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Ant colony algorithm for traffic signal timing optimization

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ABSTRACT

In order to separate the conflict of the traffic flow effectively, time delay, number of stops and traffic capacity are chosen as performance indexes, and the objective function related to the cycle time and the saturation of an intersection is established by using the weighting coefficients. Then, based on the uncertainty and convergence analysis of ant colony algorithm (ACA), computational experiments are conducted and numerical comparisons are made for the values of performance indexes achieved by the signal timing optimization problem with Webster algorithm, genetic algorithm (GA) and ACA. Numerical results show that ACA is a simple and feasible method for signal timing optimization problems.

1. Introduction

With the rapid development of economy, traffic congestion has become one of the most serious problems in many cities at present. Traditionally, the congestion problem was dealt by adding more lanes and new links to the existing transportation network [1,2]. Since such a solution can no longer be considered for limited availability of space in urban centers, greater emphasis is nowadays placed on traffic management through the implementation and operation of intelligent transportation systems such as TRANSYT, which has been widely recognized as one of the most useful tools in studying the optimization of traffic control [3,4]. Nowadays, with the development of artificial intelligence technology, ant colony algorithm (ACA) has been applied to signal timing optimization problems, as well as genetic algorithm (GA) [5].

As is known to all, the main places responsible for traffic congestion are urban intersections, and the primary reason for traffic congestion of urban intersections is the irrational cycle time of traffic lights. In order to separate the conflict of the traffic flow effectively and improve traffic capacity, how to assign the red and green time in a cycle is obviously important when dealing with traffic control problems [6]. Generally, the longer cycle time, the greater traffic capacity, but time delay and number of stops also increase with increasing of the cycle time. In other words, when the saturation of an intersection is small enough, the increase of the cycle time does not go far enough towards traffic capacity, and it only leads to the increase of time delay. Therefore, the cycle time of traffic lights should be justly distributed so as to minimize time delay and number of stops.

Ant colony algorithm (ACA), which was first brought forward by Dorigo et al. [7] in the early 90s, is a new simulated evolutionary optimization algorithm with the characteristics of positive feedback, distributed computing and strong robustness [8]. However, with the defects of an acute pheromone shortage in early period, less slow solution speed, stagnation and easy to fall in local optima, it has being obtained comprehensive attention from domestic and alien scholars. Recently, ACA has been successfully applied to many combinatorial optimization problems such as TSP, vehicle routing problem, set covering problem, graph coloring and so on [9–12]. But to our knowledge, GA has been rarely used for traffic signal timing optimization.

This paper is organized as follows. In Section 2, some basic parameters for traffic signal control are briefly described. In Section 3, ACA and its rule are both presented after the optimization model of signal timing is created. Furthermore, the uncertainty and convergence of ACA are analyzed in detail in Section 4. In Section 5, numerical results based on Webster algorithm, GA and ACA are discussed thoroughly. Finally, some conclusions are drawn by the analysis of numerical results in Section 6.

2. Basic parameters for traffic signal control

2.1. Signal phase

In traffic signal control, not all the traffic flow has the right of way in a signal cycle so as to avoid the conflict among the traffic

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flow in every direction. Generally, the traffic flow with the same colored signal lights in a signal cycle has the same signal phase.

2.2. Saturation

The saturation of an intersection, also known as the flow ratio, means the ratio of the traffic flow to the saturation flow in unit time [13]. Here, the traffic flow means the number of vehicles passing by the intersection in unit time, and the saturation flow means the maximum number of vehicles passing by the intersection when the green light is on in a signal cycle.

2.3. Time delay

During the traffic signal control, time delay has a direct bearing on the traffic access. Therefore, we often take it as a key index of traffic benefit. In recent years, traditional Webster signal intersection delay formula has been widely used in traffic field.

$$D_i = \frac{c(1-g_i)^2}{2(1-y_i)} + \frac{y_i^2}{2q_i g_i (g_i - y_i)} \tag{1}$$

where c is the cycle time, g_i is the ratio of the effective green light time to the cycle time, q_i is the traffic flow of the phase i, y_i is the saturation of the phase i.

However, Webster formula is available only when the saturation is smaller. Therefore, it is improved by Yang [14], and time delay is defined as follows:

$$D_i = \frac{cq_i(x-y_i)^2}{2x^2(1-y_i)} + \frac{x^2}{2(1-x)} \tag{2}$$

where x is the saturation of an intersection.

