

12: Polyphase Filters

- Heavy Lowpass filtering
- Maximum Decimation Frequency
- Polyphase decomposition
- Downsampled Polyphase Filter
- Polyphase Upsampler
- Complete Filter
- Upsampler Implementation
- Downsampler Implementation
- Summary

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Heavy Lowpass filtering

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● Heavy Lowpass filtering

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Filter Specification:

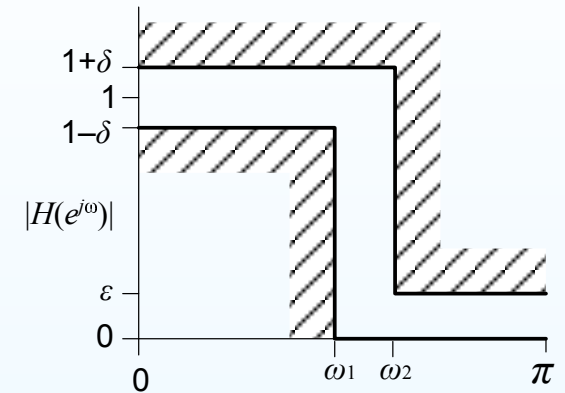
Sample Rate: 20 kHz

Passband edge: 100 Hz ($\omega_1 = 0.03$)

Stopband edge: 300 Hz ($\omega_2 = 0.09$)

Passband ripple: ± 0.05 dB ($\delta = 0.006$)

Stopband Gain: -80 dB ($\epsilon = 0.0001$)

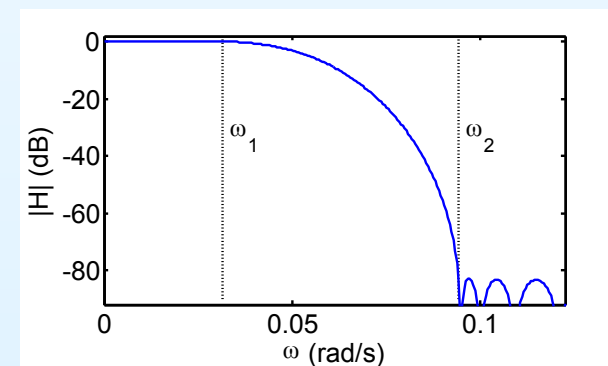
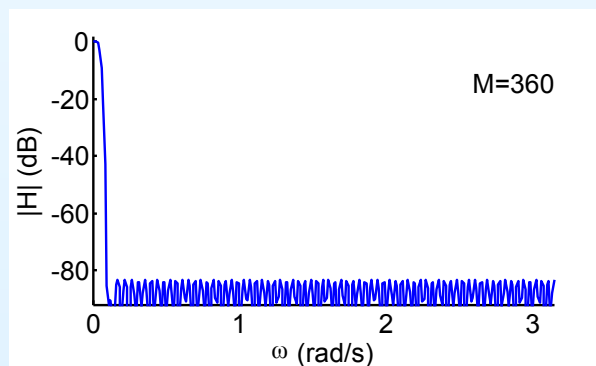


This is an extreme filter because the cutoff frequency is only 1% of the Nyquist frequency. *narrow passband \rightarrow polyphase*

Symmetric FIR Filter:

Design with Remez-exchange algorithm

Order = 360

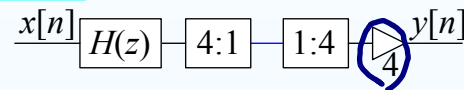


Maximum Decimation Frequency

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If a filter passband occupies only a small fraction of $[0, \pi]$, we can downsample then upsample without losing information.

