7a.1

$$(\sin x+3)y' = (y-2)\cos x \qquad y' = \frac{dy}{dx}$$

$$\Rightarrow (\sin x+3)\cdot\frac{dy}{dx} = (y-2)\cos x \quad /: (\sin x+3)\cdot(y-2) \quad \cdot dx$$

$$\Rightarrow \frac{dy}{y-2} = \frac{\cos x}{\sin x+3}\,dx \quad / \int$$

1. $$\int\frac{dy}{y-2} = \ln(y-2)$$

2. $$\int\frac{\cos x}{\sin x+3}\,dx = \left|\begin{array}{l}\text{смена: } \sin x+3=t\\ \cos x\,dx = dt\end{array}\right| = \int\frac{dt}{t} = \ln t$$

$$= \ln(\sin x+3)$$

$$\ln(y-2) = \ln(\sin x+3)+\ln C = \ln\left[C\cdot(\sin x+3)\right]$$

$$\Rightarrow y-2 = C(\sin x+3) \Rightarrow \boxed{y = C(\sin x+3)+2}$$

7.a.2 $(\operatorname{ctg} x + 4)\, y' = \dfrac{3-y}{\sin^2 x}$ $\qquad\qquad$ $y' = \dfrac{dy}{dx}$

$\Rightarrow (\operatorname{ctg} x + 4)\dfrac{dy}{dx} = \dfrac{3-y}{\sin^2 x}$ $\qquad$ $(: y-3)$

$\Rightarrow \dfrac{dy}{3-y} = \dfrac{dx}{(\operatorname{ctg} x + 4)\cdot \sin^2 x}$ $\quad / \int$

$\Rightarrow$ 1) $\displaystyle\int \frac{dy}{3-y} = -\int \frac{dy}{y-3} = \boxed{-\ln(y-3)}$

2) $\displaystyle\int \frac{dx}{(\operatorname{ctg} x + 4)\sin^2 x} = \left|\begin{array}{l} \text{смена}: \operatorname{ctg} x + 4 = t \\ -\dfrac{1}{\sin^2 x}\,dx = dt \end{array}\right.$

$\displaystyle = -\int \frac{dt}{t} = -\ln t = \boxed{-\ln(\operatorname{ctg} x + 4)}$

$C > 0$

$\Rightarrow -\ln(y-3) = -\ln(\operatorname{ctg} x + 4) \overset{-}{\cancel{+}} \ln C \quad / \cdot (-1)$

$\Rightarrow \ln(y-3) = \ln C(\operatorname{ctg} x + 4)$

$\Rightarrow y - 3 = C(\operatorname{ctg} x + 4) \Rightarrow \boxed{y = C(\operatorname{ctg} x + 4) + 3}$

7.a.3 $(1+\cos x)\,y\,y' = (3y^2-2)\sin x$ $\qquad y' = \frac{dy}{dx}$

$\Rightarrow (1+\cos x)\,y\,\frac{dy}{dx} = (3y^2-2)\sin x$

$\Rightarrow \frac{y}{3y^2-2}\,dy = \frac{\sin x}{1+\cos x}\,dx \quad / \int(\;)$

1) $\int \frac{y}{3y^2-2}\,dx = \left| \begin{array}{l} \text{смена}: 3y^2-2=t \\ 6y\,dy = dt \Rightarrow y\,dy = \frac{dt}{6} \end{array} \right|$

$= \int \frac{dt}{6y} = \frac{1}{6}\ln y = \underbrace{\ln y^{1/6}}$

2) $\int \frac{\sin x}{1+\cos x}\,dx = \left| \begin{array}{l} 1+\cos x = t \\ \sin x\,dx = -dt \end{array} \right| = -\int \frac{dt}{t} = -\ln t$

$= -\ln(1+\cos x)$

$\Rightarrow \ln y^{1/6} = -\ln(1+\cos x) + \ln C = \ln \frac{C}{1+\cos x}$

$\Rightarrow y^{1/6} = \frac{C}{1+\cos x} \Rightarrow y = \left[\frac{C}{1+\cos x}\right]^6, \; C>0$

$\underline{\underline{7.8.1}}$ $(x-5)y' - y = x^2 - 25$, (ΛΔΡ) $/ : x-5$

$$y' - \frac{1}{x-5}y = \frac{x^2-25}{x-5} = \frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}}$$

$$\Rightarrow y' + \left(-\frac{1}{x-5}\right)y = x+5$$

$$P(x) = -\frac{1}{x-5} \qquad Q(x) = x+5$$

$$\boxed{y = e^{-\int P(x)dx}\left[\int Q(x)e^{\int P(x)dx} + C\right], C \in \mathbb{R}}$$

