Introduction to Electrical Power Engineering

Final Exam

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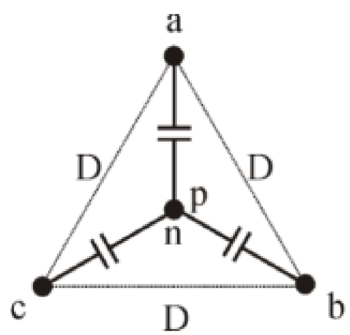
1. (20) Assume that 1. conductors are equally spaced, D, and have equal radii r. 2. $q_a + q_b + q_c = 0$ ($c_a = c_b = c_c = c$, $v_a + v_b + v_c = 0$) 3. $i_a + i_b + i_c = 0$. Find (a) c=? (b) λ_a=?



1. a.

Assume that

1. Conductors are equally spaced, D, and have equal radii r.
2. $q_a + q_b + q_c = 0$ ($c_a = c_b = c_c = c$, $v_a + v_b + v_c = 0$).

$$v_a = \frac{1}{2\pi\epsilon}\left(q_a \ln\frac{1}{r} + q_b \ln\frac{1}{D} + q_c \ln\frac{1}{D}\right) = \frac{1}{2\pi\epsilon}\left(q_a \ln\frac{1}{r} - q_a \ln\frac{1}{D}\right) = \frac{1}{2\pi\epsilon}\left(q_a \ln\frac{D}{r}\right)$$

$$C = \frac{q}{v}, \quad c_a = c_b = c_c = c = \frac{2\pi\epsilon}{\ln(D/r)} \quad \text{(F/m) to neutral}$$

$$c_a = c_b = c_c = 2\pi\epsilon \ln\frac{D_m}{r} \text{ for one line transposition}$$

$$c_a = c_b = c_c = 2\pi\epsilon \ln\frac{D_m}{R_{GMR}} \text{ for conductor bundling transposition}$$

$$D_m = (D_{12}D_{23}D_{13})^{1/3}$$

$$R_b^c = R_{GMR} = (rd_{12}d_{13}d_{1b})^{1/b}, \quad b > 1; \quad R_b^c = R_{GMR} = r, \text{ when } b = 1$$

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1. b.

Assume that

1. Conductors equally spaced D and have equal radii r.
2. $i_a + i_b + i_c = 0$.

$$\begin{aligned}
\lambda_a &= \left(\frac{\mu_0}{2\pi}\right)\{i_a[\mu_r/4 + \ln(1/r)] + i_b \ln(1/D) + i_c \ln(1/D)\} \\
&= \left(\frac{\mu_0}{2\pi}\right)\{i_a[\mu_r/4 + \ln(1/r)] - i_a \ln(1/D)\} \\
&= \left(\frac{\mu_0}{2\pi}\right)[\mu_r/4 + \ln(1/r) - \ln(1/D)] \times i_a \\
&= \left(\frac{\mu_0}{2\pi}\right)\left[\ln e^{(\mu_r/4)} + \ln(1/r) - \ln(1/D)\right] \times i_a \\
&= \left(\frac{\mu_0}{2\pi}\right)\left[\ln(1/re^{-(\mu_r/4)}) - \ln(1/D)\right] \times i_a \\
&= \left(\frac{\mu_0}{2\pi}\right)[\ln(1/r') - \ln(1/D)] \times i_a \\
&= \left(\frac{\mu_0}{2\pi}\right)[\ln(D/r')] \times i_a \\
&= l_a \times i_a
\end{aligned}$$

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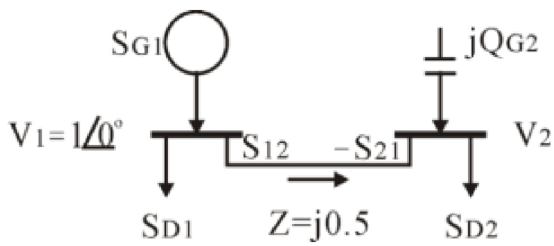
3. (20) $V_1 = 1\angle 0°$, $jQ_{G2} = j1.0$, $Z_L = j0.5$, $S_{D2} = P_{D2} + j1.0$. Find S_1 and V_2. We consider the solution as a function of P_{D2} for $P_{D2} \geqq 0$. (a) If $P_{D2} > 1$ => V_2 =? (b) If $P_{D2} = 1$ => V_2 =? (c) If $P_{D2} = 0.5$ => V_2 =?



