Ex,

large-scale fading (shadowing) $S_{dB} = 10\log_{10} S \sim N(0, \sigma_s^2)$

M: $x = 10\log_{10} S \sim N(0, \sigma_s^2)$

calculate $S = 10^{\frac{x}{10}}$ → shadowing realisation

multiple realisations of shadowing

```
for i = 1:1000
    x = randn(0, σs^2);
    S[i] = 10^(x/10);
end
```

Plot(s)

$\sigma_s^2 = 8\,dB \qquad \sigma_s^2 = 10^{\frac{8}{10}}$

Ex2:

$h_i \sim CN(0,1)$

real imag

$h_i = h_{real} + i h_{imag}$

$CN(0,1)$ $\quad$ $N(0,\frac{1}{2}) = \frac{N(0,1)}{\sqrt{2}}$ $\quad$ $N(0,\frac{1}{2})$

M:

for $i = 1:10000$

$h_R = \frac{N(0,1)}{\sqrt{2}} \sim \frac{1}{\sqrt{2}} * \text{randn}$

$h_I = \frac{N(0,1)}{\sqrt{2}}$

$h = h_R + i h_I$

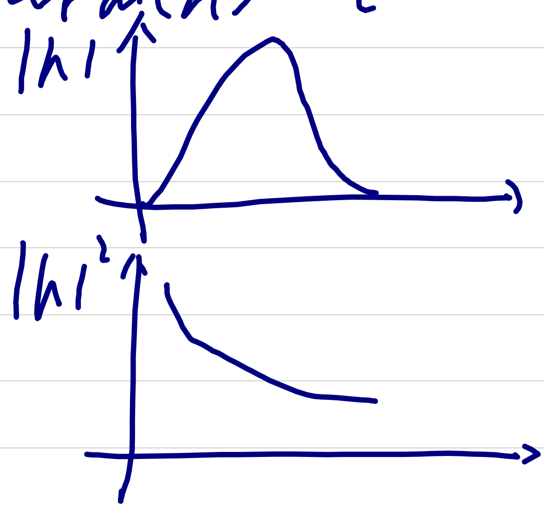
norm[i] = abs(h)

square abs.[i] = norm(h)^2

end

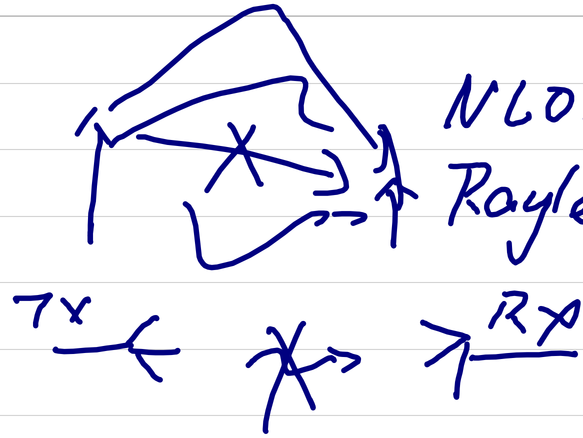
plot(norm)

plot(square abs)



① Ex2

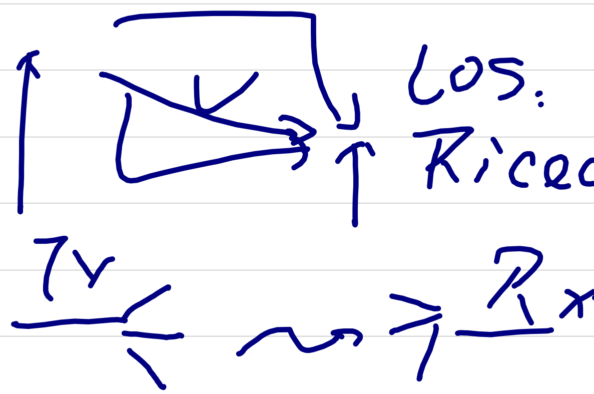
$h \sim \mathcal{CN}(0,1)$



NLOS: Rayleigh

② Ex3

$h = h_{LOS} + h_{Rayleigh}$



LOS: Ricean

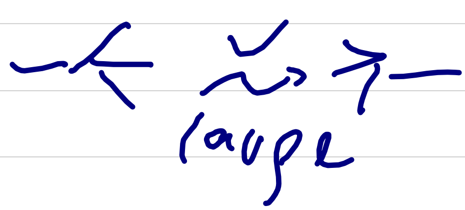
$$h = \sqrt{\frac{K}{K+1}}\,\bar{h} + \sqrt{\frac{1}{1+K}}\,\tilde{h}$$

