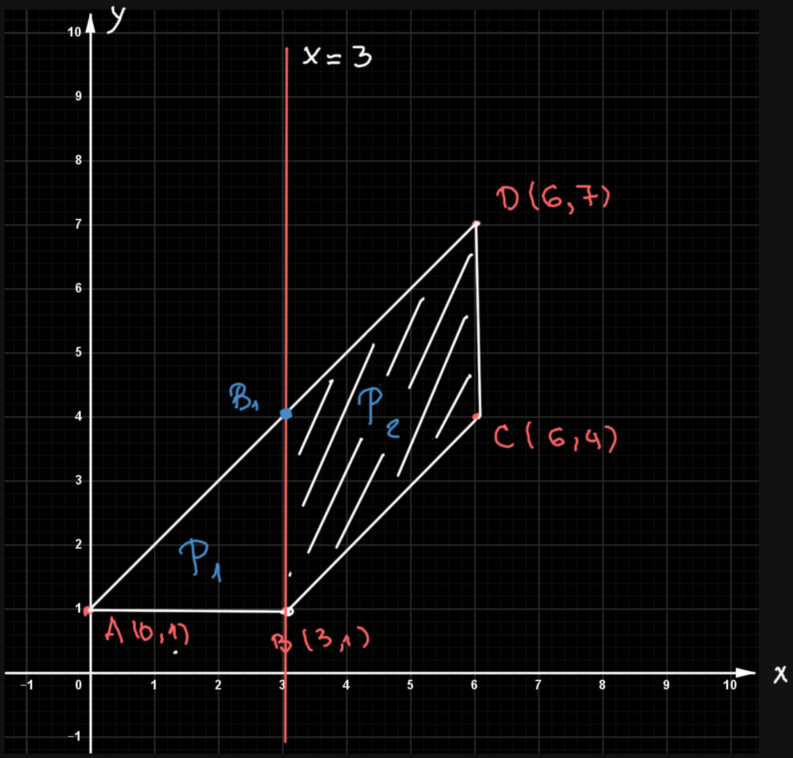
(3)



$$P = P_1 + P_2$$

$$= P_{ABB_1} + P_{BCDB_1}$$

$$= \int_0^3 \left[p_{AD}(x) - p_{AB}(x)\right] dx$$

$$+ \int_3^6 \left[p_{AD}(x) - p_{BC}(x)\right] dx$$

р-ка на права низ точки $T_1(x_1, y_1)$ и $T_2(x_2, y_2)$

$$\boxed{y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1}$$

1) $p_{AD}(x)$:

$A(\overset{x_1}{0}, \overset{y_1}{1})$ и $D(\overset{x_2}{6}, \overset{y_2}{7})$

$$y = \overset{=1}{\frac{7-1}{6-0}}(x-0) + 1$$

$$\boxed{= x + 1 = p_{AD}(x)}$$

2) $p_{AB}(x)$: A и B се со иста II коop. $\Rightarrow p_{AB}(x) = 1$ (II. коop)

или преку ф-ла : $y = \frac{1-1}{3-0}(x-0) + 1 = \underbrace{\frac{0}{3}}_{=0} \cdot x + 1 = \boxed{1 = p_{AB}(x)}$

3) $p_{BC}(x)$: $B(\overset{x_1}{3}; \overset{y_1}{1})$, $C(\overset{x_2}{6}; \overset{y_2}{4})$

$$y = \frac{4-1}{6-3}(x-3) + 1 = \overset{=1}{\frac{3}{3}}(x-3) + 1 = \boxed{x - 2 = p_{BC}(x)}$$

$$P = \int_0^3 \left[(x+1) - 1\right] dx + \int_3^6 \left[(x+1) - (x-2)\right] dx$$

$$= \int_0^3 x\,dx + \int_3^6 3\,dx = \frac{x^2}{2}\Big|_{x=0}^{x=3} + 3x\Big|_{x=3}^{x=6}$$

$$= \left(\frac{3^2}{2} - \frac{0^2}{2}\right) + \left(3\cdot 6 - 3\cdot 3\right) = 4{,}5 + 9 = 13{,}5$$

кв. ед.

4

опција 1

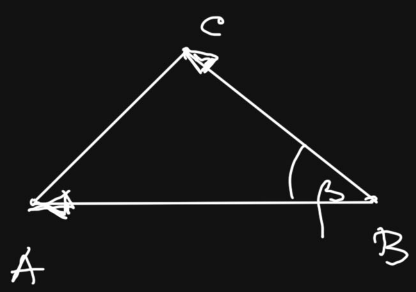
$A(-3;4,3)$

$B(-4;5,3)$

$C(-5,4;1)$

$\measuredangle B = ?$

$P_{ABC} = ?$



$$\measuredangle B = \beta = \measuredangle(\overrightarrow{BA}, \overrightarrow{BC})$$