2. a.

$z = r + j\omega l$ = series impedance per meter

$y = g + j\omega c$ = shunt admittance per meter to neutral

$\frac{dV}{dx} = zI$; $\frac{dI}{dx} = yV$

$\frac{d^2V}{dx^2} = yzV = \gamma^2 V$; $\frac{d^2I}{dx^2} = yzI = \gamma^2 I$

Propagation constant: $\gamma = (yz)^{0.5} = \alpha + j\beta$

$V = k_1 e^{\gamma x} + k_2 e^{-\gamma x} = (k_1 + k_2)\frac{(e^{\gamma x} + e^{-\gamma x})}{2} + (k_1 - k_2)\frac{(e^{\gamma x} - e^{-\gamma x})}{2}$

$$V = K_1 \cosh \gamma x + K_2 \sinh \gamma x \quad ; \quad K_1 = (k_1 + k_2) \quad , \quad K_2 = (k_1 - k_2)$$

$$\frac{dV}{dx} = K_1 \gamma \sinh \gamma x + K_2 \gamma \cosh \gamma x \quad (4.8)$$

When $x = 0$, $V = V_2 \Rightarrow V = k_1 + k_2 = K_1 = V_2$,

When $x = 0$, $I = I_2 \Rightarrow \frac{dV(0)}{dx} = zI_2 = K_2\gamma = (z/\gamma)I_2 = (z/y)^{0.5} I_2$

$Z_c = (z/y)^{0.5}$ (characteristic impedance)

$V = V_2 \cosh \gamma x + Z_c I_2 \sinh \gamma x \quad ; \quad I = I_2 \cosh \gamma x + (V_2/Z_c) \sinh \gamma x$

When $x = l$, $V = V_1$, $I = I_1 \Rightarrow$

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l;$$

$$I_1 = I_2 \cosh \gamma l + (V_2/Z_c) \sinh \gamma l$$

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2. b.

$z = r + j\omega l = 0.169 + j0.789 = 0.807\angle 77.9^\circ\ \Omega/\text{mile}$

$y = g + j\omega c = j5.38 \times 10^{-6} = 5.38 \times 10^{-6}\angle 90^\circ\ \text{mho/mile}$

$\Rightarrow Z_c = (z/y)^{0.5} = 387.3\angle -6.05^\circ\ \Omega$

$\Rightarrow \gamma l = 225(yz)^{0.5} = 0.4688\angle 83.95^\circ = 0.0494 + j0.466$

$2 \sinh \gamma l = e^{\gamma l} - e^{-\gamma l} = e^{0.0494} e^{j0.466} - e^{-0.0494} e^{-j0.466} = 1.051\angle 0.466\ \text{rad} - 0.952\angle -0.466$

$$\sinh \gamma l = 0.452\angle 84.4^\circ$$

$$2 \cosh \gamma l = e^{\gamma l} + e^{-\gamma l} = 1.051\angle 0.466\ \text{rad} + 0.952\angle -0.466\ \text{rad} = 1.790\angle 1.42^\circ$$

$$\cosh \gamma l = 0.895\angle 1.42^\circ$$

$$|V_2| = 132 \times 10^3/\sqrt{3} = 76.2\ \text{kV} \Rightarrow V_2 = 76.2\angle 0^\circ\ \text{kV}$$

$1\Phi\ P_{\text{load}} = 40 \times 10^6/3 = 13.33\ \text{MW} = 0.95|V_2||I_2| \Rightarrow |I_2| = 184.1,\ I_2 = 184.1\angle -18.195^\circ$

$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l \Rightarrow V_1 = 89.28\angle 19.39^\circ\ \text{kV} \Rightarrow V_{LL} = \sqrt{3} \times 89.28 = 154.64\ \text{kV}$

$$I_1 = I_2 \cosh \gamma l + (V_2/Z_c) \sinh \gamma l = 162.42\angle 14.76^\circ\ \text{A}$$

$$P_{12} = \text{real}(V_1 I_1^*) = 89.28 \times 10^3 \times 162.42 \cos(19.39^\circ - 14.76^\circ) = 14.45\ \text{MW}$$

Efficiency $= \frac{13.33}{14.45} = 0.92 = 92\%$

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2. (20) A 60-Hz 138-kV 3Φ transmission line is 225mile long. The distributed line parameters are r =0.169 Ω/mile , *l* = 2.093 mH/mile , c = 0.01427 μF/mile , g = 0. The transmission line delivers 40 Mwat 132 kV with 95% power factor lagging. (a) Find the sending-end voltage and current. (b) Find the transmission line efficiency.

