MATHEMATICAL MODELING

CASINOS RISK OF RUIN

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WHAT IS THE ARTICLE LOOKING FOR?

The article proposes a new way of measuring the risk of bankruptcy of a casino using a scale called ROR scale (risk of ruin scale). This scale seeks to standardize risk levels, facilitating their comparison, and provides an accurate representation of the current state of the casino's risk. (Siu et al., 2023)

OTHER APPROACHES

The article mentions several prior methods such as:

1. Discrete Time Based Risk Model: Divides time into intervals and calculates the probability of ruin at each interval. Can be unwieldy with many intervals.

2. Bayesian method: Uses prior probabilities and updates them with new information. It is useful, but more complex and requires constant updating of data.

3. Markov Chain Risk Model: Uses the current state to predict the future, but is more complex and less flexible for modeling the risk of ruin in a casino.

MODEL INTRODUCTION

ASSUMPTIONS & LIMITATIONS

A gambler with:

- Unlimited credit
- Infinite number of games

Limitations: Sum of probabilities in the model does not give 1, because not all possible outcomes of the game are being considered. This indicates that the model does not cover all possible events or states of the casino, which prevents a proper normalization of the probabilities.

INTRODUCTION TO THE MODEL:

1. Competition between the gambler and the casino (binomial random walk).
2. Poisson application: it shows the number of consecutive wins the player needs to ruin the casino.
3. House advantage

PRINCIPAL ECUATIONS

Casinos Perspective

Casino's capital

$$n \leftarrow \begin{cases} n+1, & \text{casino's win} \\ n-1, & \text{gambler's win} \end{cases}$$

How many times the casino can take the risk of losing before it goes bankrupt

Gamblers Perspective

$$p =$$

Probability that players wins in a one game

$$1 - p =$$

Probability that casino wins

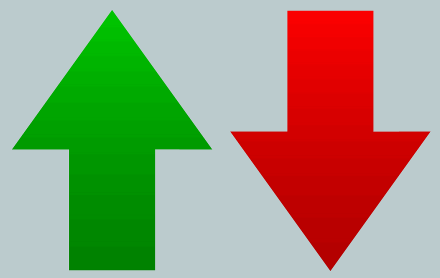
PRINCIPAL ECUATIONS

Probability of casino goes bankrupt

$$Pro(n) = \begin{cases} p^{n-k}, & k \leq n \\ 1, & k > n \end{cases}$$

Casino capital difference with victories

Probability of player wins in the next games to bankrupt casino

"n"  p < 1

Next step... Binomial analysis

Why binomial analysis? Need to determine "n" so that the gambler with the assumptions not overpass the casino

BINOMIAL ANALYSIS

$$\lim_{t\to\infty} \binom{t}{k} p^k (1-p)^{t-k}$$

Obtain the victories of the player in many test, with a constant success probability

Assumes that gamblers play “t” games (many as possible)

$p =$ Constant probability

$k =$ Victories of player

$t =$ Number of experiments

POISSON APPROACH

Now, since t is a very large value and the probability of p is low in comparison, there is a model that helps to describe this type of rare events: Poisson.

POISSON APPROACH

$$\lim_{t \to \infty} \binom{t}{k} p^k (1-p)^{t-k}$$

Having an expected value for success:

$$\mu = t \times p$$

$$\lim_{t \to \infty} \binom{t}{k} \left(\frac{\mu}{t}\right)^k \left(1 - \frac{\mu}{t}\right)^{t-k}$$

Expressed in GitHub

$$\binom{t}{k} = \frac{t!}{k!(t-k)!}$$

$$\lim_{t \to \infty} \left(1 - \frac{\mu}{t}\right)^{t-k} = e^{-\mu}$$

$$\frac{t^k}{k!} \left(\frac{\mu}{t}\right)^k e^{-\mu}$$

$$\frac{\mu^k e^{-\mu}}{k!}$$

With this, thanks to the assumption that t is large, we were able to simplify the binomial formula to make it more manageable, which allowed us to arrive at the Poisson approximation.

POISSON APPROACH

We model how many “K” rounds the player/gambler needs to break the casino. But we don't just want that, right?. We want all the probabilities in which the player breaks the casino.

$$Pro(n) = \sum_{k=0}^{\infty} \frac{\mu^k}{k!} e^{-\mu} \begin{cases} p^{n-k}, & k \leq n \\ 1, & k > n \end{cases}$$

$$e^{-\mu} \left(\underbrace{\sum_{k=0}^{n} \frac{\mu^k}{k!} p^{n-k}}_{k \leqslant n} + \underbrace{\sum_{k=n+1}^{\infty} \frac{\mu^k}{k!}}_{k > n} \right)$$

$$e^{-\mu} \left(\sum_{k=0}^{n} \frac{\mu^k}{k!} p^{n-k} + \sum_{k=n+1}^{\infty} \frac{\mu^k}{k!} \right)$$