$$P = \frac{1}{2}\left|\overrightarrow{BA} \times \overrightarrow{BC}\right|$$

$$\cos\beta = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|}$$

$$\overrightarrow{BA} = \vec{r}_A - \vec{r}_B = (-3;4;3) - (-4,5,3)$$

$$= (-3-(-4);\ 4-5;\ 3-3) = (1,-1;\ 0)$$

$$\overrightarrow{BC} = \vec{r}_C - \vec{r}_B = (-5;4,1) - (-4,5,3)$$

$$= (-5-(-4);\ 4-5;\ 1-3) = (-1;-1;-2)$$

$$\cos\beta = \frac{1\cdot(-1) + (-1)\cdot(-1) + 0\cdot(-2)}{\sqrt{1^2+(-1)^2+0^2}\cdot\sqrt{(-1)^2+(-1)^2+(-2)^2}} = \frac{-1+1+0}{\sqrt{1+1+0}\cdot\sqrt{1+1+4}}$$

$$= 0$$

$$\Rightarrow \beta = \arccos 0 = 90^\circ = \frac{\pi}{2}\text{rad}$$

за дома: $\measuredangle A = ?$ (60°)

$\measuredangle C = ?$ (30°)

$$P = \frac{1}{2} \left| \vec{BA} \times \vec{BC} \right|$$

$$\vec{BA} \times \vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ -1 & -1 & -2 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 1 & -1 \\ -1 & -1 \end{matrix}$$

$$= 2\vec{i} + 0 \cdot \vec{j} - \vec{k} - \vec{k} - 0 \cdot \vec{i} - (-2\vec{j})$$

$$= 2\vec{i} + 2\vec{j} - 2\vec{k} = (2; 2, -2)$$

$$P = \frac{1}{2}\sqrt{2^2 + 2^2 + (-2)} = \frac{1}{2}\sqrt{4+4+4} = \frac{1}{2}\sqrt{3 \cdot 4}$$

$$= \frac{1}{\cancel{2}} \sqrt{3} \cdot \underbrace{\cancel{\sqrt{4}}}_{=2} = \sqrt{3} \text{ кв. ед.}$$

НАПОМЕНА:

$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$ (р-ка на права низ 2 точки $T_1(x_1, y_1)$ и $T_2(x_2, y_2)$)

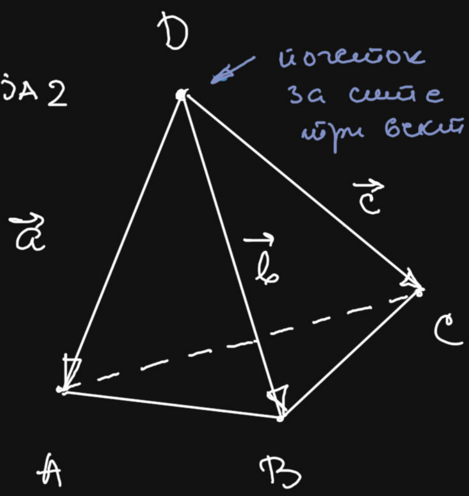
+ формули од гл. ВЕКТОРСКА АЛГЕБРА СО АНАЛИТИЧКА ГЕОМЕТРИЈА

Може да се носат на испит напишани на лист!!!

→ општа ф-ла $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

4

ОПЦИЈА 2



почеток за сите три вектори

$$V = \frac{1}{6}\left|(\vec{a}\,\vec{b}\,\vec{c})\right|$$

$$\vec{a} = \overrightarrow{DA} = \vec{r}_A - \vec{r}_D$$
$$= (0,1,4) - (2,-3,1)$$
$$= (0-2, 1-(-3), 4-1) = (-2;4,3)$$

$$\vec{b} = \overrightarrow{DB} = \vec{r}_B - \vec{r}_D$$
$$= (-2,3,5) - (2,-3,1)$$
$$= (-2-2, 3-(-3), 5-1) = (-4;6;4)$$

$$\vec{c} = \overrightarrow{DC} = \vec{r}_C - \vec{r}_D$$
$$= (7,-3;-4) - (2,-3,1)$$
$$= (7-2, -3-(-3), -4-1) = (5,0,-5)$$

$$(\vec{a}\,\vec{b}\,\vec{c}) = \left|\begin{matrix} -2 & 4 & 3 \\ -4 & 6 & 4 \\ 5 & 0 & -5 \end{matrix}\right| \begin{matrix} -2 & 4 \\ -4 & 6 \\ 5 & 0 \end{matrix}$$

$$= 60 + \cancel{80} + 0 - 90 - 0 - \cancel{80} = -30$$

$$V = \frac{1}{6}\left|-30\right| = \frac{1}{6}\cdot 30 = 5 \text{ куб. ед.}$$

⑤ $A(2;3;1)$

$$p_1:\begin{cases} 2y+z-3=0 \\ x-3y+4z=0 \end{cases} \iff \begin{cases} \overset{A_1}{0}\cdot x + \overset{B_1}{2}\cdot y + \overset{C_1}{1}\cdot z = 3 \\ \underset{A_2}{1}\cdot x + \underset{B_2}{(-3)}y + \underset{C_2}{4}\cdot z = 0 \end{cases}$$

