T1/1

$$Z = \frac{1+2i}{1-3i} = a+bi$$

$$i^2 = -1$$

$$r = \sqrt{a^2+b^2} \;;\; \cos\theta = \frac{a}{r} \;;\; \sin\theta = \frac{b}{r}$$

$a = ?,\ b = ?$

$$Z = \frac{1+2i}{1-3i} \cdot \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2-3^2\cdot i^2} = \frac{1+5i-6}{1-9\cdot(-1)}$$

$$= \frac{-5+5i}{10} = -\frac{5}{10} + \frac{5}{10}i = -\frac{1}{2} + \frac{1}{2}i$$

$$\Rightarrow a = -\frac{1}{2} \quad b = \frac{1}{2};$$

1) $r = \sqrt{a^2+b^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}}$

$= \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$ $\qquad \sqrt[3]{r} = \sqrt[3]{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt[3]{\sqrt{2}}} = \frac{1}{\sqrt[6]{2}}$

2) $\cos\theta = \frac{a}{r} = \frac{-\frac{1}{2}}{\frac{1}{\sqrt{2}}} = -\frac{\sqrt{2}}{2}$

$\sin\theta = \frac{b}{r} = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2}$

$\xRightarrow{\text{Таблица}} \boxed{\theta = \frac{3\pi}{4}}$

$$Z = \frac{1}{\sqrt[6]{2}}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$w_k = \sqrt[n]{r}\left(\cos\frac{2k\pi+\theta}{n} + i\sin\frac{2k\pi+\theta}{n}\right)$$

$(n=3) \Rightarrow k = 0, 1, 2$

$\underline{k=0}$ $\quad \dfrac{2\cdot 0\cdot\pi + \frac{3\pi}{4}}{3} = \dfrac{0+\frac{3\pi}{4}}{3} = \dfrac{3\pi}{12} = \dfrac{\pi}{4}$

$$w_0 = \frac{1}{\sqrt[6]{2}}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$\underline{k=1}$ $\quad \dfrac{2\cdot 1\cdot\pi + \frac{3\pi}{4}}{3} = \dfrac{2\pi+\frac{3\pi}{4}}{3} = \dfrac{\frac{11\pi}{4}}{3} = \dfrac{11\pi}{12}$

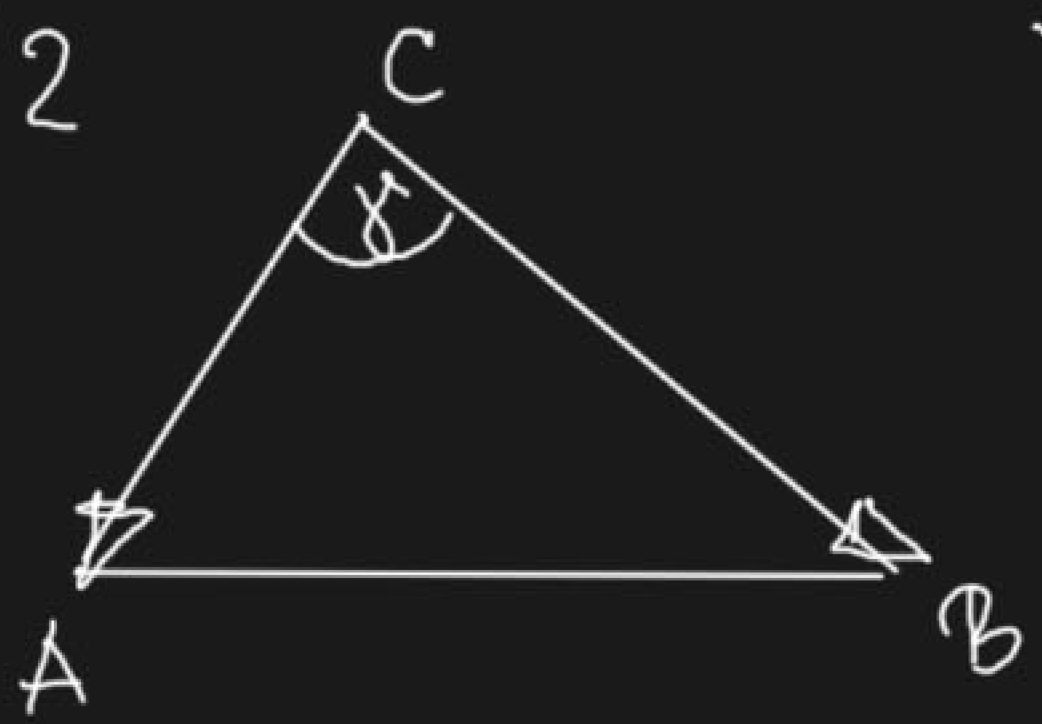
$$w_1 = \frac{1}{\sqrt[6]{2}}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$$

$\underline{k=2}$ $\quad \dfrac{2\cdot 2\pi + \frac{3\pi}{4}}{3} = \dfrac{19\pi}{12} \notin (-\pi, \pi)$ и $\dfrac{19\pi}{12} > \pi$

