SC5-c

(Four Lectures)

Positioning Architectures in Wireless Networks

by Professor A. Manikas Chair in Communications & Array Processing

References

[1] S. Guolin, C. Jie, G. Wei, and K. J. R. Liu, "Signal processing techniques in network-aided positioning: a survey of state-of-the-art positioning designs," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 12-23, 2005.
[2] A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 24-40, 2005.

ACRONYMS DESCRIPTION SECOND GENERATION OF MOBILE SYSTEMS 2G 3G THIRD GENERATION OF MOBILE SYSTEMS BS BASE STATION MS MOBILE STATION AP ACCESS POINT AMPOA AMPLITUDE OF ARRIVAL **ENHANCED 911** E911 FEDERAL COMMUNICATIONS COMMISSION FCC GLOBAL POSITIONING SYSTEM GPS LBS LOCATION BASED SERVICES MAXIMUM LIKELIHOOD ML NLOS NON-LINE-OF-SIGHT PERSONAL DIGITAL ASSISTANT PDA PSAP PUBLIC SAFETY ANSWERING POINT SINR SIGNAL-TO-INTERFERENCE-NOISE RATIO SNR SIGNAL-TO-NOISE RATIO ANGLE OF ARRIVAL AoA DIRECTION OF ARRIVAL DoA TDOA TIME DIFFERENCE OF ARRIVAL TOA TIME OF ARRIVAL **UMTS** UNIVERSAL MOBILE TELECOMMUNICATIONS SYSTEM CODE DIVISION MULTIPLE ACCESS **CDMA** WCDMA WIDEBAND CODE DIVISION MULTIPLE ACCESS WLAN WIRELESS LOCAL AREA NETWORK 2

Aims

 This course is concerned with the problem of Locating and Tracking energy emitters or reflecting sources

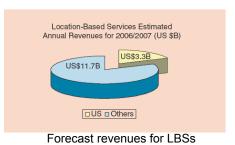
with emphasis given to applications in the area of wireless communications.

Other Applications:

Radar, Sonar, Navigation, Biomedicine, Seismology, Pollution Monitoring, Monitoring wildlife, **Location Based Services**, etc.

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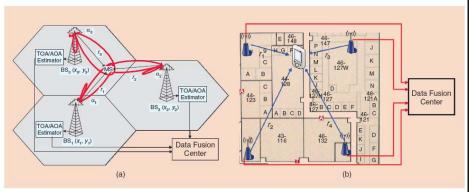
- Location Based Services (LBSs)
 - Asset Tracking,
 - Fleet Management,
 - Location Based Wireless Access Security,
 - Location Sensitive Billing,
 - Location based Advertising,
 - Etc.



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1. Classification of Positioning Systems/Architectures

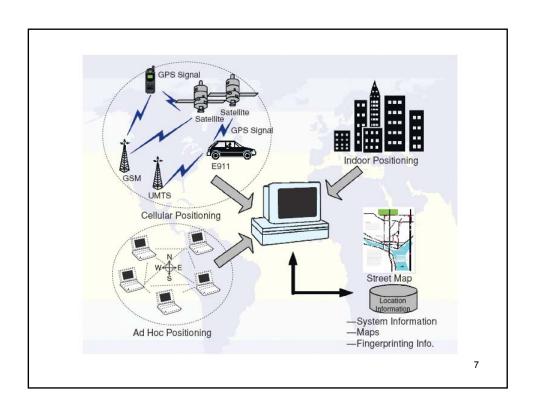
 Indoor (e.g. WLAN), and Outdoor (e.g. GPS)



- (a) Cellular Network-aided wireless Location finding (outdoor)
- (b) WLAN (indoor)

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- Or, an alternative classification:
 - Cellular Network-aided,
 - Sensor Network-aided,
 - GPS,
 - assisted GPS
 - ➤ (combination of GPS and Cellular Network-aided)



2. GPS and A-GPS

GPS	Assisted-GPS (A-GPS)
Requires minimal obstruction	Can be used even for indoor and can be much more accurate (10-50m)
Long acquisition times (30sec – 15min)	Improves acquisition time (<10sec)
Has to be synchronous	Synchronous or Asynchronous
High power consumption	More cost effective than GPS
High unit cost	Little (or no) hardware changes required in Base Stations

3. Positioning in Cellular Networks

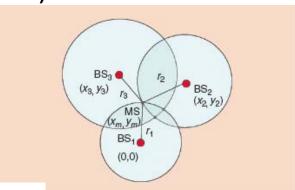
- Some of the most interesting positioning application areas have emerged in Wireless Communications. The most prominent:
 - FCC (Federal Communications Commission) which requires that the precise location of all enhanced 911 (E911) emergency calls be automatically determined.
 - FCC Mandate: 95% of all handsets sold be location compatible by the end of 2005.
 - European Recommendation E112
 - Both E911 and E112 require that wireless providers should be able to locate within tens of meters users of emergency calls.

