



UNIVERSIDADE
FEDERAL DO CEARÁ

Lista 01 - Método Gráfico

Introdução à Pesquisa Operacional
1 de março de 2019

1. Resolver pelo método gráfico o seguinte problema:

$$\text{Max}(Z) = 3x_1 + 5x_2$$

Sujeito a:

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

2. Determinar as regiões factíveis e as soluções extremas de:

- $x_1 - x_2 \leq 2, x_1 - x_2 \geq -2, x_1 + x_2 \geq 1, x_1 \geq 0, x_2 \geq 0$
- $x_1 - 2x_2 \leq 2, 2x_1 - x_2 \geq -2, x_1 \geq 0, x_2 \geq 0$
- $x_1 + x_2 \geq 4, x_1 - 2x_2 \leq 1, x_1 - x_2 \geq -2, x_1 \geq 0, x_2 \geq 0$
- $x_1 + x_2 \leq 3, -2x_1 + x_2 \leq 2, x_1 - 2x_2 \leq 0, x_1 \geq 0, x_2 \geq 0$

3. Determine todos os pontos extremos do conjunto poliédrico.

$$x_1 - x_2 + x_3 \leq 1$$

$$-x_1 + 2x_2 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

4. Resolver pelo método gráfico o seguinte problema:

$$\text{Max}(Z) = 8x_1 + 4x_2$$

Sujeito a:

$$4x_1 + 2x_2 \leq 16$$

$$x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

-
5. Resolver pelo método gráfico o seguinte problema:

$$\text{Max}(Z) = 8x_1 + 4x_2$$

Sujeito a:

$$4x_1 + 2x_2 \leq 16$$

$$x_1 + x_2 \leq 6$$

$$x_1 + x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

6. Resolver pelo método gráfico o seguinte problema:

$$\text{Max}(Z) = 5x_1 + 4x_2$$

Sujeito a:

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$-x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

7. Determine o espaço de soluções viáveis para cada uma das seguintes restrições independentes:

- $-3x_1 + x_2 \leq 6$
- $x_1 - 2x_2 \geq 5$
- $2x_1 - 3x_2 \geq 12$
- $x_1 - x_2 \geq 0$
- $-x_1 + x_2 \geq 0$

8. Identifique a função de crescimento de z em cada um dos seguintes casos:

- $\text{Max } z = x_1 - x_2$
- $\text{Max } z = -5x_1 - 6x_2$
- $\text{Max } z = -x_1 + 2x_2$

1) $\text{Max}(z) = 3x_1 + 5x_2$

Supostos a:

