

Ant Colony Optimization Approach for Optimizing Traffic Signal Timings

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1. Introduction

In urban networks, traffic signals are used to control vehicle movements so as to reduce congestion, improve safety, and enable specific strategies such as minimizing delays, improving environmental pollution, etc (Teklu et al., 2007). Due to the increasing in the number of cars and developing industry, finding optimal traffic signal parameters has been an important task in order to use the network capacity optimally. Through the last decade, developments in communications and information technologies have improved the classical methods for optimising the traffic signal timings toward the intelligent ones.

There is an important interaction between the signal timings and the routes chosen by individual road users in road networks controlled by fixed time signals. The mutual interaction leads to the framework of a leader-follower or Stackelberg game, where the supplier is the leader and the user is the follower (Fisk, 1984). Network design problem (NDP) that it may contain the signal setting problem is characterized by the so called bi-level structure. Bi-level programming problems generally are difficult to solve, because the evaluation of the upper-level objective involves solving the lower level problem for every feasible set of upper level decisions (Sun et al., 2006). On the upper level, a transport planner designs the network. Road users respond to that design in the lower level. This problem is known to be one of the most attractive mathematical problems in the optimization field because of non-convexity of feasible region that it has multiple local optima (Baskan, 2009).

Moreover, the driver's behaviours on the network should be taken into account when the traffic signal timings are optimised. When drivers follow the Wardrop's (1952) first principle, the problem is called the "user equilibrium" (UE). On the other hand, it turns to the stochastic user equilibrium (SUE) in the case that the users' face with the decision of route choice between the each Origin-Destination (O-D) pair for a given road network according to perceived travel time. The difference between SUE and UE approaches is that in SUE models each driver is meant to define 'travel costs' individually instead of using a single definition of costs applicable to all drivers. SUE traffic assignment takes into account the variability in driver's perception of cost. This is done by treating the perceived cost on

particular path as a random variable distributed across the population of users, and so a different cost for each driver can be modelled. Using the probabilistic choice models, the O-D demand is assigned to paths, where the cheapest path attracts the most flow. Road users generally use a variety of routes between their origins and destinations in urban networks based on their perception of travel time. Hence, the SUE is more appropriate than the DUE assignment (Ceylan, 2002).

A wide range of solution methods to the signal setting problem have been discussed in the literature. Allsop & Charlesworth (1977) found mutually consistent (MC) traffic signal settings and traffic assignment for a medium size road network. In their study, the signal setting and link flows were calculated alternatively by solving the signal setting problem for assumed link flows and carrying out the user equilibrium assignment for the resulting signal settings until convergence was achieved. The obtained mutually consistent signal settings and equilibrium link flows, will, however, in general be non-optimal as has been discussed by Gershwin & Tan (1979) and Dickson (1981). Abdullaal & LeBlanc (1979) reported the formulation and solution by means of the Hooke-Jeeves' method for an equilibrium network design problem with continuous variables. An unconstrained optimisation problem in which the dependence of equilibrium flows on decision variables was dealt with as one of the decision variables is solved directly by means of the convex-combination method. Suwansirikul et al. (1987) solved equilibrium network design problem that using a direct search based on the Hooke-Jeeves' method for a small test network. The direct search method, however, is computationally intensive, because frequent evaluations of deterministic (or stochastic) user equilibrium traffic assignment are required. Hence, it was emphasized that the proposed method is only suitable for small example road networks. Heydecker & Khoo (1990) proposed a linear constraint approximation to the equilibrium flows with respect to signal setting variables and solved the bi-level problem as a constraint optimisation problem. Using the linear constraint approximation method to solve the bi level problem can be carried out in a number of iterations by which the resulting equilibrium flows are regressed as the signal setting variable changes in a simple linear form.

Canteralla et al. (1991) proposed an iterative approach to solve the equilibrium network design problem, in which traffic signal settings are performed in two successive steps; green timing at each junction, and signal co-ordination on the network. Green timing calculation at each junction was based on a mixed binary linear program. Signal coordination for the whole network was performed by solving a discrete programming model with a total delay minimisation objective function. Yang & Yagar (1995) used derivatives of equilibrium flows and of the corresponding travel times to solve a bi-level program for the equilibrium network design problem for a signal control optimisation. Lee & Machemehl (1998) applied Genetic Algorithm (GA) to individual signalized intersection. Their objective function was a function of green split and the UE links flows. For stage length and cycle time optimization without considering offsets to minimise total travel time, Lee (1998) presented a comparison of GA and simulated annealing with iterative and local search algorithms and showed that different algorithms perform better for different network supply and demand scenarios. Chiou (1999) explored a mixed search procedure to solve an area traffic control optimization problem confined to equilibrium network flows, where good local optima can be effectively found via the gradient projection method. Ceylan & Bell (2004) is proposed GA approach to solve traffic signal control and traffic assignment problem is used to tackle the optimization of signal timings with SUE link flows. The system performance index is defined as the sum

of a weighted linear combination of delay and number of stops per unit time for all traffic streams. It is found that the GA method is effective and simple to solve equilibrium network design problem.