K: how much energy is in LOS

Ricean factor

$K = 0 \Rightarrow h = \tilde{h}$ (Rayleigh)

$K \to \infty \Rightarrow h \approx \bar{h} = e^{j\phi}$



fix K

for $i = 1:10000$

$h_R = \frac{N(0,1)}{\sqrt{2}}$

$h_I = \frac{N(0,1)}{\sqrt{2}}$

$\tilde{h} = h_R + i h_I \rightarrow$ Rayleigh

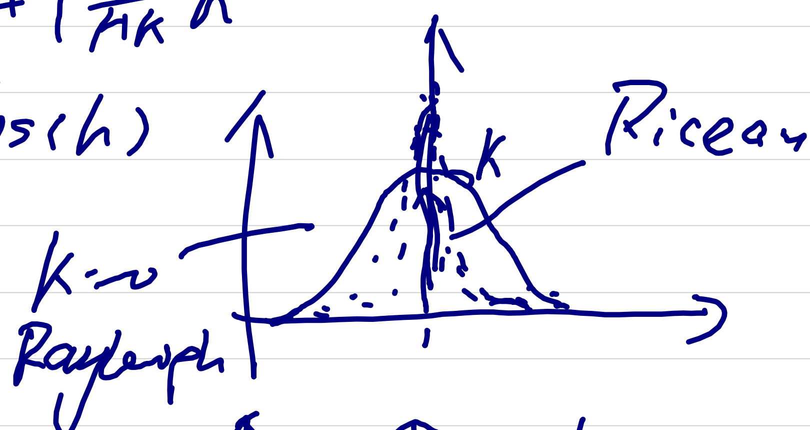
$\bar{h} = e^{j\phi} = 1$

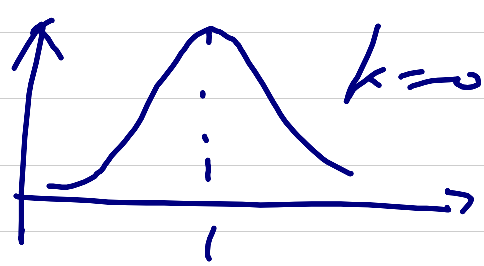
$h = \sqrt{\frac{K}{1+K}}\,\bar{h} + \sqrt{\frac{1}{1+K}}\,\tilde{h}$

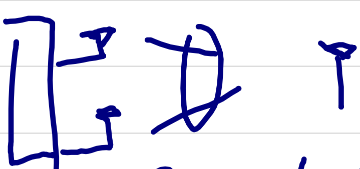
norm[i] = abs(h)

end

plot(norm)

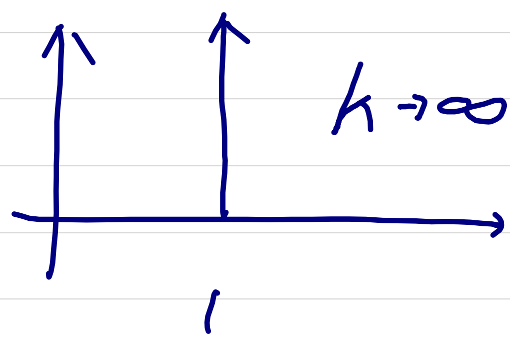






correlated

uncorrelated

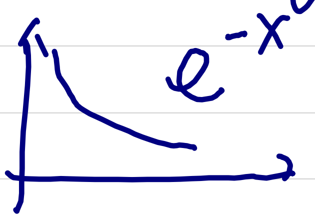


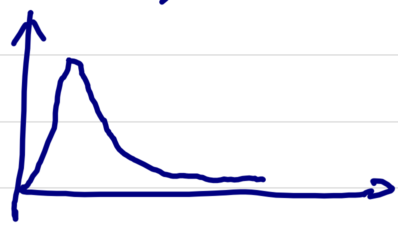
Ex4:

n complex normal RV

$X_1 . X_2 .. X_n \Rightarrow Y = |X_1|^2 + |X_2|^2 + .. + |X_n|^2$

$n=1 \Rightarrow |X_1|^2$ conj (RV) n=3





n=3

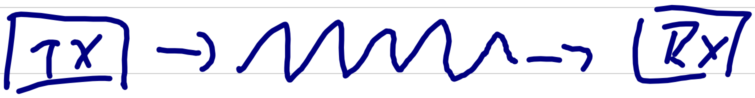
```
for i = 1:100
    hR = N(0.1)/√2
    hI = N(0.1)/√2
    h1 = hR + i hI
    :
    h2 = hR' + i h2'

sum_nom_h[i] = abs(h1)^2 + abs(h2)^2
plot(sum_nom_h)
```