2.4. Number of stops

As is known to all, number of stops is inversely proportional to the saturation. In other words, the less number of stops, the better effect of traffic control. Then, according to the research by Jindong Yang, number of stops for the phase i is defined as follows:

$$H_i = \frac{y_i(x-y_i)}{2x^2(y_i - x^2)} \tag{3}$$

2.5. Traffic capacity

Traffic capacity, as it is called, is traffic reaching. Generally, vehicles can only pass the stop line during the effective green light time. Therefore, according to the relationship between number of stops and traffic capacity, traffic capacity can be expressed as follows:

$$Q_i = s_i \cdot \left[1 - \frac{y_i(x-y_i)(1-y_i)}{1.8x^2(y_i - x^2)}\right] \tag{4}$$

where s_i is the saturation flow of the phase i.

3. Signal timing optimization based on ant colony algorithm

3.1. Optimization model of signal timing

In view of the actual traffic demand, the smallest time delay, the fewest number of stops and the largest traffic capacity are chosen as the ultimate aims. Here, time delay, number of stops and traffic capacity are all functions with respect to the cycle time c and the saturation of an intersection x. In order to maximize traffic capacity and minimize time delay and number of stops, the signal timing optimization problem can be translated into a minimization problem in the form of a fraction, thus the objective function turns into a single objective function with two design variables c and x by introducing three weighting coefficients which vary with different traffic demands. Therefore, the optimization model of signal timing can be written as:

$$\min f(x,c) = \frac{\sum_{i=1}^{n}(K_i^1 D_i + K_i^2 H_i)}{\sum_{i=1}^{n} K_i^3 Q_i} \tag{5}$$

subject to $x \in [0.6, 0.8]$, $c \in [40, 120]$.where 0.6 is to avoid the increases of time delay and number of stops caused by the smaller saturation, 0.8 is to avoid traffic congestion caused by the bigger saturation; the interval of cycle time is often defined as $[40, 120]$ [15]; K_i^1, K_i^2 and K_i^3 are respectively the weighting coefficients of time delay, number of stops and traffic capacity, and they can be calculated as follows [16]:

$$K_i^1 = 2 \cdot (1-Y) \cdot \sqrt[7]{s_i} \tag{6}$$

$$K_i^2 = \sqrt[7]{s_i} \cdot \frac{1-Y}{0.9} \tag{7}$$

$$K_i^3 = 2Y \cdot \frac{c}{3600} \tag{8}$$

where $Y = \sum_{i=1}^{n} \max y_i^j$, and y_i^j $(j = 1, 2, \ldots)$ is the saturation in a direction for the phase i. Thus it can be found that $f(x,c) \geqslant 0$ and $\min f(x,c) \geqslant 0$.

3.2. Determination of weighting coefficients

For the reason that weighting coefficients represent the tradeoff between efficiency and robustness [17], the determination of weighting coefficients K_i^1, K_i^2 and K_i^3 is also important. Here it is made under the following considerations.

(1) We hope that K_i^1 and K_i^2 decrease with increasing of the saturation y_i, but K_i^3 increases with increasing of the saturation y_i, so as to improve traffic capacity during the rush hours and reduce time delay and number of stops during the rest hours.
(2) Considering that it is more likely to cause time delay at heavy intersections during unsaturated hours, we hope that K_i^1 increases with increasing of the saturation flow s_i, so as to reduce time delay at heavier intersections.
(3) As is known to all, the longer cycle time, the larger traffic capacity. Therefore, the cycle time c is introduced into K_i^3, so as to improve traffic capacity to some extent.

3.3. Ant colony algorithm

Ant colony algorithm (ACA) [18] is a promising metaheuristic and great amount of research has been devoted to its empirical and theoretical analysis. The ants can carry on indirect communication through a chemical substance pheromone, which is accumulative and also evaporative. The ants travel a shorter path on which the pheromone accumulates faster than on the longer one. Therefore, the faster the pheromone increases on the short path, the greater the probability that the ants travel this path. The pheromone can deposit unceasingly and evaporate as time goes on. At the same time, the ants also can unceasingly secrete the pheromone in their travel process, thus the pheromone can be updated unceasingly. The pheromone on the path which few ants travel decreases more and more, but the pheromone on the path which more ants travel increases more and more. This forms a positive feedback process, and finally causes all the ants to travel the shortest path.