бидејќи sin и cos се 2π - периодични се зема $\dfrac{19\pi}{12} - 2\pi = -\dfrac{5\pi}{12}$ на местото од $\dfrac{19\pi}{12}$

$$w_2 = \frac{1}{\sqrt[6]{2}}\left(\cos\left(-\frac{5\pi}{12}\right) + i\sin\left(-\frac{5\pi}{12}\right)\right)$$

T1/2



$$\gamma = \angle(\vec{CA}, \vec{CB})$$

$$P_{\Delta ABC} = \frac{1}{2}\left|\vec{CA} \times \vec{CB}\right|$$

$$\cos\gamma = \frac{\vec{CA}\cdot\vec{CB}}{|\vec{CA}|\cdot|\vec{CB}|}$$

$$\vec{CA} = \vec{r}_A - \vec{r}_C = (8,3,4) - (6,3,2) = (2,0,2)$$
$$\vec{CB} = \vec{r}_B - \vec{r}_C = (7,4,2) - (6,3,2) = (1,1,0)$$

$P_{\Delta ABC} = ?$

$$\vec{CA}\times\vec{CB} = \left|\begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 2 \\ 1 & 1 & 0 \end{array}\right| \begin{array}{cc} \vec{i} & \vec{j} \\ 2 & 0 \\ 1 & 1 \end{array}$$

$$= 0\cdot\vec{i} + 2\vec{j} + 2\vec{k} - 0\cdot\vec{k} - 2\vec{i} - 0\cdot\vec{j}$$

$$= -2\vec{i} + 2\vec{j} + 2\vec{k} = (-2, 2, 2)$$

$$P_{\Delta ABC} = \frac{1}{2}\sqrt{(-2)^2 + 2^2 + 2^2} = \frac{1}{2}\sqrt{4+4+4}$$

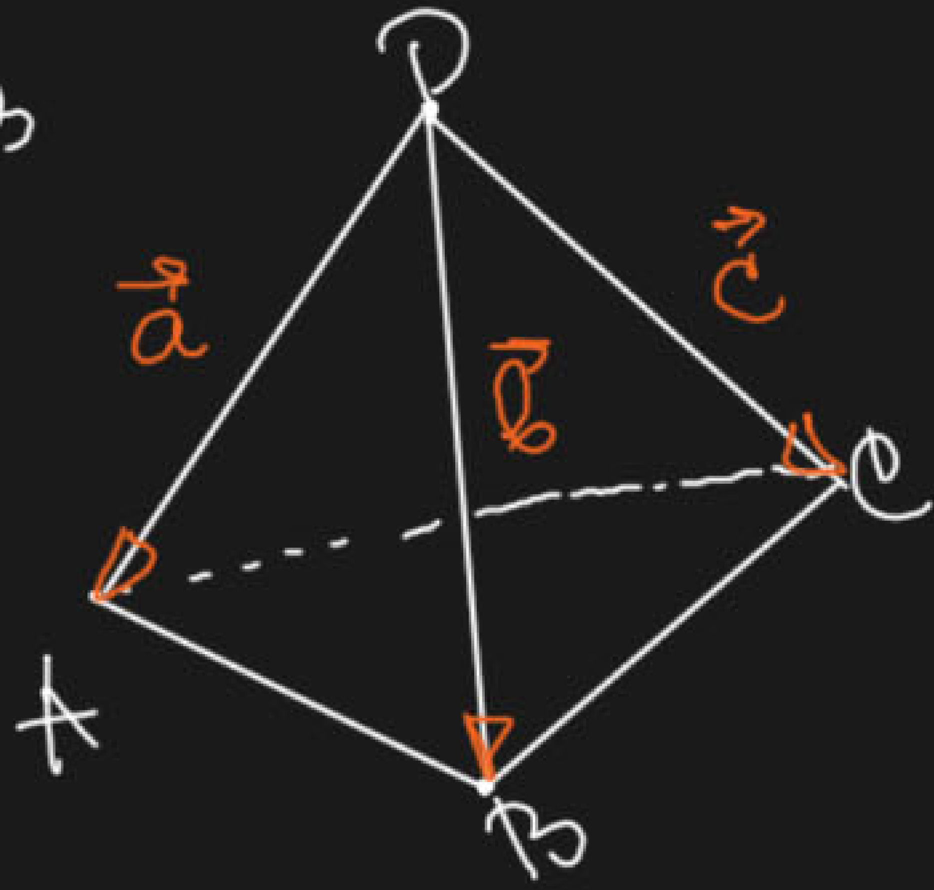
$$= \frac{1}{2}\sqrt{\underbrace{12}_{=4\cdot 3}} = \frac{1}{2}\sqrt{4}\cdot\sqrt{3} = \frac{1}{\cancel{2}}\cancel{2}\cdot\sqrt{3} = \sqrt{3}$$

кв.ед

$$\cos\gamma = \frac{2\cdot 1 + 0\cdot 1 + 2\cdot 0}{\sqrt{2^2+0^2+2^2}\cdot\sqrt{1^2+1^2+0^2}} = \frac{2+0+0}{\sqrt{4+0+4}\cdot\sqrt{1+1+0}}$$

$$= \frac{2}{\sqrt{8}\cdot\sqrt{2}} = \frac{2}{\sqrt{8\cdot 2}} = \frac{2}{\sqrt{16}} = \frac{2}{4} = \boxed{\frac{1}{2}} \Rightarrow \gamma = 60^\circ$$
$$= \pi/3 \text{ rad}$$

Т1/3



$$V = \frac{1}{6}\left|(\vec{a}\vec{b}\vec{c})\right|$$ — апсол. вред.

