170	0.8707742
## 1986	0.9941290	0.9534249	0.9746261	1.0676052	0.9711734	0.9923253	0.9305800
## 1987	1.0534553	0.7756861	1.0592555	1.0068570	1.0595081	0.9821841	1.0191159
## 1988	1.0973236	1.0524873	1.1983620	1.0839508	0.8578495	1.0623498	1.0200078
## 1989	1.4635664	0.9837017	0.8850488	1.1330369	0.9735440	0.9455808	1.0855657
## 1990	1.3208525	1.4006925	0.9888959	0.9787930	0.9664955	0.8103205	0.9077712
## 1991	0.8235804	0.7255746	0.8734605	0.8736202	1.0626606	1.2126191	0.9502077
## 1992	1.3840027	1.1476882	1.0592036	1.0207613	0.9595395	0.9713720	0.9460467
## 1993	1.1180599	1.0132669	0.7578611	0.9294204	0.9215760	1.0481809	0.9871771
## 1994	0.7199388	0.9524203	0.9699911	1.1043745	1.0150328	1.0453309	0.8637162
## 1995	0.9372192	1.0069818	1.0309249	0.8072524	1.0865204	1.0483263	0.8820168
## 1996	0.8088519	1.1005849	1.1118117	1.1019986	1.0083531	0.9798913	0.8950126
## 1997	1.2327142	1.0098547	1.1325212	1.0368377	0.8861921	0.8065462	1.0413664
## 1998	1.0946444	1.4398528	1.0505737	0.8626233	0.9780442	0.9060806	1.0048302

```

## 1999 0.9950851 1.3537305 1.0895475 0.8504985 0.9326271 1.0061377 1.0589939
## 2000 1.1666235 1.4607822 1.1133793 0.8490695 0.9694007 0.8683886 0.9030541
## 2001 0.8547821 1.1557494 1.0741728 1.1064276 0.8319030 0.8513702 1.0299973
## 2002 1.1262611 1.1119617 1.1501883 0.7560405 1.0064049 1.0593212 0.9193568
## 2003 0.9760404 1.1101135 1.0556196 0.8895945 0.9230032 0.9293001 1.0526096
## 2004 0.9234451 1.2319129 1.0497065 0.9737660 1.0038386 0.9026026 0.9479092
## 2005 0.9194371 1.3173985 1.2122649 0.9492757 0.9477469 0.9755593 0.9283084
## 2006 1.4154908 1.2413585 1.0181364 0.9936562 1.1445836 0.9205622 0.8047824
## 2007 1.3360617 1.1117636 0.9318827 1.0260241 1.1059278 1.0155582 0.9481967
## 2008 1.1761624 1.2089430 0.8541175 0.6521658 0.9459236 1.7939381 1.2473994
## 2009 0.5497102 1.1226989 1.1131466 0.7643490 0.9466699 1.2477492 0.9452941
## 2010 1.1976188 1.2699946 1.0731875 1.1145264 1.0205283 0.8541960 0.9719380
## 2011 1.6440032 0.9970020 1.0244492 0.8365151 0.8655811 0.9755037 1.1629408
## 2012 1.2250839 1.1270495 1.2687048 0.9717582 0.9836860 0.9450916 0.9861374
## 2013 1.2118897 1.5564119 1.1340115 0.8328571 0.9956884 0.7895614 1.0451313
## 2014 1.3990484 1.1593747 1.2936069 1.0909209 0.7977639 0.7831484 1.1405970
## 2015 1.0698905 0.4851928 0.8580408 1.3983810 1.0087467 1.3343392 1.0340495
## 2016 1.4925051 0.8159670 1.3300110 0.9336110 0.6273812 0.8697391 1.0714013
## 2017 1.3728545 1.2948726 1.0858430 0.7342210 0.7282622 1.2666903 1.0076989
## 2018 1.3313117 1.6363503 1.1449108 0.8370751 0.9327547 0.8828871 0.8790907
## 2019 1.6921737 1.0810233 0.7763252 1.1036269 0.7192114 0.8568512 0.7867897
## 2020 2.5456068 1.4489470 0.7822540 0.4244998 0.5759480 0.9568601 1.1246482
## 2021      NA      NA
##      Aug      Sep      Oct      Nov      Dec
## 1959 0.9083284 1.0181319 0.9646252 0.9324573 0.9931584
## 1960 1.1238675 0.9521458 1.1981940 1.2529860 0.5836981
## 1961 0.9757433 1.2788179 0.9491583 0.8866535 0.8766925
## 1962 1.1165670 0.9096897 0.8002032 0.8916241 1.1591981
## 1963 1.0127507 0.9463923 1.2207315 1.0625455 0.7708530
## 1964 1.0002215 0.8575812 1.1028471 1.0028831 1.1633733
## 1965 0.9201527 0.9952340 0.9672521 1.1333332 1.1964700
## 1966 1.1216265 0.9186923 0.6468549 0.9879251 0.8739161
## 1967 0.8297747 1.3136622 1.1844694 1.1043639 0.7469326
## 1968 1.1075499 1.1638815 0.9276045 0.8389490 0.7892920
## 1969 0.9417374 0.7197427 0.8851237 0.8143239 0.7787724
## 1970 0.8354637 0.9555929 1.0517554 1.1018682 0.8971386
## 1971 1.0468073 0.8416556 0.7958432 1.0670308 1.2434760
## 1972 1.0629390 0.9512650 1.0490366 1.1226626 1.2458947
## 1973 1.0417161 0.9267689 0.9270725 1.0348550 0.8798986
## 1974 0.9223307 1.1491922 0.9430327 0.7432724 0.8315716
## 1975 1.0225176 0.9978149 1.0443388 1.0033156 0.9126813

