Ex1

large-scale fading (shadowing) $S_{dB} = 10\log_{10} S \sim N(0, \sigma_s^2)$

M: $x = 10\log_{10} S \sim N(0, \sigma_s^2)$

calculate $S = 10^{\frac{x}{10}}$ → shadowing realisation

multiple realisations of shadowing

```
for i = 1:1000
    x = randn(0, σs^2);
    S[i] = 10^(x/10);
end
```

plot(S)

$\sigma_s^2 = 8\,dB \qquad \sigma_s^2 = 10^{\frac{8}{10}}$

Ex2:

$h_i \sim CN(0,1)$

real imag

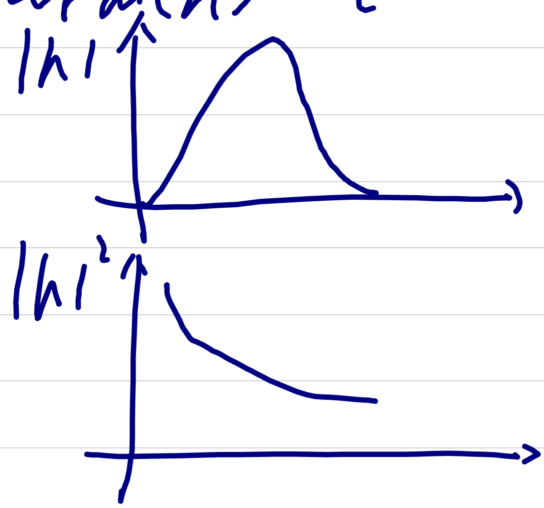
$h_i = h_{real} + i h_{imag}$

$CN(0,1)$ $N(0,\frac{1}{2})$ $N(0,\frac{1}{2})$

$= \frac{N(0,1)}{\sqrt{2}}$

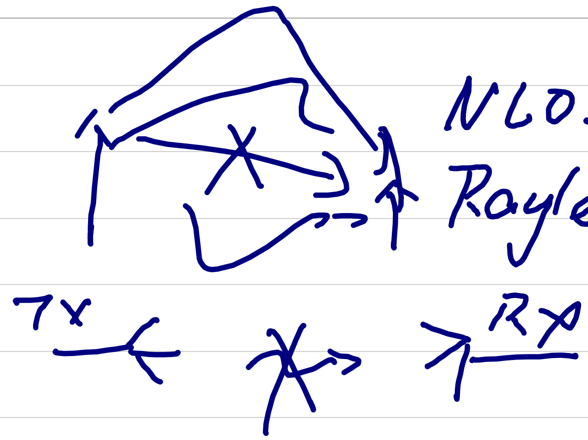
M:

```
for i = 1:10000
    hR = N(0,1)/sqrt(2) ~ 1/sqrt(2) * randn
    hI = N(0,1)/sqrt(2)
    h = hR + i hI
    norm[i] = abs(h)
    square abs.[i] = norm(h)^2
end
plot(norm)
plot(square abs)
```



① Ex2

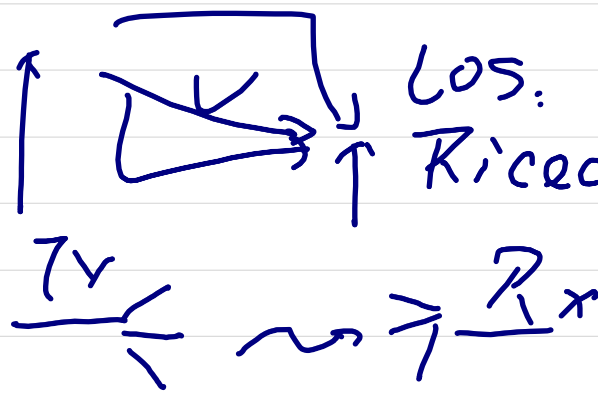
$h \sim \mathcal{CN}(0,1)$



NLOS: Rayleigh

② Ex3

$h = h_{LOS} + h_{Rayleigh}$



LOS: Ricean

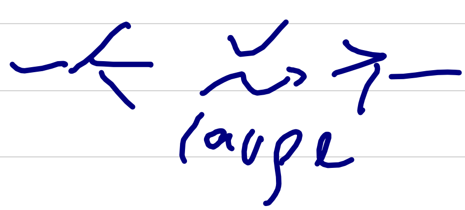
$$h = \sqrt{\frac{K}{K+1}}\,\bar{h} + \sqrt{\frac{1}{1+K}}\,\tilde{h}$$

K: how much energy is in LOS

Ricean factor

$K = 0 \Rightarrow h = \tilde{h}$ (Rayleigh)

$K \to \infty \Rightarrow h \approx \bar{h} = e^{j\phi}$



fix K

for i=1:10000

$h_R = \frac{N(0,1)}{\sqrt{2}}$

$h_I = \frac{N(0,1)}{\sqrt{2}}$

$\tilde{h} = h_R + i h_I$ → Rayleigh

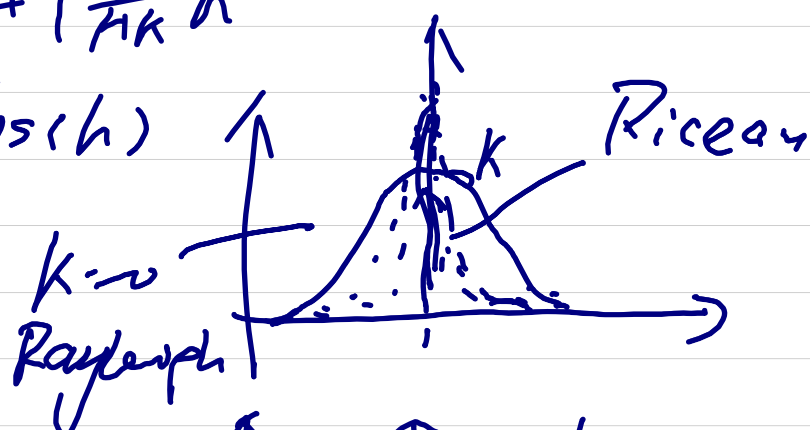
$\bar{h} = e^{j\theta} = 1$

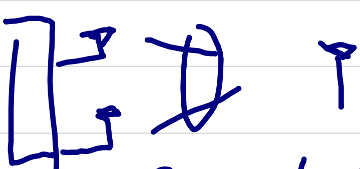
$h = \sqrt{\frac{K}{1+K}}\,\bar{h} + \sqrt{\frac{1}{1+K}}\,\tilde{h}$

