Symbols:

G: Gaussian shaped function with center at **(xGC, yGC)**. Gaussian width scales with **a1**.

PW: Plane Wave function with phase=0 at **(xGC, yGC)** and k-vector **(kx1, ky1)**

k0: Wave number in vacuum

k1: Wave number in refractive index **n1**

k2: Wave number in refractive index **n2**

E1: Field at focal plane 1

E1ft: Fourier Transform of **E1**

E2: Field at focal plane 2

(x1, y1) Real space coordinates at focal plane 1

(x2, y2) Real space coordinates at focal plane 2

(kx, ky) angular spatial frequencies for Fourier Transform

In[ ]:=
```
G := Exp[-((x1 - xGC)^2 + (y1 - yGC)^2) / a1^2] / a1^2
```

In[ ]:=
```
PW := Exp[ⅈ * (kx1 * (x1 - xGC) + ky1 * (y1 - yGC))]
```

In[ ]:=
```
E1 = E0 * PW * G
```

Out[ ]=

$$\frac{e^{i\left(\mathrm{kx1}\,(\mathrm{x1}-\mathrm{xGC})+\mathrm{ky1}\,(\mathrm{y1}-\mathrm{yGC})\right)+\frac{-(\mathrm{x1}-\mathrm{xGC})^2-(\mathrm{y1}-\mathrm{yGC})^2}{\mathrm{a1}^2}}\,\mathrm{E0}}{\mathrm{a1}^2}$$

In[ ]:=
```
E1ft = FourierTransform[E1, {x1, y1}, {kx, ky}]
```

Out[ ]=

$$\frac{1}{2}\,e^{-\frac{1}{4}\,\mathrm{a1}^2\left(\mathrm{kx}^2+2\,\mathrm{kx}\,\mathrm{kx1}+\mathrm{kx1}^2+(\mathrm{ky}+\mathrm{ky1})^2\right)+i\left(\mathrm{kx}\,\mathrm{xGC}+\mathrm{ky}\,\mathrm{yGC}\right)}\,\mathrm{E0}$$

Substitution yields E2:

In[ ]:=
```
E2 = E1ft /. {kx → -k1 * x2 / f, ky → -k1 * y2 / f}
```

Out[ ]=

$$\frac{1}{2}\,e^{-\frac{1}{4}\,\mathrm{a1}^2\left(\mathrm{kx1}^2-\frac{2\,\mathrm{k1}\,\mathrm{kx1}\,\mathrm{x2}}{f}+\frac{\mathrm{k1}^2\,\mathrm{x2}^2}{f^2}+\left(\mathrm{ky1}-\frac{\mathrm{k1}\,\mathrm{y2}}{f}\right)^2\right)+i\left(-\frac{\mathrm{k1}\,\mathrm{x2}\,\mathrm{xGC}}{f}-\frac{\mathrm{k1}\,\mathrm{y2}\,\mathrm{yGC}}{f}\right)}\,\mathrm{E0}$$

Evaluate E2 at maximum, i.e. focal point at focal plane 2, located at (x2=f*kx1/k1, y2=f*ky1/k1)

In[ ]:=
```
E2max = E2 /. {x2 → f * kx1 / k1, y2 → f * ky1 / k1}
```

Out[ ]=

$$\frac{1}{2}\,e^{i\left(-\mathrm{kx1}\,\mathrm{xGC}-\mathrm{ky1}\,\mathrm{yGC}\right)}\,\mathrm{E0}$$

Which is effectively the same as substituting kx -> -kx1, ky -> -ky1 in E1ft

In[ ]:=
```
E2maxalt = E1ft /. {kx → -kx1, ky → -ky1}
```

Out[ ]=

$$\frac{1}{2}\,e^{i\left(-\mathrm{kx1}\,\mathrm{xGC}-\mathrm{ky1}\,\mathrm{yGC}\right)}\,\mathrm{E0}$$

Side note, taking Gaussian width a1 to 0 yields point source, causing E2 to become a plane wave:

In[ ]:=
```
Limit[E2, a1 → 0]
```

Out[ ]=

$$\frac{1}{2} e^{i\left(-\frac{k1\, x2\, xGC}{f}-\frac{k1\, y2\, yGC}{f}\right)} E0$$