The optimization methods developed so far to solve signal setting problem are either calculus and mathematically lengthy or are based on the heuristic approaches. Although proposed algorithms are capable of solving signal setting problem for a road network, an efficient algorithm, which is capable of finding the global or near global optima of the upper level signal timing variables considering the equilibrium link flows under SUE conditions, is still needed. This chapter deals with the problem of optimising traffic signal timings without considering offset term taking stochastic user equilibrium (SUE) conditions into account. A bi-level technique and MC approach have been proposed and compared, in which signal setting problem is dealt with as upper level problem whilst the SUE traffic assignment is dealt with as lower level problem. In this chapter, Ant Colony Optimization (ACO) approach which is arisen from the behaviours of real ant colonies is introduced to solve the upper level problem in which traffic signal timings are optimised. Although ACO algorithms are capable of finding global or near global optimum, it may be further improved to locate better global optima for any given problem. In this study, ACO Reduced Search Space (ACORSES) algorithm is used for finding better signal timings. It differs from other approaches in that its feasible search space (FSS) is reduced with best solution obtained so far using the previous information at the each iteration (Baskan et al., 2009). At the core of ACORSES, ants search randomly the solution within the FSS to reach global optimum by jumping on each direction. At the end of the each iteration, only one ant is near to global optimum. After the first iteration, when global optimum is searched around the best solution of the previous iteration using reduced search space, the ACORSES will reach to the global optimum quickly without being trapped in bad local optimum.

In this chapter, signal timings are defined as cycle time and green time for each junction and stage, respectively. The objective function is adopted to minimise the total system cost of network as the system optimum formulation. In order to represent the route choice behaviours of drivers, the probit route choice model is used whilst SUE traffic assignment problem is solved by the method that proposed Sheffi (1985). The effectiveness of the proposed ACORSES algorithm is demonstrated through a numerical experiment. The results showed that the bi-level approach based on ACORSES is considerably effective according to MC approach in terms of the signal timings and the final values of degree of saturation to solve the problem of optimising traffic signal timings under SUE conditions. This chapter is organized as follows. The basic notations are defined in the next section. Section 3 is about the problem formulation. The solution methods for optimising signal timings under SUE conditions are given in Section 4. Numerical experiment is carried out in Section 5. Last section is about the conclusions.

2. Notations

A	matrix of signal timings
c	cycle time for each junction of a given road network
$\mathbf{c}(\mathbf{q}, \boldsymbol{\psi}) = [c_a(q_a, \boldsymbol{\psi})]$	the vector of all link travel times, where element $c_a(q_a, \boldsymbol{\psi})$ is travel time on link a as a function of flow on the link itself and the signal setting variables, $\boldsymbol{\psi} = (c, \boldsymbol{\phi})$.
C_a	perceived travel time on link a

c_a	measured travel time on link a
$\mathbf{h}=[h_p; \forall p \in \mathbf{P}_w, \forall w \in \mathbf{W}]$	vector of all path flows
I_i	intergreen time between signal stages
J	number of junctions
K	threshold value
L	number of links
M	numbers of signal stages at a signalised road network
m	numbers of signal stages for a signalised junction, $\forall m \in M$
R	colony size
s_a	degree of saturation on link a
$\mathbf{q}=[q_a; \forall a \in \mathbf{L}]$	vector of flow q_a on link a
$\mathbf{q}^*(\psi)$	vector of equilibrium link flows subject to signal parameters
TC	total cost of a given road network
$\mathbf{t}=[t_w; \forall w \in \mathbf{W}]$	vector of origin destination flows
ψ	signal setting variables
ϕ	vector of duration of green times
Ω	feasible set of signal setting variables
δ	link-path incidence matrix
Λ	OD-path incidence matrix
β	vector of search space constraints at ACO
β	variance of the perceived travel time
σ_a	standard error for each link a
α	random generated vector as initial solution for ACO

3. Formulation

The problem of optimising of signal setting variables $\psi = (c, \phi)$ without considering offset term on a road network is defined as bi-level structure. The planners aim to minimise the total cost (TC) of a given road network on the upper level whilst the SUE link flows $\mathbf{q}^*(\psi)$ on the lower level are dealt with altering signal timings. The objective function is therefore to minimise TC with respect to equilibrium link flows $\mathbf{q}^*(\psi)$ subject to signal setting constraints $\psi = (c, \phi)$. Mathematically the problem is defined as:

$$\underset{\psi \in \Omega_0}{Min} \quad TC(\psi, \mathbf{q}^*(\psi)) = \sum_a^L q_a t_a(\psi, \mathbf{q}^*(\psi)) \quad (1)$$

$$\text{subject to } \psi = (c, \phi) \in \Omega ; \left\{ \begin{array}{ll} c_{\min} \leq c \leq c_{\max} & \text{cycle time constraints for each junction} \\ \phi_{\min} \leq \phi \leq \phi_{\max} & \text{green time constraints for each stage} \\ \sum_{i=1}^m (\phi_i + I_i) = c & \forall m \in M \end{array} \right\}$$

where $\mathbf{q}^*(\psi)$ is implicitly defined by

$$\underset{\mathbf{q}}{\text{Minimise}} \quad Z(\psi, \mathbf{q})$$

$$\text{subject to} \quad \mathbf{t} = \Lambda \mathbf{h}, \quad \mathbf{q} = \delta \mathbf{h}, \quad \mathbf{h} \geq 0$$

3.1 ACORSES algorithm for optimising of signal timings (upper-level problem)

Ant algorithms were inspired by the observation of real ant colonies. Ants are social insects, that is, insects that live in colonies and whose behaviour is directed more to the survival of the colony as a whole than to that of a single individual component of the colony. Social insects have captured the attention of many scientists because of the high structuration level their colonies can achieve, especially when compared to the relative simplicity of the colony's individuals. An important and interesting behaviour of ant colonies is their foraging behaviour, and, in particular, how ants can find shortest paths between food sources and their nest (Dorigo et al., 1999). Ants are capable of finding the shortest path from food source to their nest or vice versa by smelling pheromones which are chemical substances they leave on the ground while walking. Each ant probabilistically prefers to follow a direction rich in pheromone. This behaviour of real ants can be used to explain how they can find a shortest path (Eshghi & Kasemi, 2006).