norm[i]=abs(h)

end

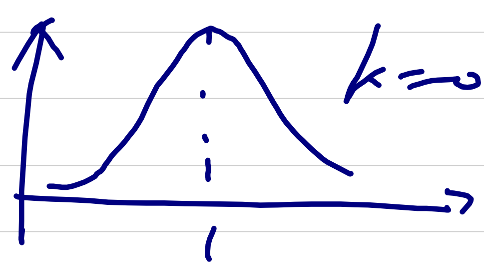
plot(norm)

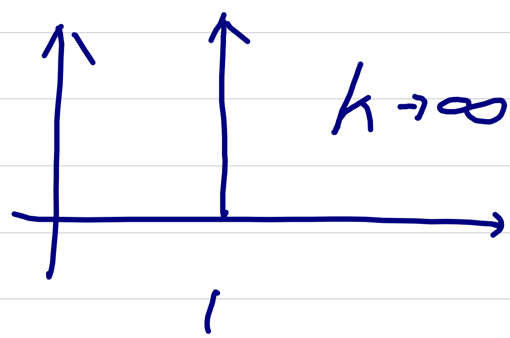




correlated

uncorrelated



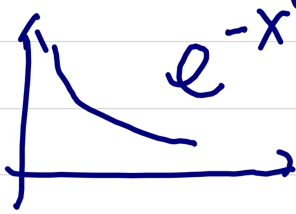


Ex 4:

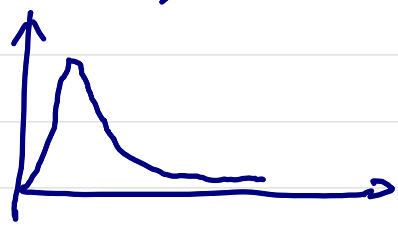
n complex normal RV

$X_1 . X_2 .. X_n \Rightarrow Y = |X_1|^2 + |X_2|^2 + .. + |X_n|^2$

$n = 1 \Rightarrow |X_1|^2$ cont'y (RV)



$n = 3$



$n = 3$

for i = 1 : 100

$h_R = \frac{N(0.1)}{\sqrt{2}}$

$h_I = \frac{N(0.1)}{\sqrt{2}}$

$h_1 = h_R + i h_I$

$\vdots$

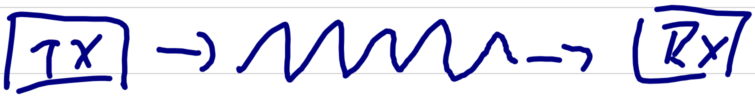
$h_2 = h_R' + i h_I'$

sum_nom_h[i] = abs(h_1)^2 + abs(h_2)^2

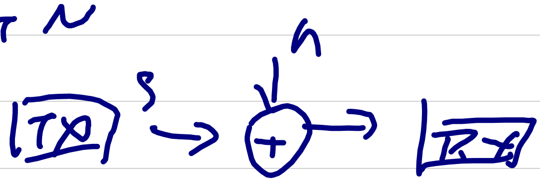
plot (sum_nom_h)

```
for i:1:100
  for j=1:n
    h=hR+jhI
    sum_norm.h[i] = sum_norm[i-1]
                    + abs(h)^2
```

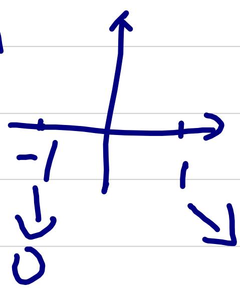
Ex 5:

