# Quantifying Marketing Performance at Channel-Partner Level by Using Marketing Mix Modeling (MMM) and Shapley Value Regression

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#### **ABSTRACT**

This paper explores the application of Shapley Value Regression in dissecting marketing performance at channel-partner level, complementing channel-level Marketing Mix Modeling (MMM). Utilizing real-world data from the financial services industry, we demonstrate the practicality of Shapley Value Regression in evaluating individual partner contributions. Although structured in-field testing along with cooperative game theory is most accurate, it can often be highly complex and expensive to conduct. Shapley Value Regression is thus a more feasible approach to disentangle the influence of each marketing partner within a marketing channel. We also propose a simple method to derive adjusted coefficients of Shapley Value Regression and compare it with alternative approaches.

Keywords: Shapley Values Regression, Game Theory, MMM, Multicollinearity, Marketing

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#### 1. Introduction

In the ever-evolving marketing landscape, organizations of varying sizes leverage a mix of online and offline channels to get their marketing messages across. Though this maximizes the outreach of the audience, it is often a challenging task to accurately disentangle the value provided by each channel. In this regard, MMM has matured to become a staple approach for cross-channel media measurement. Leading tech entities like Google and Meta have significantly contributed to advancing Marketing Mix Modeling (MMM), introducing their respective open-source packages: Lightweight [1] and Robyn [2]. Lightweight employs Bayesian statistics to incorporate business domain knowledge into media priories [3], while Robyn utilizes the frequentist approach and multiple objective functions, generating thousands of iterations. But it is often an arduous task to further disentangle partner-level contributions that can provide actionable, in-depth insights that are required for media planning and budgeting decisions. Consider an instance where the MMM highlights television (TV) as a high-performance channel. The pressing query that follows is: can we disentangle the TV partner-level (e.g., Disney, ESPN) performance to potentially reallocate investments from the underperformers to the stronger ones?

We propose a structured Designated Market Area (DMA) test as a potential approach. Inspired by cooperative game theory and first published by Shapley [4], the test can be designed in a way that makes it possible to obtain the precise partner-level contributions. But the DMA test can prove complicated if not impossible to implement especially when the number of partners is large. Alternatively, Shapley Value Regression presents itself as a viable approach to measure partner-level performance without the need of experimental tests and consequently, effectively transforming the attribution problem into a variable importance problem within regression models. A case with real data from the financial service industry is given to illustrate how Shapley Value Regression is applied. Furthermore, a straightforward method for deriving co-efficient is proposed and compared against other proposed approaches. The python program in Jupyter Notebook is attached in the appendix.

## 2. Structured DMA Test with Shapley Value

#### 2.1 Shapley Value

The theory behind Shapley Values as presented by Shapley [4] is based on cooperative game theory where a set of players, referred to as a coalition, are playing towards a common goal. The value added by each player towards achieving the goal can be quantified by considering all possible combinations of players against the outcome and thereby assessing their marginal contribution. Shapley Values are calculated as the weighted average of these marginal contributions and since the players are put through all possible scenarios, the credit attributed by averaging these values is mathematically and intuitively fair.

Shapley Value is calculated as follows:

$$\varphi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|! (n - |S| - 1)!}{n!} \left( v(S \cup \{i\}) - v(S) \right) \tag{1}$$

 $\varphi_i$ : Shapley Value of channel i

N: Total number of partners within channel i

S: Coalitions excluding channel i

v: utility function that represents contributions from a coalition

#### 2.2 DMA Tests Implementation with Shapley Value

In a hypothetical scenario that MMM indicates the TV channel is driving \$10MM sales/year with commercials aired on Disney, ESPN and CNN. The goal is to disentangle and measure the impact of each TV partner. A structured DMA test inspired by game-theory is proposed to solve the problem.

In this case the players are TV partners (Disney, ESPN and CNN). But applying the game theory approach involves more than just implementing the Shapley Value equation. Often, the definition and collection of input data may play a bigger role. To accurately calculate the contributions of each coalition, the most ideal way is through tests. Nielsen coined the term DMA [5] that divides US market into 210 areas and has become a popular way to split test and control geo-graphically in the cross-channel environment. In this case, DMA is the only feasible way to split test and control groups. Here are the steps:

Step 1. Identify 8 comparable DMAs with similar population profiles.

