1/а) I_1

$$\int\left(15x^4+\frac{4}{x}+5^x-\cos x+\overbrace{\frac{1}{x^2+25}}^{I_2}\right)dx$$

$$=15\int x^4dx+4\int\frac{dx}{x}+\int 5^x\,dx-\int\cos x\,dx+\int\frac{dx}{x^2+\underset{=5^2}{25}}$$

$$=15\cdot\frac{x^{4+1}}{4+1}+4\ln|x|+\frac{5^x}{\ln 5}-\sin x+\frac{1}{5}\operatorname{arctg}\frac{x}{5}+C$$

$$=3x^5+4\ln|x|+\frac{5^x}{\ln 5}-\sin x+\frac{1}{5}\operatorname{arctg}\frac{x}{5}+C$$

алтернативи за I_2

- $\int\frac{dx}{x^2-25}=\int\frac{dx}{x^2-5^2}=\frac{1}{25}\ln\left|\frac{x-5}{x+5}\right|$ $=\frac{1}{10}\ln\left|\frac{x-5}{x+5}\right|$
- $\int\frac{dx}{\sqrt{25-x^2}}=\int\frac{dx}{\sqrt{5^2-x^2}}=\arcsin\frac{x}{5}$
- $\int\frac{x}{x^2\pm25}dx=\frac{1}{2}\ln|x^2\pm25|$
- $\int\frac{x}{\sqrt{x^2\pm25}}dx=\sqrt{x^2\pm25}$
- $\int\frac{1}{\sqrt{x^2\pm25}}dx=\ln\left|x+\sqrt{x^2\pm25}\right|$

1/δ $\int x^{n} \underbrace{\ln x}_{u} \, dx$ [illegible]-2

[illegible] 1

$$\int \sqrt[4]{x^3} \ln x \, dx = \int x^{3/4} \ln x \, dx$$

$$= \left| \begin{matrix} \text{Π.Υ.} & \\ u = \ln x & dv = x^{3/4} dx \\ du = \frac{dx}{x} & v = \underline{\int x^{3/4} dx} = \\ & = \frac{x^{3/4+1}}{\frac{3}{4}+1} = \frac{x^{7/4}}{\frac{7}{4}} \\ & = \underbrace{\frac{4}{7} x^{7/4}} \end{matrix} \right|$$

$$= \frac{4}{7} x^{7/4} \ln x - \int \frac{4}{7} x^{\overset{3/4}{\cancel{7/4}}} \cdot \frac{dx}{\cancel{x}}$$

$$= \frac{4}{7} x^{7/4} \ln x - \frac{4}{7} \int x^{3/4} dx$$

$$= \boxed{\frac{4}{7} x^{7/4}} \ln x - \frac{4}{7} \cdot \boxed{\frac{4}{7} x^{7/4}} + C$$

$$= \frac{4}{7} \cdot x \cdot \sqrt[4]{x^3} \left[\ln x - \frac{4}{7} \right] + C$$

[illegible] 2

$$\int \frac{\ln x}{x^5} dx = \int x^{-5} \underbrace{\ln x}_{u} \, dx$$

$$= \left| \begin{matrix} \text{Π.Υ} & \\ u = \ln x & dv = x^{-5} dx \\ du = \frac{dx}{x} & v = \int x^{-5} dx = \frac{x^{-5+1}}{-5+1} = -\frac{1}{4x^4} \end{matrix} \right|$$

$$= -\frac{1}{4x^4}\ln x \textcircled{$-$} \int \left(\textcircled{$-$}\frac{1}{4x^4}\right)\cdot\frac{dx}{x}$$

$$= -\frac{1}{4x^4}\ln x \textcircled{$+$} \frac{1}{4}\int\frac{dx}{x^5}$$

$$= -\frac{1}{4x^4}\ln x + \frac{1}{4}\int x^{-5}dx$$

$$= -\frac{1}{\underline{\underline{4x^4}}}\ln x + \frac{1}{4}\cdot\left(-\frac{1}{\underline{\underline{4x^4}}}\right) + C$$