1) $\int P(x)dx = \int -\frac{dx}{x-5} = -\int \frac{dx}{x-5} = -\ln(x-5)$

$$e^{-\int P(x)dx} = e^{-(-\ln(x-5))} = e^{\ln(x-5)} = \underline{x-5}$$

$$e^{\int P(x)dx} = e^{-\ln(x-5)} = \frac{1}{x-5}$$

$$\int \frac{x+5}{x-5}\,dx = \int \frac{x-5+10}{x-5}\,dx = \int \left(1 + \frac{10}{x-5}\right) dx$$

$$= \int dx + 10 \int \frac{dx}{x-5} = 1 + 10 \ln(x-5)$$

$$y = (x-5)\left[\underbrace{1 + 10 \ln(x-5) + C_1}_{1+C_1 = C}\right], \quad C_1 \in \mathbb{R}$$

7.6.2 $\underbrace{\left(\frac{1}{\sqrt{3x-1}} + 2xy^2\right)}_{P(x,y)}dx + \underbrace{\left(\sqrt{3y-1} + 2x^2y\right)}_{Q(x,y)}dy = 0$

$$\frac{\partial P}{\partial y} \overset{?}{=} \frac{\partial Q}{\partial x}$$

$$\left.\begin{aligned}\frac{\partial P}{\partial y} &= \frac{\partial}{\partial y}\left(\frac{1}{\sqrt{3x-1}} + 2xy^2\right) = 0 + 2x\cdot 2y = 4xy \\ \frac{\partial Q}{\partial x} &= \frac{\partial}{\partial x}\left(\sqrt{3y-1} + 2x^2y\right) = 0 + 2\cdot 2x\cdot y = 4xy\end{aligned}\right\}$$

$$\Rightarrow \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$\begin{cases} A = \int P(x,y)\,dx \\ B = \int \left[Q(x,y) - \frac{\partial A}{\partial y}\right] dy \end{cases}$$

$$A + B = C,\ C \in \mathbb{R}$$

$$A = \int \left(\frac{1}{\sqrt{3x-1}} + 2xy^2 \right) dx$$

$$= \int \frac{dx}{\sqrt{3x-1}} + 2y^2 \int x \, dx$$

$$= \left| \begin{array}{l} \text{смена за I член}: \; 3x-1 = t \\ 3\,dx = dt \Rightarrow dx = \frac{dt}{3} \end{array} \right|$$

$$= \frac{1}{3} \int t^{-1/2} \, dt + \cancel{2}y^2 \cdot \frac{x^2}{\cancel{2}}$$

$$= \frac{1}{3} \cdot \frac{t^{1/2}}{1/2} + x^2y^2 = \frac{2}{3}\sqrt{3x-1} + x^2y^2$$

$$\frac{\partial A}{\partial y} = \frac{\partial}{\partial y} \left(\frac{2}{3}\sqrt{3x-1} + x^2y^2 \right) = 0 + x^2 \cdot 2y = \underline{\underline{2x^2y}}$$

$$B = \int \left[Q(x,y) - \frac{\partial A}{\partial y} \right] dy$$

$$= \int \left[\sqrt{3y-1} + \cancel{2x^2 y} - \cancel{2x^2 y} \right] dy$$

$$= \int \sqrt{3y-1}\, dy = \left| \begin{array}{l} 3y-1=t \\ 3\,dy = dt \Rightarrow dy = \frac{dt}{3} \end{array} \right)$$

$$= \int \sqrt{t} \cdot \frac{dt}{3} = \frac{1}{3} \int t^{1/2}\, dt$$

$$= \frac{1}{3} \frac{t^{3/2}}{3/2} = \frac{2}{9} t^{3/2} = \frac{2}{9} (3y-1)^{3/2}$$

$$= \boxed{\frac{2}{9} (3y-1) \sqrt{3y-1}}$$

$$\frac{2}{3} \sqrt{3x-1} + x^2 y^2 + \frac{2}{9} \ldots = C$$

$A(\overset{x_1}{-6}, \overset{y_1}{7}, \overset{z_1}{3})$

$$\ell_{AB}: \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$\underline{B} = p \cap \Sigma$

$$p: \frac{x-3}{-3} = \frac{y+1}{3} = \frac{z-5}{-4} = t$$

$$\Sigma: 3x+4y+7z-15=0$$

$$p: \begin{cases} x = -3t+3 \\ y = 3t-1 \\ z = -4t+5 \end{cases}$$

$$3(-3t+3)+4(3t-1)+7(-4t+5)-15=0$$

$$\underline{-9t}+9+\underline{12t}-4-28t+35-15=0$$

$$-25t+25=0 \Rightarrow \boxed{t=1}$$

$$\Rightarrow \begin{aligned} x_2 &= -3\cdot 1+3=0 \\ y_2 &= 3\cdot 1-1=2 \\ z_2 &= -4\cdot 1+5=1 \end{aligned}$$

$B(0,2,1)$

$$\frac{x+6}{0+6} = \frac{y-7}{2-7} = \frac{z-3}{1-3}$$

$$\Rightarrow \boxed{\frac{x+6}{6} = \frac{y-7}{-5} = \frac{z-3}{-2}}$$