Step 2. Conduct the DMA test and track the sales by DMA

DMA	Partners Aired	Sales
1	NA (baseline sales)	\$ 1000
2	Disney	\$ 10,000
3	ESPN	\$ 5,000
4	CNN	\$ 12,000
5	ESPN, CNN	\$ 15,000
6	Disney, ESPN	\$ 12,000
7	Disney, CNN	\$ 20,000
8	Disney, ESPN, CNN	\$ 25,000

Step 3. Calculate the Shapley Value for Disney based on the Shapley Value equation (1):

Coalitions	Sales	Coalitions without Disney	Sales	without Disney	Mar	ginal Contributions from Disney	Shapley Value Weight	Shap	ley Value
Disney	\$ 10,000	NA	\$	1,000	\$	9,000	0.33	\$	3,000
Disney, ESPN	\$ 12,000	ESPN	\$	5,000	\$	7,000	0.17	\$	1,167
Disney, CNN	\$ 20,000	CNN	\$	12,000	\$	8,000	0.17	\$	1,333
Disney, ESPN, CNN	\$ 25,000	ESPN,CNN	\$	15,000	\$	10,000	0.33	\$	3,333

Shapley Value for Disney \$ 8,833

Step 4. Repeat the same process for ESPN and CNN

Coalitions	Sales	Coalitions without ESPN	Sales	without ESPN	Mai	rginal Contributions from ESPN	Shapley Value Weight	Sha	apley Value
ESPN	\$ 5,000	NA	\$	1,000	\$	4,000	0.33	\$	1,333
ESPN, Disney	\$ 12,000	Disney	\$	10,000	\$	2,000	0.17	\$	333
ESPN, CNN	\$ 15,000	CNN	\$	12,000	\$	3,000	0.17	\$	500
Disney, ESPN, CNN	\$ 25,000	Disney,CNN	\$	20,000	\$	5,000	0.33	\$	1,667

Shapley Value for ESPN \$ 3,833

Coalitions	Sales	Coalitions without CNN	Sales	without CNN	Mar	ginal Contributions from CNN	Shapley Value Weight	Shaple	y Value
CNN	\$ 12,000	NA	\$	1,000	\$	11,000	0.33	\$	3,667
CNN, ESPN	\$ 15,000	ESPN	\$	5,000	\$	10,000	0.17	\$	1,667
CNN, Disney	\$ 20,000	Disney	\$	10,000	\$	10,000	0.17	\$	1,667
Disney, ESPN, CNN	\$ 25,000	Disney, ESPN	\$	12,000	\$	13,000	0.33	\$	4,333

Shapley Value for CNN \$ 11,333

Step 5. Calculate the % contribution

Partners	Shap	oley Value	%Contribution
ESPN	\$	3,833	16%
Disney	\$	8,833	37%
CNN	\$	11,333	47%
Total	\$	24,000	100%

Step 6.

Extrapolate the DMA test result to the national level. There we can derive the \$ contribution by partner.

Partners	%Contribution	\$Contribution			
ESPN	16%	\$	1,597,222		
Disney	37%	\$	3,680,556		
CNN	47%	\$	4,722,222		
Total	100%	\$	10,000,000		

Although testing is considered the golden standard to detect the casual relationship due to concerns of the Simpson's paradox [6], there are unique challenges in this space:

- 1. The number of testing DMAs will increase exponentially as the number of partners increases. For example, 6 partners require 64 comparable DMAs. The execution of such test is complicated and prone to error, not to mention 64 comparable DMAs are impossible to find.
- 2. Unlike in-channel creative testing, certain cross-contaminations between testing and control DMAs are inevitable as people travel across DMAs.

Hence, the Shapley Value Regression is a more practical approach to address the problem.

### 3. Shapley Value Regression

Expanding upon the game-theory and in the context of regression, Shapley Value Regression, initially proposed by Lindeman et al. [7] and later popularly presented by Kruskal [8][9], treats the independent variables in the regression analysis as players that collaboratively aim to predict the dependent variable, mirroring the cooperative nature of a game.