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- Other applications of wireless positioning (besides E911 and E112 services)
 - Vehicle navigation
 - Network optimisation
 - Resource allocation
 - Automatic billing
 - Ubiquitous computing
 - ➤ e.g. for accessing personal info, corporate data, share resources, anywhere
 - location-aware computing

- Localisation Algorithms
 - Time-Of-Arrival (TOA) based
 - Time-Difference-Of-Arrival (TDOA) based
 - Direction-of-Arrival (DoA) based
 also known as Angle-of-Arrival (AoA) based
 - Received Signal Strength (RSS) based
- The above localisation algorithms can be
 - Cooperative localisation algorithms
 - · Centralised Algorithms
 - Distributed Algorithms
 - Non-cooperative algorithms

3.1. Localisation with TOA (Time Of Arrival) Data Fusion



 $t^0={
m known}$ (MS) r $t_1,\,t_2,\,t_3={
m measured\ TOA\ BSs)}$ $r_1,\,r_2,\,r_3={
m estimated\ using}$

 $r_i = (t_i - t^0)c$

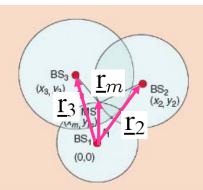
for i=1,2,3

Note:

$$r_1 = \|\underline{\mathbf{r}}_{1} - \underline{\mathbf{r}}_{m}\| = \|\underline{\mathbf{r}}_{m}\|$$

$$r_2 = \|\underline{\mathbf{r}}_2 - \underline{\mathbf{r}}_m\|$$

$$r_3 = \|\mathbf{r}_3 - \mathbf{r}_m\|$$



BS1, BS2, BS3 coordinates=known

$$\underline{\mathbf{r}}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \underline{\mathbf{r}}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}; \underline{\mathbf{r}}_3 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

mobile coordinates:
$$\underline{\mathbf{r}}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix} = \text{unknown}$$

Equation: $\mathbb{H} \underline{\mathbf{r}}_m = \underline{b}_{\text{\tiny TOA}}$

$$\Rightarrow \underline{\mathbf{r}}_m = (\mathbb{H}^T \mathbb{H})^{-1} \mathbb{H}^T \underline{b}_{\scriptscriptstyle \mathrm{TOA}}$$

where

$$\mathbb{H} = \left[\underline{\mathbf{r}}_2, \underline{\mathbf{r}}_3\right]^T = \begin{bmatrix} x_2, & y_2 \\ x_3, & y_3 \end{bmatrix}$$

$$\underline{b}_{\scriptscriptstyle{\mathrm{TOA}}} = \frac{1}{2} \left[\frac{\left\|\underline{\mathbf{r}}_{2}\right\|^{2} - r_{2}^{2} + r_{1}^{2}}{\left\|\underline{\mathbf{r}}_{3}\right\|^{2} - r_{3}^{2} + r_{1}^{2}} \right]$$

3.2. TOA Requirements

- TOA approach requires accurate synchronisation between the BSs and MS clocks.
 - This requirement ensures that the estimated r_1 , r_2 , r_3 are good approximations of the actual distances.

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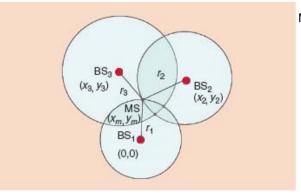
3.3. TOA: An Important Comment

- Many of the current wireless system standards only require tight timing synchronisation amongst BSs.
 - The MS clock itself might have a drift that can be even a few microseconds
 - This drift directly produces errors in r_1 , r_2 , r_3 and, consequently, errors in the location estimate of the TOA method.

3.4. Localisation with TDOA (Time Difference of Arrival) Data Fusion

 TDOA method does not suffer from MS clock synchronisation errors

BSs: t_1, t_2, t_3 = measured TOA

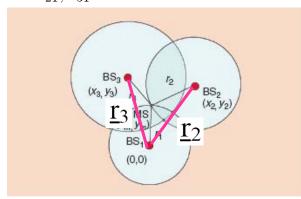


 $MS: t^0 = unknown$

 $r_i = (t_i - t^0)c$ cannot be used to estimate r_1, r_2, r_3

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However $r_{21}, r_{31} = \text{can be estimated}$



using
$$r_{i1} = r_i - r_1 = (t_i - t^0)c - (t_1 - t^0)c$$

$$\Rightarrow$$
 $r_{i1} = (t_i - t_1)c$

The TDOA associated with the BS_i is $t_i - t_1$. That is, the difference between the TOA of the MS signal at the BS_i and the BS_1 .

Equation:
$$\mathbb{H} \underline{\mathbf{r}}_m = \underline{b}_{\text{\tiny TDOA}}$$

$$\Rightarrow \underline{\mathbf{r}}_m = (\mathbb{H}^T \mathbb{H})^{-1} \mathbb{H}^T \underline{b}_{\text{\tiny TDOA}}$$

where

$$\underline{b}_{\text{TDOA}} = \frac{1}{2} \left[\frac{\|\underline{\mathbf{r}}_2\|^2 - r_{21}^2 - 2r_{21}r_1}{\|\underline{\mathbf{r}}_3\|^2 - r_{31}^2 - 2r_{31}r_1} \right]$$

To be estimated firstly (positive root of quadr. Equ.)

r,2= |rm|2

3.5. TDOA Requirements

- TDOA approach requires accurate synchronisation only amongst the BSs clocks.
 - i.e it requires a "synchronous" network
 - This requirement ensures that the estimated r_{12} , r_{23} , r_{13} are good approximations of the actual distances.