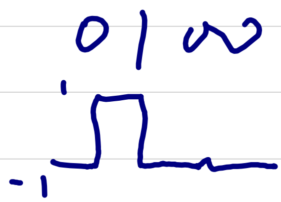
BPSK 

① AWGN



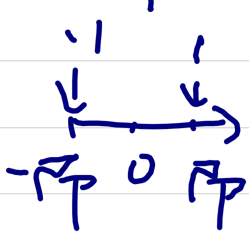
BPSK



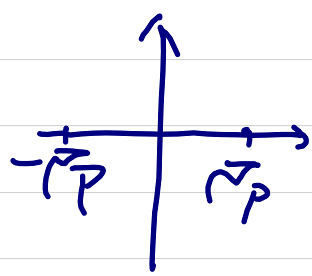


Power $= |s|^2$

$110100 \to \{1, 1, -1, 1, -1, -1\}$

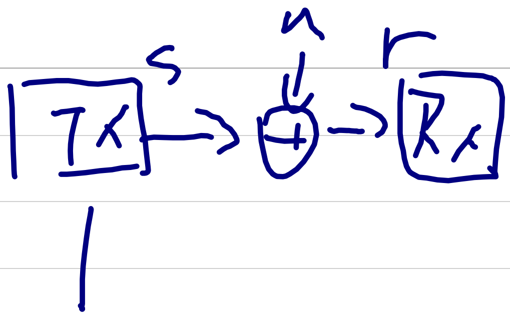


BPSK with power $P \Rightarrow$ symbols



$P = 10dB \Rightarrow 10 \Rightarrow 3..$

$P = 20dB \Rightarrow 100 \Rightarrow 10$

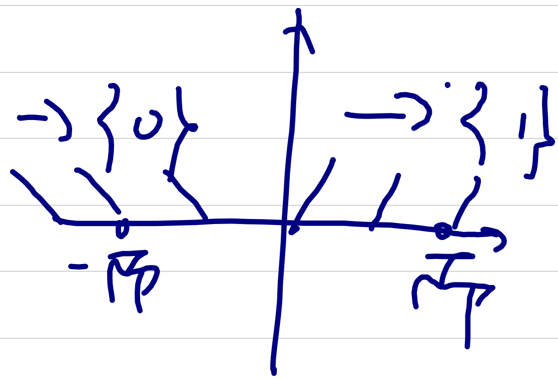


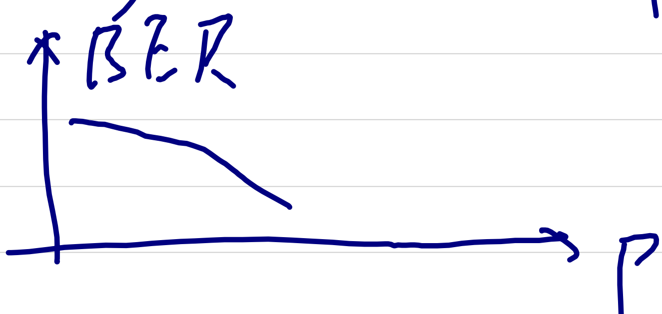
generate 10000 random bits

$0 \to (-1) \to -\sqrt{P}$

$1 \to \sqrt{P}$

$r = s + n \sim \mathcal{N}(0, 1)$





$b_i \in \{0, 1\} \mapsto s_i \in \{-\sqrt{P}, \sqrt{P}\}$

$\oplus \leftarrow n$

$\begin{cases} >0 \to 1 \\ <0 \to 0 \end{cases} \leftarrow \mathrm{Re}\{y_i\} \leftarrow y_i = s_i + n$

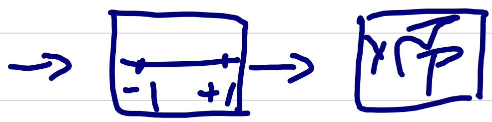
Ex5. 1)

BPSK/QPSK on AWGN channel

① BPSK

bit/data generation   mapping   power=SNR   Tx

$\underbrace{1\,0\,1 \ldots 0\,1\,1}_{1e^4}$ →



eg: $1 \to 1 \to \sqrt{P}$

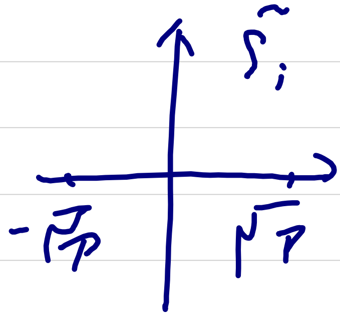
$0 \to -1 \to -\sqrt{P}$

channel

CN

$s_i + CN(0,1)$

→



$\tilde{s}_i \to \begin{cases} 0. & Re\{\tilde{s}_i\} < 0 \\ 1. & Re\{\tilde{s}_i\} > 0 \end{cases}$

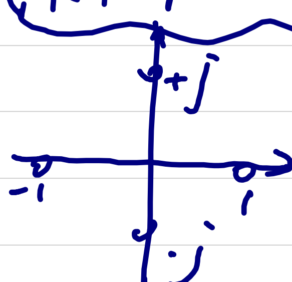
Rx

$$P_e = \frac{\#error}{\#bits}$$

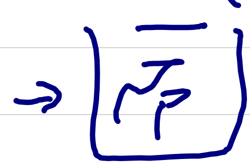
② QPSK

Rx knows h_i

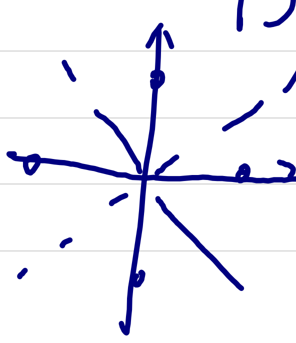
$\underbrace{00}_{1}\ \underbrace{10}_{-j}\ \underbrace{01}_{+j}\ \underbrace{00}_{1}\ \underbrace{10}_{-j}$ →