Shapley Value Regression stands out in the cross-channel partner measurement domain due to several key characteristics:

- 1. The sum of total partner contributions will always sum up to the channel contribution.
- 2. A partner that makes no marginal contribution to any coalition will get zero credit but not negative credit. This is aligned with the prevalent assumption that unlike political marketing, commercial marketing efforts typically yield a positive impact on sales or, at the least, maintain status quo, rather than negatively affecting outcomes.
- 3. It mitigates multicollinearity problem, which is commonly seen in cross-channel-partner marketing.

The following sub-sections detail the Shapley Value Regression approach to better understand partner efficiencies through a real-world data set sourced from the financial services industry.

#### 3.1 Understanding the Data

Similar to the hypothetical example, TV is the marketing channel under study in the following example and partners are the different TV network partners where advertisements were aired from the period of July 2020 to December 2022. Gross rating point (GRP) is used to represent the viewership of each partner and is considered the independent variables. The dependent variable is the attributed TV sales from MMM. To re-iterate the problem, the goal is to further attribute the overall TV sales to each TV partner.

Table 1. A snapshot of the time-series data with the dependent and independent variables

Week	Partner A	Partner B	Partner C	Partner D	Partner E	TV Sales from MMM
6/1/2020	1.713	-0.677	0.100	-0.458	1.109	0.520
6/8/2020	1.995	-0.973	0.210	-0.471	1.172	0.607
6/15/2020	2.145	-0.811	0.147	-0.398	0.977	0.667
6/22/2020	2.276	-0.940	-0.031	-0.292	0.839	0.636
6/29/2020	-0.973	-0.973	-0.973	-0.973	-0.973	-0.773
7/6/2020	-0.973	-0.973	-0.836	-0.973	-0.973	-1.176
7/13/2020	1.168	-0.818	0.272	0.153	1.704	0.073
7/20/2020	1.216	0.260	0.498	1.489	1.115	1.072
7/27/2020	1.882	-0.486	0.133	0.871	1.244	1.168
8/3/2020	3.301	-0.756	-0.001	0.494	1.825	1.582
8/10/2020	4.677	-0.268	0.433	0.576	1.578	2.320
8/17/2020	1.693	-0.407	0.289	1.078	1.360	1.724
8/24/2020	-0.973	-0.787	-0.970	-0.973	-0.973	-0.412
8/31/2020	1.027	-0.604	1.502	0.662	0.645	0.653
9/7/2020	1.241	-0.569	1.727	2.068	0.710	1.503
9/14/2020	0.699	-0.544	1.749	2.437	0.544	1.696
9/21/2020	-0.973	-0.973	-0.973	-0.973	-0.973	-0.500
9/28/2020	-0.851	-0.973	-0.645	-0.973	-0.973	-0.969
10/5/2020	0.147	-0.593	-0.204	1.071	0.061	-0.282
10/12/2020	0.540	0.011	-0.849	1.178	0.234	0.079
10/19/2020	0.715	0.602	-0.664	0.817	0.417	0.398

Note: Data has been scaled and normalized to keep confidentiality

It is worthy to note that if the MMM was built at a channel-partner level instead of the channel level, performing Shapley Value Regression could have been avoided. But since MMM will be at high risk of overfitting if all the media channels are at partner level, Shapley Value Regression can prove to be extremely beneficial in such scenarios.

#### 3.2 Relative Importance of Marketing Partners

 $R^2$ , or the coefficient of determination, is the proportion of the variation in the dependent variable that is explained by the independent variables. As proposed by Lindeman et al. [7], the relative importance of marketing partners is calculated by performing linear regression on every possible coalitions of independent variables and averaging the marginal increase in  $R^2$  values.

Table 2 details all the possible coalitions of the predictors and the corresponding  $R^2$  resulting from their linear regressions. For the use case being discussed in this paper, the intercept in linear regression is forced to zero with the business intuition that the TV sales indicated by MMM would be 100% attributed back to TV partners within that channel.