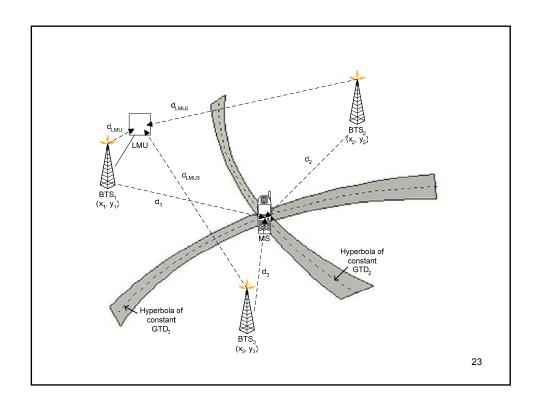
3.6. TDOA: An Important Comment

- A GSM network is not a "synchronous" network.
 - Therefore, TDOA can not be used directly (it suffers from synchronisation errors amongst BSs)
 - GSM employs a quite expensive solution known as E-OTD (Enhanced Observed Time Difference)

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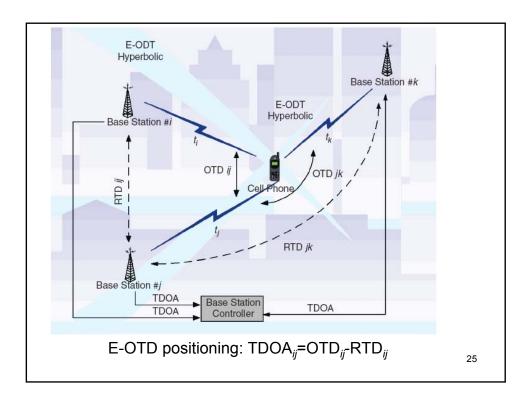
3.7. Enhanced Observed Time Difference (E-OTD)

- The real time differences (RTD) between pairs of BSs are measured by an LMU (Location Measuring Unit) device
 - computes the clock difference between BSs and send this information to the corresponding BSs.



 In addition the Observed Time Differences (OTD) are measured between pairs of BSs

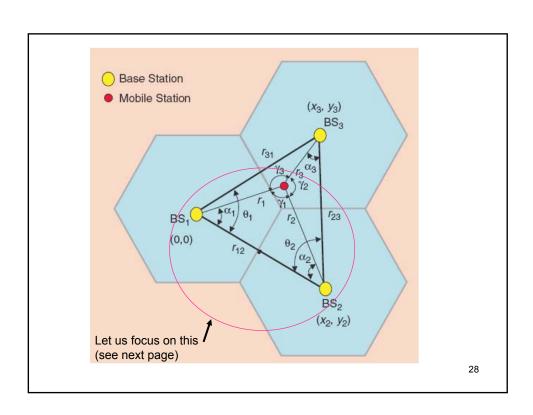
(The measured time difference between one pair of BS, is referred to as OTD)

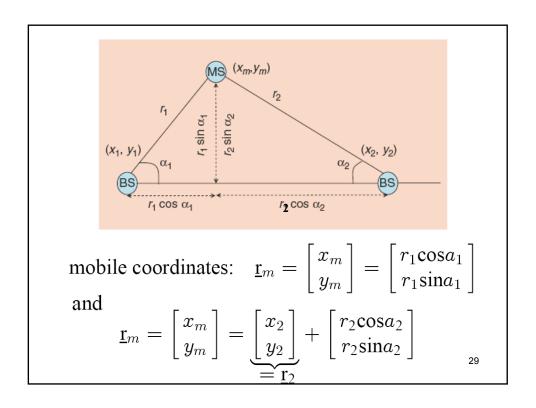


3.8. Localisation with DoA (Direction-of-Arrival) Data Fusion

- A better approach that does not require BS or MS clock synchronisation is the DoA (Direction-of-Arrival)
 - However, antenna array structures do not currently exist in 2G but the use of antenna arrays is planned for 3G

- Consider that each BS (using antenna arrays) estimates the Direction-of-Arrival of the MS signal.
 (e.g using MUSIC algorithm)
 - For 3 BSs: estimated directions a_1, a_2, a_3 (say)
 - By combining the DoA estimates from two different BSs (e.g. a_1,a_2 see next two figures) an estimate of the MS position can be obtained





and for any other BS_i

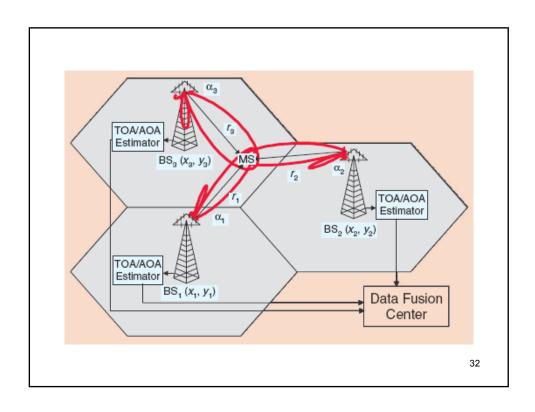
$$\underline{\mathbf{r}}_{m} = \begin{bmatrix} x_{m} \\ y_{m} \end{bmatrix} = \underbrace{\begin{bmatrix} x_{i} \\ y_{i} \end{bmatrix}}_{=\mathbf{r}_{i}} + \begin{bmatrix} r_{i} \cos a_{i} \\ r_{i} \sin a_{i} \end{bmatrix}$$
Equation: $\mathbf{H} \underline{\mathbf{r}}_{m} = \underline{b}_{\text{DoA}} \Rightarrow \underline{\mathbf{r}}_{m} = (\mathbf{H}^{T}\mathbf{H})^{-1}\mathbf{H}^{T}\underline{b}_{\text{DoA}}$
where
$$\mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \underline{b}_{\text{DoA}} = \begin{bmatrix} r_{1} \cos a_{1} \\ r_{1} \sin a_{1} \\ x_{2} + r_{2} \cos a_{2} \\ y_{2} + r_{2} \sin a_{2} \\ \vdots \\ x_{n} + r_{n} \cos a_{n} \\ y_{n} + r_{n} \sin a_{n} \end{bmatrix}_{30}$$