1) $x_1 \leq 4$

2) $2x_2 \leq 12$

3) $3x_1 + 2x_2 \leq 18$

4) $x_1, x_2 \geq 0$

2) $2x_2 \leq 12$

$x_2 \leq 6$

3) $3x_1 + 2x_2 \leq 18$

Encontrar P_1

$x_1 = 0$

$3 \cdot 0 + 2x_2 \leq 18$

$x_2 = 9$

$P_1(0, 9)$

Encontrar P_2

$x_2 = 0$

$3x_1 + 2 \cdot 0 \leq 18$

$x_1 = 6$

$P_2(6, 0)$

\Rightarrow Função objetivo

$\text{Max}(z) = 36$

$x_1 = 0$

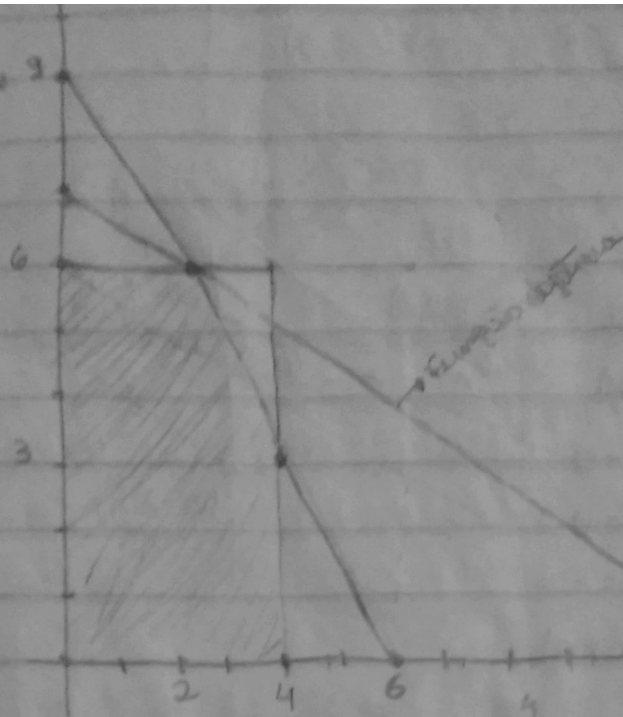
$3 \cdot 0 + 5x_2 = 36$

$x_2 = 7,2 \Rightarrow P(0, 7,2)$

$x_2 = 0$

$3x_1 + 5 \cdot 0 = 36$

$x_1 = 12 \Rightarrow P(12, 0)$



\Rightarrow Descobrimos P_3

$x_2 \leq 6$

$3x_1 + 2 \cdot 6 \leq 18$

$x_1 = 2$

$P_3(2, 6)$

\Rightarrow maximização

$\text{Max}(z) = 3x_1 + 5x_2$

$x_1 = 0; x_2 = 0; \text{Max}(z) = 0$

$x_1 = 0; x_2 = 6; \text{Max}(z) = 30$

$x_1 = 4; x_2 = 0; \text{Max}(z) = 12$

$x_1 = 2; x_2 = 6$

$\text{Max}(z) = 3 \cdot 2 + 5 \cdot 6 = 36$

$x_1 = 4; x_2 = 3$

$\text{Max}(z) = 3 \cdot 4 + 5 \cdot 3 = 27$

\Rightarrow Solução Ótima

2) 1) $x_1 - x_2 \leq 2$

2) $x_1 - x_2 \geq -2$

3) $x_1 + x_2 \geq 1$

4) $x_1 \geq 0$

5) $x_2 \geq 0$

1) $x_1 = 0$ $x_2 = 0$

$0 - x_2 \leq 2$ $x_1 - 0 \leq 2$

$x_2 = -2$ $x_1 = 2$

$P_1(0; -2)$ $P_2(2; 0)$

2) $x_1 = 0$ $x_2 = 0$

$0 - x_2 \geq -2$ $x_1 - 0 \geq -2$

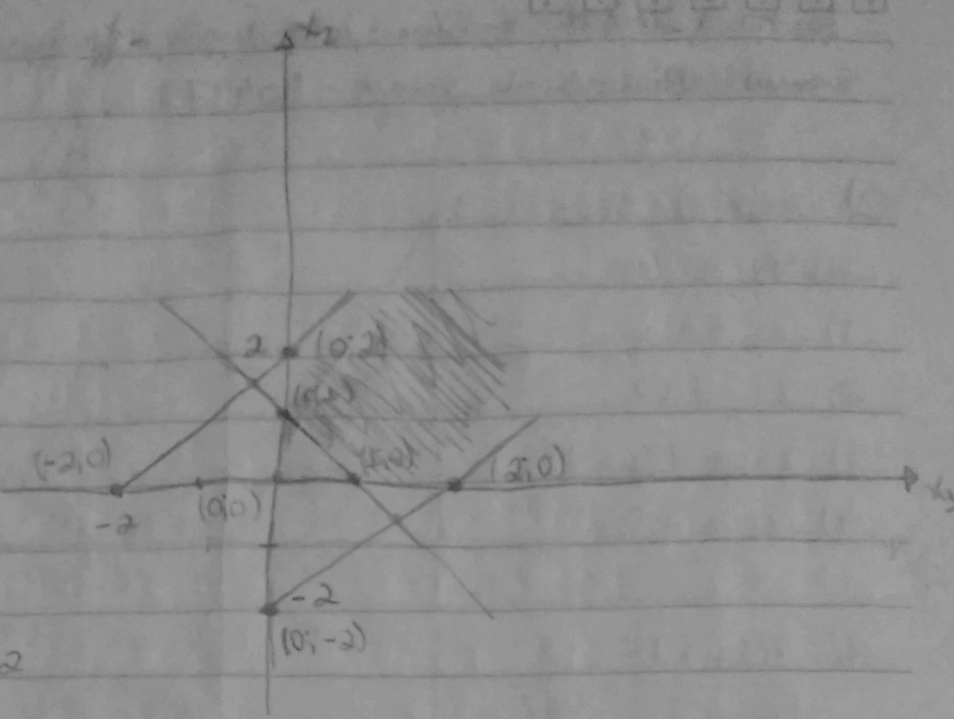