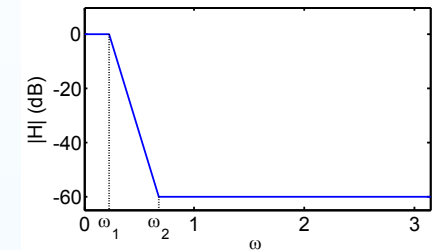


Downsample: aliased components at offsets of $\frac{2\pi}{K}$ are almost zero because of $H(z)$

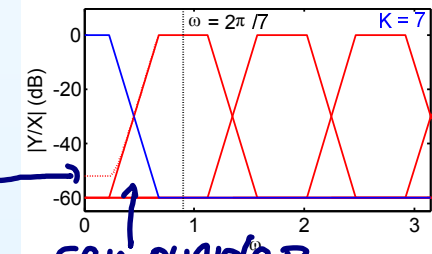
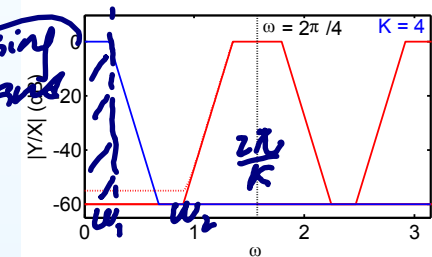
Upsample: Images spaced at $\frac{2\pi}{K}$ can be removed using another low pass filter

To avoid aliasing in the passband, we need

$$\frac{2\pi}{K} - \omega_2 \geq \omega_1 \Rightarrow K \leq \frac{2\pi}{\omega_1 + \omega_2}$$



no aliasing in passband

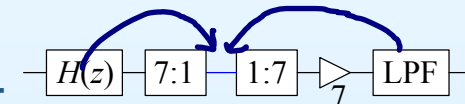


accumulated noise

can overlap

Centre of transition band must be \leq intermediate Nyquist freq, $\frac{\pi}{K}$

! We must add a **lowpass filter** to remove the images:



Passband noise = noise floor at output of $H(z)$ plus $10 \log_{10} (K - 1)$ dB.

Polyphase decomposition

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$$\frac{\pi}{K} \approx \frac{\omega_1 + \omega_2}{2}$$

For our filter: ^{intermediate} original Nyquist frequency = 10 kHz and transition band centre is at 200 Hz so we can use $K = 50$.

We will split $H(z)$ into K filters each of order $R - 1$. For convenience, assume $M + 1$ is a multiple of K (else zero-pad $h[n]$).

Example: $M = 399, K = 50 \Rightarrow R = \frac{M+1}{K} = 8$

$$H(z) = \sum_{m=0}^M h[m] z^{-m}$$

$$= \sum_{m=0}^{K-1} h[m] z^{-m} + \sum_{m=0}^{K-1} h[m+K] z^{-(m+K)} + \dots \quad [R \text{ terms}]$$

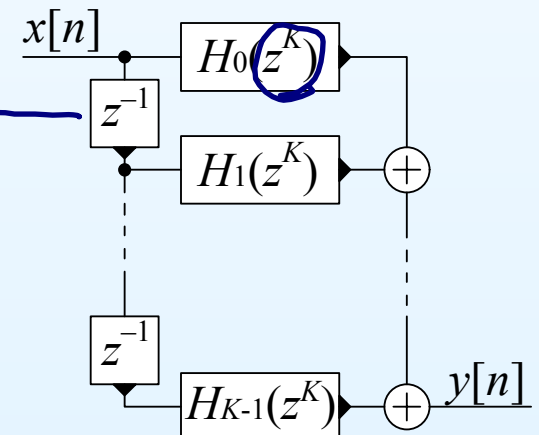
$$= \sum_{r=0}^{R-1} \sum_{m=0}^{K-1} h[m+Kr] z^{-m-Kr}$$

$$= \sum_{m=0}^{K-1} z^{-m} \sum_{r=0}^{R-1} h_m[r] z^{-Kr}$$

$$= \sum_{m=0}^{K-1} \underbrace{z^{-m}}_{\text{different delays}} \underbrace{H_m(z^K)}_{\text{same filter. sparse.}} \quad \text{where } h_m[r] = h[m+Kr]$$