Substracted the first n+1 terms from k=0 to k=n

Expressed in GitHub

$$\sum_{k=n+1}^{\infty} \frac{\mu^k}{k!} = \sum_{k=0}^{\infty} \frac{\mu^k}{k!} - \sum_{k=0}^{n} \frac{\mu^k}{k!}$$

Exponential definition

$$e^{-\mu} \left(\sum_{k=0}^{n} \frac{\mu^k}{k!} p^{n-k} + e^{\mu} - \sum_{k=0}^{n} \frac{\mu^k}{k!} \right)$$

POISSON APPROACH

Group the summatory terms

$$= \sum_{k=0}^{n} \frac{\mu^k}{k!} p^{n-k} - \sum_{k=0}^{n} \frac{\mu^k}{k!} = \sum_{k=0}^{n} \frac{\mu^k}{k!} \left(p^{n-k} - 1\right)$$

$$= -\sum_{k=0}^{n} \frac{\mu^k}{k!} \left(1 - p^{n-k}\right)$$

Take out the common factor

$$= e^{-\mu} e^{\mu} - e^{-\mu} \sum_{k=0}^{n} \frac{\mu^k}{k!} \left(1 - p^{n-k}\right)$$

$$Pro(n) = 1 - e^{-\mu} \sum_{k=0}^{n} \frac{\mu^k}{k!} \left(1 - p^{n-k}\right)$$

Adding...

HOUSE EDGE

It is always a > 0. Measured as a percentage and depends on the game you are playing:

1. Blackjack about 1%.
2. Craps greater than 1%
3. Poker is less than 1%.
4. Keno around 25%.
5. Sic bo around 15%.
6. Slot machines around 10%.

HOUSE EDGE

$$n \leftarrow \begin{cases} n + (1 + a), & \text{casino's win} \\ n - 1, & \text{gambler's win} \end{cases}$$

$$Pro(n) = \sum_{k=0}^{\infty} \frac{\mu^k}{k!} e^{-\mu} \begin{cases} p^{n(1+a)-k}, & k \leq n \\ 1, & k > n \end{cases}$$

$$Pro(n) = 1 - e^{-\mu} \sum_{k=0}^{n} \frac{\mu^k}{k!} \left(1 - p^{n(1+a)-k}\right)$$

WHY WE APPLY HOUSE EDGE?

Casinos advantage is more than 1 because of the house edge “a”.
Means that a long-term in the race vs the gambler it has further advance

That’s why we apply (1+a) to the equations, which represents this “set distance”.

Basic effect: Player need to win more times to reach “n”.

MONOTONICITY AND BOUNDARIES

$$\forall n_1, n_2 \in \mathbb{Z}^+, \quad n_1 < n_2 \Rightarrow Pro(n_1) > Pro(n_2)$$

For all $n1$ and $n2$ in the positive integers, if $n1 < n2$, then $Pro(n1) > Pro(n2)$. (determine its monotonicity)

$$Pro(n_1) = 1 - e^{-\mu} \sum_{k=0}^{n_1} \frac{\mu^k}{k!} (1 - p^{n_1(1+a)-k})$$

$$Pro(n_2) = 1 - e^{-\mu} \left[\sum_{k=0}^{n_1} \frac{\mu^k}{k!} (1 - p^{n_2(1+a)-k}) + \sum_{k=n_1+1}^{n_2} \frac{\mu^k}{k!} (1 - p^{n_2(1+a)-k}) \right]$$

Subtract and factorize with power properties

$$Pro\,(n_1) - Pro\,(n_2) = e^{-\mu} \left(\left(p^{n_1(1+a)} - p^{n_2(1+a)}\right) \sum_{k=0}^{n_1} \frac{\mu^k}{k!} \frac{1}{p^k} + \sum_{k=n_1+1}^{n_2} \frac{\mu^k}{k!} \frac{1}{p^k} \left(p^k - p^{n_2(1+a)}\right) \right)$$

As we now “” range (0, 1), p^x tell us: x rise the value will be decreasing

$$n_1\,(1 + a) < n_2\,(1 + a) \Rightarrow p^{n_1(1+a)} > p^{n_2(1+a)}$$

Theorem was proved

and boundaries will be:

$$1.0 > Pro\,(n) > 0.0$$

CONCLUSIONS

This equation is directly associated with the problem of quantifying the probability that a casino will be bankrupted.
Combines:

- Binomial approach
- Poisson Process
- Expected value(media)
- House advantage

Mathematics is essential in this context because the problem is inherently probabilistic and dynamic. The goal is not just to estimate single outcomes but to understand and quantify risk over time, under uncertainty.

CONCLUSIONS

The final formula provides a simple yet effective way to estimate the Risk of Ruin the probability that a casino could go bankrupt due to an extremely lucky winning streak by a gambler. It combines the Poisson distribution with the concept of house edge, making the model more realistic and applicable.

This tool can help casinos or game designers assess potential financial risk based on win probabilities, betting behavior, and built-in advantages. However, the model has limitations, as it assumes each round is independent and does not account for human factors like changing strategies, emotional decisions, or specific game rules, all of which can affect the actual outcome.

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