University of Edinburgh

School of Mathematics

Bayesian Data Analysis, 2022/2023, Semester 2

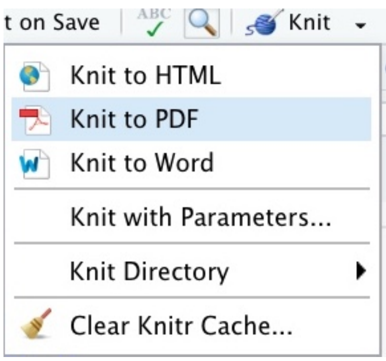
Assignment 1

IMPORTANT INFORMATION ABOUT THE ASSIGNMENT

In this paragraph, we summarize the essential information about this assignment. The format and rules for this assignment are different from your other courses, so please pay attention.

1) Deadline: The deadline for submitting your solutions to this assignment is the 6 March 12:00 noon Edinburgh time.

2) Format: You will need to submit your work as 2 components: a PDF report, and your R Markdown (.Rmd) notebook. There will be two separate submission systems on Learn: Gradescope for the report in PDF format, and a Learn assignment for the code in Rmd format. You need to write your solutions into this R Markdown notebook (code in R chunks and explanations in Markdown chunks), and then select Knit/Knit to PDF in RStudio to create a PDF report.



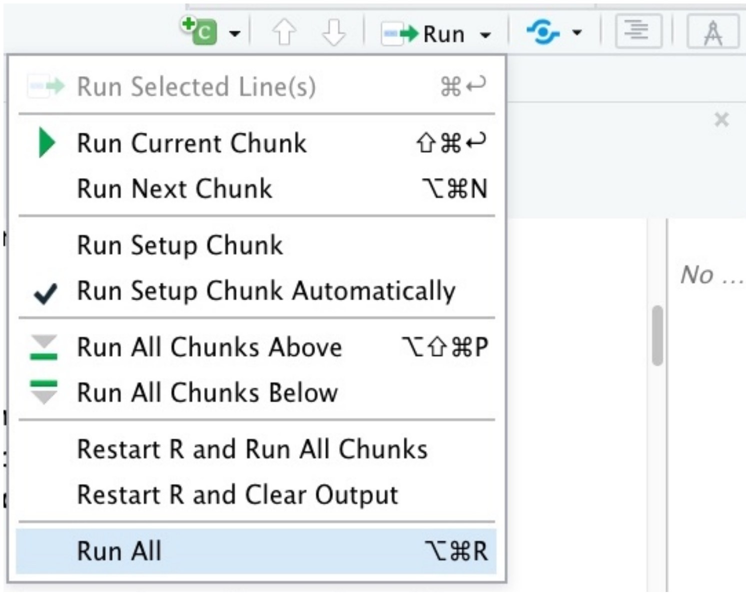
The compiled PDF needs to contain everything in this notebook, with your code sections clearly visible (not hidden), and the output of your code included. Reports without the code displayed in the PDF, or without the output of your code included in the PDF will be marked as 0, with the only feedback "Report did not meet submission requirements".

You need to upload this PDF in Gradescope submission system, and your Rmd file in the Learn assignment submission system. You will be required to tag every sub question on Gradescope.

Some key points that are different from other courses:

a) Your report needs to contain written explanation for each question that you solve, and some numbers or plots showing your results. Solutions without written explanation that clearly demonstrates that you understand what you are doing will be marked as 0 irrespectively whether the numerics are correct or not.

b) Your code has to be possible to run for all questions by the Run All in RStudio, and reproduce all of the numerics and plots in your report (up to some small randomness due to stochasticity of Monte Carlo simulations). The parts of the report that contain material that is not reproduced by the code will not be marked (i.e. the score will be 0), and the only feedback in this case will be that the results are not reproducible from the code.



c) Multiple Submissions are allowed BEFORE THE DEADLINE are allowed for both the report, and the code.
However, multiple submissions are NOT ALLOWED AFTER THE DEADLINE.
YOU WILL NOT BE ABLE TO MAKE ANY CHANGES TO YOUR SUBMISSION AFTER THE DEADLINE.
Nevertheless, if you did not submit anything before the deadline, then you can still submit your work after the deadline, but late penalties will apply. The timing of the late penalties will be determined by the time you have submitted BOTH the report, and the code (i.e. whichever was submitted later counts).