The ACO is the one of the most recent techniques for approximate optimization methods. The main idea is that it is indirect local communication among the individuals of a population of artificial ants. The core of ant's behavior is the communication between the ants by means of chemical pheromone trails, which enables them to find shortest paths between their nest and food sources. This behaviour of real ant colonies is exploited to solve optimization problems. The general ACO algorithm is illustrated in Fig. 1. The first step consists mainly on the initialization of the pheromone trail. At beginning, each ant builds a complete solution to the problem according to a probabilistic state transition rules. They depend mainly on the state of the pheromone.

Step 1:	Initialize Pheromone trail
Step 2:	Iteration Repeat for each ant Solution construction using pheromone trail Update the pheromone trail Until stopping criteria

Fig. 1. A generic ant algorithm

Once all ants generate a solution, then global pheromone updating rule is applied in two phases; an evaporation phase, where a fraction of the pheromone evaporates, and a reinforcement phase, where each ant deposits an amount of pheromone which is proportional to the fitness. This process is repeated until stopping criteria is met. In this study, ACORSES algorithm proposed by Baskan et al. (2009) to solve upper-level problem is used to tackle the optimization of signal timings with stochastic equilibrium link flows. The ACORSES algorithm is based on each ant searches only around the best solution of the

previous iteration with reduced search space. It is proposed for improving ACO's solution performance to reach global optimum fairly quickly. The ACORSES is consisted of three main phases; Initialization, pheromone update and solution phase. All of these phases build a complete search to the global optimum. At the beginning of the first iteration, all ants search randomly to the best solution of a given problem within the Feasible Search Space (FSS), and old ant colony is created at initialization phase. After that, quantity of pheromone is updated. In the solution phase, new ant colony is created based on the best solution from the old ant colony. Then, the best solutions of two colonies are compared. At the end of the first iteration, FSS is reduced by β and best solution obtained from the previous iteration is kept. β guides the bounds of search space during the ACORSES application, where β is a vector, β_j ($j=1,2,\dots,n$), and n is the number of variables. The range of the β may be chosen between minimum and maximum bounds of any given problem. Optimum solution is then searched in the reduced search space during the algorithm progress. The ACORSES reaches to the global optimum as ants find their routes in the limited space.

Let the objective function given in Eq. (1) take a set of ψ signal timing variables, $\psi = (c_1, \varphi_1, \dots, c_n, \varphi_n)$. On the assumption that each decision variable ψ can take values from a domain $\Omega = [\psi_{\min}, \psi_{\max}]$ for all $\psi \in \Omega$. The presentation of signal timing variables, $\psi = (c, \varphi)$, based on ACO approach is given in Eq. (2).

$$A = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \dots & c_{1J} & \varphi_{11} & \varphi_{12} & \varphi_{13} & \dots & \varphi_{1M} \\ c_{21} & c_{22} & c_{23} & \dots & c_{2J} & \varphi_{21} & \varphi_{22} & \varphi_{23} & \dots & \varphi_{2M} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_{R1} & c_{R2} & c_{R3} & \dots & c_{RJ} & \varphi_{R1} & \varphi_{R2} & \varphi_{R3} & \dots & \varphi_{RM} \end{bmatrix} \quad (2)$$

where M , J and R are the number of signal stages, the number of junctions at a signalised road network and the value of colony size, respectively. In order to optimise the objective function in ACO real numbers of decision variables are used instead of coding them as in GA. This is one of the main advantage of ACO approach that it provides to optimise the signal timings at a road network with less mathematically lengthy. Moreover, ACORSES algorithm has ability to reach to the global optimum quickly without being trapped in bad local optimum because it uses the reduced search space and the values of optimum signal timings are then searched in the reduced search space during the algorithm progress. The ACORSES reaches to the global optimum or near global optimum as ants find their routes in the limited space. In that algorithm, A consists of the cycle and green timings for each junction and stage at a given road network. In order to provide the constraint of cycle time for each junction, the green timings can be distributed to the all signal stages in a road network as follows (Ceylan & Bell, 2004):

$$\varphi_i = \varphi_{\min,i} + \frac{\varphi_i}{\sum_{k=1}^m \varphi_i} \left(c_i - \sum_{k=1}^m I_k - \sum_{k=1}^m \varphi_{\min,k} \right) \quad i=1,2,\dots,m \quad (3)$$

3.2 The solution method for the lower level problem

In this study, probit stochastic user equilibrium (PSUE) model is used to solve lower level problem. Probit model has the advantage of being able to represent perceptual differences in utility of alternatives on a road network. Although it behaves like the logit model for statistically independent paths, it has many advantages in the case of existing correlated paths at a network. For example, two paths that overlap for virtually their whole length are likely to be perceived very similarly by an individual since they have many links in common, but this cannot be captured by a logit model. This can lead to unrealistically high flows being assigned to the common parts of these paths of a given road network. The PSUE model is able to overcome these drawbacks, by supposing path costs are formed from a sum of link costs, with the error distribution (Clark & Watling, 2002). Therefore, PSUE model proposed by Sheffi (1985) is used to find path choice probabilities in order to overcome this drawback. To the best of our knowledge, this is the first time to date that the PSUE model is used to solve SUE traffic assignment problem at the lower level whilst the optimization of signal timings is dealt with upper level problem.