$x_2 = 2$ $x_1 = -2$

$P_1(0; 2)$ $P_2(-2; 0)$

3) $x_1 = 0$ $x_2 = 0$

$x_2 = 1$ $x_1 = 1$

$P_1(0; 1)$ $P_2(1; 0)$



6) 1) $x_1 - 2x_2 \leq 2$

$-2x_2 \leq 2 - x_1$

2) $2x_1 - x_2 \geq -2$ $x_2 = (2 + x_1)/2$

3) $x_1, x_2 \geq 0$

1) $x_1 = 0$ $x_2 = 0$

$0 - 2x_2 = 2$ $x_1 - 2 \cdot 0 = 2$

$x_2 = -1$ $x_1 = 2$

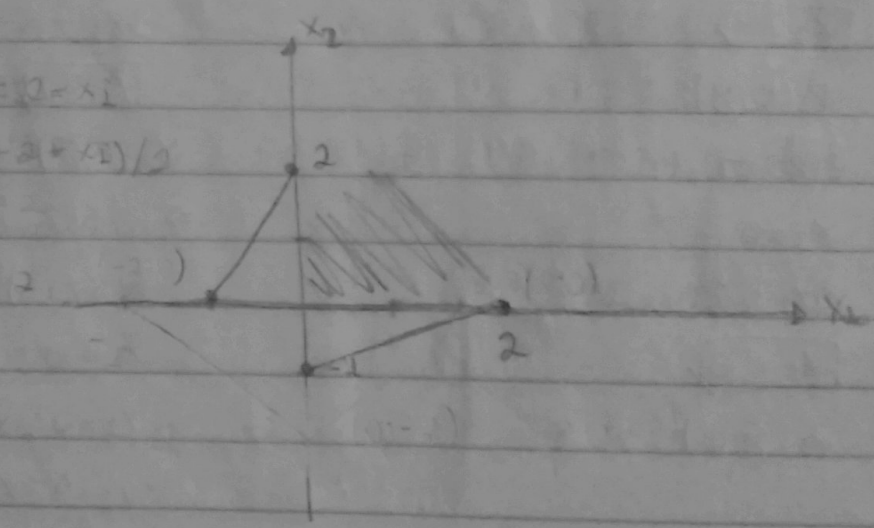
$P_1(0; -1)$ $P_2(2; 0)$

2) $x_1 = 0$ $x_2 = 0$

$2 \cdot 0 - x_2 = -2$ $2x_1 - 0 = -2$

$x_2 = 2$ $x_1 = -1$

$P_3(0; 2)$ $P_4(-1; 0)$



c) 1) $x_1 + x_2 \geq 4$

2) $x_1 - 2x_2 \leq 2$ $-2x_2 = 2 - x_1$

3) $x_1 - x_2 \geq -2$ $x_2 = (-1 - x_1)/2$

1) $x_1 = 0$ $x_2 = 0$

$x_2 = 4$ $x_1 = 4$

$P_1(0; 4)$ $P_2(4; 0)$

2) $x_1 = 0$ $x_2 = 0$

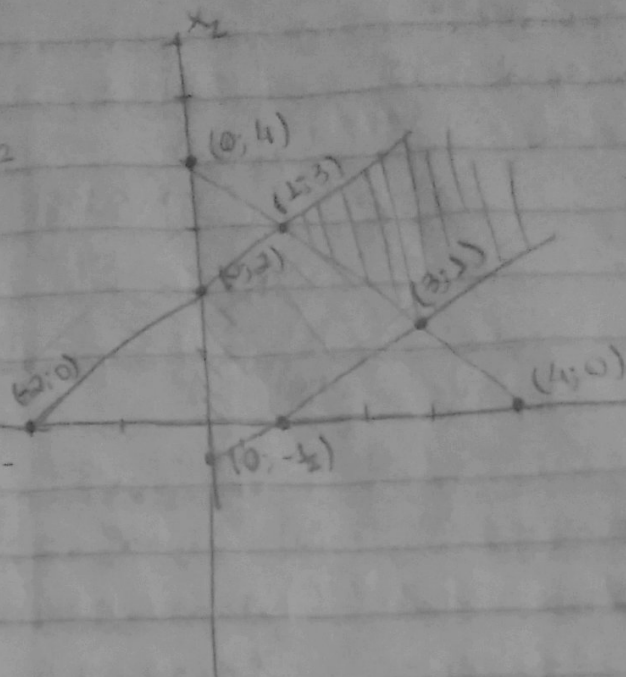
$x_2 = -\frac{1}{2}$ $x_1 = 1$

$P_3(0; -\frac{1}{2})$ $P_4(1; 0)$

3) $x_1 = 0$ $x_2 = 0$

$x_2 = 2$ $x_1 = -2$

$P_5(0; 2)$ $P_6(-2; 0)$



b) $x_1 - x_2 \geq -2$

1) $x_1 + x_2 \geq 4$

$2 + x_1 = 4 - x_1$

$P(3; 1)$

$-x_2 = -2 - x_1$

$x_2 = 4 - x_1$

$x_1 + x_1 = 4 - 2$

$x_2 = 2 + x_1$

$x_2 = 4 - 1$

$2x_1 = 2$

$x_2 = 3$

$x_1 = 1$

(2) $x_1 - 2x_2 \leq 1$

(3) $x_1 + x_2 \geq 4$

$1 + 2x_2 = 4 - x_2$

$P(1; 1)$

$-x_1 = 1 + 2x_2$

$x_1 = 4 - x_2$

$3x_2 = 3$

$x_1 = 3$

$x_2 = 1$