Table 2. All coalitions of Partner A, B, C, D and E

S. No.	Coalition	$R^2$
0		0
1	[Partner A]	0.562
2	[Partner A, Partner B]	0.647
3	[Partner A, Partner C]	0.751
4	[Partner A, Partner D]	0.756
5	[Partner A, Partner E]	0.739
6	[Partner A, Partner B, Partner C]	0.799
7	[Partner A, Partner B, Partner D]	0.793
8	[Partner A, Partner B, Partner E]	0.812
9	[Partner A, Partner C, Partner D]	0.843
10	[Partner A, Partner C, Partner E]	0.818
11	[Partner A, Partner D, Partner E]	0.793
12	[Partner A, Partner B, Partner C, Partner D]	0.870
13	[Partner A, Partner B, Partner C, Partner E]	0.869
14	[Partner A, Partner B, Partner D, Partner E]	0.840
15	[Partner A, Partner C, Partner D, Partner E]	0.856
16	[Partner A, Partner B, Partner C, Partner D, Partner E]	0.889

The following step calculates the Shapley Value of each partner using the formula below.

Let there be *n* channel partners  $\{x_1, x_2, \ldots, x_n\}$  working in a coalition *N*, the Shapley Value of an individual partner *i*, represented by  $\varphi_i(v)$ , can be calculated as:

$$\varphi_{i}(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|! (n - |S| - 1)!}{n!} \Big( v \Big( S \cup \{i\} - v(S) \Big) \Big)$$
 (1)

In this formula, marginal contribution is the value added by each partner which is calculated as:  $(v(S \cup \{i\}) - v(S))$ 

Since  $R^2$  is the chosen utility function in this analysis, the marginal contribution of a partner(s) can be calculated as the increments in  $R^2$  value of the coalition that contains the partner compared to the coalition where that partner is absent.

*Table 3. The marginal contribution for Partner A.* 

S. No.	Coalition	Coalition Excluding Partner A	R <sup>2</sup> of Coalition	R <sup>2</sup> of Coalition Excluding Partner A	Marginal Contribution from Partner A
1	[Partner A]	[]	0.562	0.000	0.562
2	[Partner A, Partner B]	[Partner B]	0.647	0.281	0.366

3	[Partner A, Partner C]	[Partner C]	0.751	0.559	0.192
4	[Partner A, Partner D]	[Partner D]	0.756	0.510	0.247
5	[Partner A, Partner E]	[Partner E]	0.739	0.605	0.134
6	[Partner A, Partner B, Partner C]	[Partner B, Partner C]	0.799	0.656	0.142
7	[Partner A, Partner B, Partner D]	[Partner B, Partner D]	0.793	0.600	0.193
8	[Partner A, Partner B, Partner E]	[Partner B, Partner E]	0.812	0.731	0.081
9	[Partner A, Partner C, Partner D]	[Partner C, Partner D]	0.843	0.716	0.127
10	[Partner A, Partner C, Partner E]	[Partner C, Partner E]	0.818	0.732	0.086
11	[Partner A, Partner D, Partner E]	[Partner D, Partner E]	0.793	0.662	0.130
12	[Partner A, Partner B, Partner C, Partner D]	[Partner B, Partner C, Partner D]	0.870	0.767	0.102
13	[Partner A, Partner B, Partner C, Partner E]	[Partner B, Partner C, Partner E]	0.869	0.812	0.056
14	[Partner A, Partner B, Partner D, Partner E]	[Partner B, Partner D, Partner E]	0.840	0.754	0.085
15	[Partner A, Partner C, Partner D, Partner E]	[Partner C, Partner D, Partner E]	0.856	0.769	0.087
16	[Partner A, Partner B, Partner C, Partner D, Partner E]	[Partner B, Partner C, Partner D, Partner E]	0.889	0.829	0.060

$$\frac{|S|! (n-|S|-1)!}{n!}$$

This part of the equation can be understood as the weight. It is calculated as follows:

Table 4. The calculated weights for Partner A.