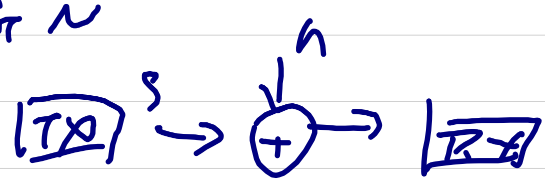
```
for i=1:1000
  for j=1:n
    h=hR+jhI
    sum_norm.h[i] = sum_norm[i-1]
                              + abs(h)^2
```

Ex 5:

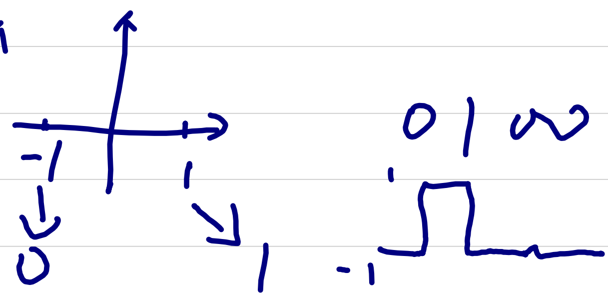
BPSK



① AWGN

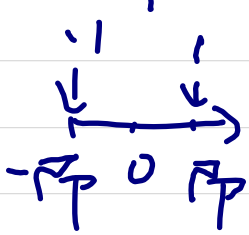


BPSK

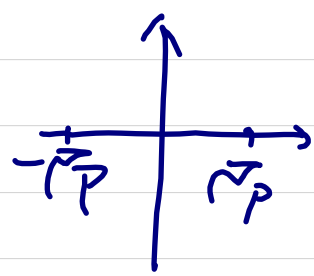


Power $= |s|^2$

$110100 \to \{1, 1, -1, 1, -1, -1\}$

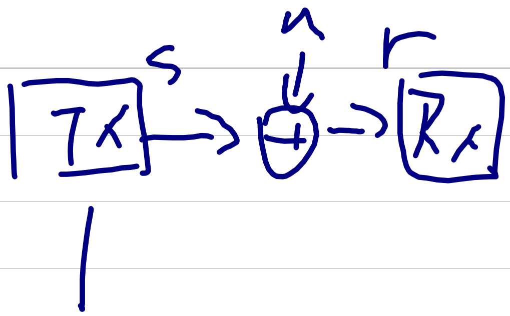


BPSK with power $P \Rightarrow$ symbols



$P = 10\,dB \Rightarrow 10 \Rightarrow 3..$

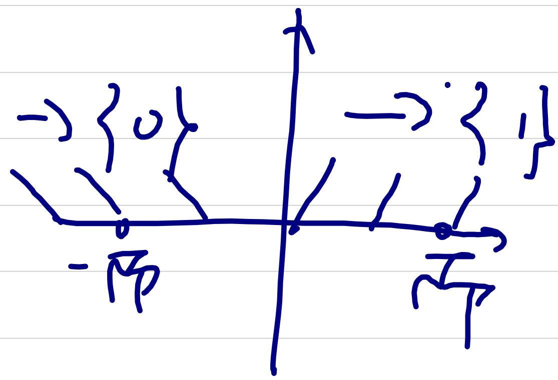
$P = 20\,dB \Rightarrow 100 \Rightarrow 10$



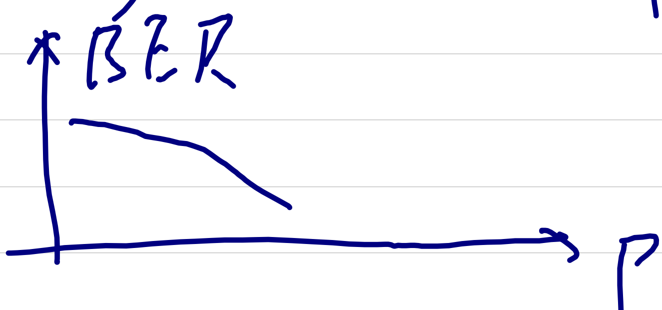
generate 10000 random bits

$0 \to (-1) \to -\sqrt{p}$

$1 \to \sqrt{p}$



$r = s + n \sim \mathcal{N}(0, 1)$



$b_i \in \{0, 1\} \to s_i \in \{-\sqrt{p}, \sqrt{p}\}$

$\oplus \leftarrow n$

$\begin{cases} >0 \to 1 \\ <0 \to 0 \end{cases} \leftarrow \mathrm{Re}\{y_i\} \leftarrow y_i = s_i + n$