Monte-Carlo simulation method is adopted to obtain probit choice probabilities so as to none of the analytical approximation methods can be practically applied to medium or large scale networks due to probit choice probabilities cannot be written in closed form. The advantage of the simulation method is that it does not require the sampling of perceived path travel times only perceived link travel times are sampled at every iteration thus avoiding path enumeration. The underlying assumption of probit model, the random error term of each alternative is assumed normally distributed. The notation $\xi \sim \text{MVN}(\mu, \Sigma)$ indicates that the vector of error terms ξ is multivariate normal (MVN) distributed with mean vector μ and covariance matrix Σ . The algorithm to solve PSUE is given as follows (Sheffi, 1985):

Step 0. Initialization. Set $n=1$

Step 1. Sampling phase. Sample $C_a^{(n)}$ from $C_a \sim N(c_a, \beta c_a)$ for each link a .

Step 2. Based on $C_a^{(n)}$, assign t_w to the shortest path connecting each O-D pair w . Obtain the set of link flows, $q_a^{(n)}$

Step 3. Link flow averaging. Let $q_a^{(n)} = [(n-1)q_a^{(n-1)} + q_a^{(n)}] / n \quad \forall a$

Step 4. Stopping criterion (standard error). Let $\sigma_a^{(n)} = \sqrt{\frac{1}{n(n-1)} \sum_{m=1}^n (q_a^{(m)} - q_a^{(n)})^2} \quad \forall a$

If $\max_a \left\{ \frac{\sigma_a^{(n)}}{q_a^{(n)}} \right\} \leq K$, stop. The solution is $q_a^{(n)}$. Otherwise, set $n=n+1$ and go to step 1.

where K is the predetermined threshold value. In PSUE algorithm, perceived link travel time for each link is random variable that is assumed to be normally distributed with mean equal to the measured link travel time and with variance of related link. According to this assumption, perceived link travel times for each link are sampled. Then, demand between each O-D pair $w \in W$ assigned to the shortest path to obtain set of link flows, $a \in L$. The link flows have to be averaged at every iteration in order to compute the standard error to test for convergence of algorithm. The PSUE algorithm is terminated when the stopping criterion is met.

4. Solution methods

4.1 The Mutually Consistent (MC) solution for optimising signal timings

The MC solution process was proposed by Allsop (1974) and Gartner (1974). It is an iterative optimisation and assignment procedure that is very similar in nature to other iterative procedures in the literature. In this method, the upper level problem is solved by keeping the flows fixed, and then the traffic assignment problem is solved by keeping the signal timings fixed. MC calculation of signal timings and the corresponding equilibrium flows for bi-level problem starts with an initial assignment and tries to reach a MC solution by solving sequentially an equilibrium assignment problem and a traffic signal setting problem until two successive flow patterns or signal timings are close enough within a specified tolerance. If convergence is reached in a finite number of iterations, the solution is considered to be mutually consistent. This means that the signal settings generate a set of link costs which determine flow pattern such that these settings are optimal for it (Ceylan, 2002). The MC calculation is performed in the following steps.

- Step 0. Set $k=0$ for given signal timings $\psi^{(k)}$, find the corresponding equilibrium flows $q^*(\psi^{(k)})$ by solving the problem of PSUE at the lower level.
- Step 1. Solve the upper level problem to obtain the optimal signal timings $\psi^{(k+1)}$ for the flows $q^*(\psi^{(k)})$.
- Step 2. Update the travel time function of all links according to obtained signal timings $\psi^{(k+1)}$.
- Step 3. Calculate the corresponding equilibrium flows $q^*(\psi^{(k+1)})$ by solving the PSUE problem.

At Step 3, method of successive averages (MSA) (Sheffi, 1985) smoothing is applied to the equilibrium link flows in the iterative process in order to overcome fluctuations on equilibrium link flows. The MSA smoothing approach is carried out using the following relationship.

$$q_a^{(k+1)} = \frac{1}{k} q_a^{(k-1)} + \left(1 - \frac{1}{k}\right) q_a^{(k)}$$

where k is the iteration number and a is a set of links in L

- Step 4. Solve the upper level problem again to obtain the optimal signal timings $\psi^{(k+2)}$ given by the $q^*(\psi^{(k+1)})$.
- Step 5. Compare the values of $\psi^{(k+2)}$ and $\psi^{(k+1)}$, if there is no change between $\psi^{(k+1)}$ and $\psi^{(k+2)}$ then go to Step 6; otherwise, $k=k+1$ and go to Step 2.
- Step 6. Stop: $\psi^{(k+1)}$ and $q^*(\psi^{(k+1)})$ are the mutually consistent signal timings and equilibrium flows.

The MC calculation process is terminated when the difference in values of signal timings or of PSUE flows between successive iterations is smaller than a predetermined threshold value.