We illustrate these rules by some examples:

Alice has spent a lot of time and effort on her assignment for BDA. Unfortunately she has accidentally introduced a typo in her code in the first question, and it did not run using Run All in RStudio. - Alice will get 0 for the whole assignment, with the only feedback "Results are not reproducible from the code".

Bob has spent a lot of time and effort on his assignment for BDA. Unfortunately he forgot to submit his code. - Bob will get no personal reminder to submit his code. Bob will get 0 for the whole assignment, with the only feedback "Results are not reproducible from the code, as the code was not submitted."

Charles has spent a lot of time and effort on his assignment for BDA. He has submitted both his code and report in the correct formats. However, he did not include any explanations in the report. Charles will get 0 for the whole assignment, with the only feedback "Explanation is missing."

Denise has spent a lot of time and effort on her assignment for BDA. She has submitted her report in the correct format, but thought that she can include her code as a link in the report, and upload it online (such as Github, or Dropbox). - Denise will get 0 for the whole assignment, with the only feedback "Code was not uploaded on Learn."

3) Group work: This is an INDIVIDUAL ASSIGNMENT, like a 2 week exam for the course. Communication between students about the assignment questions is not permitted. Students who submit work that has not been done individually will be reported for Academic Misconduct, that can lead to serious consequences. Each problem will be marked by a single instructor, so we will be able to spot students who copy.

4) Piazza: During the periods of the assignments, the instructor will change Piazza to allow messaging the instructors only, i.e. students will not see each others messages and replies.

Only questions regarding clarification of the statement of the problems will be answered by the instructors. The instructors will not give you any information related to the solution of the problems, such questions will be simply answered as "This is not about the statement of the problem so we cannot answer your question."

THE INSTRUCTORS ARE NOT GOING TO DEBUG YOUR CODE, AND YOU ARE ASSESSED ON YOUR ABILITY TO RESOLVE ANY CODING OR TECHNICAL DIFFICULTIES THAT YOU ENCOUNTER ON YOUR OWN.

5) Office hours: There will be two office hours per week (Monday 14:00-15:00, and Wednesdays 15:00-16:00) during the 2 weeks for this assignment. The links are available on Learn / Course Information. I will be happy to discuss the course/workshop materials. However, I will only answer questions about the assignment that require clarifying the statement of the problems, and will not give you any information about the solutions. Students who ask for feedback on their assignment solutions during office hours will be removed from the meeting.

6) Late submissions and extensions: NO EXTENSIONS ARE ALLOWED FOR THIS ASSIGNMENT, AND THERE IS NO SUCH OPTION PROVIDED IN THE ESC SYSTEM. Students who have existing Learning Adjustments in Euclid will be allowed to have the same adjustments applied to this course as well, but they need to apply for this BEFORE THE DEADLINE on the website

https://www.ed.ac.uk/student-administration/extensions-special-circumstances

by clicking on "Access your learning adjustment". This will be approved automatically.

Students who submit their work late will have late submission penalties applied by the ESC team automatically (this means that even if you are 1 second late because of your internet connection was slow, the penalties will still apply). The penalties are 5% of the total mark deduced for every day of delay started (i.e. one minute of delay counts for 1 day). The course instructors do not have any role in setting these penalties, we will not be able to change them.

```r
rm(list = ls(all = TRUE))
#Do not delete this!
#It clears all variables to ensure reproducibility
```