→



$CN(0,1)$



ML decoding

$\tilde{s}_i = s_i * h_i$

$\min\, \mathcal{D}_i$

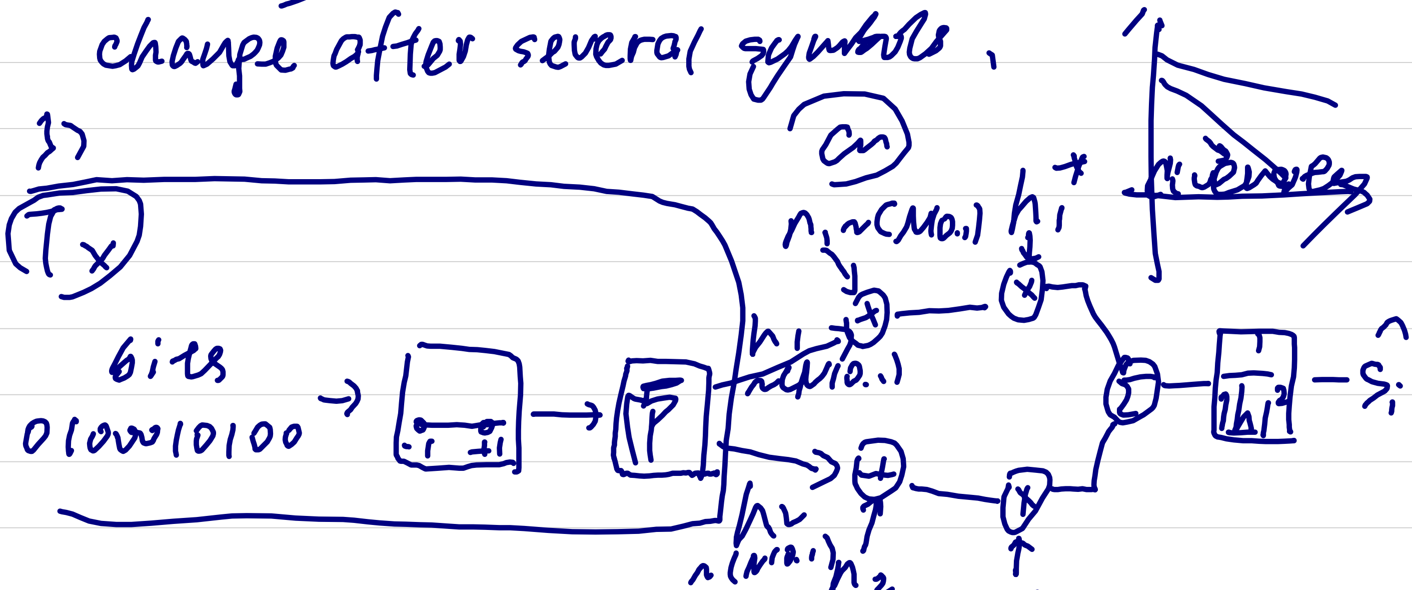
$$\hat{s}_i = \frac{\tilde{s}_i}{h_i}$$

$P_e(\text{symbol}) = \dfrac{\#\text{ wrong symbol}}{\#\text{ Tx symbol}}$

2) channel: $h_i s_i + CN(0,1)$. $h_i \sim CN(0,1)$

change after several symbols.

3)



6) MRT @ Rx not to do anything

\# noise = \# receiver

MRT $\equiv$ MRC

(Tx need to know CN)

$\begin{bmatrix} h_1^* s_i \\ h_2^* s_i \end{bmatrix}$ 2

s_i: $h_1^* \to \boxed{\sqrt{P}} \to$, $h_2^* \to \boxed{\sqrt{P}} \to$

$|\sqrt{P} h_1^* s_1|^2 + |\sqrt{P} h_2^* s_2|^2$

$= P(t)(|h_1|^2 + |h_2|^2)$

$\quad\;\; \| \quad 1$

5)

000110111010

$s_6\ s_5\ s_4\ s_3\ s_2\ s_1$

2x symbols each time

(s_2, s_1)

Alamouti: get diversity gain w/o CSI

$$s_1, s_2 \to \begin{array}{c|c} -s_2^* & s_1 \\ s_1^* & s_2 \\ ② & ① \end{array} \quad \frac{1}{\sqrt{2}} \quad h_1,\ h_2$$

$$① \Rightarrow y^{①} = \frac{1}{\sqrt{2}}(h_1 s_1 + h_2 s_2) + n^{①}$$

$$y^{②} = \frac{1}{\sqrt{2}}(h_1(-s_2^*) + h_2 s_1^*) + n^{②}$$

$$[s_1\ s_2]\begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix}^* = 0 \qquad \Rightarrow y^{①} = \frac{1}{\sqrt{2}}(h_1 s_1 + h_2 s_2) + n^{①}$$

orthogonal

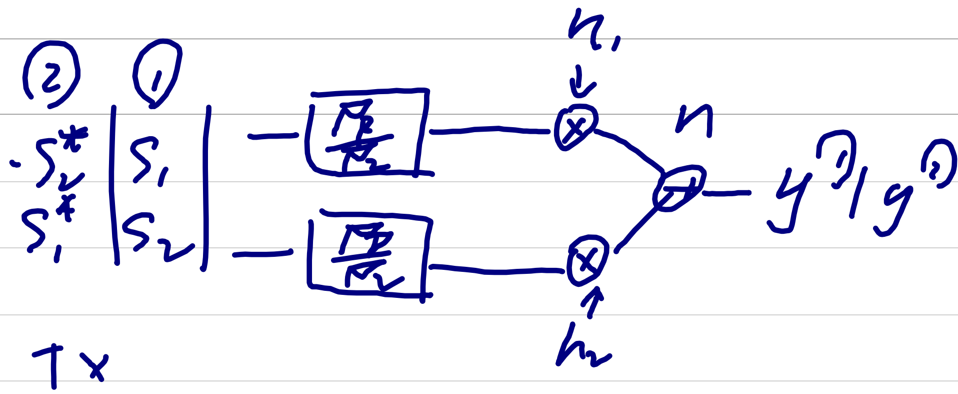
$$y^{②*} = \frac{1}{\sqrt{2}}(-h_1^* s_2 + h_2^* s_1) + n^{②}$$

$$\begin{bmatrix} y^{①} \\ y^{②*} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n^{①} \\ n^{②} \end{bmatrix}$$

$$\hat{s}_1 = [h_1^*\ h_2]\begin{bmatrix} y^{①} \\ y^{②} \end{bmatrix} = \frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)s_1$$

$$\hat{s}_2 = [h_2^*\ -h_1]\begin{bmatrix} y^{①} \\ y^{②} \end{bmatrix} = \underbrace{\frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)}_{\text{diversity gain}} s_2$$

$\frac{1}{\sqrt{2}} \to 3dB$



$$\Rightarrow \begin{cases} \hat{s}_1^{pre} = (h_1^* \ \ h_2)(y^{(1)}, y^{(2)}) \\ \hat{s}_2^{pre} = [h_2^* \ \ -h_1] \end{cases}$$

$$\Rightarrow \hat{s}_1 = \frac{\hat{s}_1^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