4.2 The bi-level solution for optimising signal timings

The problem of optimizing signal timings can be seen as a particular case of the NDP in which the signal timings play a critical role to optimize the performance of the network while the network topological characteristics are fixed. It is also a difficult problem because the evaluation of the upper level objective involves solving the lower level problem for every feasible set of upper level decisions. The complexity of the problem comes from the non-convexity of objective functions and constraints at both levels. In this chapter, the ACORSES algorithm is used to overcome this drawback. Although ACO algorithms are similar to other stochastic optimization techniques, the ACORSES differs from that it uses reduced search space technique to prevent being trapped in bad local optimum (Baskan et al., 2009). The solution steps for the bi-level solution are:

- Step 0. *Initialisation*, $k=1$. Set the user-specified ACO parameters (β , R , α); represent the decision variables Ψ within the range Ψ_{\min} and Ψ_{\max} .
- Step 1. If $k=1$ generate the initial random population of signal timings Ψ_k as shown in (2). Else generate random population of signal timings according to best signal timings obtained in Step 8 and β vector of search space constraints. This step plays a critical role because the ACORSES uses the random generated population within the reduced search space at each iteration to prevent being trapped in bad local optimum.
- Step 2. Solve the lower level problem by solving the PSUE. This gives an equilibrium link flows for each link a in L .
- Step 3. Calculate the value of objective function using Eq. (1) in order to obtain old ant colony (A_{old}) in which signal timings are presented, for resulting signal timings at Step 1 and the equilibrium link flows resulting in Step 2.
- Step 4. Carry out the pheromone evaporation and updating phases.
- Step 5. Determine the search direction and generate the matrix of length of jump.
- Step 6. According to search direction and generated matrix of length of jump, produce the new ant colony (A_{new}).
- Step 7. Calculate the new values of objective function using produced new ant colony.
- Step 8. Compare the values of objective functions relating to the old and new ant colonies, then we have best decision variables, Ψ_{best} .
- Step 9. If the difference between the values of Ψ_k and Ψ_{k+1} is less than predetermined threshold value, the algorithm is terminated. Else go to Step 1.

5. Numerical example

The test network is chosen that is used by Allsop & Charlesworth (1977) in order to show the performance of the ACORSES to optimise signal setting variables. The network topology and stage configurations are given in Fig. 2a and 2b, where figures are adapted from Ceylan & Bell (2004). Travel demands for each O-D are given in Table 1. This network has 20 O-D pairs and 20 signal setting variables at six signal-controlled junctions. The signal timing constraints are given as follows:

$$c_{\min}, c_{\max} = 60, 100 \text{ sec} \quad \text{cycle time for each junction}$$

$$\phi_{\min} = 7 \text{ sec} \quad \text{minimum green time for signal stages}$$

Origin/Destination	A	B	D	E	F	Origin totals
A	--	250	700	30	200	1180
C	40	20	200	130	900	1290
D	400	250	--	50*	100	800
E	300	130	30*	--	20	480
G	550	450	170	60	20	1250
Destination totals	1290	1100	1100	270	1240	5000

* where the travel demand between O-D pair D and E are not included in this numerical test which can be allocated directly via links 12 and 13

Table 1. Travel demands for the test network

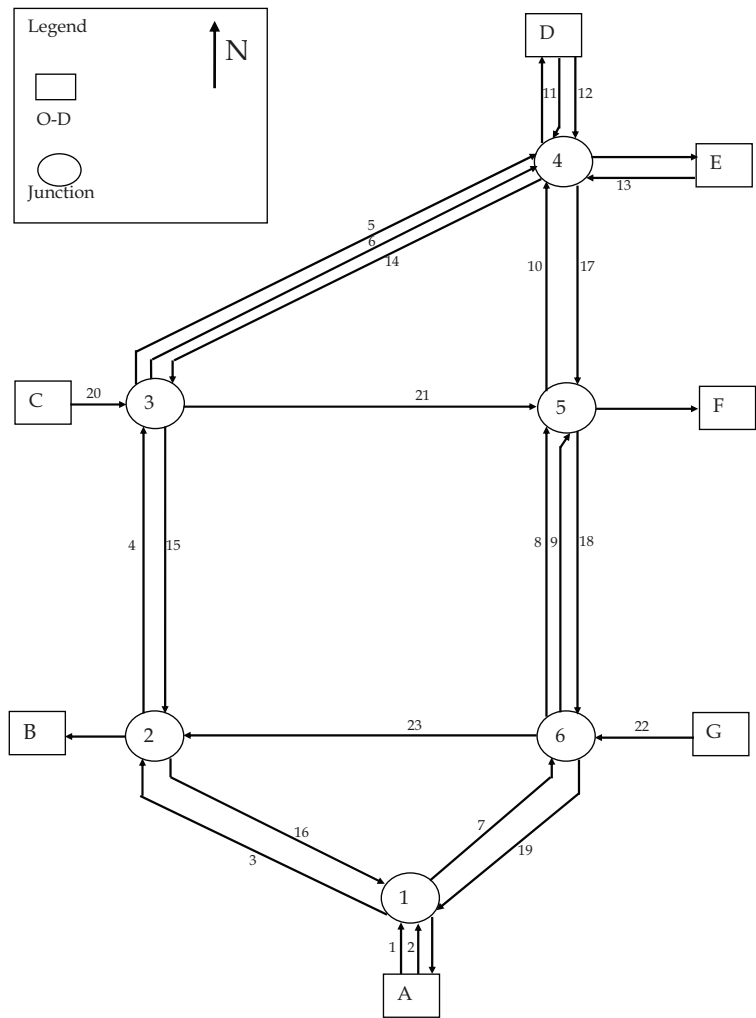


Fig. 2. (a) Layout for the test network (Ceylan & Bell, 2004)