Problem 1

In this problem, we study a dataset about currency exchange rates. The exrates dataset of the stochvol package contains the daily average exchange rates of 24 currencies versus the EUR, from 2000-01-03 until 2012-04-04.

```
require(stochvol)
```

```
## Loading required package: stochvol
```

```
data("exrates")
#You may need to set the working directory first before loading the dataset
#setwd("location of Assignment 1")
#The first 6 rows of the dataframe
print.data.frame(exrates[1:6,])
```

```
##                AUD    CAD    CHF    CZK    DKK    GBP    HKD     IDR    JPY
## 2000/01/03 1.5346 1.4577 1.6043 36.063 7.4404 0.6246 7.8624 7133.32 102.75
## 2000/01/04 1.5677 1.4936 1.6053 36.270 7.4429 0.6296 8.0201 7265.16 105.88
## 2000/01/05 1.5773 1.5065 1.6060 36.337 7.4444 0.6324 8.0629 7437.97 107.34
## 2000/01/06 1.5828 1.5091 1.6068 36.243 7.4441 0.6302 8.0843 7495.52 108.72
## 2000/01/07 1.5738 1.5010 1.6079 36.027 7.4436 0.6262 8.0030 7398.88 108.09
## 2000/01/10 1.5587 1.4873 1.6089 35.988 7.4448 0.6249 7.9471 7320.49 107.26
##                KRW    MXN    MYR    NOK    NZD    PHP    PLN    RON     RUB
## 2000/01/03 1140.02 9.6105 3.8422 8.0620 1.9331 40.424 4.1835 1.8273 27.7548
## 2000/01/04 1157.32 9.7453 3.9188 8.1500 1.9745 40.992 4.2423 1.8858 28.3594
## 2000/01/05 1176.08 9.8969 3.9393 8.2060 1.9956 41.637 4.2627 1.8979 28.3006
## 2000/01/06 1191.90 9.9751 3.9498 8.2030 2.0064 42.148 4.2593 1.9000 28.5111
## 2000/01/07 1169.52 9.8368 3.9100 8.1945 1.9942 41.493 4.1897 1.8822 28.2526
```

```
## 2000/01/10 1157.84 9.6449 3.8824 8.1900 1.9783 41.226 4.1567 1.8729 28.7609
##                SEK    SGD     THB      TRY    USD       date
## 2000/01/03 8.5520 1.6769 37.2793 0.546131 1.0090 2000-01-03
## 2000/01/04 8.6215 1.7047 38.2078 0.552354 1.0305 2000-01-04
## 2000/01/05 8.6415 1.7159 38.5375 0.555329 1.0368 2000-01-05
## 2000/01/06 8.6445 1.7291 39.0734 0.555674 1.0388 2000-01-06
## 2000/01/07 8.6450 1.7096 38.4299 0.554980 1.0284 2000-01-07
## 2000/01/10 8.6570 1.6975 38.0328 0.552469 1.0229 2000-01-10
```

```r
cat(paste("Data from ", min(exrates$date)," until ",max(exrates$date)))
```

```
## Data from  2000-01-03  until  2012-04-04
```

As we can see, not all dates are included in the dataset. Some are missing, such as weekends, and public holidays.

In this problem, we are going to fit a various stochastic volatility models on this dataset (see e.g. https://www.jstor.org/stable/1392251).

a)[10 marks] Consider the following leveraged Stochastic Volatility (SV) model.

$$
\begin{aligned}
y_t &= \beta_0 + \beta_1 y_{t-1} + \exp(h_t/2)\epsilon_t \quad \text{for} \quad 1 \le t \le T,\\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma\eta_t \quad \text{for} \quad 0 \le t \le T, \quad h_0 \sim N(\mu, \sigma^2/(1-\phi^2)),\\
(\epsilon_t, \eta_t) &\sim N(0, \Sigma_\rho) \quad \text{for} \quad \Sigma_\rho = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}.
\end{aligned}
$$

Here t is the time index, y_t are the observations (such as daily USD/EUR rate), h_t are the log-variance process, ϵ_t is the observation noise, and η_t is the log-variance process noise (which are correlated, but independent for different values of \$t\$). The hyperparameters are $\beta_0, \beta_1, \mu, \phi, \sigma, \rho$.

For stability, it is necessary to have $\phi \in (-1, 1)$, and by the definition of correlation matrices, we have $\rho \in [-1, 1]$.

Implement this model in JAGS or Stan on the first 3 months of USD/EUR data from the dataset, i.e. from dates 2000-01-03 until 2000-04-02.

Explain how did you choose priors for all parameters. Explain how did you take into account the days without observation in your model.

