Physics 104

Taylor Expansion:

$$f(a+h) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} h^k, \qquad h = (x-a)$$

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$e^{ix} = \cos(x) + i \sin x$$

Euler's Equations:

$$\left.\begin{array}{ll} \textbf{a.} & f = f(y', x) \\ \textbf{a1.} & f = f(y') \end{array}\right\} \frac{\partial f}{\partial y} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial y'} = 0$$

$$\textbf{b.} \quad f = f(y', x)\} \; \frac{\partial f}{\partial x} - \frac{\mathrm{d}}{\mathrm{d}x}\left[f - y'\frac{\partial f}{\partial y'}\right] = 0$$

Cylindrical $\mathrm{d}s = \sqrt{\mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + \mathrm{d}z^2}$

explicit constraints (Lagrange Multipliers):

$$\frac{\partial f}{\partial y_i} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial y_i'} + \sum_i \lambda \frac{\partial g}{\partial y} = 0$$

The Lagrange Equation:

$$\mathcal{L} = T - U$$

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0$$

The Hamiltonian Equation:

$$\mathcal{H} = \sum_k \dot{q}_k P_k - \mathcal{L}, \quad P_k = \frac{\partial \mathcal{L}}{\partial \dot{q}_k}$$

$$\frac{\partial \mathcal{H}}{\partial P_k} = \dot{q}_k, \qquad -\frac{\partial \mathcal{H}}{\partial q_k} = \dot{P}_k$$

E is conserved if $\mathcal{L}$ has no time dependence

Reference Frames:

$$\begin{array}{ll} m\vec{a}_r = \vec{F} & \\ \quad -m\ddot{\vec{R}}_T & \text{(Trans. Accel.)} \\ \quad -m\dot{\vec{\omega}} \times \vec{r} & \text{(Rot. Accel.)} \\ \quad -2m\vec{\omega} \times \vec{v}_r & \text{(Coriolis Effect)} \\ \quad -m\vec{\omega} \times (\vec{\omega} \times \vec{r}) & \text{(Centrifugal)} \end{array}$$

Rigid Body Motion:

$$I_{ij} = \sum_\alpha m_\alpha(\delta_{ij})$$

(come back to this one)