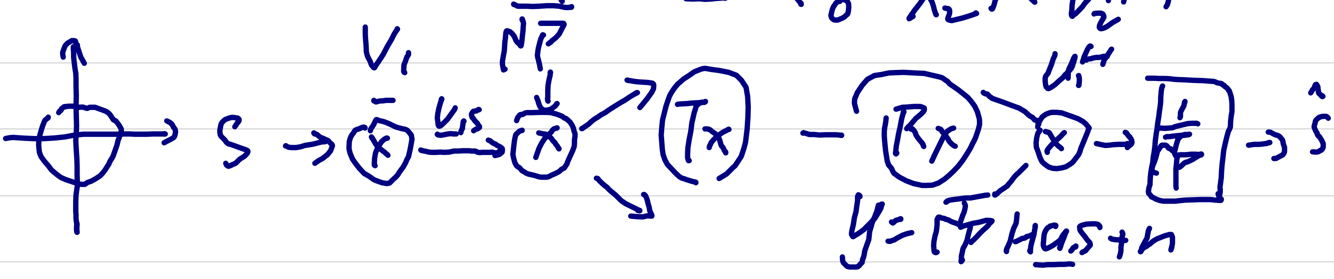
$$\hat{s}_2 = \frac{s_2^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

6. a) Dominant eigenmode transmission (DET)

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

Rx　　H　　Tx

$$\xrightarrow{SVD} U \Lambda V^H = (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \underline{v}_1^H \\ \underline{v}_2^H \end{pmatrix}$$



$$\underline{y} = \sqrt{P} H \underline{v}_1 s + \underline{n}$$

2x2 DET:
array gain = 4
div. gain = 4

Rx1, Rx2 $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \sqrt{P}\, U \Lambda \underline{V^H v_1} s + n$

$$= \begin{pmatrix} v_1^H \\ v_2^H \end{pmatrix} v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

2x2 Alamouti:
array gain = 2 (no CSIT: 3 dB loss)
d.v. gain = 4

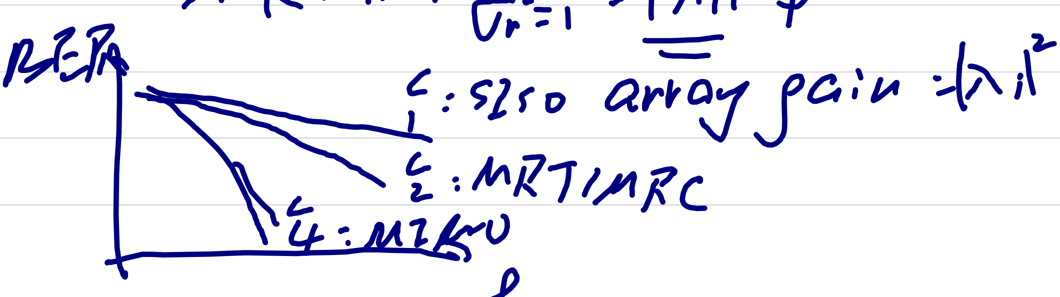
$$= \sqrt{P} (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} s$$

$$= \sqrt{P} (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 s \\ 0 \end{pmatrix} = \sqrt{P}\, \underline{u}_1 \lambda_1 s$$

Rx: $y = \sqrt{P} H \underline{u}_1 s + n$

$$u_1^H y = \sqrt{P} H \underline{u}_1^H u_1 \lambda_1 s + u_1^H n = \sqrt{P} \lambda_1 s + \underline{n}$$

$$SNR = |\lambda_1|^2 \frac{P}{\sigma_n^2 = 1} = \underline{\underline{|\lambda_1|^2 P}}$$



b)

$-s_2^* \quad s_1$

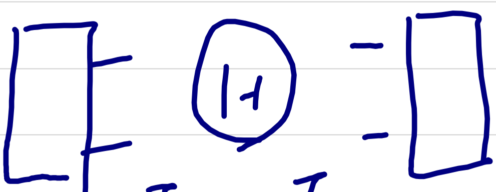
$s_1^* \quad s_2$

$$y(1) = h_1 s_1 + h_2 s_2 + n$$
$$y(2) = -h_1 s_2^* + h_2 s_1^* + n$$
$$\Rightarrow \underbrace{\begin{bmatrix} y(1) \\ y^*(2) \end{bmatrix}}_{y} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\hat{s}_1 = [h_1^* \;\; h_2]^T y \qquad \text{div. gain} = 2$$
$$\hat{s}_2 = [h_2^* \;\; -h_1]^T y$$

$-s_2^* \quad s_1$

$s_1^* \quad s_2$



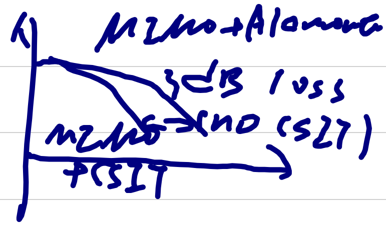
$$H = \begin{array}{c c} & \begin{array}{cc} Tx_1 & Tx_2 \end{array} \\ \begin{array}{c} Rx_1 \\ Rx_2 \end{array} & \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \end{array} = (\underline{h}_1 \,|\, \underline{h}_2)$$

$$\underline{y}(1) = \underline{h}_1 s_1 + \underline{h}_2 s_2 + \underline{n}(1)$$
$$\underline{y}(2) = -\underline{h}_1 s_2^* + \underline{h}_2 s_1^* + \underline{n}(2)$$
$$\Rightarrow \begin{bmatrix} \underline{y}(1) \\ \underline{y}^*(2) \end{bmatrix} = \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \\ \underline{h}_2^* & -\underline{h}_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \underline{n}(1) \\ \underline{n}(2) \end{bmatrix}$$

$\hat{s}_1 = [\underline{h}_1^* \; \underline{h}_2^T]\, y$

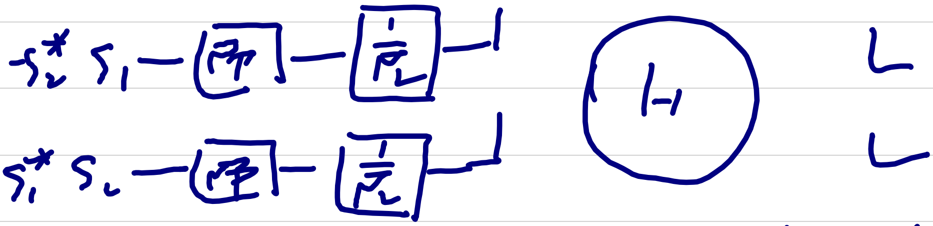
$\hat{s}_2 = [\underline{h}_2^* \; -\underline{h}_1^T]\, y$