```

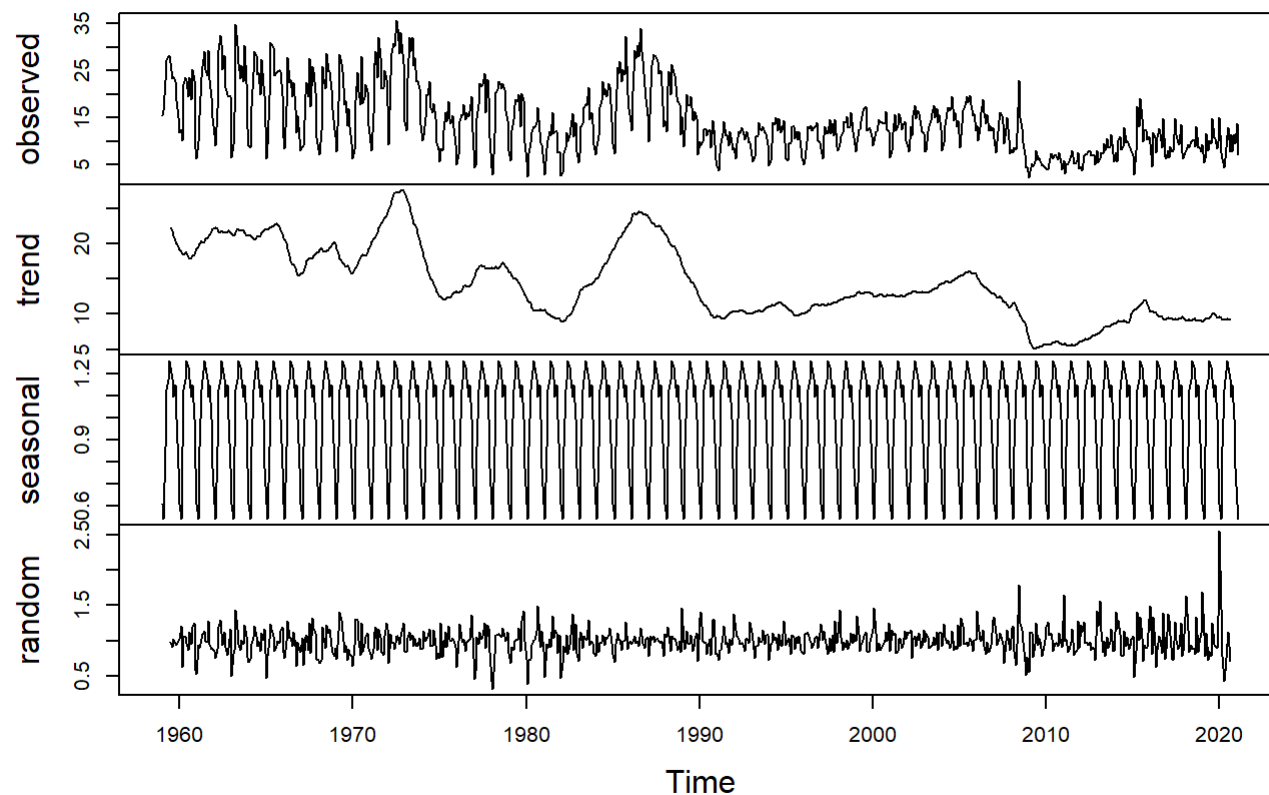
```
## 1976 0.9818417 1.0037486 1.0113995 1.3698771 1.0549328
## 1977 1.2434095 0.8957325 1.2256658 1.1750214 0.8373037
## 1978 1.0574752 1.1753886 1.0691203 0.9992960 1.1266895
## 1979 1.1722882 0.8529358 1.2823621 1.4182851 0.9902674
## 1980 1.1336199 1.4852423 0.9963193 0.9117771 1.1312694
## 1981 1.0344114 1.1495146 1.1097900 0.8184532 1.0856822
## 1982 1.0840896 1.3750103 0.8703802 1.2326538 0.8314374
## 1983 1.2918734 0.9411750 0.9922167 0.9633893 0.8188736
## 1984 1.0472913 1.0270639 1.0799051 1.1756558 0.9712480
## 1985 0.9991678 1.0014932 1.2613800 0.8873959 0.9627395
## 1986 1.1827129 1.0868584 0.9731809 0.9012203 1.0219468
## 1987 1.0212914 0.9917444 0.9709411 0.9972186 0.7873673
## 1988 1.0404178 0.9393872 0.8689338 0.9942266 0.8419090
## 1989 0.8544262 0.9137441 1.0212474 1.1061545 0.7172601
## 1990 1.0252018 0.7961026 1.2883649 1.3056572 0.8047977
## 1991 0.9868548 0.8268263 1.0198681 0.9427947 1.0420881
## 1992 0.8546585 1.0176323 1.0380423 0.9661491 1.2636718
## 1993 1.0017742 1.1474138 0.9565240 1.0744411 1.0834233
## 1994 1.0914357 0.9331944 0.9273644 1.2791345 1.1133536
## 1995 1.0521696 0.9736525 1.0625449 0.8622724 0.8381496
## 1996 0.9714050 1.0437733 1.0604491 0.8938370 1.1141182
## 1997 0.8592591 1.0747350 0.8421504 1.2724674 0.9622762
## 1998 0.8885441 0.9608319 1.1117603 0.9621369 1.0936831
## 1999 1.1245069 0.8498089 0.8397305 0.9785974 1.0961661
## 2000 0.9338484 1.0140694 1.1214067 0.9900558 1.2433386
## 2001 1.0322763 0.9002261 0.9374237 1.1361728 1.0205817
## 2002 1.0711434 1.1089500 0.8893552 0.9185005 1.1511482
## 2003 0.9348517 1.1174273 0.8723170 1.1056804 1.1272004
## 2004 1.1330522 0.8739415 0.9280865 0.8763783 1.2168014
## 2005 1.0436186 1.0111227 0.8492703 1.0129996 1.0128569
## 2006 0.9767152 0.8386191 0.8774175 0.9433833 1.2954949
## 2007 0.6901901 0.9930826 1.1502553 0.9594398 0.8669640
## 2008 1.1490533 0.9813050 0.7105804 0.5142442 0.7714313
## 2009 1.0382531 0.9898499 0.7807570 0.8568142 1.0114956
## 2010 0.9708293 0.9261796 1.1047971 0.9611499 0.9047859
## 2011 0.7957607 0.8353252 0.8641338 1.2754893 0.8896752
## 2012 0.8632236 0.9077917 0.8868967 0.7261437 1.4243671
## 2013 0.9017643 0.9785123 1.1596278 0.8602536 0.8531166
## 2014 0.9208521 1.0507792 0.8758028 0.9195789 1.0302836
## 2015 0.7141712 0.9390820 0.8967572 0.9156857 1.3115221
## 2016 1.0241299 0.8265225 1.3878276 0.7412307 0.8789807
```

```
## 2017 0.8783001 0.9035328 1.2435550 0.7883362 0.8470111
## 2018 0.8157186 1.3290569 0.8357005 0.9733074 1.1044812
## 2019 1.2437617 0.9547566 0.9296512 0.8953976 1.3441953
## 2020 0.7150369      NA      NA      NA      NA
## 2021
##
## $figure
## [1] 0.6117873 0.5427180 0.8683181 1.1276672 1.1700935 1.2574701 1.2320798
## [8] 1.1747646 1.0969375 1.1497942 0.9944642 0.7739055
##
## $type
## [1] "multiplicative"
##
## attr(,"class")
## [1] "decomposed.ts"
```