$$= -\frac{1}{4x^4}\left(\ln x - \frac{1}{4}\right) + C$$

$a>0,\ b\neq 0$

Инт 3 $\int (2x-1)\sin x\,dx$ $\left(\int (ax+b)\sin x\,dx\right)$

общ от одник

$$= \left| \begin{array}{ll} \text{П.И :} & \\ u = 2x-1 & dv = \sin x\,dx \\ du = 2dx & v = \int \sin x\,dx = -\cos x \end{array} \right|$$

$$= (2x-1)\cdot(-\cos x) - \int -\cos x\cdot 2\,dx$$

$$= -(2x-1)\cos x + 2\int\cos x\,dx$$

$$= -(2x-1)\cos x + 2\sin x + C$$

тип 4 :

$$\int (4x+5)\cos x\,dx \qquad \left[\ \overset{\text{општи облик}}{\int (ax+b)\cos x\,dx} \quad a>0,\ b\neq 0\right.$$

$$= \left| \begin{array}{ll} \text{П.И.} & \\ u = 4x+5 & dv = \cos x\,dx \\ du = 4\,dx & v = \int \cos x\,dx = \sin x \end{array} \right|$$

$$= (4x+5)\sin x - \int \sin x \cdot 4\,dx$$

$$= (4x+5)\sin x - 4\underbrace{\int \sin x\,dx}_{-\cos x}$$

$$= (4x+5)\sin x + 4\cos x + C$$

Задача 2 :

$$\int_0^{+\infty} \frac{dx}{(2x+9)^5} = \lim_{\beta\to+\infty} \int_0^{\beta} \frac{dx}{(2x+9)^5}$$

$$= \left| \begin{array}{l} \text{смена} :\ 2x+9=t \\ \quad 2\,dx = dt \quad \Rightarrow \quad dx = \frac{dt}{2} \\ x=0 \quad \Rightarrow \quad t = 2\cdot 0+9 = 9 \\ x=\beta \quad \Rightarrow \quad t = 2\beta+9 \end{array} \right|$$

$$= \lim_{\beta\to+\infty} \int_9^{2\beta+9} \frac{\frac{dt}{2}}{t^5} = \frac{1}{2}\lim_{\beta\to+\infty}\int_9^{2\beta+9} t^{-5}\,dt$$

$$= \frac{1}{2} \lim_{\beta \to +\infty} \boxed{\frac{t^{-5+1}}{-5+1}} \Bigg|_{t=9}^{t=2\beta+9}$$

$$= -\frac{1}{4t^4}$$

$$= \frac{1}{2} \cdot \left(-\frac{1}{4}\right) \lim_{\beta \to +\infty} \frac{1}{t^4} \Bigg|_{t=9}^{t=2\beta+9}$$

$$= - \frac{1}{8} \lim_{\beta \to +\infty} \left[\overset{\to 0}{\cancel{\frac{1}{(2\beta+9)^4}}} - \frac{1}{9^4} \right]$$

$$= \frac{1}{8} \cdot \frac{1}{9^4} \neq \pm\infty \Rightarrow$$ интегралот конвергира

пример 2

$$\int_0^{+\infty} \frac{dx}{\sqrt[4]{(3x+1)^3}} = \int_0^{+\infty} \frac{dx}{(3x+1)^{3/4}}$$

$$= \lim_{\beta \to +\infty} \int_0^{\beta} \frac{dx}{(3x+1)^{3/4}} =$$

смена : $3x + 1 = t$

$3\,\cancel{dx} = dt \Rightarrow dx = \frac{dt}{3}$

$x = 0 \Rightarrow t = 3 \cdot 0 + 1 = 1$

$x = \beta \Rightarrow t = 3\beta + 1$

$$= \lim_{\beta \to +\infty} \int_1^{3\beta+1} \frac{\frac{dt}{3}}{t^{3/4}} = \frac{1}{3} \lim_{\beta \to +\infty} \int_1^{3\beta+1} t^{-3/4}\, dt$$

$$= \frac{1}{3} \lim_{p \to +\infty} \frac{t^{-3/4+1}}{-3/4+1} \Bigg|_{t=1}^{t=3p+1}$$

$$= \frac{1}{3} \lim_{\beta \to +\infty} \frac{t^{1/4}}{\frac{1}{4}} \Bigg|_{t=1}^{t=3\beta+1}$$