That is to say, ACA makes full use of simple agents composed of ants and iteratively constructs solutions to the optimization problem. Guided by pheromone, an individual ant constructs a complete solution by starting with the null solution and iteratively adding solution components. After the construction of a solution, each ant gives feedback by depositing pheromone for each solution component [19].

Basic steps and detailed explanations of ACA are presented as follows:

① Initialization. The initial values of some parameters are given, and then the fitness and the pheromone are respectively calculated.

② Probability adjustment. The state transition probability for ant k is defined as [20]

$$P_{ij}^{k}(t) = \frac{\tau_{ij}(t)}{\sum_{r=1}^{m}\tau_{ir}(t)} \tag{9}$$

where $\tau_{ij}(t)$ is the pheromone on $edge(i,j)$ at time t.

③ Pheromone updating rule. After travelling once, the pheromone on all the paths should be adjusted as follows:

$$\tau_{ij}(t+1) = (1-\rho)\cdot\tau_{ij}(t) + Q\cdot\Delta\tau_{ij} \tag{10}$$

$$\Delta\tau_{ij} = (f_{\max} - f)/(f_{\max} - f_{\min}) \tag{11}$$

where ρ is the residual degree of pheromone; Q is a constant, and $Q > 0$; f is the objective function value at that time, $f_{\max}$ and $f_{\min}$ are respectively the maximum and minimum of the objective function.

④ Crossover and mutation operation. To improve the ability of random search, the new individuals are generated by adding a random operator, and the main method can be listed as follows:

$$r = \max(rand(), 0.618) \tag{12}$$

$$x' = LB + r^{*}(UB - LB) \tag{13}$$

where x' is the new individual generated by the operation above; in order to avoid less obvious new individuals caused by a smaller coefficient r, $rand()$ is compared with the golden section ratio 0.618, and $\max(rand(), 0.618)$ is used as the ultimate coefficient; LB is a 2 by 1 matrix composed of the lower bounds of two design variables c and x; UB is also a 2 by 1 matrix composed of the upper bounds of two design variables c and x.

4. Numerical analysis

4.1. Uncertainty analysis

(1) Pheromone updating method. There are usually two types of pheromone updating methods, that is [21]: ① The pheromone is renewed gradually when ants take every step; ② The pheromone on the whole paths is renewed after all the ants travel once. Different updating methods have different effects on the convergence rate of ACA. Experiment results show that the second updating method can improve the convergence rate, so it is often adopted to assure much faster convergence rate.

(2) Random quality of random numbers. Generally, random numbers should follow uniform distribution, and the random quality of random numbers may have a direct effect on the accuracy of results, especially when ants are in the stage of random selection. For the reason that identical random numbers may be continuously generated within a certain time period, Eq. (12) is used to avoid this problem.

(3) The number of ants and iteration times. The performance of ACA can be improved with the number of ants, but the running time is obviously prolonged. In addition, iteration times have a direct effect on the results to a certain extent. That is, the more iteration times, the closer the solution obtained by ACA is to the optimal solution. Therefore, how to choose iteration times and the number of ants is of great importance when solving traffic signal timing optimization problems by ACA.

4.2. Convergence analysis

Suppose that ρ_1 is the initial value of ρ, denote $A = \{\rho_1, \rho_2, \ldots, \rho_k\}$, where $\rho_1 \geqslant \rho_2 \geqslant \ldots \geqslant \rho_k \geqslant \rho_{\min}$. Let x^* be the optimal solution, denote $g(x^*) = \max(Q * \Delta\tau_{ij})$.

Lemma 1. For any τ_{ij}, if there exist $\tau_{\min}$ and $\tau_{\max}$ such that $\tau_{\min} \leqslant \tau_{ij} \leqslant \tau_{\max}$, then $\tau_{\max} = \frac{1}{\rho_1} g(x^*)$.

Proof.

(i) For the reason that there exist maximum and minimum for the objective function, and by using Eqs. (10) and (11), it can be readily verified that there exists a minimum for the pheromone. That is to say, $\tau_{ij} \geqslant \tau_{\min}$.

(ii) Based on the pheromone updating rule, the pheromone on each edge will increase at most $g(x^*)$ after every iteration. Therefore, for any $edge(i,j)$, after one iteration, we have $\tau_{ij}(1) \leqslant (1-\rho_1)\tau_0 + g(x^*)$.