gráficos

```
plot(decomposicao_mult)
```

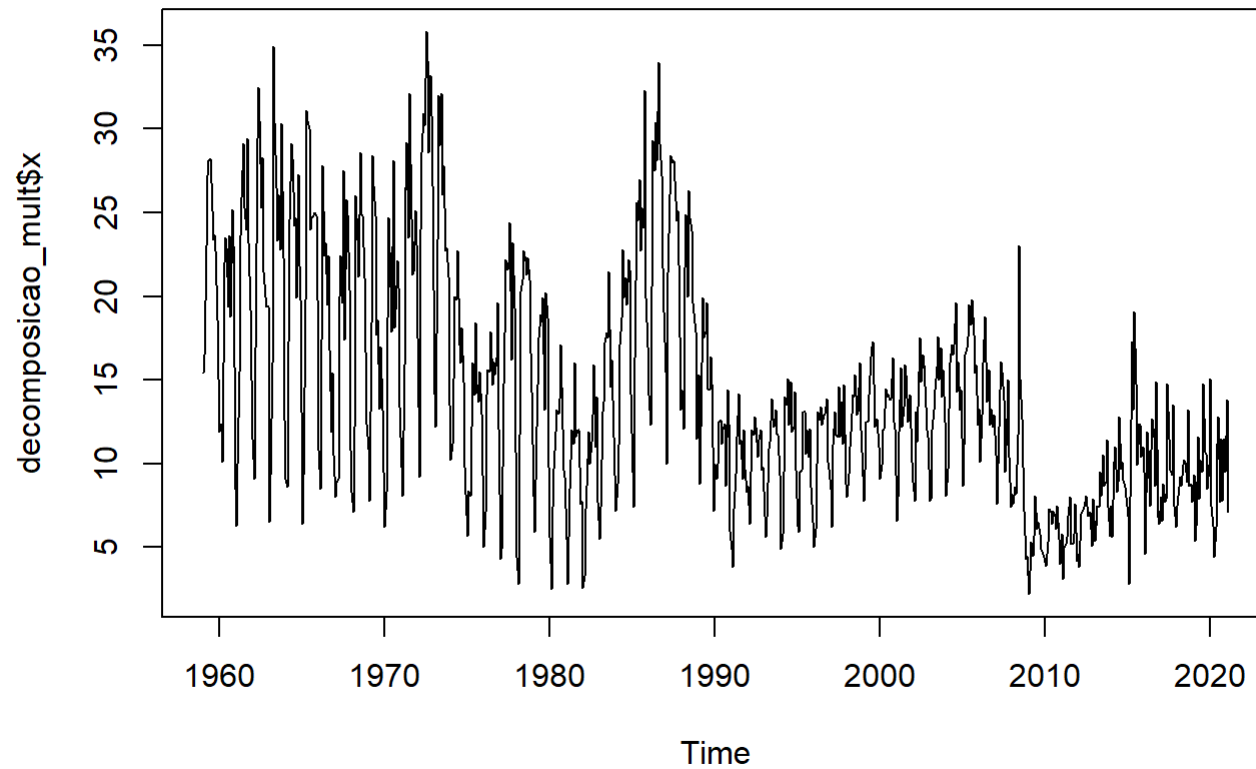
## Decomposition of multiplicative time series