S.	Coalition	Coalition Excluding	S	N	S !(n- S -1)!
No.		Partner A			<i>n</i> !
1	[Partner A]	[]	0	5	0.2
2	[Partner A, Partner B]	[Partner B]	1	5	0.05
3	[Partner A, Partner C]	[Partner C]	1	5	0.05
4	[Partner A, Partner D]	[Partner D]	1	5	0.05
5	[Partner A, Partner E]	[Partner E]	1	5	0.05
6	[Partner A, Partner B, Partner C]	[Partner B, Partner C]	2	5	0.033333
7	[Partner A, Partner B, Partner D]	[Partner B, Partner D]	2	5	0.033333
8	[Partner A, Partner B, Partner E]	[Partner B, Partner E]	2	5	0.033333
9	[Partner A, Partner C, Partner D]	[Partner C, Partner D]	2	5	0.033333
10	[Partner A, Partner C, Partner E]	[Partner C, Partner E]	2	5	0.033333
11	[Partner A, Partner D, Partner E]	[Partner D, Partner E]	2	5	0.033333
12	[Partner A, Partner B, Partner C, Partner D]	[Partner B, Partner C, Partner D]	3	5	0.05
13	[Partner A, Partner B, Partner C, Partner E]	[Partner B, Partner C, Partner E]	3	5	0.05
14	[Partner A, Partner B, Partner D, Partner E]	[Partner B, Partner D, Partner E]	3	5	0.05

15	[Partner A, Partner C, Partner D,	[Partner C, Partner D,	3	5	0.05
	Partner E]	Partner E]			
16	[Partner A, Partner B, Partner C,	[Partner B, Partner C,	4	5	0.2
	Partner D, Partner E]	Partner D, Partner E]			

From equation (1) the Shapley Value can now be calculated as the product of weight and marginal contribution summed over all the coalitions that Partner A belongs to.

Table 5 shows the calculation of the Shapley Value for Partner A. The Shapley Values for the rest of the partners are calculated in a similar fashion and the results are shown in Table 6.

Table 5. Shapley Values for Partner A

S.	Coalition	Coalition Excluding	S !(n- S -1)!	Marginal	Shapley
No.		Partner A	n!	Contribution	Value
1	[Partner A]	[]	0.2	0.562	0.1123
2	[Partner A, Partner B]	[Partner B]	0.05	0.366	0.0183
3	[Partner A, Partner C]	[Partner C]	0.05	0.192	0.0096
4	[Partner A, Partner D]	[Partner D]	0.05	0.247	0.0123
5	[Partner A, Partner E]	[Partner E]	0.05	0.134	0.0067
6	[Partner A, Partner B, Partner C]	[Partner B, Partner C]	0.033333	0.142	0.0047
7	[Partner A, Partner B, Partner D]	[Partner B, Partner D]	0.033333	0.193	0.0064
8	[Partner A, Partner B, Partner E]	[Partner B, Partner E]	0.033333	0.081	0.0027
9	[Partner A, Partner C, Partner D]	[Partner C, Partner D]	0.033333	0.127	0.0042
10	[Partner A, Partner C, Partner E]	[Partner C, Partner E]	0.033333	0.086	0.0029
11	[Partner A, Partner D, Partner E]	[Partner D, Partner E]	0.033333	0.130	0.0043
12	[Partner A, Partner B, Partner C, Partner D]	[Partner B, Partner C, Partner D]	0.05	0.102	0.0051
13	[Partner A, Partner B, Partner C, Partner E]	[Partner B, Partner C, Partner E]	0.05	0.056	0.0028
14	[Partner A, Partner B, Partner D, Partner E]	[Partner B, Partner D, Partner E]	0.05	0.085	0.0043
15	[Partner A, Partner C, Partner D, Partner E]	[Partner C, Partner D, Partner E]	0.05	0.087	0.0043
16	[Partner A, Partner B, Partner C, Partner D, Partner E]	[Partner B, Partner C, Partner D, Partner E]	0.2	0.060	0.0121

ley Values for Partner A 0.213	Shapley Values for Partner A
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The Shapley Values shown in Table 6 provide an idea of the relative importance of each of the independent variables in predicting the target variable. The Shapley Values can be normalized to find the share of sales driven by each partner.

Table 6. Shapley Value for all partners

S. No.	Parent Owner	Shapley Value	Normalized Share
1	Partner A	0.213	24%
2	Partner B	0.105	12%
3	Partner C	0.203	23%
4	Partner D	0.166	19%
5	Partner E	0.203	23%
	Total	0.889	100%

From Table 6, it is also observed that Shapley Values of each partner obtained by using the Shapley Value Regression method are always positive if not 0, and total to the  $R^2$  when all the partners work in a coalition.