d) 1) $x_1 + x_2 \leq 3$

2) $-2x_1 + x_2 \leq 2$

3) $x_1 - 2x_2 \leq 0$

4) $x_1, x_2 \geq 0$

(1) $x_1 = 0$ $x_2 = 0$

$x_2 = 3$ $x_1 = 3$

$P(0; 3)$ $P(3; 0)$

(3) $x_1 = 0$

$0 - 2x_2 = 0$

$x_2 = 0$

$P_3(0; 0)$

(4) $x_2 = 0$

$x_1 - 2 \cdot 0 = 0$

$x_1 = 0$

$P_4(0; 0)$

(2) $x_1 = 0$

$x_2 = 0$

$-2 \cdot 0 + x_2 = 2$ $-2x_1 + 0 = 2$

$x_2 = 2$

$x_1 = -1$

$P_5(0; 2)$

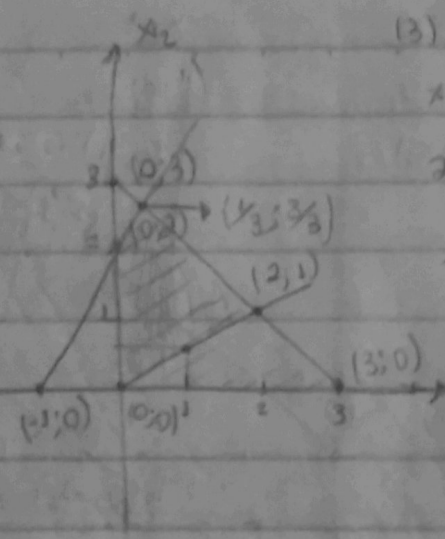
$P_6(-1; 0)$

(3) $x_1 - 2x_2 \leq 0$

Supondo $x_1 = 2$

$2 - 2x_2 = 0$

$x_2 = \frac{1}{2}$



(3) $x_1 - 2x_2 \leq 0$

$x_1 = 2x_2$

$2x_2 = 3 - x_2$

$x_2 = 1$

$x_1 = 3 + x_2$

$x_1 = 3 - x_2$

$x_1 = 2$

P(2,1)

(1) $x_1 + x_2 \leq 3$

$x_2 = 3 - x_1$

(2) $-2x_1 + x_2 \leq 2$

$x_2 = 2 + 2x_1$

$x_2 = 2 + 2 \cdot \frac{1}{3}$

$x_2 = \frac{8}{3}$

$3 - x_1 = 2 + 2x_1$

$-x_1 - 2x_1 = 2 - 3$

$-3x_1 = -1$

$x_1 = \frac{1}{3}$

④ $\text{Max}(Z) = 8x_1 + 4x_2$

1) $4x_1 + 2x_2 \leq 16$

2) $x_1 + x_2 \leq 6$

3) $x_1, x_2 \geq 0$

(1) $x_1 = 0$ $x_2 = 0$

$2 \cdot 0 + x_2 = 8$ $2x_1 + 0 = 8$

$x_2 = 8$ $x_1 = 4$

$P_1(0; 8)$ $P_2(4; 0)$

(2) $x_1 = 0$ $x_2 = 0$

$x_2 = 6$ $x_1 = 6$

$P_3(0; 6)$ $P_4(6; 0)$

Maximum Z occurs //

• $x_1 = 0; x_2 = 6$

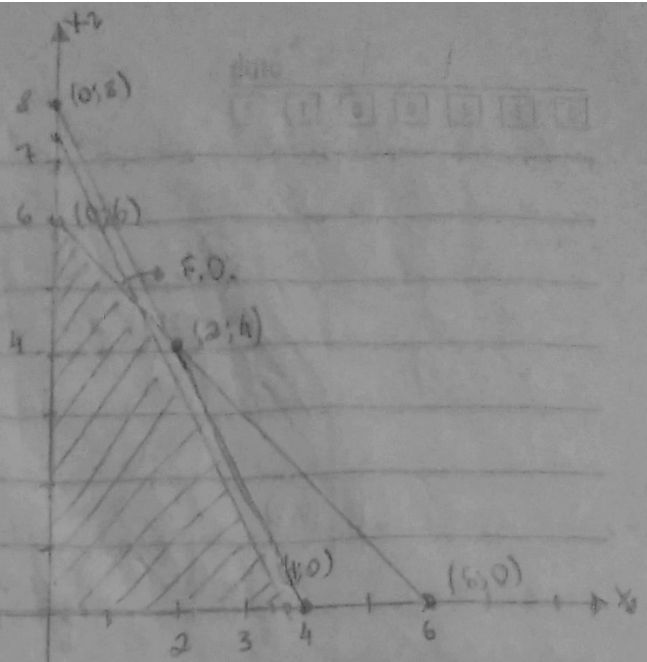
$\text{Max}(Z) = 8 \cdot 0 + 4 \cdot 6 = 24$