$\Rightarrow SNR = \frac{|h_1|^2 + |h_2|^2}{2} \Rightarrow$



array gain = 1

diversity gain = 4



$\underline{y}(1) \mid \underline{y}(2)$

$2\times 1 \quad 2\times 1$

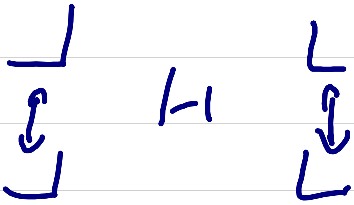
$\underline{y} = \begin{pmatrix} y(1) \\ y(2)^* \end{pmatrix} \quad 4\times 1$

$P_{tot} = \left(\frac{\sqrt{P}}{\sqrt{2}}\right)^2 \cdot 2 = P$

$\hat{s}_1 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_1^H \; \underline{h}_2^T]\, \underline{y}$

$\hat{s}_2 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_2^H \; -\underline{h}_1^T]\, \underline{y}$

7.8



$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

spacing $> \frac{\lambda}{2}$: h uncorrelated.

$< \frac{\lambda}{2}$: h correlated.

How correlation impact the performance?

Kronecker model — correlation at Rx. Tx (1.2)

$$R_r = \begin{bmatrix} 1 & r^* \\ r & 1 \end{bmatrix} \quad R_t = \begin{bmatrix} 1 & t^* \\ t & 1 \end{bmatrix}$$

$$H = R_r^{\frac{1}{2}} H_w R_t^{\frac{1}{2}}$$

no Rx correlation. only Tx correlation $\Rightarrow$ $R_r = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R_r^{\frac{1}{2}}$, $R_t = \begin{bmatrix} 1 & t \\ t & 1 \end{bmatrix}$

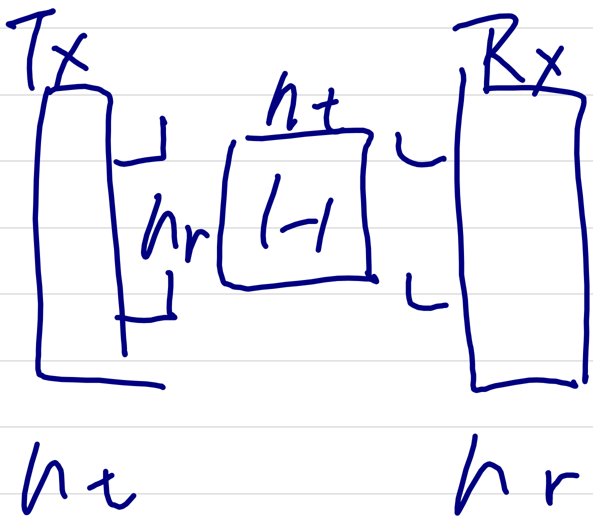
$$H = H_w R_t^{\frac{1}{2}}$$

task: repeat the previous exercise with channel $H = H_w R_t^{\frac{1}{2}}$ ($t = 0, 0.9$)

$$E\{HH^H\} = R_r \otimes R_t$$

$$R_t^{\frac{1}{2}} \neq \begin{pmatrix} 1 & \sqrt{t} \\ \sqrt{t} & 1 \end{pmatrix}$$

ensure at least 100 errors

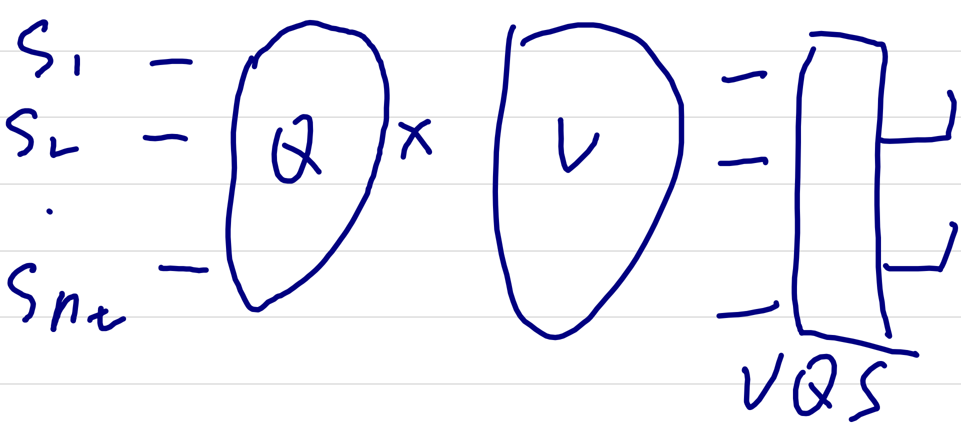


$$I(H,Q) = \log_2 \det(I + \rho H Q H^H)$$

$$C = \max_{Q} I(H,Q)$$

Q (obtained by WF)

$$\underset{n_r \times n_t}{H} = \underset{n_r \times n_r}{U}\ \overset{n_r \times n_t}{\Lambda}\ \underset{n_t \times n_t}{V^H} \qquad Q = \begin{bmatrix} Q_1 & & \\ & \ddots & \\ & & Q_{n_t} \end{bmatrix} \quad Tr(Q) = P$$



$$\underline{J} = U\Lambda V^H V Q S = U\Lambda Q S$$

$$\underbrace{U^H U}_{I}\Lambda \underbrace{V^H V}_{I} Q S + n = \Lambda Q S + n$$