This inherent property of Shapley Values is very useful when deriving partner-level insights from a highly correlated dataset by providing the ability to isolate the performance of each partner.

The normalized Shapley share  $\varphi_i(v)_{norm}$  for the  $i^{th}$  channel partner obtained from above can be multiplied with the response variable column (Y) to determine the sales attributed to that partner, represented by O.

$$O_i = \sum(Y) * \varphi_i(v)_{norm} \tag{2}$$

Leveraging any additional information such as the marketing spend on that partner and the GRPs received by them, it is possible to arrive at an estimated cost per sales and the partner efficiencies can be determined in the method that is beneficial for the use case.

## 4. Calculating Coefficients in Shapley Value Regression

The coefficients are not necessary to quantify the overall partner level attribution. But if the coefficients are available, it is beneficial to

- 1. plot the week-over-week predicted values and visually see how well they fit against the actual data.
- 2. forecast future channel-partner level sales

The best approach to calculate the co-efficient is being debated. Lipovetsky [11] found the Sharply Value net effect and adjusted the coefficients by decomposing the  $R^2$  and forming a quadratic equation to minimize its square distance with Sharply Value. Building on that approach, Mishra [10] provided different optimization techniques such as Host-Parasite Co-Evolutionary (HPC).

Gromping and Landau [12] criticized Lipovetsky's approach being misleading in certain cases. Their central argument is if there is a strong negative causal relationship between an independent variable and dependent variable, Lipovetsky's approach can assign a large positive coefficient. Gromping then offered an OLS with non-negativity constraints to possibly trim the variable when the relationship is negative. However, we believe that Gromping's argument rather reveals the fundamental flaw of Shapley Value Regression itself. His non-negativity constraints approach is nothing related to the game theory and will suffer the same multi-collinearity issue as the regular regression.

In this paper we propose a very straightforward approach to calculate the coefficients in Shapley Value Regression. The regression equation can be written as

$$Y = \sum_{1}^{n} (\beta_{i} * \sum_{1}^{m} x_{i,j})$$
 (3)

 $\beta_i$  is the coefficient for player  $x_i$ 

i is the ith player

n is the total number of players

j is the jth data points in the time-series

m is the total number of data points in the time-series

Note there is no intercept in the equation assuming Y is contributed 100% by the sum of  $x_i$ . Given that total contributed sales by partner i has been calculated as  $o_i$ , the regression equation can also be written as

$$Y = \sum_{i=1}^{n} O_i \tag{4}$$

Given (3) and (4), the beta coefficients could be technically calculated as:

$$\beta_i = \frac{O_i}{\sum_{1}^{m} x_{i,j}}$$

#### 4.1 Comparison of Co-efficient Deduction Approaches in Shapley Value Regression

In Mishra's paper [10], the author calculated the coefficients using Shapley Values by using quadratic optimization.

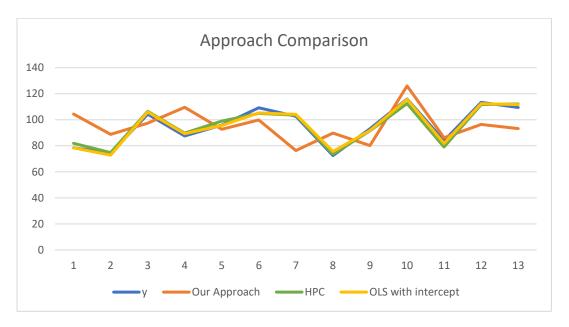
Mishra used a dummy dataset and compared linear regression and the coefficients produced by HPC. The dataset is below:

Tal	Table 7. The Portland Cement Dataset (cf. Woods, Steinour and Starke, 1932)												
	1	2	3	4	5	6	7	8	9	10	11	12	13
$x_1$	7	1	11	11	7	11	3	1	2	21	1	11	10
$x_2$	26	29	56	31	52	55	71	31	54	47	40	66	68
$x_3$	6	15	8	8	6	9	17	22	18	4	23	9	8
$x_4$	60	52	20	47	33	22	6	44	22	26	34	12	12
y	78.5	74.3	104.3	87.6	95.9	109.2	102.7	72.5	93.1	115.9	83.8	113.3	109.4

Using the dataset above, we found the coefficients using our approach and compared them to Mishra's paper and linear regression (OLS).