Fit the model, do convergence diagnostics, print out the summary of the results, and discuss them.

Make sure that the Effective Sample Size is at least 1000 for all 6 hyperparameters (you need to choose burn-in and number of steps appropriately for this).

Explanation: (Write your explanation here)

b)[10 marks] In practice, one often encounters outliers in exchange rates. These can be sometimes modeled by assuming Student's t distribution in the observation errors (i.e. ϵ_t). The robust leveraged SV model can be expressed as

$$
\begin{aligned}
y_t &= \beta_0 + \beta_1 y_{t-1} + \exp(h_t/2)\epsilon_t \quad \text{for} \quad 1 \le t \le T,\\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma\eta_t \quad \text{for} \quad 0 \le t \le T, \quad h_0 \sim N(\mu, \sigma^2/(1-\phi^2)),\\
\eta_t &\sim N(0,1)\\
\epsilon_t | \eta_t &\sim t_\nu(\rho\eta_t, 1).
\end{aligned}
$$

Here ν is the degrees of freedom parameter (unknown).

Implement this model in JAGS or Stan on the first 3 months of USD/EUR data from the dataset.

Explain how did you choose priors for all parameters. Explain how did you take into account the days without observation in your model.

Fit the model, do convergence diagnostics, print out the summary of the results, and discuss them.

Make sure that the Effective Sample Size is at least 1000 for all 6 hyperparameters (you need to choose burn-in and number of steps appropriately for this).

Explanation: (Write your explanation here)

c)[10 marks]

Perform posterior predictive checks on both models a) and b). Explain how did you choose the test functions.

Discuss the results.

Explanation: (Write your explanation here)

d)[10 marks]

Based on your models a) and b), plot the posterior predictive densities of the USD/EUR rate on the dates 2000-04-03, 2020-04-04 and 2020-04-05 (the next 3 days after the period considered). Compute the posterior means and 95% credible intervals. Discuss the results.

Explanation: (Write your explanation here)

e)[10 marks]

In this question, we are going to look use a multivariate stochastic volatility model with leverage to study the USD/EUR and GBP/EUR exchange rates jointly. The model is described as follows,

$$\begin{aligned}
\boldsymbol{y}_t &= \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 \boldsymbol{y}_{t-1} + \exp(h_t/2)\boldsymbol{\epsilon}_t \quad \text{for} \quad 1 \le t \le T, \\
\boldsymbol{h}_{t+1} &= \boldsymbol{\phi}(\boldsymbol{h}_t) + \boldsymbol{\eta}_t \quad \text{for} \quad 0 \le t \le T, \quad h_0 \sim N(0, I), \\
(\epsilon_t, \eta_t) &\sim N(0, \Sigma).
\end{aligned}$$

Here I denotes the 2 x 2 identity matrix, $\boldsymbol{y}_t, \boldsymbol{\beta}_0, \boldsymbol{h}_t, \boldsymbol{\eta}_t, \boldsymbol{\epsilon}_t$ are 2 dimensional vectors, $\boldsymbol{\beta}_1$ and $\boldsymbol{\phi}$ are 2 x 2 matrices, $\boldsymbol{\Sigma}$ is a 4 x 4 covariance matrix. At each time step t, the two components of y_t will be used to model the USD/EUR and GBP/EUR exchange rates, respectively.

Implement this model in JAGS or Stan.

Discuss your choices for priors for every parameter [Hint: you can use Wishart or scaled Wishart priors for $\boldsymbol{\Sigma}$, **see https://www.stats.ox.ac.uk/~nicholls/MScMCMC15/jags_user_manual.pdf , https://mc-stan.org/docs/2_19/functions-reference/wishart-distribution.html].**

Fit the model, do convergence diagnostics, print out the summary of the results, and discuss them.

Explanation: (Write your explanation here)

Problem 2 - NBA data

In this problem, we are going to construct a predictive model for NBA games.

We start by loading the dataset.

```
games<-read.csv("games.csv")
teams<-read.csv("teams.csv")
```

games.csv contains the information about games such as GAME_DATE, SEASON, HOME_TEAM_ID, VISITOR_TEAM_ID, PTS_home (final score for home team) and PTS_away (final score for away team).

teams.csv contains the names of each team, i.e. the names corresponding to each team ID.