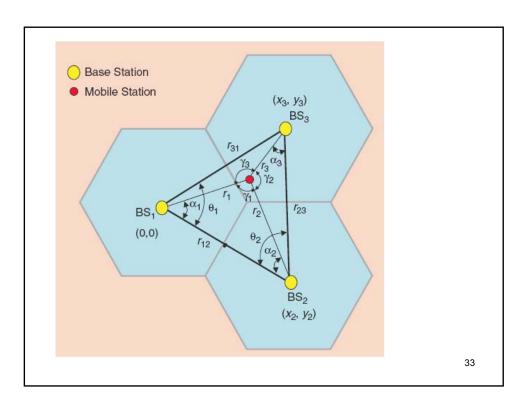
3.9. Localisation with Hybrid Data Fusion

- In TOA, TDOA and DoA approaches two or more Bs are employed in the MS location estimation. In some situations

 e.g.
 - » MS much closer to one BS,
 - » BS antenna array is surrounded by many scatterers

an alternative procedure may be used which combines DoA and TOA estimates using a 3-step procedure.





STEP-1:

The least-squares estimate of $\underline{\mathbf{r}}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix}$ using TOA measurements :

TOA- Equation:
$$\mathbb{H}_{\text{TOA}} \underline{\mathbf{r}}_m = \underline{b}_{\text{TOA}}$$

$$\Rightarrow \underline{\mathbf{r}}_m^{\text{(TOA)}} = \left(\mathbb{H}_{\text{TOA}}^T \mathbb{H}_{\text{TOA}}\right)^{-1} \mathbb{H}_{\text{TOA}}^T \underline{b}_{\text{TOA}}$$

STEP-2:

The least-squares estimate of $\underline{\mathbf{r}}_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix}$ using DoA measurements :

DoA Equation:
$$\mathbb{H}_{\text{DoA}} \underline{\mathbf{r}}_{m} = \underline{b}_{\text{DoA}}$$

$$\Rightarrow \underline{\mathbf{r}}_{m}^{\text{(DoA)}} = \left(\mathbb{H}_{\text{DoA}}^{T} \mathbb{H}_{\text{DoA}}\right)^{-1} \mathbb{H}_{\text{DoA}}^{T} \underline{b}_{\text{DoA}}$$
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STEP-3:

Finally, the MS location could be taken as a linear combination of the two estimates

e.g.
$$\underline{\mathbf{r}}_m = \eta \, \underline{\mathbf{r}}^{\text{(TOA)}} + (1 - \eta) \, \underline{\mathbf{r}}^{\text{(DoA)}}$$

where $0 < \eta < 1$ is chosen depending on the relative accuracy of the TOA and DoA measurements

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4. Overview of Cellular Network Positioning

- There are two main classifications of Cellular Network-aided Positioning Systems/Architectures
 - "standard"
 - > 2G: GSM with E-OTD (Existing Observed Time Difference)
 - > 2.5G: CDMA/GPRS with A-GPS
 - MS-assisted GPS
 - BS-based GPS
 - > 3G: WCDMA with OTDOA (Observed TDOA)
 - > Cellular ID
 - "non-standard"
 - > Architectures based on Antenna Arrays
 - Hybrid Positioning Using Data Fusion
 - > Pattern Matching Positioning

STANDARD

- GSM (with E-OTD)
 - Accuracy 50-125m
 - Slow (~5sec)
 - Software change is needed
- CDMA/GPRS (with A-GPS)
 - MS needs an A-GPS Rx
 - Accuracy <10m
- WCDMA
 - Not as Accurate as A-GPS (50m)
 - Needs to be visible to at least 3 BSs
 - Requires changes in the BS (IPDL, TA-IPDL, OTDOA-PE)
- Cellular ID
 - No Air-interface needed
 - Accuracy depends on sector size
 - Accuracy can be improved by Hybridization with other methods

NON-STANDARD

- Antenna Arrays (Smart Antennas)
 - Potential to be very accurate
 - No changes in the handset (MS)
- Hybrid Positioning Using Data Fusion
 - Hybrid TOA/TDOA/DoA can improve accuracy
 - GSP+CDMA can also improve accuracy and coverage
- Pattern Matching For Positioning
 - Only server BS required
 - Software solution with hardware modification

Note:

IPDL: Idle Period DownLink
(it is a modification of OTDOA)
TA-IPDL: synchronised IPDL
OTDOA-PE: OTDA Positioning Elegrents

5. Localisation: Sources of Errors

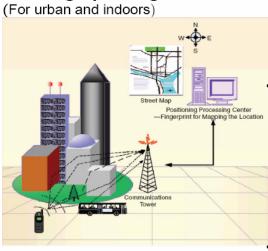
- Multipaths
 - RSS: 30-40dB variation over distances in the order of one halfwavelength.
 - DoA: scattering near and around MS & BS will affect the measurements
 - TOA and TDOA: results in a shift in the peak of the correlator
- NLoS (signal takes a longer path)
 - TOA based 400-700m error
- MAI (CDMA systems)
- Co-channel interference

5.1. Comments

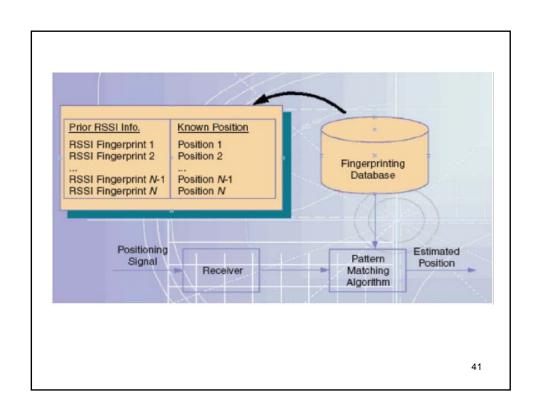
- TOA, TDOA, DOA, Hybrid belong to the family of "triangle" positioning algorithms
- These algorithms assume LOS propagation and use measurements to establish geometric equations (their solution will provide the mobile location)
- "triangle"-type algorithms
 - Deterministic
 - Probabilistic