Table 8	. Regression	Coefficients	by Method
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		Co-eff	Intercept/Constant		
	$x_1$	$x_2$	$x_3$	$x_4$	intercept/Constant
Our Approach	3.24006	0.5875	1.11326	0.99515	0
HPC - Regular	0.82883	0.33204	-0.6289	-0.3136	90.1
OLS	1.551	0.51	0.102	-0.144	62.4



From there, we compared how each coefficient performed when predicting values. The blue line is the actual dependent variable. The HPC approach is close to OLS, both of which unarguably fit the actual dependent variable better. Although our approach underperforms HPC and OLS for fitness, both HPC and OLS would suffer interpretability as the negative co-efficient  $(x_3, x_4)$  violates the basic assumption in our use case, which is marketing partners will not have a negative impact on business KPIs. The HPC approach surprisingly gave negative co-efficient despite it is based on

Shapley Value Regression, because the cost function proposed by Lipovetsky is trying to minimize the sum of square difference of the Shapley Value of each player, and the contribution of each player to  $R^2$ , but it doesn't guarantee the Shapley Value of each player will remain positive. The approach that Gromping proposed will not only suffer multi-collinearity, but is also not practical in the business context when multiple partners (especially the top invested partners) are trimmed due to the negative sign, indicating none of them contributing anything to the channel.

It is possible to improve Lipovetsky's approach with additional restrictions to make it more practical in the marketing measurement space:

- 1. The sign of each coefficient >0
- 2. The magnitude of each coefficient should be in the same order as the Shapley Value

However, with these restrictions that aim to improve the interpretability, the  $R^2$  or the goodness of fit is likely to be compromised.

### 5. A Python Program in Jupyter Notebook

A Python program in Jupyter Notebook is attached in the Appendix. The input data is a time-series data that is stored as an Excel file with column headers. The first column is the date (in weeks), the last column is the dependent variable while the rest are independent variables. Only basic python packages are used to produce the Shapley Value Regression and the co-efficient following the steps illustrated in the above section. Please note this is not to be confused with packages like SHAP that aim to explain the variable importance in machine learning models.

#### 6. Conclusion

In conclusion, Shapley Value Regression offers a promising avenue to unravel partner-level performance within marketing channels. Its ability to dissect contributions without succumbing to multicollinearity presents a valuable tool for marketers. The discussion on coefficient calculation methods underscores the need for a balanced approach between interpretability and model accuracy.