Ex5. 1)

BPSK/QPSK on AWGN channel

① BPSK

bit/data generation — mapping — power: SNR — Tx

$\underbrace{101\ldots011}_{1e^4} \rightarrow \boxed{-1 \quad +1} \rightarrow \boxed{\times\sqrt{P}}$

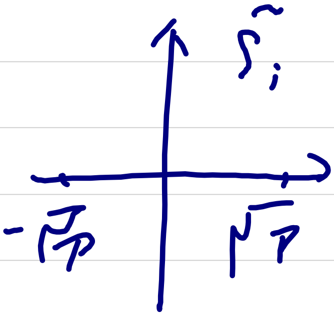
eg: $1 \rightarrow 1 \rightarrow \sqrt{P}$

$0 \rightarrow -1 \rightarrow -\sqrt{P}$

channel

$\rightarrow \boxed{s_i \oplus} \quad$ CN $\rightarrow$

$s_i + CN(0,1)$



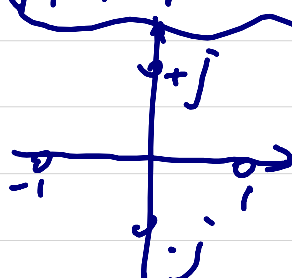
$\tilde{s}_i \rightarrow \begin{cases} 0. & Re\{\tilde{s}_i\} < 0 \\ 1. & Re\{\tilde{s}_i\} > 0 \end{cases}$ Rx

$P_e = \frac{\#error}{\#bits}$

Rx knows h_i

② QPSK

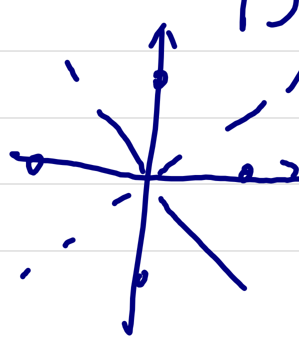
$\underbrace{00}_{1}\ \underbrace{10}_{-j}\ \underbrace{01}_{+j}\ \underbrace{00}_{1}\ \underbrace{10}_{-j} \rightarrow$



$\rightarrow \boxed{\sqrt{P}}$

$CN(0,1)$

$\rightarrow \oplus \rightarrow$



ML decoding

$\tilde{s}_i = s_i * h_i$

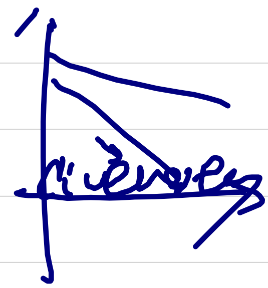
$\min d_i$

$\hat{s}_i = \frac{\tilde{s}_i}{h_i}$

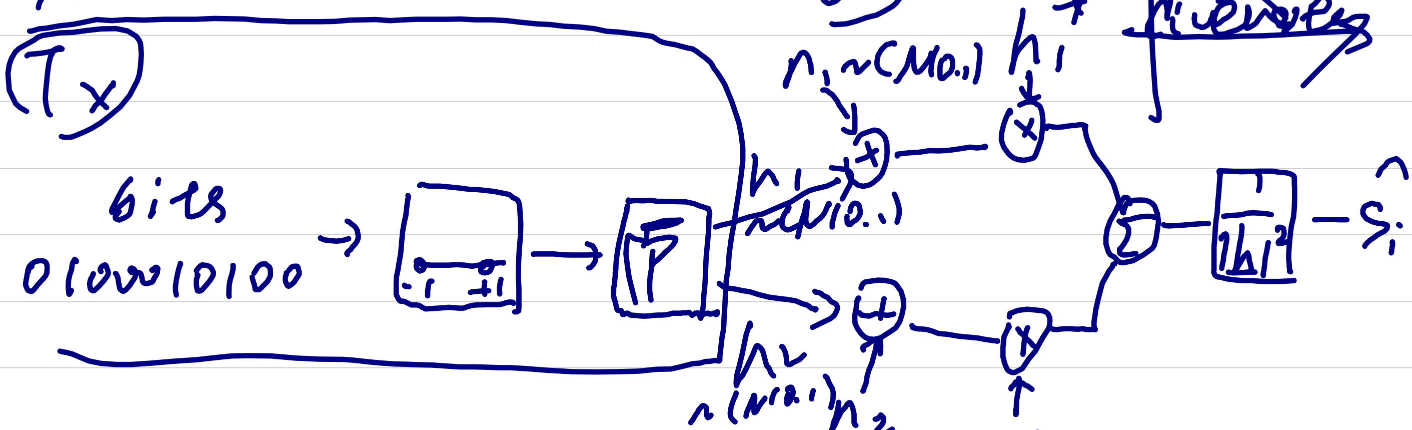
$P_e(\text{symbol}) = \dfrac{\#\text{ wrong symbol}}{\#\text{ Tx symbol}}$

2) channel: $h_i s_i + CN(0,1)$. $h_i \sim CN(0,1)$

change after several symbols.