за тип 3

ако $\overset{3}{a}\;{}^{4}$

$$-\frac{4}{3}+1$$

$$= -\frac{1}{3}$$

$$\frac{1}{t^{1/3}}$$

$$= \frac{4}{3} \lim_{\beta \to +\infty} t^{1/4} \Big|_{1}^{3\beta+1}$$

$$= \frac{4}{3} \lim_{\beta \to +\infty} \left[\cancel{(3\beta+1)^{1/4}} \xrightarrow{} +\infty - 1^{1/4} \right]$$

$$= \frac{4}{3}(+\infty - 1) = \frac{4}{3} \cdot (+\infty) = +\infty$$

$\Rightarrow$ интегралот дивергира

$$\int_0^{+\infty} \frac{dx}{\sqrt[\boxed{3}]{(3x+1)^{\boxed{4}}}}$$

тип 3

$$\boxed{\int_1^{+\infty} \frac{dx}{x^p}}$$

Во материјали за неправи интегр.

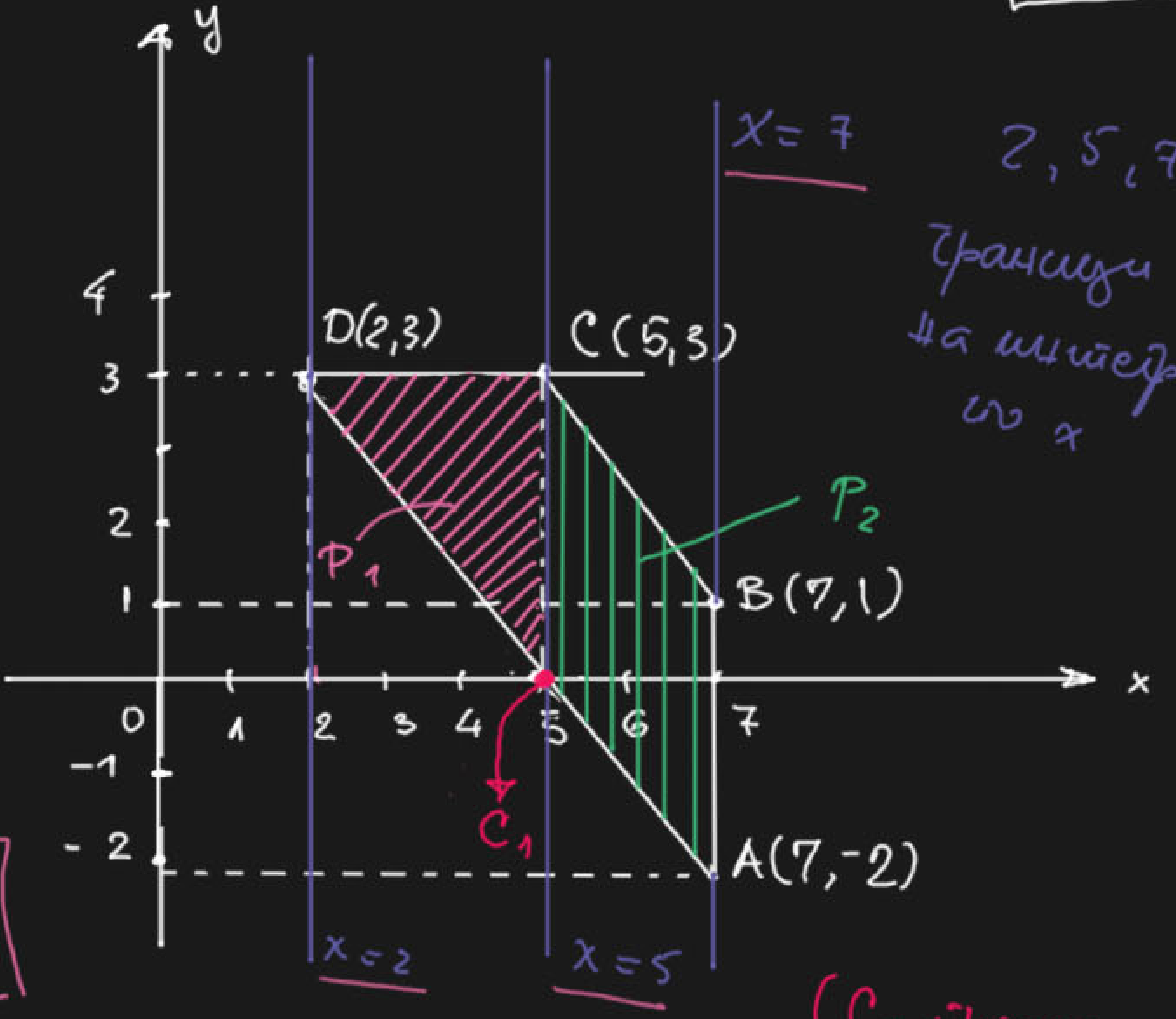
Задача 3 | 2

$A(7,-2)$

$B(7,1)$

$C(5,3)$

$D(2,3)$



2, 5, 7 граници на интеграција по x

интеграција по x

$$\boxed{P = P_1 + P_2}$$

(C₁ пресек на правата x=5 со p_{AD})

$$P_1 = \int_2^5 \left[p_{CD}(x) - p_{AD}(x)\right] dx$$

$$P_2 = \int_5^7 \left[p_{CB}(x) - p_{AD}(x)\right] dx$$

1) $p_{CD}(x) = 3$ - паралелна со x-оска

↪ променлива со која се интегрира

$$\boxed{\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}}$$