Example: $M = 399, K = 50, R = 8$

$$h_3[r] = [h[3], h[53], \dots, h[303], h[353]]$$



This is a polyphase implementation of the filter $H(z)$

Downsampled Polyphase Filter

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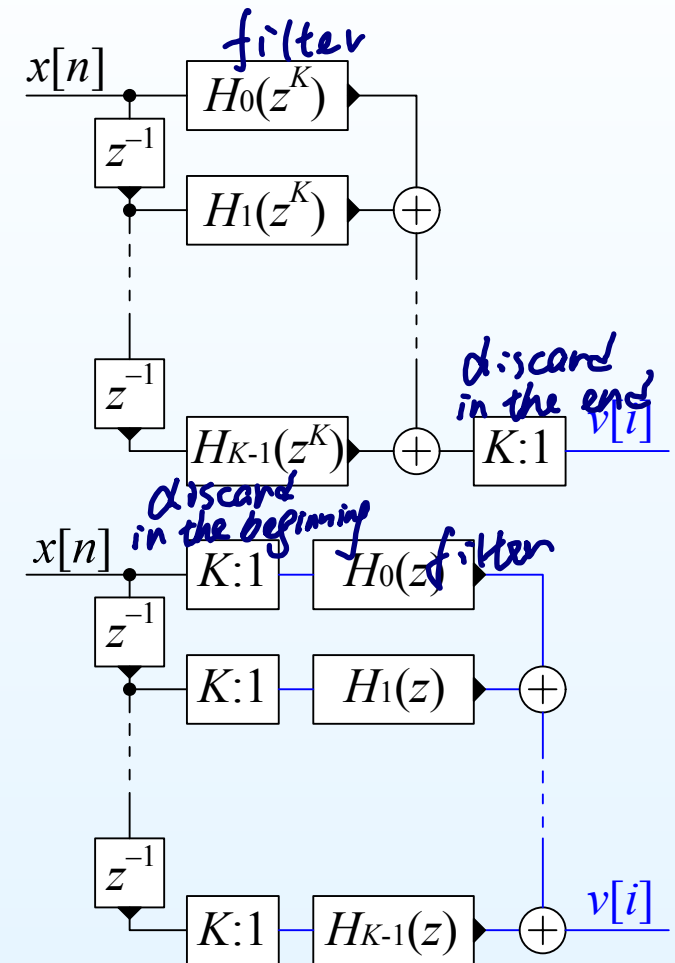
$H(z)$ is low pass so we downsample its output by K without aliasing.

The number of multiplications per input sample is $M + 1 = 400$.

Using the Noble identities, we can move the resampling back through the adders and filters. $H_m(z^K)$ turns into $H_m(z)$ at a lower sample rate. !

We still perform 400 multiplications but now only once for every K input samples. !

Multiplications per input sample = 8 (down by a factor of 50 😊) but $v[n]$ has the wrong sample rate (😞).



Polyphase Upsampler

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To restore sample rate: upsample and then lowpass filter to remove images

We can use the same lowpass filter, $H(z)$, in polyphase form:

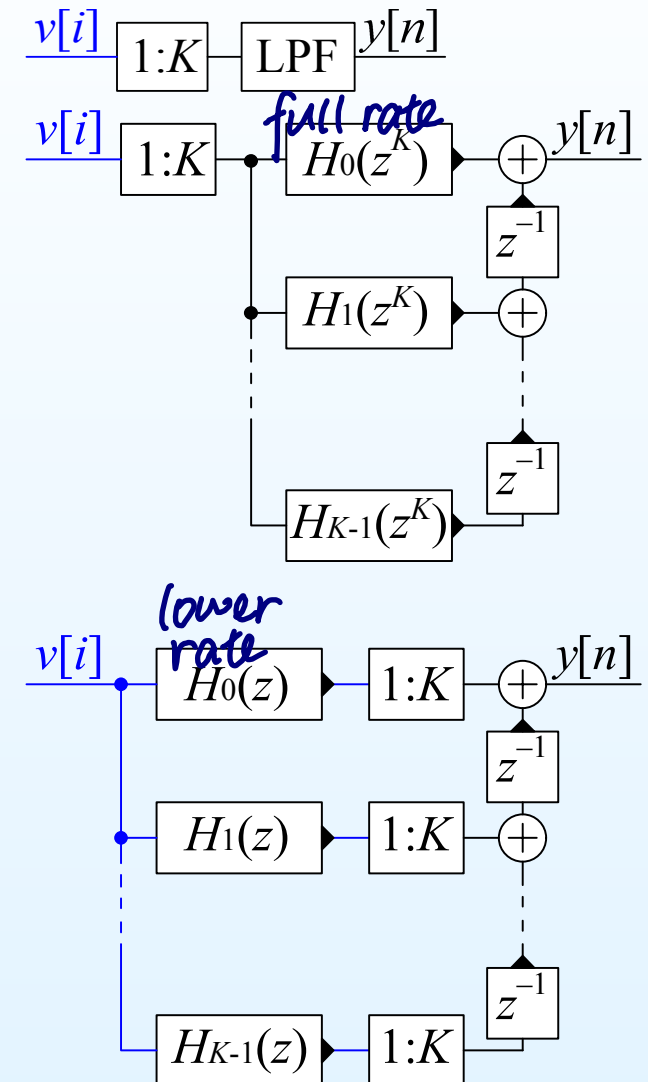
$$\sum_{m=0}^{K-1} z^{-m} \sum_{r=0}^{R-1} h_m[r] z^{-Kr}$$

This time we put the delay z^{-m} after the filters.

Multiplications per output sample = 400

Using the Noble identities, we can move the resampling forwards through the filters. $H_m(z^K)$ turns into $H_m(z)$ at a lower sample rate.

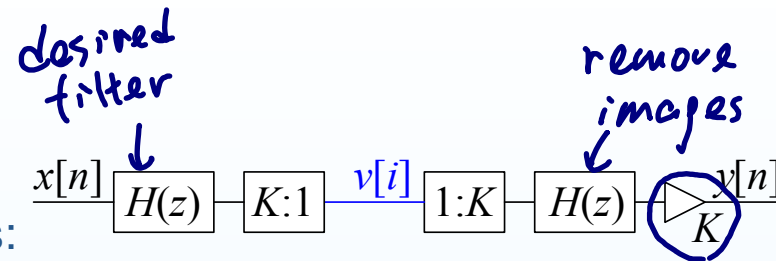
Multiplications per output sample = 8
(down by a factor of 50 😊).



Complete Filter

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The overall system implements:

Need an extra gain of K to compensate for the downsampling energy loss.

Filtering at downsampled rate requires 16 multiplications per input sample (8 for each filter). Reduced by $\frac{K}{2}$ from the original 400.

$H(e^{j\omega})$ reaches -10 dB at the downsampler

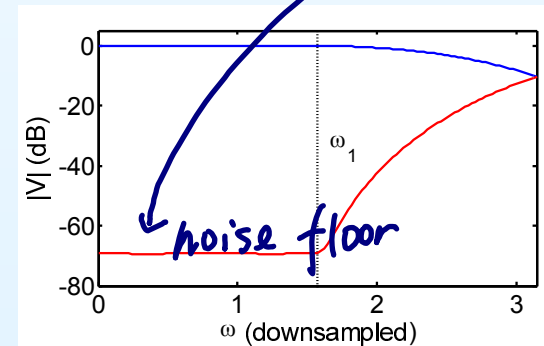
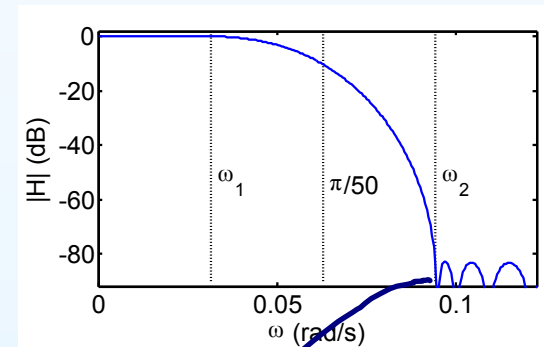
Nyquist frequency of $\frac{\pi}{K}$.