3. a. b. c.

In this case, the capacitor injects a specified power, while the voltage is not controlled. Thus bus2 is a P, Q bus. In fact,

$$S_2 = S_{21} = S_{G2} - S_{D2} = j1.0 - (P_{D2} + j1.0) = -P_{D2}$$

$$S_{12} = |V_1|^2 e^{j\angle Z}/|Z| - |V_1||V_2|e^{j\angle Z}e^{j\theta_{12}}/|Z|$$

$$S_{21} = |V_2|^2 e^{j\angle Z}/|Z| - |V_1||V_2|e^{j\angle Z}e^{-j\theta_{12}}/|Z|$$

$$R = 0, \quad Z = jX = j0.5 \Rightarrow \angle Z = 90^\circ, \quad e^{j\angle Z} = j, \quad |V_1| = 1$$

$$S_{21} = j2|V_2|^2 - j2|V_2|e^{-j\theta_{12}} = -P_{D2}$$

We draw a receiving-end circle:

$$(2|V_2|^2)^2 + (P_{D2})^2 = (2|V_2|)^2 \Rightarrow 4|V_2|^4 - 4|V_2|^2 + (P_{D2})^2 = 0$$

$$|V_2|^2 = \left[1 \pm (1 - (P_{D2})^2)^{0.5}\right]/2$$

If $P_{D2} > 1 \Rightarrow |V_2|$ has no solution,

If $P_{D2} = 1 \Rightarrow |V_2| = 0.707,\ \theta_{12} = 45^\circ$
If $0 \leq P_{D2} \leq 1 \Rightarrow |V_2|$ has **two solutions**

We can find θ_{12} and $S_1 = S_{12}$

If $P_{D2} = 0.5 \Rightarrow V_2 = 0.97\angle - 15^\circ$ (OK) and $V_2 = 0.26\angle - 75^\circ$ (not OK)

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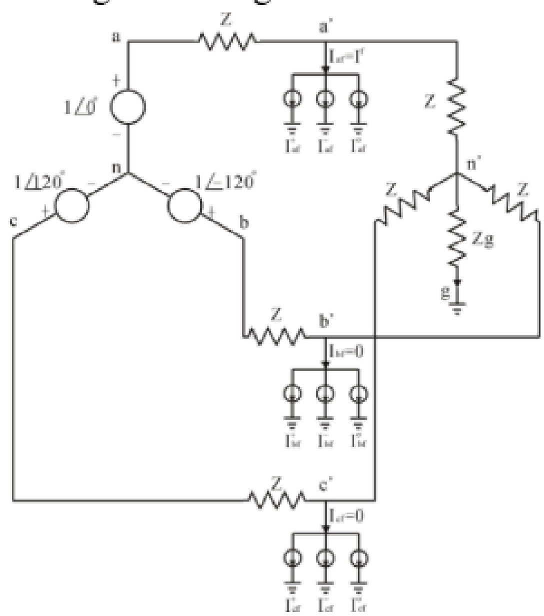
4. (20) Fault current: $I^f = [I_{af}\ I_{bf}\ I_{cf}] = [I^f \quad 0 \quad 0]$. Find the symmetrical components of single line-to-ground faults currents.