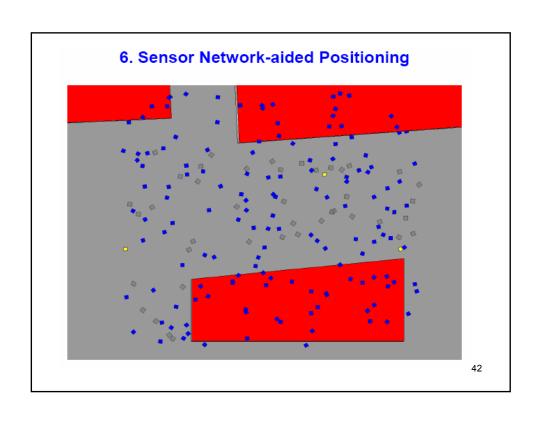
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- One way of solving the multipath problem is to consider multipath signal patterns characteristics as "fingerprinting" of MS
- Comparing received by a BS signal patterns/characteristics with signals characteristics stored in a Database (using pattern matching algorithms known as fingerprinting)
 - e.g. signal levels, time delays, multipath delay profile;
- Needs one BS with several multipath copies





- Sensor networks are, in general, asynchronous networks.
 Therefore, very difficult to estimate node-positions.
- (Due to the ad hoc nature of sensor networks, it is important to extract location information from data collected for location aware routing and from information dissemination protocols and query processing in a sensor network.)
- Most localization methods in sensor networks are based on RF signals.

6.1. Ultra Wide Band Techniques

- UWB techniques are quite promising for indoor positioning.
- The UWB technique is a viable approach for
 - future gigabit indoor communications and
 - geolocation problems .
- A UWB signal is a series of very short base band pulses with time durations of only a few nanoseconds that exist on all frequencies simultaneously, resembling a blast of electrical noise.
- The fine time resolution of UWB signals makes them promising for use in high-resolution ranging.
- A representative technique: A generalized maximum-likelihood (ML) detector for multipaths in UWB propagation measurement.

6.2. Systems Taxonomy

- · Spatial and Temporal Scale
 - Spatial Density of sensors relative to significant events (stimulus)
 Data rates
- Variability
 - Ad hoc versus engineered system structure
 - System task variability
 - variability in space (mobility)
- Resource constraints
 - EUE, BUE
 - storage, computation, transmission

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6.3. Load/Events Models

- Frequency of events
 - temporal and spatial density of events
- Locality
 - spatial and temporal correlation
- Mobility
 - rate and pattern

6.4. Metrics

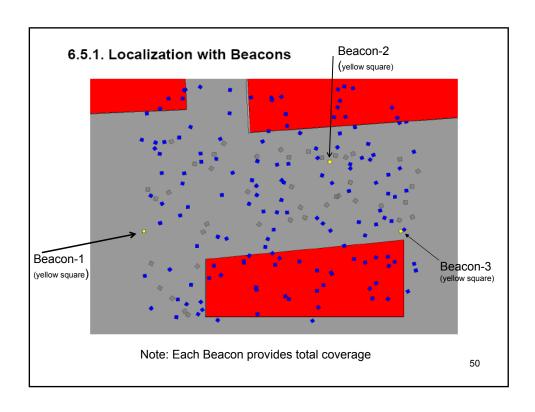
- Distributed design
 - every node is capable of estimating its own location
- Localised design
 - every node gets info from neighbouring nodes
- · asymptotic convergence design
 - computation stops when a certain degree of accuracy has been achieved.
- Self organizing design
 - node functioning is <u>independent</u> of global infrastructure
- · Robust design
 - node failure and range errors (and other environmental dynamics) can be tolerated

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- · Cost-effective and energy-efficient design
 - the algorithm requires little computational etc overheads)
- · Resolution/Fidelity
- Latency
 - response time
- Scalability
 - over space and time

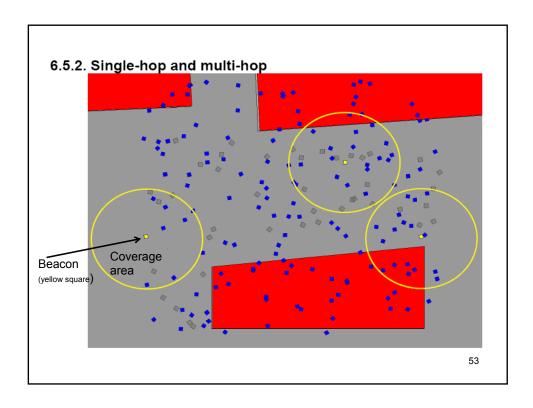
6.5. Localisation Taxonomy in Ad hoc Sensor Networks

- In terms of systems, the types of localization solutions can generally be classified into three categories:
 - localization with beacons
 - ➤Single-hop and multi-hop
 - localization with moving beacons,
 - beacon-free localization