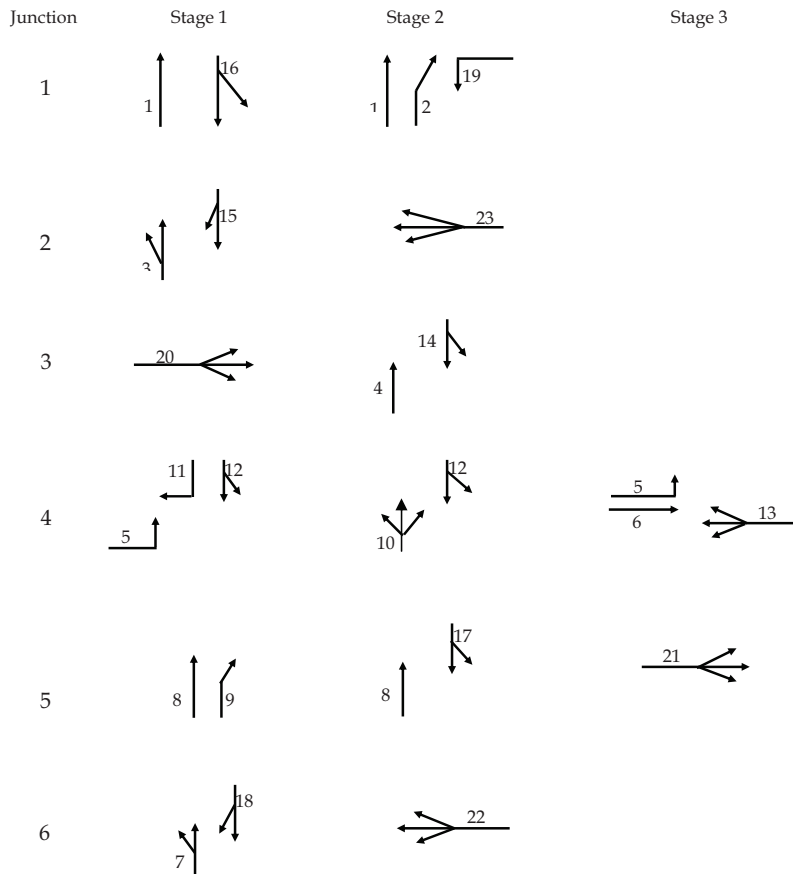


Fig. 2 (b) Stage configurations for the test network (Ceylan & Bell, 2004)

The ACORSES is performed with the following user-specified parameters. β vector should be chosen according to constraints of cycle time as proposed by Baskan et al. (2009).

Colony size (R) is 50.

Search space constraint for ACORSES is chosen as $\beta = [100, 100, \dots, 100]$.

The length of jump is chosen as $\alpha = 1/\text{random}(10)$.

5.1 The bi-level solution for the test network

This numerical test attempted to show that the ACORSES is able to prevent being trapped in bad local optimum although the bi-level programming is non-convex. In order to overcome this non-convexity, the ACORSES starts with a large base of solutions, each of which provided that the solution converges to the optimum and it also uses the reduced search space technique. In ACORSES, new ant colony is created according to randomly generated α value. In this reason, any of the newly created solution vectors may be outside the reduced search space. Therefore, created new ant colony prevents being trapped in bad local optimum. The ACORSES is able to achieve global optimum or near global optimum to

optimise signal timings because it uses concurrently the reduce search technique and the orientation of all ants to the global optimum.

The application of the ACORSES to the test network can be seen in Fig. 3, where the convergence of the algorithm and the evaluation of the objective function are shown. As shown Fig. 3, the ACORSES starts the solution process according to signal timing constraints. It was found that the value of objective function is 133854 veh-sec at first iteration. The ACORSES keeps the best solution and then it uses the best solution to the optimum in the reduced search space. Optimum solution is then searched in the reduced search space during the algorithm progress. The significant improvement on the objective function takes place in the first few iteration because the ACORSES starts with randomly generated ants in a large colony size. After that, small improvements to the objective function takes place since the pheromone updating rule and new created ant colony provide new solution vectors on the different search directions. Finally, the number of objective function reached to the value of 124587 veh-sec.