4.

$$(13.5) \quad \begin{bmatrix} I_{af} & I_{bf} & I_{cf} \end{bmatrix} = A \begin{bmatrix} I_{af}^0 & I_{af}^+ & I_{af}^- \end{bmatrix}$$

$$(13.6) \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

Fault current:

$$I^f = \begin{bmatrix} I_{af} & I_{bf} & I_{cf} \end{bmatrix} = \begin{bmatrix} I^f & 0 & 0 \end{bmatrix}$$

$$I_{af} = I_{af}^0 + I_{af}^+ + I_{af}^- = I^f$$

$$I_{bf} = I_{bf}^0 + I_{bf}^+ + I_{bf}^- = \left(\frac{I^f}{3}\right) \begin{bmatrix} 1 & \alpha^2 & \alpha \end{bmatrix} = 0$$

$$I_{cf} = I_{cf}^0 + I_{cf}^+ + I_{cf}^- = \left(\frac{I^f}{3}\right) \begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix} = 0$$

$$A^{-1} = \left(\frac{1}{3}\right) \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

$$\begin{bmatrix} I_{af}^0 & I_{af}^+ & I_{af}^- \end{bmatrix} = A^{-1} \begin{bmatrix} I^f & 0 & 0 \end{bmatrix} = \left(\frac{I^f}{3}\right) \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$I_{af}^+ = I_{af}^- = I_{af}^0 = \left(\frac{I^f}{3}\right)$$

Using Superposition:

- Positive Sequence
- Negative Sequence
- Zero Sequence

5.(20) $v_a = 180 \cos \omega t$, $v_b = 180 \cos (\omega t-120°)$, $v_c = 180 \cos (\omega t+120°)$ (a) Find abc Reference Frame to Stationary Reference Frame (b) Stationary Reference Frame to Synchronous Reference Frame

5. a.

$$\begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ 0 & -1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/2 & -\sqrt{3}/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix}$$

3-phase 220V$_{rms}$ (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377 \text{ rad/sec.}$

$v_a + v_b + v_c = 0$, balanced system, ($v_a + v_b + v_c \neq 0$, unbalanced system)

$$v_a = 180 \cos \omega t, \quad v_b = 180 \cos(\omega t - 120^\circ), \quad v_c = 180 \cos(\omega t + 120^\circ)$$

abc Reference Frame to Stationary Reference Frame

$$v_{qs} = \left(\frac{2}{3}\right) v_a + \left(-\frac{1}{3}\right) v_b + \left(-\frac{1}{3}\right) v_c = \left(\frac{2}{3}\right) v_a + \left(\frac{1}{3}\right) v_a = v_a = 180 \cos \omega t$$

$$v_{ds} = \left(-\frac{1}{\sqrt{3}}\right) v_b + \left(\frac{1}{\sqrt{3}}\right) v_c = \left(-\frac{1}{\sqrt{3}}\right) [v_a + 2v_b] = -180 \sin \omega t$$

Stationary Reference Frame to abc Reference Frame

$$v_a = v_{qs}, \quad v_b = \left(-\frac{1}{2}\right) v_{qs} + \left(-\frac{\sqrt{3}}{2}\right) v_{ds} = 180 \cos(\omega t - 120^\circ)$$

$$v_c = \left(-\frac{1}{2}\right) v_{qs} + \left(\frac{\sqrt{3}}{2}\right) v_{ds} = 180 \cos(\omega t + 120^\circ)$$

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5. b.

$$\begin{bmatrix} v_{qe} \\ v_{de} \end{bmatrix} = \begin{bmatrix} \cos \omega t & -\sin \omega t \\ \sin \omega t & \cos \omega t \end{bmatrix} \begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix}$$

$$\begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} = \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix} \begin{bmatrix} v_{qe} \\ v_{de} \end{bmatrix}$$

3-phase 220V$_{rms}$ (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377 \text{ rad/sec.}$

$v_{qs} = 180 \cos \omega t, \quad v_{ds} = -180 \sin \omega t$

Stationary Reference Frame to Synchronous Reference Frame

$$v_{qe} = \cos \omega t \; v_{qs} - \sin \omega t \; v_{ds} = 180$$

$$v_{de} = \sin \omega t \; v_{qs} + \cos \omega t \; v_{ds} = 0$$

Synchronous Reference Frame to Stationary Reference Frame

$$v_{qs} = \cos \omega t \; v_{qe} + \sin \omega t \; v_{de} = 180 \cos \omega t$$

$$v_{ds} = -\sin \omega t \; v_{qe} + \cos \omega t \; v_{de} = -180 \sin \omega t$$