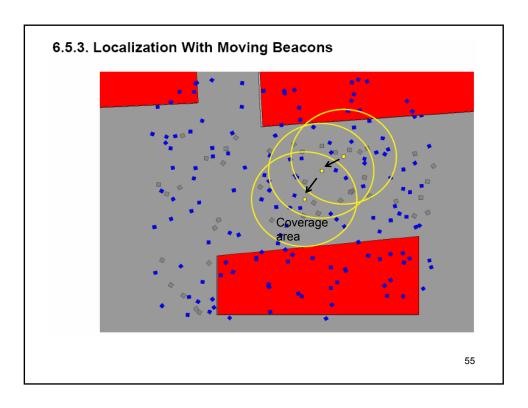


- some nodes of the sensor network, are equipped with special positioning devices that are aware of their locations (e.g., equipped with a GPS receiver).
 - These nodes are called beacons.
 - Other nodes that do not initially know their locations are called unknowns.
- When these systems perform localization, the *unknowns* are located using ranging or connectivity (also known as proximity).
 Generally, an *unknown* can estimate its location if three or more beacons are available in its 2-D coverage.
- Once an unknown has estimated its position, it becomes a beacon and other unknowns can use it in their position estimations.

- The major challenge in localization with beacons is to make localization algorithms as robust as possible using as few beacons as possible.
- The resulting design consumes little energy and few radio resources
- Limitations: they need a GPS or another positioning scheme to bootstrap the beacon node positioning algorithms
- · Are unsuitable for use indoors



- In multihop positioning systems, nodes typically do not receive beacon nodes' signals directly.
- Given the influence of single-hop positioning systems such as cellular networks and WLANs, it is not surprising that the first multihop localization algorithms tried to adapt single-hop technologies (i.e. TDOA, RSS, and/or AOA information is collected, and the position of each node is then computed using triangulation.)
- Example: It first determines the hop distance (called gradient) to the beacons (called seeds) and, as a function of the average node density, calculates the actual average hop distance to a beacon.



- Using moving beacons in a system design can significantly reduce power consumption and cost.
- In this type of system, nodes determine their own locations by estimating their distance from moving beacons (also referred to as mobile observers) in a coordinated fashion by applying a transform to the range estimations to determine each node's position within a global coordinate system.
- Sensor nodes receiving beacon packets infer their distance from a mobile beacon and use these measurements as constraints to construct and maintain position estimates.
- Also, once localized, network nodes can localize and track a mobile object (robot) and guide its navigation. (Note: The mobility of targets can be used to significantly enhance position estimation accuracy of nodes, even when the number of reference nodes is small.)

6.5.4. Beacon-Free Localization.

- In non-urban outdoor environments, localization may be achieved using several beacons equipped with GPS. However, equipping sensors with GPS does not work in indoor or urban environments. In addition, the use of beacons, even assuming that sensors are scattered randomly at the start, increases the cost of building a sensor network.
- In practice, a larger network may be designed to operate without beacons, which is known as beacon-free design.
 Such a design determines the position of every node via local nodeto-node communication.

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 Beacon-free positioning should be a fully decentralized solution: all nodes start from a random initial coordinate assignment. Then, they cooperate with each other using only local distance estimations to figure out a coordinate assignment. The resulting coordinate assignment has both translation and orientation degrees of freedom and has to be correctly scaled. A post-process is needed to convert the translation and orientation coordinate assignment to absolute position information based on reference information, such as information from GPS.

6.5.5. Some Commenst

1) BEACON-BASED ALGORITHMS

Assume that a certain minimum number of nodes know their own positions through manual configuration or GPS. Individual nodes' location could then be determined by referring to a beacon's position.

All beacon-based positioning algorithms, however, have their limitations because they need another positioning scheme to bootstrap the beacon node positions, and they cannot be easily employed in environments where other location systems are unavailable; thus, they are unsuitable for use indoors.

In practice, a large number of beacon nodes are required to achieve an acceptable level of position error.

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2) **BEACON-FREE ALGORITHMS**

use local distance information to attempt to determine each node's relative coordinates without relying on beacons that are aware of their positions.

Of course, any algorithm that does not use beacon nodes can be easily converted to one that uses a small number of beacon nodes by adding a final step in the procedure, in which all node coordinates are transformed using three (in 2-D positioning) or four (in 3-D positioning) beacon nodes.

With three or more beacons, the absolute coordinates of all the nodes can be determined at the same time.

6.5.6. Incremental and Concurrent

1) Incremental algorithms begin with only three or four core nodes being aware of their own coordinates.

They then recursively add appropriate nodes to this set by calculating each node's coordinates using measured relative distances from nodes with previously known coordinates.

These coordinate calculations are based on either simple triangle algorithms or on local optimization Schemes.

Limitations: they propagate measurement error, resulting in poor overall network localization. Some incremental algorithms can thus be applied in later stages of global optimization to reduce propagation error. Escaping from local minima and reaching global minima in the incremental stage continues to be a major challenge.

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2) In **concurrent algorithms**, all nodes calculate and then refine their coordinates in parallel using local information.

Some of these algorithms use iterative optimization schemes that reduce the differences between measured distances and calculated distances based on current coordinate estimates.

Concurrent optimization algorithms have a better chance of avoiding local minima than incremental schemes.

They can also avoid error propagation by continuously reducing global errors.

6.5.7. RANGE-BASED AND RANGE-FREE

Many localization algorithms rely on the distances between nodes: this is known as range-based localization.

This type of distance estimation is usually implemented using signal strength decay, TOA, or TDOA for internode range estimation.

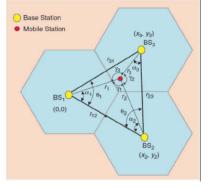
While range-based algorithms require absolute point-to-point distance estimation (range) or angle estimation for positioning, range free algorithms do not require this information.