• $x_1 = 2; x_2 = 4$

$\text{Max}(Z) = 8 \cdot 2 + 4 \cdot 4 = 32$

• $x_1 = 4; x_2 = 0$

$\text{Max}(Z) = 8 \cdot 4 + 4 \cdot 0 = 32$



(1) $2x_1 + x_2 \leq 8$

$x_2 = 8 - 2x_1$

$8 - 2x_1 = 6 - x_1$

$-2x_1 + x_1 = 6 - 8$

$-x_1 = -2$

$x_1 = 2$

(2) $x_1 + x_2 \leq 6$

$x_2 = 6 - x_1$

$x_2 = 6 - 2$

$x_2 = 4$

Empire Objetiva //

$3x_1 + 4x_2 = 30$

$x_1 = 0$

$3 \cdot 0 + 4x_2 = 30$

$x_2 = 7,5$

$P_1(0; 7,5)$

$x_2 = 0$

$3x_1 + 4 \cdot 0 = 30$

$x_1 = 3,75$

$P_2(3,75; 0)$

⑤ $\text{Max}(Z) = 8x_1 + 4x_2$

1) $4x_1 + 2x_2 \leq 16$

2) $x_1 + x_2 \leq 6$

3) $x_1 + x_2 \geq 1$

4) $x_1, x_2 \geq 0$

(1) $x_1 = 0$

$x_2 = 0$

$2 \cdot 0 + x_2 = 8$

$2x_1 + 0 = 8$

$x_2 = 8$

$x_1 = 4$

$P_1(0; 8)$

$P_2(4; 0)$

(2) $x_1 = 0$

$x_2 = 0$

$x_2 = 6$

$x_1 = 6$

$P_3(0; 6)$ $P_4(6; 0)$

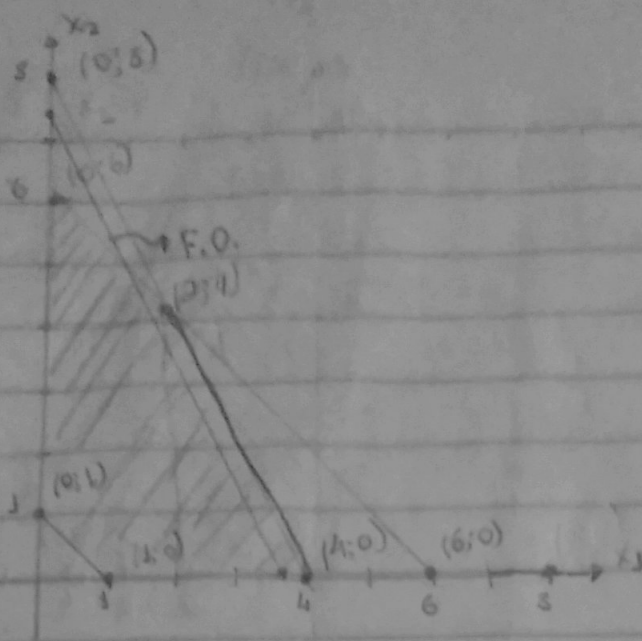
(3) $x_1 = 0$

$x_2 = 0$

$x_2 = 1$

$x_1 = 1$

$P_5(0; 1)$ $P_6(1; 0)$



$$(1) \quad 2x_1 + x_2 \leq 6$$

$$x_2 = 6 - 2x_1$$

$$(2) \quad x_1 + x_2 \leq 6$$

$$x_2 = 6 - x_1$$

$$x_2 = 4$$

$$3 - 2x_1 = 6 - x_1$$

$$-2x_1 + x_1 = 6 - 3$$

$$x_1 = -3$$

$$P(2; 4)$$

Maximização

$$\text{Max}(Z) = 3x_1 + 4x_2$$

$$\bullet x_1 = 0; x_2 = 6$$

$$\text{Max}(Z) = 3 \cdot 0 + 4 \cdot 6 = 24$$

$$\bullet x_1 = 2; x_2 = 4$$

$$\text{Max}(Z) = 3 \cdot 2 + 4 \cdot 4 = 22$$

$$\bullet x_1 = 4; x_2 = 0$$

$$\text{Max}(Z) = 3 \cdot 4 + 4 \cdot 0 = 12$$

$$\bullet x_1 = 0; x_2 = 1$$

$$\text{Max}(Z) = 3 \cdot 0 + 4 \cdot 1 = 4$$

$$\bullet x_1 = 1; x_2 = 0$$

$$\text{Max}(Z) = 3 \cdot 1 + 4 \cdot 0 = 3$$

Função objetivo

$$3x_1 + 4x_2 = 30$$

$$x_1 = 0$$

$$x_2 = 0$$

$$3 \cdot 0 + 4x_2 = 30$$

$$3x_1 + 4 \cdot 0 = 30$$

$$x_2 = 7,5$$

$$x_1 = 7,5$$

$$P(7,5; 7,5)$$