Similarly, after two iterations, we get $\tau_{ij}(2) \leqslant (1-\rho_1)^2\tau_0 + (1-\rho_1)g(x^*) + g(x^*)$.

And so on, after k iterations, we obtain $\tau_{ij}(k) \leqslant (1-\rho_1)^k\tau_0 + \sum_{i=1}^{k}(1-\rho_1)^{i-1}g(x^*)$.

With $\rho_1 \in (0,1)$ and $(1-\rho_1) \in (0,1)$, we have $\tau_{\max} = \lim\limits_{k\to\infty}[(1-\rho_1)^k\tau_0 + \sum_{i=1}^{k}(1-\rho_1)^{i-1}g(x^*)] = \frac{1}{\rho_1}g(x^*)$, which proves the conclusion. □

In the following, the convergence of ACA will be proved in detail.

Let $P(n)$ be the probability that the optimal solution is found at least once during n iterations; the probability that the optimal solution can be obtained is denoted as P, and the minimum of P is denoted as $P_{\min}$.

Theorem 1. For an arbitrarily small constant $\varepsilon > 0$, if n is big enough, then $P(n) \geqslant 1-\varepsilon$ and $\lim\limits_{n\to\infty} P(n) = 1$.

Proof. For $\forall edge(i,j) \in l^*$, the worst condition when solving problems by ACA is listed as follows:

$$\tau_{ij} = \begin{cases} \tau_{\min}, edge(i,j) \in l^* \\ \tau_{\max}, edge(i,j) \notin l^* \end{cases} \tag{14}$$

where l^* is the optimal path.

According to Eq. (9), we can get

$$P_{\min} \geqslant \frac{\tau_{\min}}{m * \tau_{\max} + \tau_{\min}} > 0 \tag{15}$$

where m is the number of ants.

Then, we have $P(n) = 1-(1-P)^n \geqslant 1-(1-P_{\min})^n$.

Let $\varepsilon = (1-p_{\min})^n$, then $P(n) \geqslant 1-\varepsilon$, and $\lim\limits_{n\to\infty} P(n) = \lim\limits_{n\to\infty}[1-(1-p_{\min})^n] = 1$, which proves our assertion. □

5. Numerical results

For an intersection with two phases (as shown in Fig. 1), the traffic flow and the saturation flow of each entrance are given in

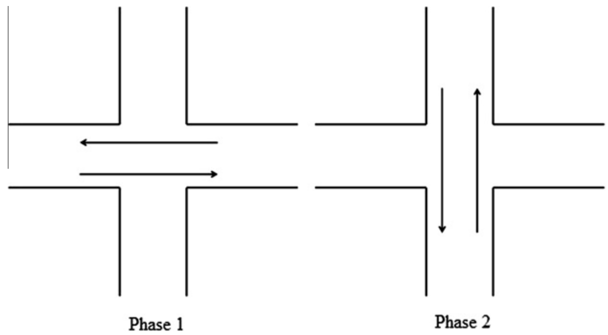

Fig. 1. Intersections with two phases.

Table 1. Suppose the green light interval is 7 s, the yellow light time is 3 s and the lost time when vehicles start is 3 s, then the results based on Webster algorithm can be obtained as follows.

Now, GA is applied to the signal timing optimization problems. Here, its program is written under MATLAB environment based on the Sheffield GA toolbox, and the settings of each parameter for GA are listed as follows: the number of individuals is 100, the maximum number of generations is 20, the precision of variables is 20, the generation gap is 0.9, the crossover probability is 0.9. Then, we can get the optimal solutions based on GA, as shown in Fig. 2.

Finally, ACA is used for solving the signal timing optimization problems, and the parameters are set as follows: the number of ants is 20, iteration times 20, the crawl velocity of ants is 0.3, $\rho = 0.85$, $Q = 0.8$. Then, write a program in MATLAB based on the rule of ACA, and we can get its optimal solutions, as shown in Fig. 3.

From Fig. 2 it can be seen that GA has a fast convergence rate and the optimal solutions can be obtained after four iterations. However, Fig. 3 shows that ACA has a faster convergence rate and better stability compared with GA, and smaller optimal solutions can be obtained after one iteration, which is consistent with expected. For the reason that ACA makes full use of random selection agents composed of ants, its iteration history varies with the initial position of each ant. The closer the initial position is to the corresponding position of the optimal solution, the more pheromone accumulated on the path, thus the optimal solutions can be found for a long time, which results in a constant development from iteration 1 to 12. With the evaporation of pheromone on the path, few ants can find the optimal solutions, which leads to their random travel. Therefore, there is a sharp skip from iteration 12 to 13. However, with increasing of the number of ants, the pheromone accumulated on these paths is increasing, thus more and more ants travel on them, which causes a constant development after 13 iterations.