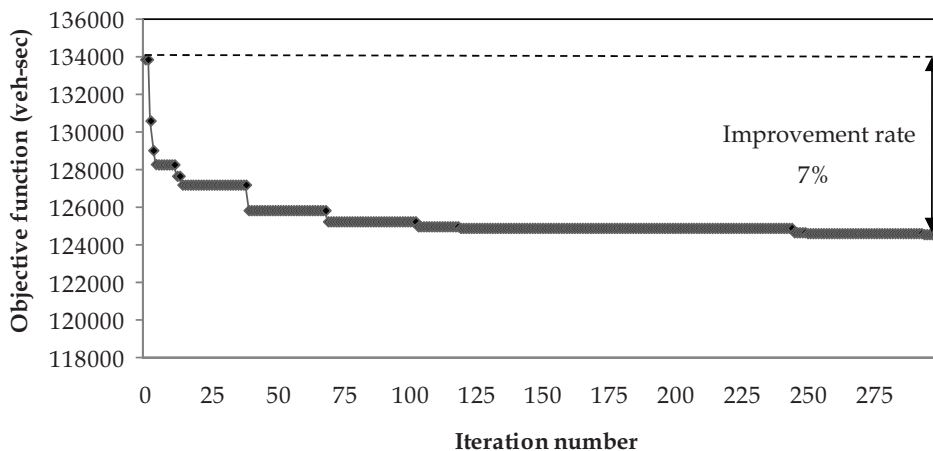


Fig. 3. Convergence behaviour of the bi-level solution

The ACORSES is performed for the 297th iteration, where the difference between the values of ψ_k and ψ_{k+1} is less than 1%, and the number of objective function is obtained for that is 124587 veh-sec. The improvement rate is 7% according to the initial solution of objective function. Table 2 shows the signal timings and final value of objective function.

Objective function (veh-sec)	Junction number	Cycle time c (sec)	Green timings in seconds		
			Stage 1	Stage 2	Stage 3
124587	1	86	7	79	-
	2	76	32	44	-
	3	69	38	31	-
	4	62	11	30	21
	5	74	11	35	28
	6	80	50	30	-

Table 2. The final values of signal timing derived from the bi-level solution

The degrees of saturation are given in Table 3. No links are over-saturated and the final values of degree of saturation of them are not greater than 95%.

S ₁	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇	S ₈	S ₉	S ₁₀	S ₁₁	S ₁₂
30	39	45	25	57	23	51	52	84	58	76	40
S ₁₃	S ₁₄	S ₁₅	S ₁₆	S ₁₇	S ₁₈	S ₁₉	S ₂₀	S ₂₁	S ₂₂	S ₂₃	
60	25	38	74	93	68	81	84	76	93	33	

Table 3. The final values of degree of saturation (%) obtained from the bi-level solution

5.2 MC solution for the test network

The MC calculations were carried out with random generated initial set of signal timings for first iteration. It was found that the value of objective function is 129500 veh-sec at first iteration. The value of objective function decreases steadily as from the second iteration. As can be seen in Figure 4, after MC solution process, the value of objective function decreased to 125216 veh-sec from 129500 veh-sec. The improvement rate was also found 3% according to initial value of objective function.

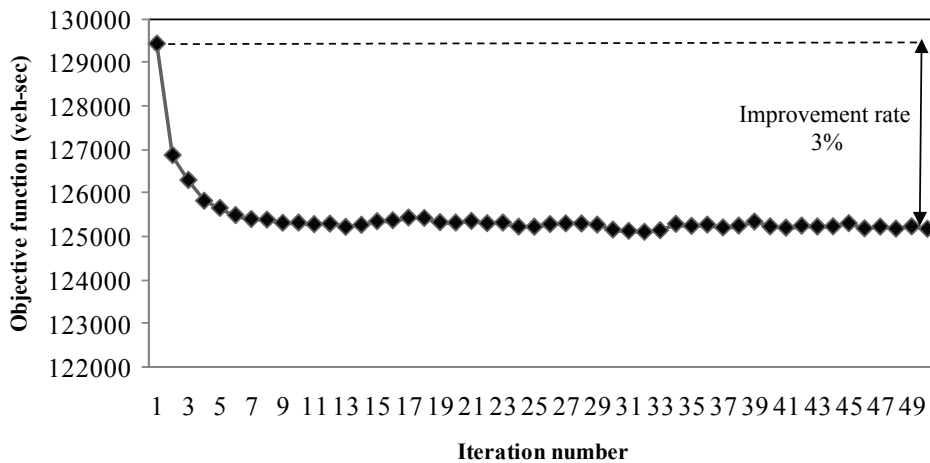


Fig. 4. Convergence behaviour of the MC solution

The final value of objective function and corresponding signal timings for the MC solution are given in Table 4. As can be seen in Table, the MC solution for optimize signal timings produces greater cycle times for each junction compared with the bi-level approach.

Objective function (veh-sec)	Junction number	Cycle time c (sec)	Green timings in seconds		
			Stage 1	Stage 2	Stage 3
125216	1	89	8	81	-
	2	100	50	50	-
	3	74	47	27	-
	4	81	17	41	23
	5	100	14	50	36
	6	91	58	33	-

Table 4. The final values of signal timing derived from the MC solution

Table 5 shows the degree of saturation for the MC solution. No links are over-saturated but some of them approach the critical degree of saturation (i.e. 100%).

s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}
28	43	35	27	54	26	54	56	95	60	65	37
s_{13}	s_{14}	s_{15}	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	
72	32	33	70	87	65	81	73	81	96	39	

Table 5. The final values of degree of saturation (%) obtained from the MC solution

6. Conclusions

In this chapter, ACORSES algorithm was used to optimize signal timings on a given road network without considering offset term using bi-level approach and MC solution procedures. The Allsop & Charlesworth's test network was used to denote the performance of the ACORSES in terms of the value of objective function and the degree of saturation on links. According to the results, the final values of degree of saturation from the bi-level solution were not greater than 95% while no links are over-saturated but some of them approach the critical degree of saturation (i.e. 100%) for the MC solution. The ACORSES algorithm is performed on PC Core2 Toshiba machine and each iteration for this test network was not greater than 12.3 sec of CPU time in Visual Basic code. On the other hand, the computation effort for the MC solution on the same machine was carried out for each iteration in less than 15.4 sec of CPU time.