componentes plotados separadamente

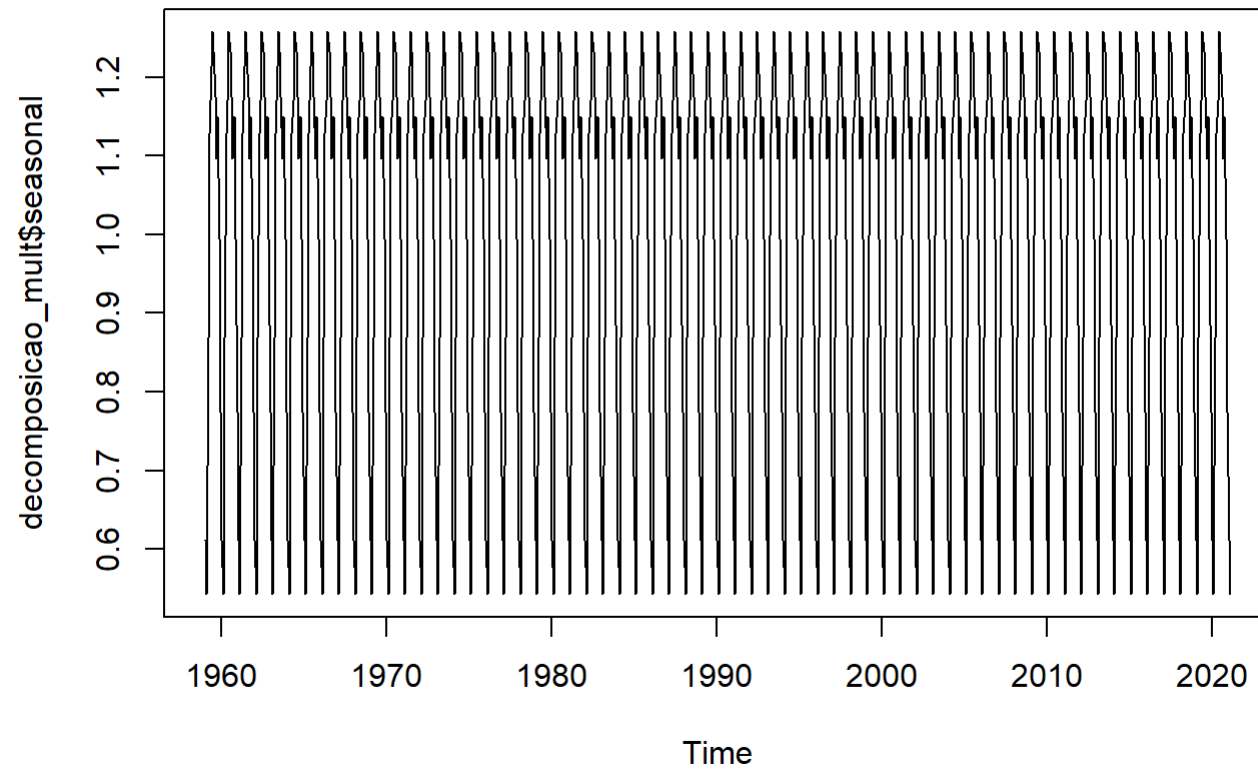
### Série original

```
plot(decomposicao_mult$x)
```