$$\textcircled{6} \quad \text{Max}(Z) = 5x_1 + 4x_2$$

$$(1) \quad 3x_1 + 2x_2 \leq 12$$

$$1) \quad 6x_1 + 4x_2 \leq 24$$

$$x_1 = 0$$

$$x_2 = 0$$

$$2) \quad x_1 + 2x_2 \leq 6$$

$$3 \cdot 0 + 2x_2 = 12$$

$$3x_1 + 2 \cdot 0 = 12$$

$$3) \quad -x_1 + x_2 \leq 1$$

$$x_2 = 6$$

$$x_1 = 4$$

$$4) \quad x_2 \leq 2$$

$$P_1(0; 6)$$

$$P_2(4; 0)$$

$$5) \quad x_1, x_2 \geq 0$$

(2) $x_1 + 2x_2 \leq 6$

$x_1 = 0$ $x_2 = 0$

$x_2 = 3$ $x_1 = 6$

$P_3(0; 3)$ $P_4(6; 0)$

(3) $-x_1 + x_2 \leq 1$

$x_1 = 0$ $x_2 = 0$

$x_2 = 1$ $x_1 = -1$

$P_5(0; 1)$ $P_6(-1; 0)$

(3) $-x_1 + x_2 \leq 1$

$x_2 = 2$

$-x_1 + 2 = 1$

$x_1 = 1$

$P(1; 2)$

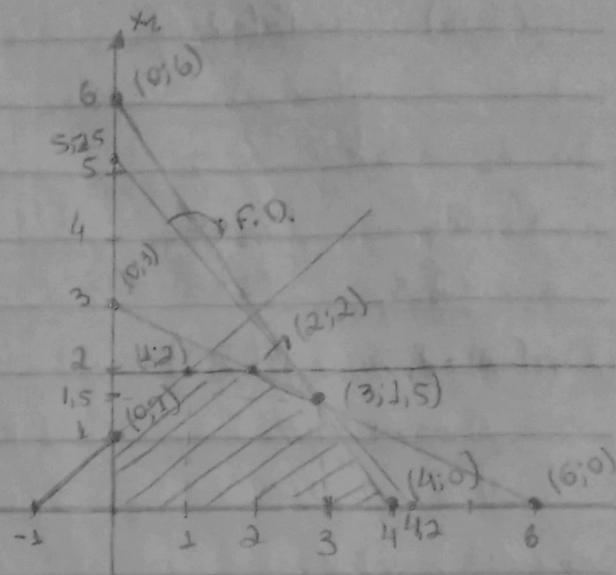
(2) $x_1 + 2x_2 \leq 6$

$x_2 = 2$

$x_1 + 2 \cdot 2 = 6$

$x_1 = 2$

$P(2; 2)$



(1) $6x_1 + 4x_2 \leq 24$

$3x_1 + 2x_2 \leq 12$

$x_1 = 4 - \frac{2}{3}x_2$

(2) $x_1 + 2x_2 \leq 6$

$x_1 = 6 - 2x_2$

$x_1 = 6 - 2 \cdot 1.5$

$x_1 = 3$

$P(3; 1.5)$

$4 - \frac{2}{3}x_2 = 6 - 2x_2$

$-\frac{2}{3} + 2x_2 = 6 - 4$

$\frac{4}{3}x_2 = 2$

$x_2 = 2 \cdot \frac{3}{4} \Rightarrow x_2 = 1.5$

maximização //

$\text{Max}(z) = 5x_1 + 4x_2$

$x_1 = 0; x_2 = 1$

$\text{Max}(z) = 5 \cdot 0 + 4 \cdot 1 = 4$

$x_1 = 1; x_2 = 2$

$\text{Max}(z) = 5 \cdot 1 + 4 \cdot 2 = 13$

$x_1 = 2; x_2 = 2$

$\text{Max}(z) = 5 \cdot 2 + 4 \cdot 2 = 18$

$x_1 = 3; x_2 = 1.5$

$\text{Max}(z) = 5 \cdot 3 + 4 \cdot 1.5 = 21$

$x_1 = 4; x_2 = 0$

$\text{Max}(z) = 5 \cdot 4 + 4 \cdot 0 = 20$

Função objetiva //

$5x_1 + 4x_2 = 21$

$x_1 = 0$

$x_2 = 0$

$5 \cdot 0 + 4x_2 = 21$

$5x_1 + 4 \cdot 0 = 21$

$x_2 = 5.25$

$x_1 = 4.2$

$P(4.2; 5.25)$

7) a) $-3x_1 + x_2 \leq 6$

$x_1 = 0$ $x_2 = 0$

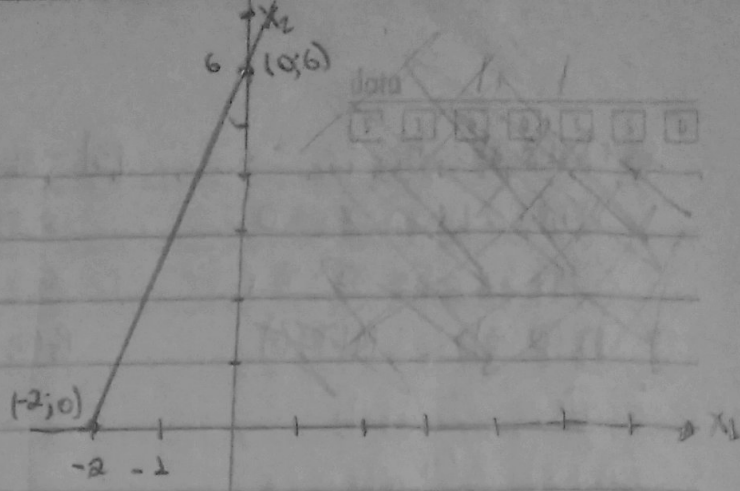
$-3 \cdot 0 + x_2 \leq 6$ $-3x_1 + 0 \leq 6$

