Battleships

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1 Introduction

Battleships also known as *Bimaru* is a logic puzzle in a square grid of 10 x 10 cells with simple rules and challenging solutions.

The rules of Battleships are simple:
You have to find the location of the battleships hidden in the grid.
The armada includes one battleship four squares long, two cruisers three squares long, three destroyers two squares long, and four submarines one square in size.
Each ship occupies a number of contiguous squares on the grid, arranged horizontally or vertically.
The ships are placed so that no ship touches any other ship, not even diagonally.
The numbers outside the grid show the number of cells occupied by battleships on that row/column.

2 Modeling

2.1 Variables

2.1.1 Boats

$n \in [\![1, 4]\!]$ where n is the size of the boat.
$k \in [\![1, 10]\!]$ is a boat such that :

- k $\in [\![1, 4]\!] \implies$ boat is of size n = 1
- k $\in [\![5, 7]\!] \implies$ boat is of size n = 2
- k $\in [\![8, 9]\!] \implies$ boat is of size n = 3
- $k = 10 \implies$ boat is of size n = 4

$B_{i,j,k}$: The upper left corner of the boat k at the coordinate (i,j)

2.1.2 Orientation

O_k : the boat k is vertical
$\neg O_k$: the boat k is horizontal

2.1.3 Cell

$X_{i,j,k}$: cell (i,j) occupied by boat k

2.2 Constraints

2.2.1 Boat Existence

The boat k occupies the cells such that:

$$(B_{i,j,k} \wedge O_k \iff \bigwedge_{l=0}^{n-1} X_{i+l,j,k}) \wedge (B_{i,j,k} \wedge \neg O_k \iff \bigwedge_{l=0}^{n-1} X_{i,j+l,k})$$

There is at least one boat k :

$$\bigvee_{(i,j)} B_{i,j,k}$$

There is at most one boat k :

$$\bigwedge_{(i,j)\neq(i',j')} \neg B_{i,j,k} \vee \neg B_{i',j',k}$$

2.2.2 In every cell there is only one boat or section of boat.

$$\bigvee_{i,j,k} X_{i,j,k_1} X_{i,j,k_2} = 0$$

2.2.3 The numbers outside the grid show the number of cells occupied by battleships on that row/column.

$$\binom{10}{M}$$

where M is the number of occupied cells in each line or column.

2.2.4 The boats are placed so that no boat touches any other boat, not even diagonally.

$X_{i,j}$ is the initial placement of the boat.
Orientation = Horizontal

$$(B_{i,j,k} \wedge \neg O_k) \iff (\bigwedge_{k\in\{i-1,i+1\}} \bigwedge_{l\in[\![j-1,j+n]\!]} \neg X_{k,l}) \wedge \neg X_{i,j-1} \wedge \neg X_{i,j+n}$$

Orientation = Vertical

$$(B_{i,j,k} \wedge O_k) \iff \Big(\bigwedge_{k \in \{j-1, j+1\}} \bigwedge_{l \in [\![i-1, i+n]\!]} \neg X_{l,k} \Big) \wedge \neg X_{i-1,j} \wedge \neg X_{i+n,j}$$