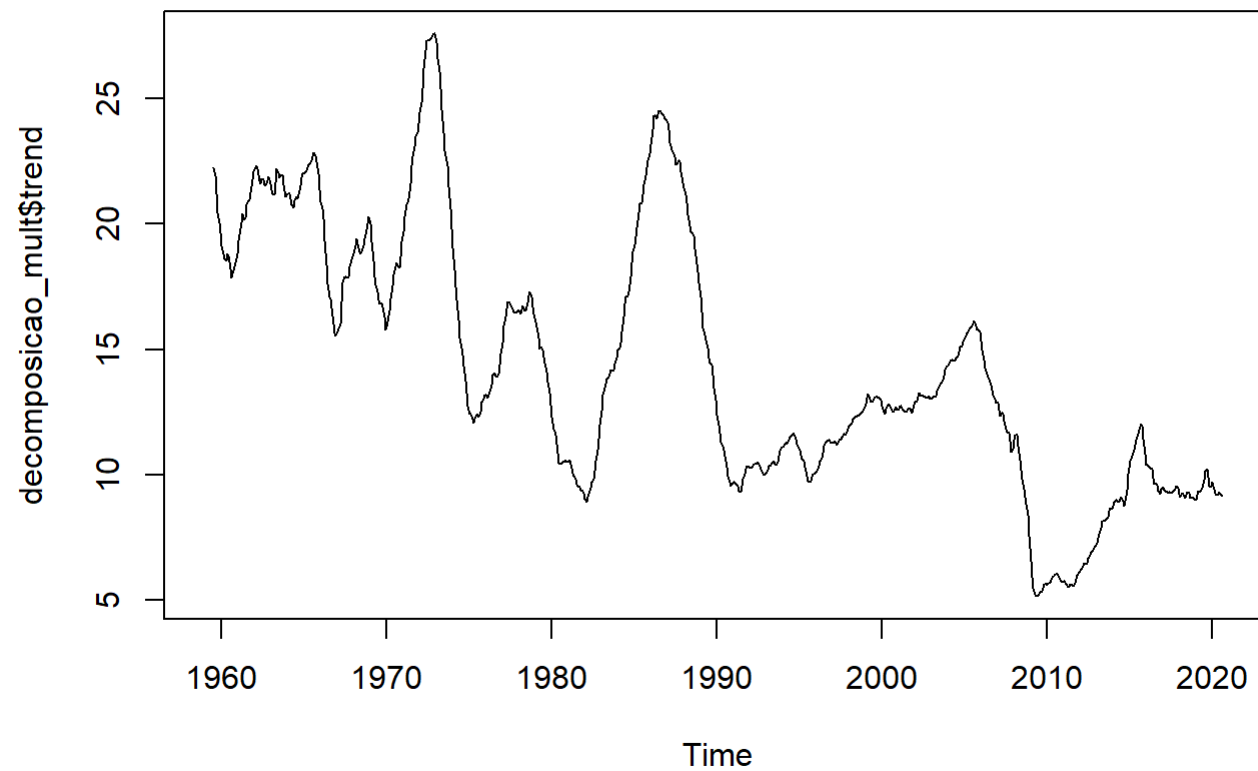
### Sazonalidade multiplicativa

```
plot(decomposicao_mult$seasonal)
```