$$\vec{p}_1 = (l_1, m_1, n_1)$$

$$l_1 = \begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix} = 2\cdot 4 - (-3)\cdot 1 = 8+3 = 11$$

$$m_1 = \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix} = 1\cdot 1 - 4\cdot 0 = 1$$

$$n_1 = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} = \begin{vmatrix} 0 & 2 \\ 1 & -3 \end{vmatrix} = 0\cdot(-3) - 1\cdot 2 = -2$$

$$\vec{p}_1 = (11, 1, -2)$$

$$p_2: \frac{x+1}{-3} = \frac{y}{2} = \frac{z-4}{0} \qquad \vec{p}_2 = (l_2, m_2, n_2) = (-3; 2, 0)$$

$$0 = \left|\begin{array}{ccc|cc} x-2 & y-3 & z-1 & x-2 & y-3 \\ 11 & 1 & -2 & 11 & 1 \\ -3 & 2 & 0 & -3 & 2 \end{array}\right.$$

$$= \overset{=0}{\cancel{0\cdot(x-2)}} + 6(y-3) + 22(z-1) - (-3)(z-1) - (-4)(x-2)$$
$$- \overset{=0}{\cancel{0\cdot(y-3)}}$$

$$= 4(x-2) + 6(y-3) + 25(z-1)$$

$$= 4x - 8 + 6y - 18 + 25z - 25$$

$$= 4x + 6y + 25z - 51$$

$$\Rightarrow \boxed{4x + 6y + 25z - 51 = 0}$$

⑥ $$\sum_{n=1}^{\infty}\frac{(2x-7)^n}{7n+2}=\sum_{n=1}^{\infty}\frac{\left[2\left(x-\frac{7}{2}\right)\right]^n}{7n+2}=\sum_{n=1}^{\infty}\frac{2^n}{7n+2}\left(x-\frac{7}{2}\right)^n$$

$\left[\sum_{n=0}^{\infty}a_n(x-a)^n\right.$ - општ облик на степенски ред

$a_0=0$

$a_n=\dfrac{2^n}{7n+2},\ n\in\mathbb{N}\ (n\geq 1)$

$a=\dfrac{7}{2}$ - центар на редот

радиус на конвергенција :

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{\frac{2^n}{7n+2}}{\frac{2^{n+1}}{7(n+1)+2}}\right|$$

$$=\lim_{n\to\infty}\frac{\cancel{2^n}(7n+9)}{2^{\cancel{n+1}}(7n+2)}=\lim_{n\to\infty}\frac{7n+9}{2(7n+2)}=\frac{1}{2}\lim_{n\to\infty}\frac{7n+9}{7n+2}$$

$$=\frac{1}{2}\lim_{n\to\infty}\frac{7+\frac{9}{n}\to 0}{7+\frac{2}{n}\to 0}=\frac{1}{2}\cdot\frac{7}{7}=\boxed{\frac{1}{2}}=R$$

интервал на конвергенција :

$$(a-R;a+R)=\left(\frac{7}{2}-\frac{1}{2};\frac{7}{2}+\frac{1}{2}\right)=\left(\frac{6}{2};\frac{8}{2}\right)=(3;4)$$

конвергенција во крајни граници:

за $x=3$ се добива : $\displaystyle\sum_{n=1}^{\infty}\frac{(2\cdot 3-7)^n}{7n+2}=\sum_{n=1}^{\infty}\frac{(-1)^n}{7n+2}$ | $b_n=\dfrac{1}{7n+2}$

за $x=4$ се добива : $\displaystyle\sum_{n=1}^{\infty}\frac{(2\cdot 4-7)^n}{7n+2}=\sum_{n=1}^{\infty}\frac{1}{7n+2}$

За редот $\sum\limits_{n=1}^{\infty} \frac{1}{7n+2}$

$$7n+\overset{=2\cdot 1\le 2\cdot n}{2} \le 7n+2\cdot n = 9n \qquad \forall n\in\mathbb{N}$$

$$\Rightarrow \frac{1}{7n+2} \ge \frac{1}{9n}, \quad \forall n\in\mathbb{N}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{7n+2} \ge \sum_{n=1}^{\infty} \frac{1}{9n} = \frac{1}{9}\underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}_{\substack{\text{Хармониски ред} \\ \text{(кој е дивергентен)}}}$$

$$\Rightarrow \boxed{\sum_{n=1}^{\infty} \frac{1}{7n+2} \quad \text{дивергира}} \qquad (*)$$

За $b_n = \frac{1}{7n+2}$

1) $b_n > 0 \quad \forall n\in\mathbb{N}$

2) $7n+2 < 7(n+1)+2 \Rightarrow \frac{1}{7(n+1)+2} < \frac{1}{7n+2} \quad \forall n\in\mathbb{N}$

$\Rightarrow b_n$ строго опаѓа

3) $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \frac{1}{7n+2}$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}\to 0}{7+\frac{2}{n}\to 0} = \frac{0}{7+0} = 0$$

Од 1)-3) според Лајбницовиот критериум следи дека редот $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{7n+2}$ конвергира но, заради $(*)$, само условно (т.е не и апсолутно)