Spectral components $> \frac{\pi}{K}$ will be aliased down in frequency in $V(e^{j\omega})$.

For $V(e^{j\omega})$, passband gain (blue curve) follows the same curve as $X(e^{j\omega})$.

Noise arises from K aliased spectral intervals.

Unit white noise in $X(e^{j\omega})$ gives passband noise floor at -69 dB (red curve) even though stop band ripple is below -83 dB (due to $K - 1$ aliased stopband copies).



polyphase: higher noise floor

Upsampler Implementation

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We can represent the upsampler compactly using a commutator.

Sample $y[n]$ comes from $H_k(z)$ where $k = n \bmod K$.

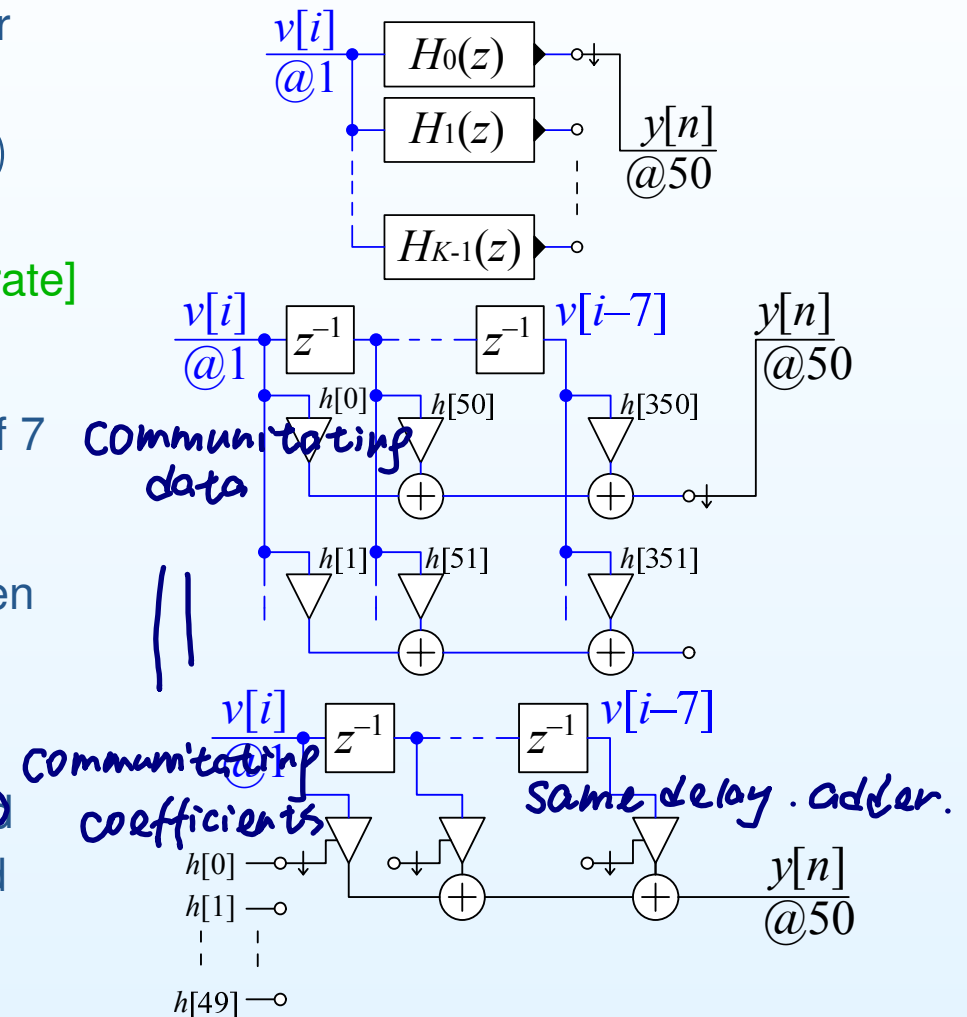
["@f" indicates the sample rate]

$H_0(z)$ comprises a sequence of 7 delays, 7 adders and 8 gains.