$\vec{a}, \vec{b}$ и $\vec{c}$ се в-ри со почеток во иста точка теме на тетраедарот

$$\vec{a} = \vec{DA} = \vec{r}_A - \vec{r}_D = (1,2,2) - (9,2,6) = (-8, 0, -4)$$

$$\vec{b} = \vec{DB} = \vec{r}_B - \vec{r}_D = (8,1,3) - (9,2,6) = (-1,-1,-3)$$

$$\vec{c} = \vec{DC} = \vec{r}_C - \vec{r}_D = (5,0,2) - (9,2,6) = (-4,-2,-4)$$

$$(\vec{a}\vec{b}\vec{c}) = \begin{vmatrix} -8 & 0 & -4 \\ -1 & -1 & -3 \\ -4 & -2 & -4 \end{vmatrix} \begin{matrix} -8 & 0 \\ -1 & -1 \\ -4 & -2 \end{matrix}$$

$$= -32 + 0 - 8 - (-16) - (-48) - 0 = 24$$

$$V = \frac{1}{6}|24| = \frac{1}{6}24 = 4 \text{ куб. ед.}$$

T1/5 $\sum\limits_{n=1}^{\infty} a_n(x-a)^n$ $\underset{a}{\longrightarrow}$ центар на конвергенција

$$\sum_{n=1}^{\infty}\frac{(3x+5)^n}{5n+3}=\sum_{n=1}^{\infty}\frac{\left[3\left(x+\frac{5}{3}\right)\right]^n}{5n+3}$$

$$=\sum_{n=1}^{\infty}\frac{3^n}{5n+3}\cdot\left(x+\frac{5}{3}\right)^n$$

со спротивен знак

$$a_n=\frac{3^n}{5n+3}\qquad n=1,2,\dots;\quad \boxed{a=-\frac{5}{3}}$$

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{\frac{3^n}{5n+3}}{\frac{3^{n+1}}{5(n+1)+3}}\right|$$

$$=\lim_{n\to\infty}\frac{\cancel{3^n}(5n+8)}{3^{\cancel{n+1}}(5n+3)}=\frac{1}{3}\lim_{n\to\infty}\frac{5n+8}{5n+3}$$

$$=\frac{1}{3}\lim_{n\to\infty}\frac{5+\cancel{\frac{8}{n}}\to 0}{5+\cancel{\frac{3}{n}}\to 0}=\frac{1}{3}\cdot\frac{5}{5}=\boxed{\frac{1}{3}=R}$$

интервал на конвер. $(a-R,\ a+R)$

$$=\left(-\frac{5}{3}-\frac{1}{3},\ -\frac{5}{3}+\frac{1}{3}\right)=\left(-\frac{6}{3},\ -\frac{4}{3}\right)=\left(-2;\ -\frac{4}{3}\right)$$

$x = -2$

$$\sum_{n=1}^{\infty} \frac{(3\cdot(-2)+5)^n}{5n+3} = \sum_{n=1}^{\infty} \frac{(-6+5)^n}{5n+1} =$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{5n+3} = \sum_{n=1}^{\infty} (-1)^n b_n$$

каде $b_n = \frac{1}{5n+3}$ $\begin{cases} 1.\ \text{строго опаѓа !} \\ 2.\ \text{тежи кон } 0 \end{cases}$

$x = -\frac{4}{3}$

$$\sum_{n=1}^{\infty} \frac{\left(\cancel{3}\cdot\left(-\frac{4}{\cancel{3}}\right)+5\right)^n}{5n+3} = \sum \frac{\overbrace{(-4+5)^n}^{=1^n=1}}{5n+3}$$

$$= \sum_{n=1}^{\infty} \frac{1}{5n+3}$$

→ ако овој ред конвергира ќе конвергира и горниот (и тоа апсолутно

$$5n+3 \le 5n+3n = 8n$$

$3 = 3\cdot 1 \quad n \ge 1$

$$\Rightarrow \frac{1}{5n+3} \ge \frac{1}{8n} \Rightarrow \boxed{\sum_{n=1}^{\infty} \frac{1}{5n+3}} \ge \sum_{n=1}^{\infty} \frac{1}{8n} = \frac{1}{8} \boxed{\sum_{n=1}^{\infty} \frac{1}{n}}$$

Хармониски ред кој е дивергентен дополнително

редот $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+3}$ не може да конвергира апсолутно, но (бидејќи $b_n = \frac{1}{5n+3}$ е строго опаѓачка низа која конвергира кон 0) конвергира условно.