### Tendência

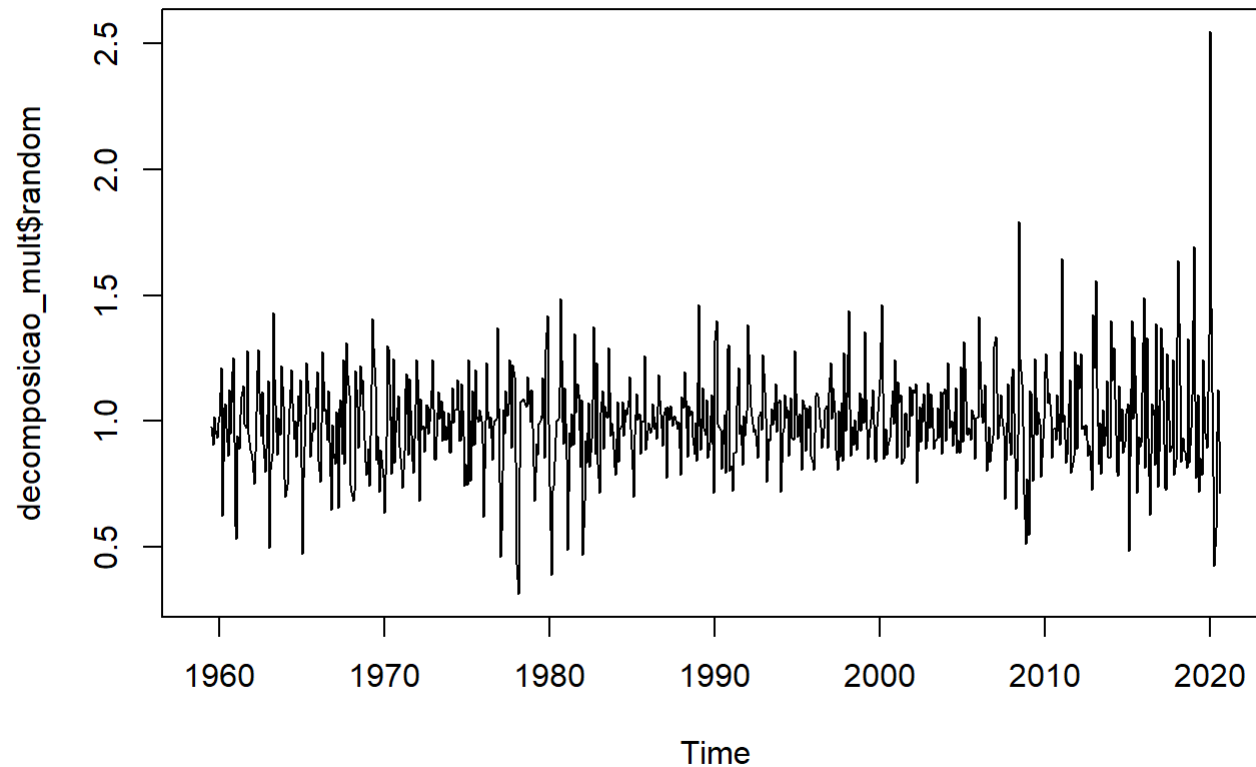
```
plot(decomposicao_mult$trend)
```



### erros

```
plot(decomposicao_mult$random)
```



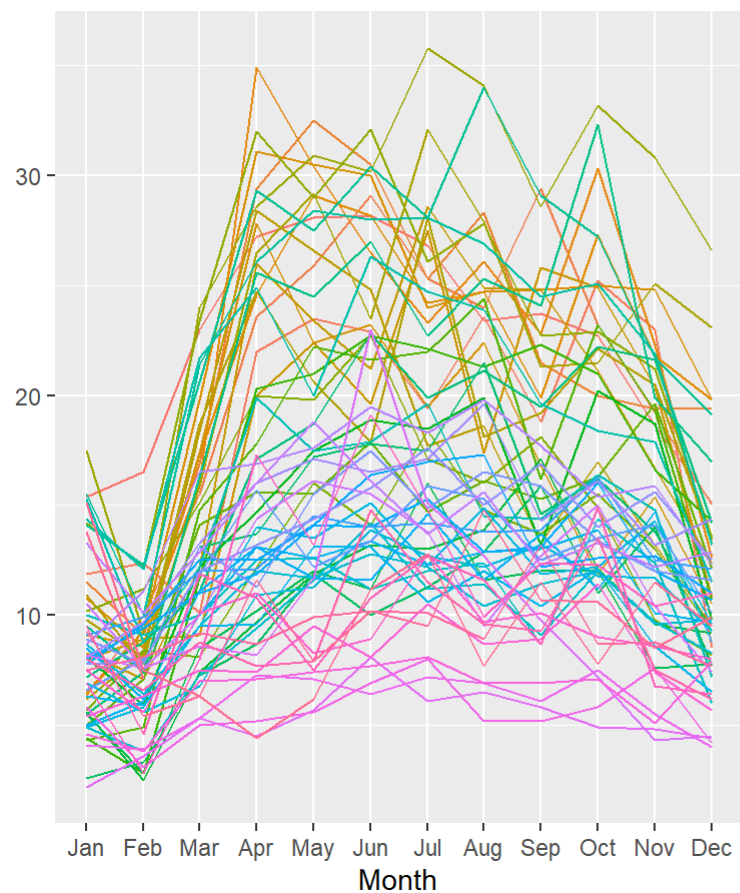


Vemos que para essa série a decomposição com sazonalidade multiplicativa gerou resíduos menores.

A sazonalidade também pode ser visualizada com outros gráficos. Abaixo, nesse gráfico será gerado as sazonalidade de cada ano. Já podemos notar que para anos mais recentes a sazonalidade é menor do que nos anos iniciais da série.

```
ggseasonplot(serie$y)
```