Table 1
Flow and saturation of each entrance.

Variables	East entrance	West entrance	South entrance	North entrance
The traffic flow q	370	320	864	720
The saturation flow s	1000	1000	2400	2400
The saturation of phases y_i	0.37	0.32	0.36	0.30
$\max(y_i^1, y_i^2)$	0.37		0.36	
$Y = \sum_{i=1}^{2} \max(y_i^1, y_i^2)$	0.73			

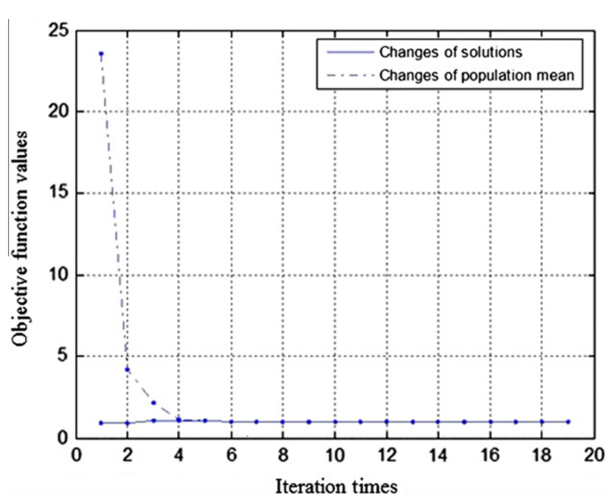

Fig. 2. Iterative history of GA.

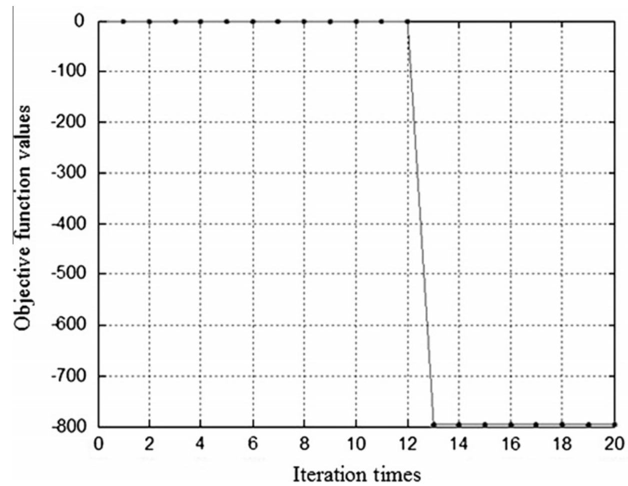

Fig. 3. Iterative history of ACA.

Table 2
Numerical comparisons among Webster algorithm, GA and ACA.

Algorithms	Cycle time	Total time delay $\sum cD_iq_i$	Total stops $\sum H_iq_i$	Total capacity $\sum Q_i$
Webster	97	186,80,329	3585	2895
GA	82	89,96,074	88	6806
ACA	63	75,08,309	35	6755

Table 2 shows the numerical comparisons among Webster algorithm, GA and ACA. Judging by the results in Table 2, compared with Webster algorithm, the cycle time, total time delay and total number of stops obtained by ACA are significantly reduced, and its total traffic capacity is obviously increased, which is correspondent with the expected results. As can be seen, total time delay and total number of stops based on ACA decrease by more than one order of magnitude. In addition, compared with GA, total traffic capacity obtained by ACA increases, but the cycle time, total time delay and total number of stops obtained by ACA also decrease to a certain extent. Therefore, we can achieve better performance by ACA, which can well meet the actual traffic demand. Consequently, ACA is an efficient and feasible method when solving signal timing optimization problems.

6. Conclusions

In this paper, a more efficient algorithm based on ACA is introduced and applied to the traffic signal timing optimization, which can preferably make time delay smaller, number of stops fewer and traffic capacity larger. Moreover, the results show that ACA has more superiority than Webster algorithm and GA when dealing with signal timing optimization problems, and the optimal solutions can be obtained effectively. Therefore, ACA has broad application prospects in the field of traffic signal control.

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