Tx

$$\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{n_t} \end{pmatrix} \begin{pmatrix} q_1 & & \\ & \ddots & \\ & & q_{n_t} \end{pmatrix} \begin{pmatrix} s_1 \\ \vdots \\ s_{n_t} \end{pmatrix}$$

$$C = \sum_{i=1}^{n_t} \log_2(1 + \lambda_i^2 q_i P)$$

$$\begin{pmatrix} \lambda_1 q_1 s_1 \\ \lambda_2 q_2 s_2 \\ \vdots \\ \lambda_{n_t} q_{n_t} s_{n_t} \end{pmatrix}$$

$$H = U\Lambda V^H$$

$$H H^H = U\Lambda \underbrace{V^H V}_{I} \Lambda^H U^H = U\Lambda^2 U^H$$

$Tx \rightarrow V$

$Rx \rightarrow U^H$

$\boxed{2\times 2}$

$\boxed{2\times 4}$

$\boxed{4\times 2}$

WF

$$C = \log_2(1+\lambda_1^2 q_1) + \log(1+\lambda_2^2 q_2)$$

$$H = U\Lambda V^H \rightarrow (\lambda_1, \lambda_2)$$

$$q_1 + q_2 = P \Rightarrow \begin{cases} q_1 = x_1 P \\ q_2 = x_2 P \\ x_1 + x_2 = 1 \end{cases}$$

$N_s = \min(n_t, n_r) = 2$

streams

$A_{shade} = 1$

$SVD(H)$: 2

$SVD(H^H H)$: no square

$$\left(\mu_i - \frac{1}{P\lambda_1^2}\right)^+ + \left(\mu_i - \frac{1}{P\lambda_2^2}\right)^+ = 1$$

μ^* such that

$$\max\left\{\mu^* - \frac{1}{P\lambda_1^2}, 0\right\} + \max\left\{\mu^* - \frac{1}{P\lambda_2^2}, 0\right\} = 1$$

$$H = U\Sigma V^H$$

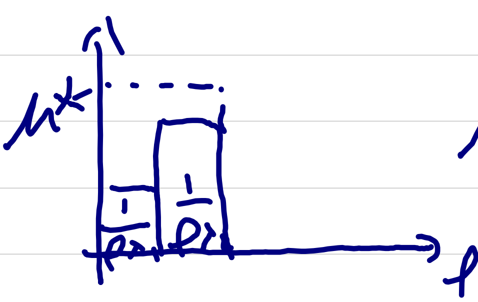
$$\Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

$$\downarrow$$

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \qquad q_1 + q_2 = 1$$

$$\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} \to \boxed{Q} \to \boxed{V} \to \underset{Tx}{\boxed{\vec{p}}}$$

$$C = \log_2(1 + Pq_1\underbrace{\sigma_1^2}_{=\lambda_1}) + \log_2(1 + Pq_2\underbrace{\sigma_2^2}_{=\lambda_2})$$



μ^* such that

$$\left\{\mu^* - \frac{1}{\rho\lambda_1}\right\}^+ + \left\{\mu^* - \frac{1}{\rho\lambda_2}\right\}^+ = 1$$

① generate matrix H

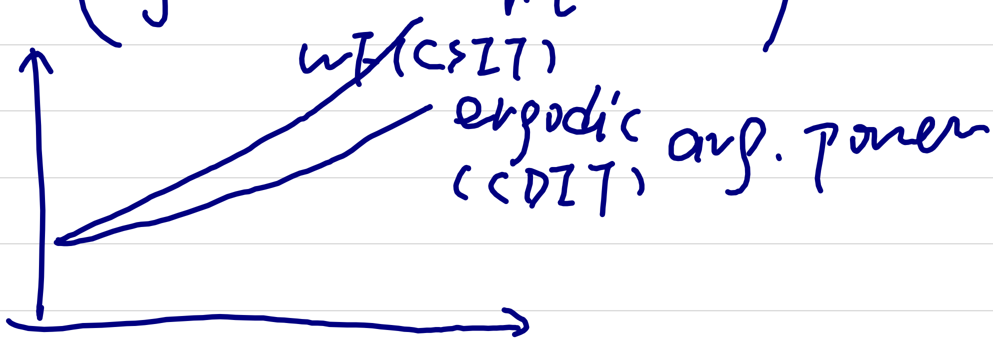
② SVD $H = U\Sigma V^H$ ③ $\Sigma = \begin{pmatrix}\sigma_1 & \\ & \sigma_2\end{pmatrix} \to \Lambda = \begin{pmatrix}\lambda_1 & \\ & \lambda_2\end{pmatrix}$

④ 

$$\left(\mu^* - \frac{1}{\rho\lambda_1}\right)^+ + \left(\mu^* - \frac{1}{\rho\lambda_2}\right)^+ = 1$$

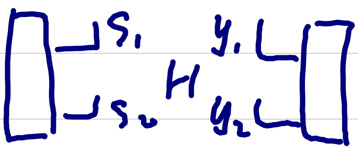
⑤ calculate $C = \log_2(1+\rho\lambda_1 s_1) + \log_2(1+\rho\lambda_2 s_2)$

$$C = E\left(\log_2\det\left(I + \frac{\rho}{n_t} HH^H\right)\right)$$



11.

2) ML receiver



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$\{\hat{s}_1, \hat{s}_2\}$: all possible combinations

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = H \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} \quad \arg\min_{\hat{s}_1, \hat{s}_2} \| y - \hat{y} \|^2$$

3) ZF

$$y = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{pmatrix} = H^{-1} y$$

4) MMSE

ZF $T = H^{-1}$

$$\tilde{s} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{bmatrix} = T y$$

MMSE $T = \sqrt{\frac{2}{E_s}} H^{H} \left(H H^{H} + \frac{2}{P} I \right)^{-1}$

5) unordered

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = (h_1 | h_2) \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + n$$

$$= h_1 s_1 + h_2 s_2 + n$$

ZF-SIC: estimate one, remove one . . .