## Seasonal plot: serie\$y



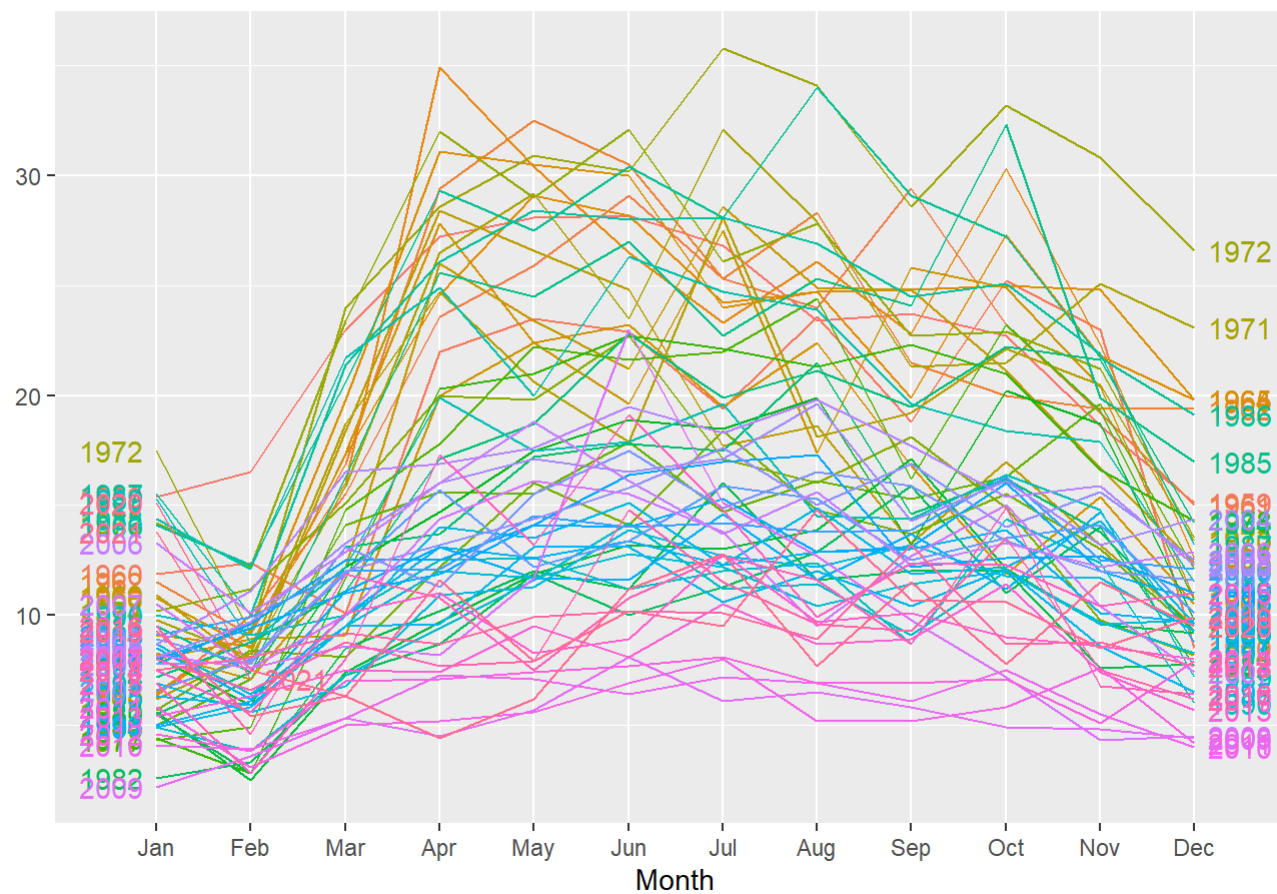
year

1959	1975	1991	2007
1960	1976	1992	2008
1961	1977	1993	2009
1962	1978	1994	2010
1963	1979	1995	2011
1964	1980	1996	2012
1965	1981	1997	2013
1966	1982	1998	2014
1967	1983	1999	2015
1968	1984	2000	2016
1969	1985	2001	2017
1970	1986	2002	2018
1971	1987	2003	2019
1972	1988	2004	2020
1973	1989	2005	2021
1974	1990	2006	

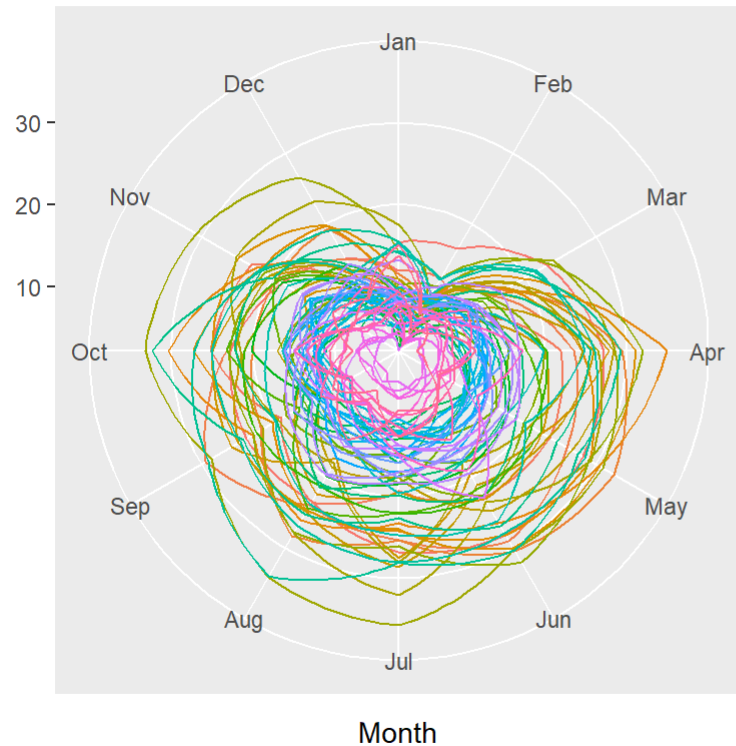
Inserindo rótulo das datas.

```
#Como temos muitos anos, a visualizacao não está muito boa
ggseasonplot(serie$y, year.labels = TRUE, year.labels.left = TRUE)
```

