7 а) $y'(3\sin x + 2) = (y+5)\cos x$ $\qquad y' = \dfrac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx}\cdot(3\sin x+2) = (y+5)\cos x \quad \Big/ \; 1): (3\sin x+2)\cdot(y+5) \quad 2)\cdot dx$$

$$\Rightarrow \frac{dy}{y+5} = \frac{\cos x}{3\sin x+2}\,dx$$ – променливите се раздвоени $/\int(\cdot)$

$$\Rightarrow \int\frac{dy}{y+5} = \int\frac{\cos x}{3\sin x+2}\,dx$$

Т $\displaystyle\int\frac{dx}{x+a} = \ln|x+a| + C$

1) $\displaystyle\int\frac{dy}{y+5} = \ln(y+5)$

2) $$\int\frac{\cos x}{3\sin x+2}\,dx = \left|\begin{array}{c} \text{смена: } 3\sin x+2 = t \\ 3\cos x\,dx = dt \\ \cos x\,dx = \frac{dt}{3}\end{array}\right|$$

$$= \int\frac{\frac{dt}{3}}{t} = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln t = \ln t^{1/3} = \ln\sqrt[3]{t}$$

$$= \ln\sqrt[3]{3\sin x+2}$$

$\underrightarrow{1) \text{ и } 2)}$ $$\ln(y+5) = \ln\sqrt[3]{3\sin x+2} + \ln C \qquad (C>0)$$
$$= \ln C\cdot\sqrt[3]{3\sin x+2}$$

$$\Rightarrow y+5 = C\cdot\sqrt[3]{3\sin x+2}$$

$$\Rightarrow y = C\cdot\sqrt[3]{3\sin x+2} - 5 \qquad (C>0)$$

б) $(x-4)y' + y = x^2 - 16 \quad / : (x-4)$ $\qquad y' + P(x)y = Q(x)$

$$y' + \underbrace{\frac{1}{x-4}}_{=P(x)} y = \frac{x^2-16}{x-4} = \frac{(x-4)(x+4)}{x-4} = \underbrace{x+4}_{=Q(x)}$$

Општо решение: (може да се допишe на листови)

$$y = e^{-\int P(x)dx}\left[\int Q(x) e^{\int P(x)dx} dx + C\right], \quad C \in \mathbb{R}$$

1) $\int P(x)dx = \int \frac{1}{x-4}dx = \int \frac{dx}{x-4} = \ln(x-4)$

2) $e^{\int P(x)dx} = e^{\ln(x-4)} = x-4$

3) $\int Q(x) e^{\int P(x)dx} dx = \int (x+4)(x-4)dx = \int (x^2-16)dx$
$= \int x^2 dx - 16\int dx = \frac{x^3}{3} - 16x$

$$\overset{1.)-3.)}{\Longrightarrow} \quad y = \frac{1}{x-4}\left(\frac{x^3}{3} - 16x + C\right), \quad C \in \mathbb{R}$$

алтернатива на б): $(x+5)y' - y = x^2 - 25 \quad / : (x+5)$

$$\Rightarrow y' - \frac{1}{x+5}y = \frac{x^2-25}{x+5} = \frac{(x-5)\cancel{(x+5)}}{\cancel{x+5}}$$

$$\Rightarrow y' + \underbrace{\left(-\frac{1}{x+5}\right)}_{=P(x)} y = \underbrace{x-5}_{Q(x)}$$

1) $\int P(x)dx = \int -\frac{1}{x+5}dx = -\int \frac{dx}{x+5} = -\ln(x+5)$

2) $e^{\int P(x)dx} = e^{-\ln(x+5)} = \frac{1}{e^{\ln(x+5)}} = \frac{1}{x+5}$

3) $\int Q(x) e^{\int P(x)dx} dx = \int (x-5)\cdot \frac{1}{x+5}\, dx = \int \frac{x-5}{x+5}\, dx$

$$= \int \frac{(x+5)-5-5}{x+5}\, dx = \int \left(1 - \frac{10}{x+5}\right) dx$$

$$= \int dx - 10 \int \frac{dx}{x+5} = x - 10 \ln(x+5)$$

$1.)-3.) \Longrightarrow \quad y = \frac{1}{\frac{1}{x+5}} \left[x - 10\ln(x+5) + C \right]$

$$= (x+5)\left[x - 10 \ln(x+5) + C\right], \quad C \in \mathbb{R}$$

б) $\underbrace{\left(\sqrt[7]{2x+5} + 7xy^2\right)}_{= P(x,y)} dx + \underbrace{\left(\frac{1}{\sqrt[7]{5y+2}} + 7x^2y\right)}_{= Q(x,y)} dy = 0$

проверка дали $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$?