$x_2 = 6$

$x_1 = -2$

$P_1(0; 6)$

$P_2(-2; 0)$



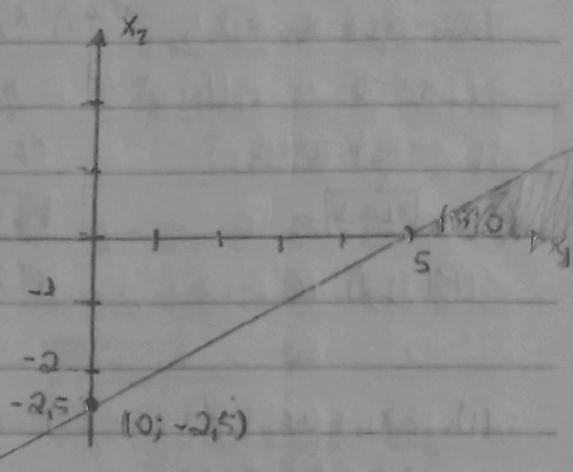
b) $x_1 - 2x_2 \geq 5$

$x_1 = 0$ $x_2 = 0$

$x_2 = -2,5$ $x_1 = 5$

$P_1(0; -2,5)$

$P_2(5; 0)$



c) $2x_1 - 3x_2 \leq 12$

$x_1 = 0$ $x_2 = 0$

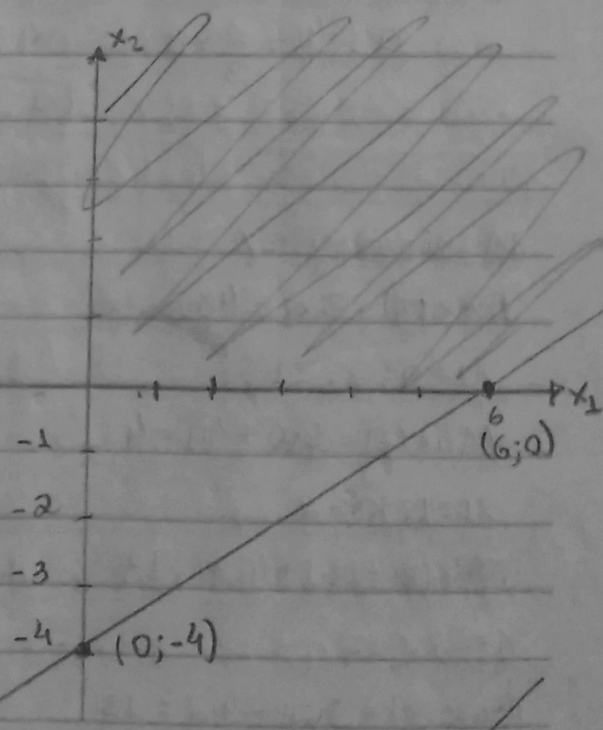
$2 \cdot 0 - 3x_2 = 12$ $2x_1 - 3 \cdot 0 = 12$