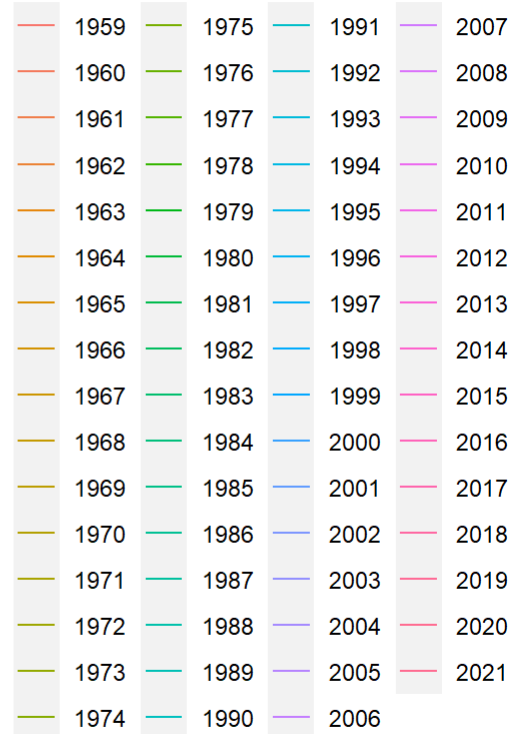
## Seasonal plot: serie\$y



Seasonal plot: serie\$y

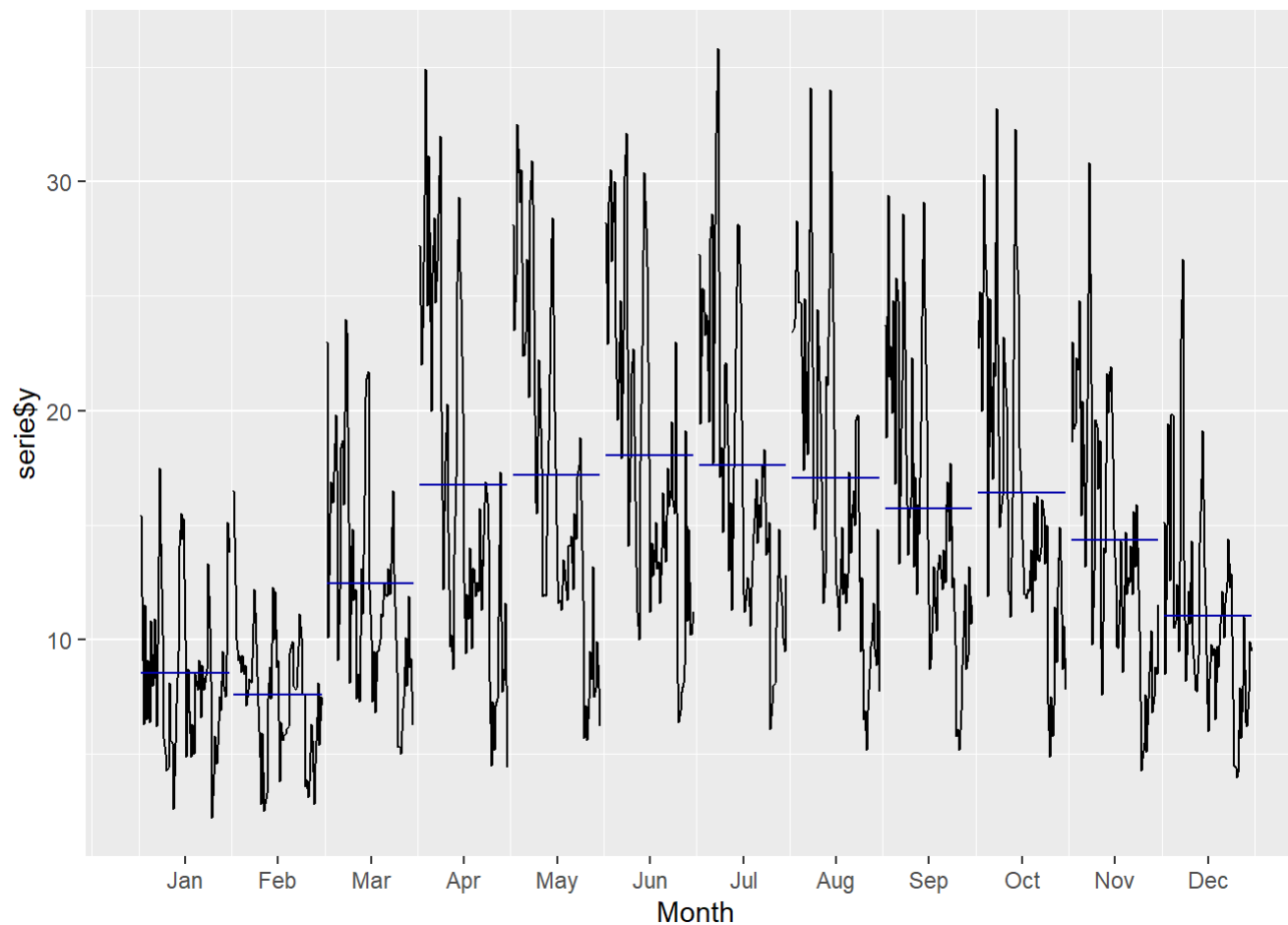


year



Por fim, podemos plotar gráficos que enfatizam os padrões sazonais e onde os dados para cada estação são coletados juntos em mini gráficos temporais separados.

```
ggsubseriesplot(serie$y)
```



Conforme o gráfico acima há um padrão de sazonalidade em todos os meses, caso não houvesse as linhas azuis estariam alinhadas formando uma linha reta. A sazonalidade para os meses do começo e final do ano são menores; e entre os meses Abril a Setembro são bem semelhantes.