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}\Big(\underbrace{\sqrt[7]{2x+5}}_{\text{зависи само од } x} + 7xy^2 \Big) = 0 + 7x \cdot 2y = 14xy$$

$$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}\Big(\underbrace{\frac{1}{\sqrt[7]{5y+2}}}_{\text{зависи само } y} + 7x^2 y \Big) = 0 + 7\cdot 2x \cdot y = 14xy$$

$\Rightarrow$ дадената ДР е во тотален диференцијал

$\Rightarrow$ општо решение е од облик $A + B = C$, $C \in \mathbb{R}$ и

1) $A = \int P(x,y)\,dx = \int \left(\sqrt[7]{2x+5} + 7xy^2\right) dx$

$$= \int (2x+5)^{1/7} + 7y^2 \int x\, dx$$

$$= \left| \begin{array}{l} \text{смена за I интеграл: } 2x+5 = t \\ 2\,dx = dt \implies dx = \frac{dt}{2} \end{array} \right|$$

$$= \int t^{1/7}\,\frac{dt}{2} + 7y^2\cdot\frac{x^2}{2} = \frac{1}{2}\int t^{1/7}\,dt + \frac{7}{2}x^2y^2$$

$$= \frac{1}{2}\cdot\frac{t^{1/7+1}}{\underbrace{\frac{1}{7}+1}_{=\frac{8}{7}}} + \frac{7}{2}x^2y^2 = \frac{1}{2}\cdot\frac{7}{8}\,t^{8/7} + \frac{7}{2}x^2y^2$$

$$= \boxed{\frac{7}{16}(2x+5)^{8/7} + \frac{7}{2}x^2y^2}$$

2) $$\frac{\partial A}{\partial y} = \frac{\partial}{\partial y}\left(\frac{7}{16}(2x+5)^{8/7} + \frac{7}{2}x^2y^2\right) = 0 + \frac{7}{\cancel{2}}\cdot x^2\cdot\cancel{2}y$$

$$= \underline{7x^2y}$$

$$B = \int\left[Q(x,y) - \frac{\partial A}{\partial y}\right]dy = \int\left(\frac{1}{\sqrt[7]{5y+2}} + \cancel{7x^2y} - \cancel{7x^2y}\right)dx$$

$$= \int (5y+2)^{-1/7}\,dy = \left| \begin{array}{l} \text{смена: } 5y+2 = t \\ 5\,dy = dt \\ dy = \frac{dt}{5} \end{array} \right|$$

$$= \int t^{-1/7}\,\frac{dt}{5} = \frac{1}{5}\cdot\frac{t^{-1/7+1}}{\underbrace{-\frac{1}{7}+1}_{=6/7}} = \frac{1}{5}\cdot\frac{7}{6}\,t^{6/7}$$

$$= \boxed{\frac{7}{30}(5y+2)^{6/7}}$$

$$\underline{\underline{1),\,2)}} \implies \frac{7}{16}(2x+5)^{8/7} + \frac{7}{2}x^2y^2 + \frac{7}{30}(5y+2)^{6/7} = C,$$

$$C \in \mathbb{R}$$

алтернативна на в) : $\underbrace{(8x^7+3x^2y^2)}_{=P(x,y)}dx + \underbrace{(5y^4+2x^3y)}_{=Q(x,y)}dy = 0$

$$\left.\begin{aligned}\frac{\partial P}{\partial y} &= \frac{\partial}{\partial y}(8x^7+3x^2y^2) = 0+3x^2\cdot 2y = 6x^2y\\ \frac{\partial Q}{\partial x} &= \frac{\partial}{\partial x}(5y^4+2x^3y) = 0+2\cdot 3x^2\cdot y = 6x^2y\end{aligned}\right\} =$$

=> дадената ДР е ДР. во тотален диференцијал

=> општо решение е од облик $A+B=C,\ C\in\mathbb{R}$

1) $A = \int P(x,y)dx = \int (8x^7+3x^2y^2)dx$

$= 8\int x^7dx + 3y^2\int x^2dx = \cancel{8}\cdot\frac{x^{7+1}}{\cancel{7+1}} + \cancel{3}y^2\cdot\frac{x^3}{\cancel{3}}$

$= \boxed{x^8+x^3y^2}$

2) $\frac{\partial A}{\partial y} = \frac{\partial}{\partial y}(x^8+x^3y^2) = x^3\cdot 2y = 2x^3y$

$B = \int\left[Q(x,y) - \frac{\partial A}{\partial y}\right]dy = \int\left[(5y^4+\cancel{2x^3y}) - \cancel{2x^3y}\right]dx$

$= \int 5y^4dy = 5\int y^4dy = \cancel{5}\cdot\frac{y^5}{\cancel{5}} = \boxed{y^5}$

$\xrightarrow{1)\text{ и }2)}\quad x^8+x^3y^2+y^5 = C,\quad C\in\mathbb{R}$