We can share the delays between all 50 filters.

We can also share the gains and adders between all 50 filters and use commutators to switch the coefficients.

We now need 7 delays, 7 adders and 8 gains for the entire filter.



Downsampler Implementation

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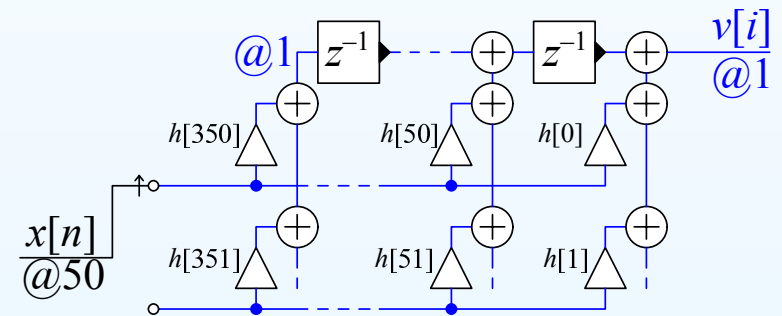
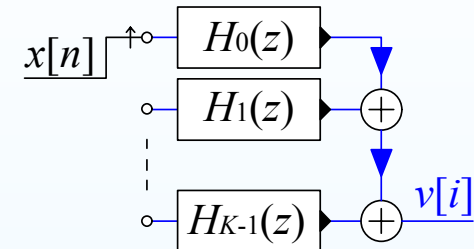
We can again use a commutator. The outputs from all 50 filters are added together to form $v[i]$.

We use the transposed form of $H_m(z)$ because this will allow us to share components.

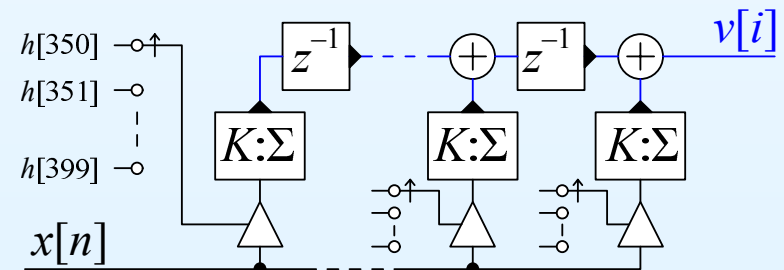
We can sum the outputs of the gain elements using an accumulator which sums blocks of K samples.

Now we can share all the components and use commutators to switch the gain coefficients.

We need 7 delays, 7 adders, 8 gains and 8 accumulators in total.



$$u[n] \xrightarrow{K:\Sigma} w[i] \quad w[i] = \sum_{r=0}^{K-1} u[Ki - r]$$



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- Complexity $\sim f_s^2$
- **Filtering** should be performed at the **lowest possible sample rate**
 - reduce filter computation by K
 - actual saving is only $\frac{K}{2}$ because you need a second filter
 - downsampled Nyquist frequency $\geq \max(\omega_{\text{passband}}) + \frac{\Delta\omega}{2}$
 $\frac{\pi}{K} \geq \frac{\omega_1 + \omega_2}{2}$
 - **Polyphase decomposition:** split $H(z)$ as $\sum_{m=0}^{K-1} z^{-m} H_m(z^K)$
 - each $H_m(z^K)$ can operate on subsampled data $H(z) \rightarrow H_m(z^K)$
 - combine the filtering and down/up sampling
 - **Noise floor is higher** because it arises from K spectral intervals that are aliased together by the downsampling.
 - **Share components between the K filters**
 - multiplier gain coefficients switch at the original sampling rate
 - need a new component: accumulator/downsampler ($K : \Sigma$)

For further details see Harris 5.