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#### **Appendix**

```
#package installations
import pandas as pd
import numpy as np
import itertools
import math
from matplotlib import pyplot as plt
import sklearn
from sklearn.linear model import LinearRegression
from sklearn.preprocessing import StandardScaler
#function to read input data
def read_data(path, sheet_name):
      input data = pd.read excel(path, sheet name=sheet name, engine='openpyx1')
      input_data = input_data.loc[:, ~input_data.columns.str.contains('^Unnamed')]
      input_data.fillna(0, inplace = True)
    return input data
#function to form all combinations of independent variables
def form combinations(all columns, dependent var, excluded vars):
      independent_vars = list(set(all_columns) - set(excluded_vars) -
set(dependent_var))
      independent_vars.sort()
      all_combinations = []
      for var in range(len(independent_vars) + 1):
             for var_subset in itertools.combinations(independent_vars, var):
                   all_combinations.append(var_subset)
      return independent vars, all combinations
```

```
#function to run Linear Regression for each combination of independent variables
def form_coalitions(X, y, x_var_combinations):
      coalitions = pd.DataFrame(columns=["Independent Variables", "R Squared",
"Coefficients"])
      for independent_var_combo in x_var_combinations:
             if independent_var_combo != ():
                   X_curr = X[list(independent_var_combo)]
                   scaler = StandardScaler()
                   scaler.fit(X_curr)
                   X_std = scaler.transform(X_curr)
                   X_std = pd.DataFrame(X_std)
                   X_std.columns = X_curr.columns
                   scaler.fit(pd.DataFrame(y))
                   Y_std = scaler.transform(pd.DataFrame(y))
                   model = LinearRegression(fit_intercept=False).fit(X_std,Y_std)
                   r_sq = model.score(X_std, Y_std)
                          coeff = model.coef
                   coalition_result = {
                   "Independent Variables": list(independent_var_combo),
                    "R Squared": r sq, "Coefficients": coeff
                   coalitions = coalitions.append(coalition_result,
                                                    ignore_index = True)
             else:
                   r_sq = 0
```

```
"Independent Variables": list(independent_var_combo),
                   "R Squared": r_sq, "Coefficients": [0]
                   }
                   coalitions = coalitions.append(coalition result,
                                                     ignore_index = True)
    coalitions['R Squared'] = coalitions['R Squared'].astype(float)
    return coalitions, X_std, Y_std
#function to calculate Shapley value for each coalition
def get_shapley_values(independent_vars, coalitions):
      shapley_values = pd.DataFrame(columns = ["Coalition", "Current Partner",
                                 "Other Partners", "R Squared", "S", "N",
             "Weight", "Marginal Contribution", "Shapley Value"])
      for var in independent_vars:
             for index, row in coalitions.iterrows():
                   var_combo_list = row['Independent Variables']
                   if var in row['Independent Variables']:
                          other_channels = list(filter(lambda x: x not in
                                              set([var]),var_combo_list))
                          r_sq = row['R Squared']
                          s = len(var combo list) - 1
                          n = len(independent_vars)
                          weight = math.factorial(abs(s))*math.factorial(n-
                                              abs(s)-1)/math.factorial(n)
                          marginal contribution = 0
                          shapley_value = 0
                          shapley_values = shapley_values.append({'Coalition':
                                 var_combo_list, 'Current Partner': var,
```

coalition result = {

```
r_sq, 'S': s, 'N':n , "Weight": weight, 'Marginal
                                 Contribution': marginal_contribution, 'Shapley
                                 Value': shapley_value}, ignore_index = True)
      for shapley_index, shapley_row in shapley_values.iterrows():
             for coalition index, coalition row in coalitions.iterrows():
                    if shapley_row['Other Partners'] == coalition_row['Independent
                                                                  Variables']:
                          marginal contribution = shapley row['R Squared'] -
                                                     coalition_row['R Squared']
                          shapley values.loc[shapley index, 'Marginal Contribution']
                                                           = marginal contribution
                          shapley_values.loc[shapley_index, 'Shapley Value'] =
                          shapley_row['Weight'] * marginal_contribution
      return shapley values
#function to get Shapley adjusted and regular Linear Regression coefficients
def get coefficients(shapley values, y, X, dependent var):
    total_shap = shapley_values['Shapley Value'].sum()
    norm_shapley_values = shapley_values.groupby("Current Partner")[['Shapley
                                                           Value']].sum()
    norm_shapley_values['Normalized Shapley Value'] = norm_shapley_values['Shapley
                                                     Value']/total shap*100
    norm shapley values = norm shapley values.reset index()
    norm_shapley_values = norm_shapley_values.rename(columns={'Current Partner':
                                                     'Parent Owner'})
    total y = sum(y[dependent var])
    norm_shapley_values["Reattributed Visits"] = norm_shapley_values['Normalized
                                              Shapley Value' | *total y/100
    reattr_values = norm_shapley_values['Normalized Shapley Value']*total_y/100
    shap_reg_coeff = []
    k=0
    for i in X train.columns:
```

'Other Partners': other channels, 'R Squared':

```
temp = []
for j in range(0, len(reattr_values)):
    i_sum = sum(X[i])
    temp.append(reattr_values[j]/i_sum)
    shap_reg_coeff.append(temp[k])
    k=k+1
norm_shapley_values["Shapley Coefficients"] = shap_reg_coeff

lin_reg = LinearRegression(fit_intercept = False).fit(X, y)
lin_reg_r = lin_reg.score(X,y)
lin_reg_coeff = lin_reg.coef_
norm_shapley_values["Regression Coefficients"] = lin_reg_coeff[0]
return norm_shapley_values
```