$x_2 = -4$

$x_1 = 6$

$P_1(0; -4)$

$P_2(6; 0)$



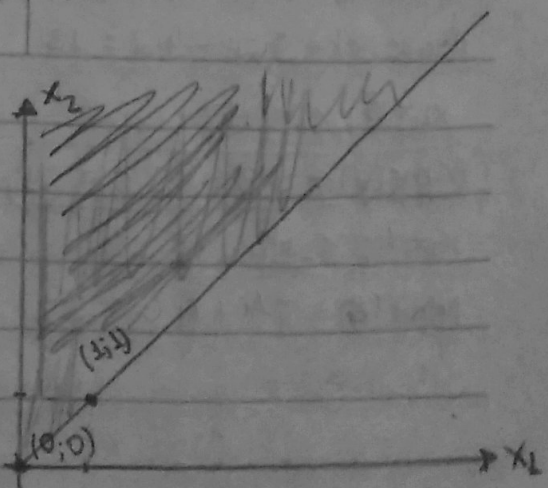
d) $x_1 - x_2 \leq 0$

$x_1 = 0$ $x_1 = 1$

$x_2 = 0$ $x_2 = 1$

$P_1(0; 0)$

$P_2(1; 1)$

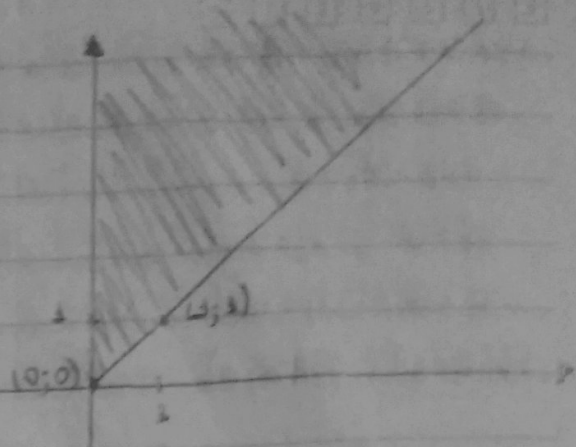


$$1) -x_1 + x_2 \geq 0$$

$$x_1 = 0 \quad x_2 = 1$$

$$x_2 = 0 \quad x_1 = 1$$

$$P_1(0;0) \quad P_2(1;1)$$



$$5) a) \text{Max } z = x_1 - x_2$$

$$\text{Admissível } z = 2$$

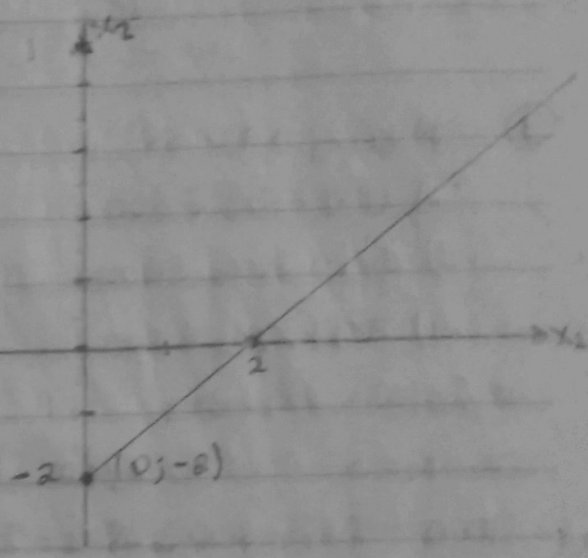
$$x_1 - x_2 = 2$$

$$x_1 = 0 \quad x_2 = 0$$

$$x_2 = -2 \quad x_1 = 2$$

$$x_1, x_2 \geq 0$$

$$P_1(0;-2) \quad P_2(2;0)$$



$$b) z = -5x_1 - 6x_2$$

$$\text{Admissível } z = -30$$

$$-5x_1 - 6x_2 = 30$$

$$x_1 = 0 \quad x_2 = 0$$

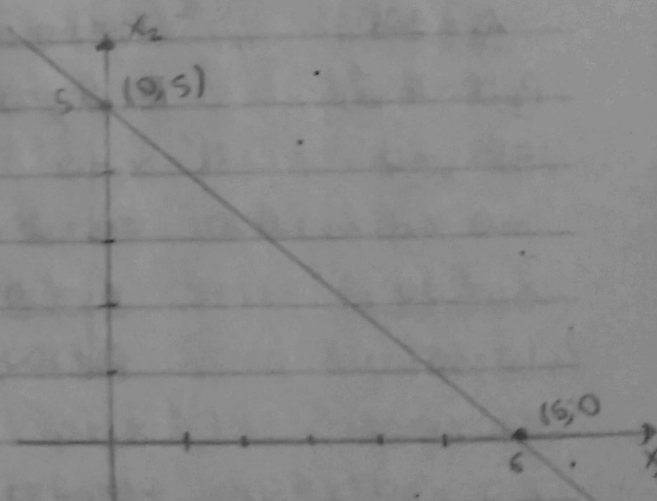
$$-5 \cdot 0 - 6x_2 = -30 \quad -5x_1 - 6 \cdot 0 = -30$$

$$x_2 = 5$$

$$x_1 = 6$$

$$P_1(0;5)$$

$$P_2(6;0)$$



$$c) z = -x_1 + 2x_2$$

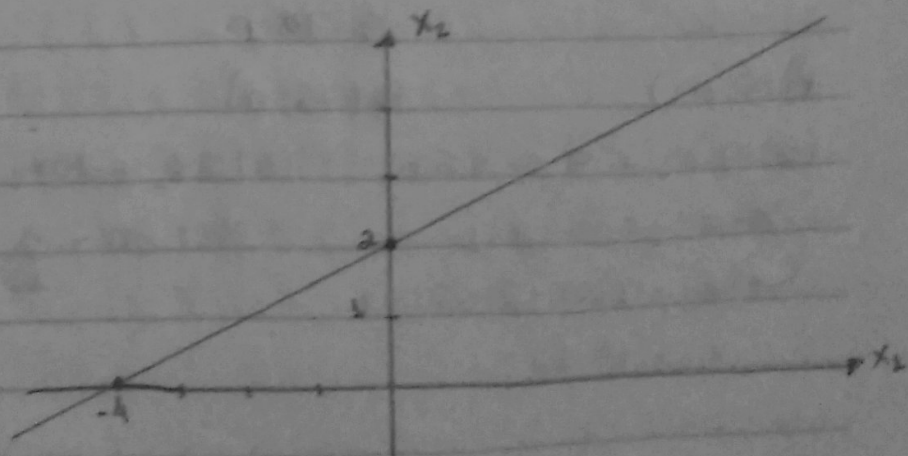
$$\text{Admissível } z = 4$$

$$-x_1 + 2x_2 = 4$$

$$x_1 = 0 \quad x_2 = 0$$

$$x_2 = 2 \quad x_1 = -4$$

$$P_1(0;2) \quad P_2(-4;0)$$



d) $z = -3x_1 + x_2$

Admissibles $z = 6$

$$-3x_1 + x_2 = 6$$

$$x_1 = 0 \quad x_2 = 6$$

$$x_2 = 6 \quad x_1 = -2$$

$$P_1(0; 6) \quad P_2(-2; 0)$$