The ACORSES algorithm was found considerably successful in terms of the signal timings and the final values of degree of saturation. Although the MC solution gives the similar results in terms of the value of objective function, it produces greater cycle times than the bi-level solution. Moreover, the MC solution was also dependent on the initial set of signal timings and its solution was sensitive to the initial assignment.

7. References

Allsop, R. E. (1974). Some possibilities of using traffic control to influence trip distribution and route choice. in: Transportation and Traffic Theory. *Proceedings of the 6th*

- International Symposium on Transportation and Traffic Theory*, Elsevier, 345-374, Amsterdam.
- Abdulaal, M. & LeBlanc, L. J. (1979). Continuous equilibrium network design models. *Transportation Research*, 13B(1), 19-32.
- Allsop, R. E. & Charlesworth J. A. (1977). Traffic in a signal-controlled road network: an example of different signal timings including different routings. *Traffic Engineering Control*, 18(5), 262-264.
- Baskan, O. (2009) *Network design using ant colony optimization*. PhD Thesis, the Graduate School of Natural and Applied Sciences, Pamukkale University, Turkey, (in Turkish).
- Baskan, O., Haldenbilen, S., Ceylan, H. & Ceylan, H. (2009). A new solution algorithm for improving performance of ant colony optimization. *Applied Mathematics and Computation*, 211(1), 75-84.
- Cantarella, G. E., Improta, G. & Sforza, A. (1991) Iterative procedure for equilibrium network traffic signal setting. *Transportation Research*, 25A(5), 241-249.
- Ceylan, H. (2002). *A Genetic Algorithm Approach to the Equilibrium Network Design Problem*. PhD Thesis, University of Newcastle upon Tyne.
- Ceylan, H. & Bell, M. G. H. (2004). Traffic signal timing optimisation based on genetic algorithm approach, including drivers' routing. *Transportation Research*, 38B (4), 329-342.
- Chiou, S. W. (1999). Optimizatoin of area traffic control for equilibrium network flows. *Transportation Science*, 33(3), 279-289.
- Clark, S. D. & Watling, D. P. (2002). Sensitivity analysis of the probit-based stochastic user equilibrium assignment model. *Transportation Research*, 36B, 617-635.
- Dickson, T. J. (1981). A note on traffic assignment and signal timings in a signal-controlled road network. *Transportation Research*, 15B(4), 267-271.
- Dorigo, M., Di Caro, G. & Gambardella, L. M. (1999). *Ant Algorithms for Discrete Optimization*. *Artificial Life*, MIT press.
- Eshghi, K. & Kazemi, M. (1999). Ant colony algorithm for the shortest loop design problem. *Computers & Industrial Engineering*, 50, 358-366.
- Fisk, C. (1984). Optimal signal controls on congested networks, *In: 9th International Symposium on Transportation and Traffic Theory*, VNU Science Press., 197-216.
- Gartner, N. H. (1974). Area traffic control and network equilibrium methods, *In: Traffic Equilibrium Methods*, (M. Florian, Ed.), 274-297, Springer-Verlag, Berlin.
- Gershwin, S. B. & Tan, H. N (1979). Hybrid optimisation: optimal static traffic control constrained by drivers' route choice behaviour, *Laboratory for Information and Decision System Report LIDS-p-870*, Massachusetts Institute of Technology.
- Heydecker, B. G. & T.K. Khoo (1990). The equilibrium network design problem, *Proceedings of AIRO'90 conference on Models and methods for Decision Support*, Sorrento, 587-602.
- Lee, C. (1998). *Combined traffic signal control and traffic assignment: Algorithms, implementation and numerical results*. Ph.D. dissertation, the University of Texas at Austin, Texas.
- Lee, C. & Machemehl, R. B. (1998). Genetic algorithm, local and iterative searches for combining traffic assignment and signal control, *Traffic and Transportation Studies: In: Proceedings of ICTTS 98*, 489-497.
- Sheffi, Y. (1985). *Urban Transportation networks: Equilibrium Analysis with Mathematical Programming Methods*. MIT Prentice-Hall, Inc. New Jersey.

- Sun, D., Benekohal, R. F. & Waller, S. T. (2006). Bi-level programming formulation and heuristic solution approach for dynamic traffic signal optimization. *Computer-Aided Civil and Infrastructure Engineering*, 21, 321-333.
- Suwansirikul, C., Friesz, T. L. & Tobin, R. L. (1987). Equilibrium decomposed optimisation: a heuristic for the continuous equilibrium network design problem. *Transportation Science*, 21(4), 254-263.
- Teklu, F., Sumalee, A. & Watling, D. (2007). A genetic algorithm approach for optimizing traffic control signals considering routing. *Computer-Aided Civil and Infrastructure Engineering*, 22, 31-43.
- Wardrop, J. G. (1952). Some theoretical aspects of road research, *Proc. Inst. Civ. Engineers (Part II)*, 1(2), 325-362.
- Yang, H. & Yagar, S. (1995). Traffic assignment and signal control in saturated road networks. *Transportation Research*, 29A(2), 125-139.