We are going to fit some Bayesian linear regression models on the scores of each team.

You can use either INLA, JAGS or Stan.

a)[10 marks]

The dataset contains data from 20 seasons, but we are going to focus on only one, the 2021 season.
Please only keep games where SEASON is 2021 in the dataset, and remove all other seasons.
Please order the games according to the date of occurrence (they are not ordered like that in the dataset).

The scores are going to be assumed to follow a linear Gaussian model,

$$S_g^H \sim N(\mu_g^H, \sigma^2), \quad S_g^A \sim N(\mu_g^A, \sigma^2).$$

Here S_g^H denotes the final score of the home team in game g, and S_g^A denotes the final score of the away team in game g.

Note that the true scores can only take non-negative integer values, so the Gaussian distribution is not perfect, but it can still be used nevertheless.

The means for the scores are going to be modeled as a combination of three terms: attacking strength, defending ability, and whether the team is playing at home, or away. For each team, we denote their attacking strength parameter by a_{team}, their defending strength parameter by d_{team}, and the effect of playing at home as h. This quantifies the effect of playing at home on the expected number of goals scored. Our model is the following (μ_g^H is for the goals scored by the home team, and is μ_g^A is for the away team):

$\mu_g^H = \beta_0 + a_{home.team} + d_{away.team} + h$

$\mu_g^A = \beta_0 + a_{away.team} + d_{home.team}$

Implement this model. Select your own prior distributions for the parameters, and discuss the reason for using those priors.

Obtain the summary statistics for the posterior distribution of the model parameters.

Evaluate the root mean square error (RMSE) of your posterior means versus the true scores.

Interpret the results.

Explanation: (Write your explanation here)

b)[10 marks] In part a), the model assumed that the home effect is the same for each team. In this part, we consider a team-specific home effect $h_{home.team}$,

$\mu_g^H = \beta_0 + a_{home.team} + d_{away.team} + h_{home.team}$

$\mu_g^A = \beta_0 + a_{away.team} + d_{home.team}$

Implement this model. Select your own prior distributions for the parameters, and discuss the reason for using those priors.

Obtain the summary statistics for the posterior distribution of the model parameters.

Evaluate the root mean square error (RMSE) of your posterior means versus the true scores.

Interpret the results.

Explanation: (Write your explanation here)

c)[10 marks] Propose an improved linear model using the information in the dataset before the game (you cannot use any information in the same row as the game, as this is only available after the game). Hint: you can try incorporating running averages of some covariates specific to each team, by doing some pre-processing.

Implement your model. Select your own prior distributions for the parameters, and discuss the reason for using those priors.

Obtain the summary statistics for the posterior distribution of the model parameters.

Evaluate the root mean square error (RMSE) of your posterior means versus the true scores.

Interpret the results.

Explanation: (Write your explanation here)

d)[10 marks] Perform posterior predictive checks on all 3 models a), b), and c). Explain how did you choose the test functions.

Discuss the results.

Explanation: (Write your explanation here)

e)[10 marks] In the previous questions, we were assuming a model of the form.

$$S_g^H \sim N(\mu_g^H, \sigma^2), \quad S_g^A \sim N(\mu_g^A, \sigma^2).$$

It is natural to model these two results jointly with a multivariate normal,

$$(S_g^H, S_g^A) \sim N\left(\begin{pmatrix}\mu_g^H \\ \mu_g^A\end{pmatrix}, \Sigma\right),$$

where Σ is a 2 times 2 covariance matrix.

Implement such a model. The definition of μ_g^H and μ_g^A can be either one of a), b), or c), you just need to implement one of them.

Explain how did you choose the prior on Σ [Hint: you can use a Wishart prior, or express this a product of diagonal and correlation matrices and put priors on those terms].

Obtain the summary statistics for the posterior distribution of the model parameters.

Evaluate the root mean square error (RMSE) of your posterior means versus the true scores.

Interpret the results.

Explanation: (Write your explanation here)