3)



6) MRT @ Rx not to do anything

#noise = #receiver

MRT = MRC (Tx need to know CN)

$|\sqrt{P} h_1^* s_1|^2 + |\sqrt{P} h_2^* s_2|^2$

$= P(E)(|h_1|^2 + |h_2|^2)$

$= 1$

5)

000110111010

$s_6\ s_5\ s_4\ s_3\ s_2\ s_1$

2xsymbols each time

(s_2, s_1)

Alamouti: get diversity gain w/o CSI

$$s_1, s_2 \to \begin{array}{c|c} -s_2^* & s_1 \\ s_1^* & s_2 \\ (2) & (1) \end{array} \quad \boxed{\frac{1}{\sqrt{2}}} \quad h_1,\ h_2 \quad (+) \Rightarrow \begin{array}{l} y^{(1)} = \frac{1}{\sqrt{2}}(h_1 s_1 + h_2 s_2) + n^{(1)} \\ y^{(2)} = \frac{1}{\sqrt{2}}(h_1(-s_2^*) + h_2 s_1^*) + n^{(1)} \end{array}$$

$$[s_1\ s_2]\begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix}^* = 0 \qquad \Rightarrow \begin{array}{l} y^{(1)} = \frac{1}{\sqrt{2}}(h_1 s_1 + h_2 s_2) + n^{(1)} \\ y^{(2)*} = \frac{1}{\sqrt{2}}(-h_1^* s_2 + h_2^* s_1) + n^{(2)} \end{array}$$

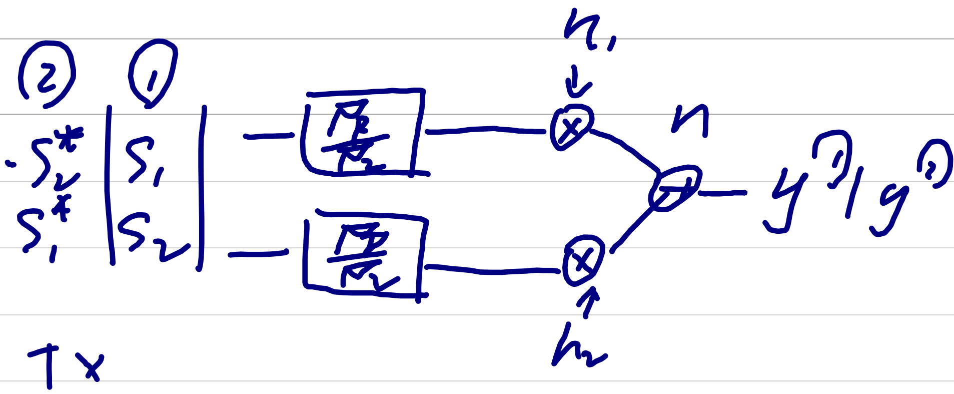
orthogonal

$$\begin{bmatrix} y^{(1)} \\ y^{(2)*} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n^{(1)} \\ n^{(2)} \end{bmatrix}$$

$$\hat{s}_1 = [h_1^*\ h_2]\begin{bmatrix} y^{(1)} \\ y^{(2)} \end{bmatrix} = \frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2) s_1$$

$$\hat{s}_2 = [h_2^*\ -h_1]\begin{bmatrix} y^{(1)} \\ y^{(2)} \end{bmatrix} = \underbrace{\frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)}_{\text{diversity gain}} s_2$$

$\frac{1}{\sqrt{2}} \to 3\text{dB}$



$$\Rightarrow \begin{cases} \hat{s}_1^{pre} = (h_1^* \;\; h_2)(y^{(1)}, y^{(2)}) \\ \hat{s}_2^{pre} = [h_2^* \;\; -h_1](y^{(1)}, y^{(2)}) \end{cases}$$

$$\Rightarrow \hat{s}_1 = \frac{\hat{s}_1^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

$$\hat{s}_2 = \frac{s_2^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$