In addition to measuring range information, range-free localization algorithms achieve position estimation by solving a convex optimization problem using a connectivity matrix of sensor nodes.

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7. An Array Based Localisation Architecure (Ref: Prof Manikas' Research)

- can be used for Positioning in Cellular Networks as well as in Ad-hoc Sensor Networks
- It is based on considering a number of BSs or a number of sensor nodes as an Array System



7.1 Propagation Modelling

Consider a single spherical wave

$$\sqrt{P}\exp(j2\pi F_c t)$$

propagating from the MS to the receiver Rx of the 1st BS through our 3D real space.

This spherical wave arrives at the BS1 receiver's antenna and produces a constant-amplitude voltage

$$\sqrt{P}\left(\frac{1}{r_1}\right)^a \exp(j2\pi F_c(t-\tau_1)) =$$

$$\sqrt{P} \mathrm{exp}(j2\pi F_c t) \left(\tfrac{1}{r_1} \right)^a \mathrm{exp} \big(- j2\pi F_c \tfrac{r_1}{c} \big)$$

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Furthermore, this spherical wave arrives at the BS_i (i = 2, 3, 4 etc) receiver's antenna and produces a constant-amplitude voltage $\sqrt{P}\left(\frac{1}{r_i}\right)^a \exp(j2\pi F_c(t-\tau_i)) =$

$$\sqrt{P}\left(\frac{1}{r_i}\right)^{\bar{a}} \exp(j2\pi F_c(t-\tau_i)) =$$

$$\sqrt{P}\exp(j2\pi F_c t) \left(\frac{1}{r_i}\right)^a \exp\left(-j2\pi F_c \frac{r_i}{c}\right) = (r_i = r_1 + \Delta r_{i1})$$

$$\sqrt{P} \exp(j2\pi F_c t) \left(\frac{r_1}{r_1} \frac{1}{r_i}\right)^a \exp\left(-j2\pi F_c \frac{r_1 + \Delta r_{i1}}{c}\right)$$

= signal at BS1 (array reference point)

= signal at BST (array reference point)
$$\frac{\sqrt{P}\exp(j2\pi F_c t)}{\text{Tx signal}} \frac{\left(\frac{1}{r_1}\right)^a \exp\left(-j2\pi F_c \frac{r_1}{c}\right)}{\text{fading coeff.}} \frac{\left(\frac{r_1}{r_i}\right)^a \exp\left(-j2\pi F_c \frac{\Delta r_{i1}}{c}\right)}{i^{th} \text{element of the manifold vector}}$$

or, the baseband vector-signal (using all N basestations) is

$$\sqrt{P}\beta_{1}\begin{bmatrix} \left(\frac{r_{1}}{r_{1}}\right)^{a} \exp\left(-j2\pi F_{c}\frac{r_{1}-r_{1}}{c}\right) \\ \dots \\ \left(\frac{r_{1}}{r_{i}}\right)^{a} \exp\left(-j2\pi F_{c}\frac{r_{1}-r_{i}}{c}\right) \\ \dots \\ \left(\frac{r_{1}}{r_{N}}\right)^{a} \exp\left(-j2\pi F_{c}\frac{r_{1}-r_{N}}{c}\right) \end{bmatrix} = \sqrt{P}\begin{bmatrix} \left(\frac{1}{r_{1}}\right)^{a} \exp\left(-j2\pi F_{c}\frac{r_{1}}{c}\right) \\ \dots \\ \left(\frac{1}{r_{N}}\right)^{a} \exp\left(-j2\pi F_{c}\frac{r_{N}}{c}\right) \\ \dots \\ \left(\frac{1}{r_{N}}\right)^{a} \exp\left(-j2\pi F_{c}\frac{r_{N}}{c}\right) \end{bmatrix}$$

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7.2. Spherical Array Manifold Vector

Consider an array of N-elements.

The complex array response, can be expressed using the spherical wave propagation model, as

$$\underline{S}_i = \left((r_i \cdot \underline{1}_N \oslash \underline{d}_i)^a \odot \exp\left(-j \cdot \frac{2 \cdot \pi \cdot f_e}{c} \cdot (r_i \cdot \underline{1}_N - \underline{d}_i) \right) \right) \tag{1}$$

a is a constant scalar

 $\text{where} \begin{cases} \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1}{2N} \text{ is a N-dimensional column vector of ones} \\ \frac{1$

with the vector $(r_i \cdot \underline{1}_N \oslash \underline{d}_i)^a$ denoting the amplitude path loss attenuation at the N array elements normalised with respect to the path loss attenuation associated with the reference point.

Furthermore, in Equation 1 the vector \underline{d}_i is defined as:

$$\underline{d}_i = \underline{d}(\theta_i, \, \phi_i, \, r_i, \, \underline{\underline{\mathbf{r}}}_i) = \sqrt{r_i^2 \cdot \underline{\mathbf{1}}_N + diag(\underline{\underline{\mathbf{r}}}_i^T \cdot \underline{\underline{\mathbf{r}}}_i) - \frac{r_i \cdot c}{\pi \cdot f_c} \cdot \underline{\underline{\mathbf{r}}}_i^T \cdot \underline{k}(\theta_i, \, \phi_i)} \quad (2)$$

with

$$\underline{k}(\theta_i, \ \phi_i) = \frac{2 \cdot \pi \cdot f_e}{c} \cdot [\cos(\phi_i)\cos(\theta_i), \ \cos(\phi_i)\sin(\theta_i), \ \sin(\phi_i)]^T$$

representing the wavenumber vector.