$T_1(x_1,y_1)$ $x_1 \neq x_2$

$T_2(x_2,y_2)$ $y_1 \neq y_2$

$$p_{T_1T_2}(x) = y = \frac{y_2-y_1}{x_2-x_1}(x-x_1) + y_1$$

$p_{AD}(\overset{x}{\cancel{y}})$ $A(7,-2)$ $D(2,3)$

x_1, y_1 x_2, y_2

$$p_{AD}(\overset{x}{\cancel{y}}) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)+y_1$$

$$= \frac{3-(-2)}{2-7}(x-7)+(-2)$$

$$= \frac{5}{-5}(x-7)-2 = -x+7-2$$

$$-1 =$$

$$= \boxed{-x+5}$$

$p_{CB}(\overset{x}{\cancel{y}})$: $C(5,3)$ $B(7,1)$

x_1, y_1 x_2, y_2

$$p_{CB}(\overset{x}{\cancel{y}}) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)+y_1$$

$$= \frac{1-3}{7-5}(x-5)+3 = \frac{-2}{2}(x-5)+3$$

$$= -(x-5)+3 = \boxed{-x+8}$$

$$P_1 = \int_2^5 \left[3 - (-x+5)\right] dx$$

$$= \int_2^5 (x-2)dx = \left(\frac{x^2}{2} - 2x\right)\Bigg|_{x=2}^{x=5}$$

$$= \left(\frac{25}{2} - 10\right) - \left(\frac{4}{2} - 4\right) = 12,5 - 10 - 2 + 4$$

$$= \underbrace{4,5}$$

$$P_2 = \int_5^7 \left[(-\cancel{x}+8) - (\cancel{-x}+5)\right] dx$$

$$= \int_5^7 3\,dx = 3x\Big|_{x=5}^{x=7} = 3(7-5) = 6$$

$$P = P_1 + P_2 = 4,5 + 6 = 10,5 \text{ кв. ед.}$$