Note that $\underline{\mathbf{r}}_{i} = [\underline{\mathbf{r}}_{x_i}, \ \underline{\mathbf{r}}_{y_i}, \ \underline{\mathbf{r}}_{z_i}]^T$ is the 3 \times N matrix containing the Cartesian coordinates of the array elements with respect to the reference point.

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On the basis of the above modelling it is clear that the manifold vector \underline{S}_i depends on the array reference point relative to which the bearings and the range of the source as well as the Cartesian coordinates of the array configuration are measured.

By denoting with

$$\underline{S}_1 = \underline{S}(\theta_1, \, \phi_1, \, r_1, \, \underline{\underline{\mathbf{r}}}_1, \, f_c)$$

and

$$\underline{S}_2 = \underline{S}(\theta_2, \, \phi_2, \, r_2, \, \underline{\underline{\mathbf{r}}}_2, \, f_c)$$

the manifold vectors with respect to two reference points 1 and 2, it can be proved, that

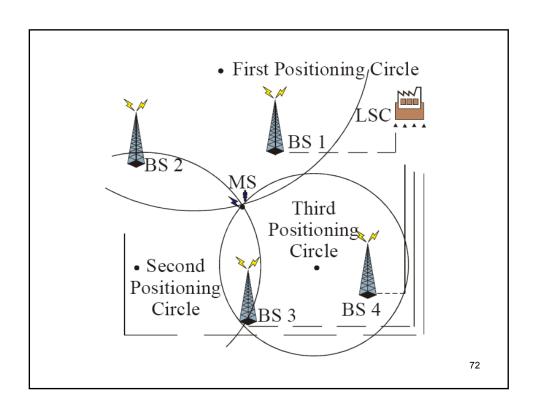
$$\underline{d}_1 = \underline{d}_2 = \begin{bmatrix} r_1 \\ r_2 \\ \dots \\ r_N \end{bmatrix}$$

and therefore the manifold vectors \underline{S}_1 and \underline{S}_2 are interrelated by the following expression:

$$\underline{S}_2 = \left(\frac{r_2}{r_1}\right)^a \cdot \exp\left(-j \cdot \frac{2 \cdot \pi \cdot f_c}{c} \cdot (r_2 - r_1)\right) \cdot \underline{S}_1 \tag{3}$$

The above implies that the effect of a change in the reference point on the manifold vector \underline{S} is simply a change in the norm of the vector, (which depends only on the range of the mobile with respect to the two reference points).

Consequently, the subspace spanned by the manifold vector remains unchanged.



Four measurements

$$\mathbb{R}_{xx}^{(1)} \Rightarrow \gamma_1 = \max\left(\operatorname{eig}\,\mathbb{R}_{xx}^{(1)}\right)$$

$$\mathbb{R}_{xx}^{(2)} \Rightarrow \gamma_2 = \max\left(\operatorname{eig}\,\mathbb{R}_{xx}^{(2)}\right)$$

$$\mathbb{R}_{xx}^{(3)} \Rightarrow \gamma_3 = \max\Bigl(\operatorname{eig}\,\mathbb{R}_{xx}^{(3)}\Bigr)$$

$$\mathbb{R}_{xx}^{(4)} \Rightarrow \gamma_4 = \max\Bigl(\operatorname{eig}\,\mathbb{R}_{xx}^{(4)}\Bigr)$$

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Estimate noise power $\hat{\sigma}_n^2$ and then

$$\lambda_1 = \gamma_1 - \hat{\sigma}_n$$

$$\lambda_2 = \gamma_2 - \hat{\sigma}_n \qquad \Rightarrow \text{form } \frac{\lambda_2}{\lambda_1} = \left(\frac{r_2}{r_1}\right)^{2a}$$

$$\lambda_3 = \gamma_3 - \hat{\sigma}_n \qquad \Rightarrow \text{form } \frac{\lambda_3}{\lambda_1} = \left(\frac{r_3}{r_1}\right)^{2a}$$

$$\lambda_4 = \gamma_4 - \hat{\sigma}_n \qquad \Rightarrow \text{form } \frac{\lambda_4}{\lambda_1} = \left(\frac{r_4}{r_1}\right)^{2a}$$

etc.

Equation:
$$\mathbb{H} \, \underline{\mathbf{r}}_m = \underline{b}_{\mathrm{IC}} \Rightarrow \underline{\mathbf{r}}_m = \left(\mathbb{H}^T \mathbb{H}\right)^{-1} \mathbb{H}^T \underline{b}_{\mathrm{IC}}$$

where
$$\underline{b}_{\mathrm{IC}} = \frac{1}{2} \begin{bmatrix} \|\underline{\mathbf{r}}_2\|^2 + \left(1 - \sqrt[4]{\frac{\lambda_2}{\lambda_1}}\right) r_1^2 \\ \|\underline{\mathbf{r}}_3\|^2 + \left(1 - \sqrt[4]{\frac{\lambda_3}{\lambda_1}}\right) r_1^2 \\ \|\underline{\mathbf{r}}_4\|^2 + \left(1 - \sqrt[4]{\frac{\lambda_4}{\lambda_1}}\right) r_1^2 \end{bmatrix}$$
 $\mathbb{H} = [\underline{\mathbf{r}}_2, \underline{\mathbf{r}}_3, \underline{\mathbf{r}}_4]^T = \begin{bmatrix} x_2, & y_2 \\ x_3, & y_3 \\ x_4, & y_4 \end{bmatrix}$

To be estimated firstly (positive root of quadr. Equ.)