$$G = H^{-1} \quad G = \begin{pmatrix} G(1,:) \\ G(2,:) \end{pmatrix}$$

$$\tilde{s}_1 = G(1,:)\underline{y}$$

$$\tilde{s}_1 \to \hat{s}_1$$

$$\underline{\hat{y}} = \underline{y} - \underline{h}_1 s_1 = \underline{h}_2 s_2 + n \overset{MRC}{\Longrightarrow} \hat{s}_2 = \frac{\underline{h}_2^H}{\|\underline{h}_2\|} \hat{y}$$

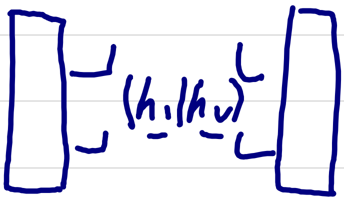
6) ordered

$$G = \begin{pmatrix} G(1,:) \\ G(2,:) \end{pmatrix}$$

Choose $G(i,:)$ such that $|G(i,:)|$ is min

$\min\{|G(1,:)|, |G(2,:)|\}$

7) Alamouti



$$\underline{y}(1) = H\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \underline{h}_1 s_1 + \underline{h}_2 s_2 + n_1$$

$$\underline{y}(2) = H\begin{pmatrix} -s_2^* \\ s_1^* \end{pmatrix} = -\underline{h}_1 s_2^* + \underline{h}_2 s_1^* + n_2$$

$$\begin{array}{cc} \begin{pmatrix} -s_2^* & s_1 \\ s_1^* & s_2 \end{pmatrix} \\ \uparrow\ \ \ \ \ \uparrow \\ 2\ \ \ \ \ 1 \end{array} \qquad \begin{bmatrix} y(1) \\ y(2)^* \end{bmatrix} = \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \\ \underline{h}_2^* & -\underline{h}_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \underline{n}_1 \\ \underline{n}_2 \end{bmatrix}$$

$$\tilde{s}_1 = [\underline{h}_1^H \ \underline{h}_2^T] \begin{bmatrix} \underline{y}(1) \\ \underline{y}(2)^* \end{bmatrix}$$

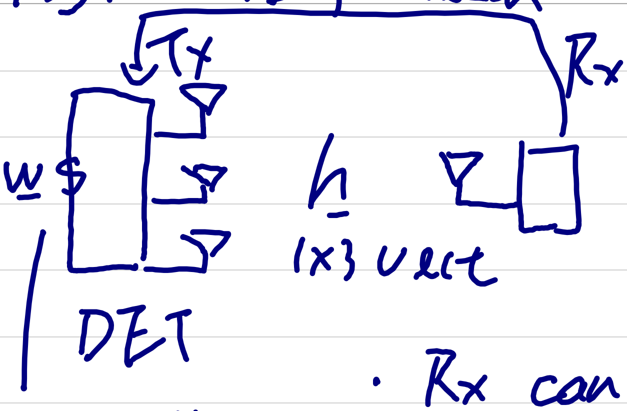
$$\tilde{s}_2 = [\underline{h}_2^H \ -\underline{h}_1^T] \begin{bmatrix} y(1) \\ y(2) \end{bmatrix}$$

12.

$$H = R_{T_x}^{i} H_w R_{R_x}^{i}$$

418 bits

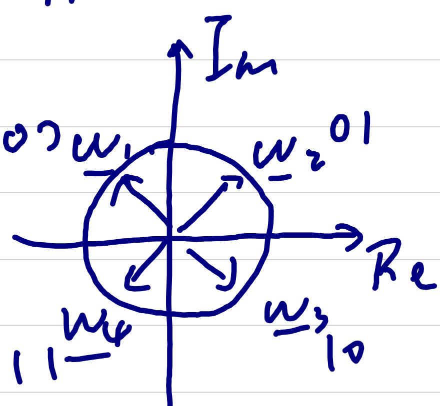
13. CSI feedback



CSIT: how possible?

$\underline{w} = \underline{h}^H$

- Rx can estimate $\underline{h}$.
- Rx has to feedback $\underline{h}$ to Tx.



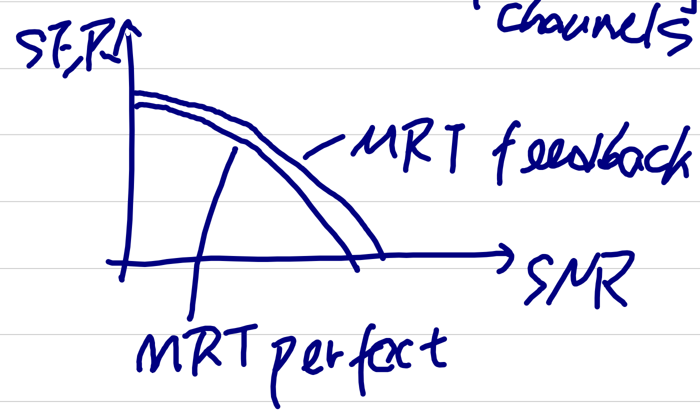
Rx can only feedback 2 bits

1. Rx estimate $\underline{h}$
2. Rx choose $\hat{\underline{h}}$

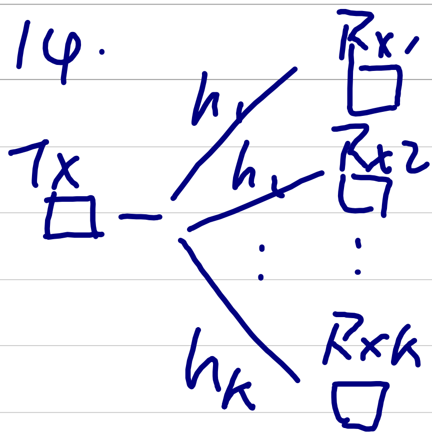
$$\underline{w} = \frac{\underline{h}^H}{|\underline{h}|}$$

quantised precoding $\quad \underline{w}_1 \dots \underline{w}_{n_p} \quad n_p$ precoding channels

$$w = \max_{i \in 1,2 \dots n_p} \|h w_i\|^2$$



14.



$q \in \{1 . 2 . . K\}$

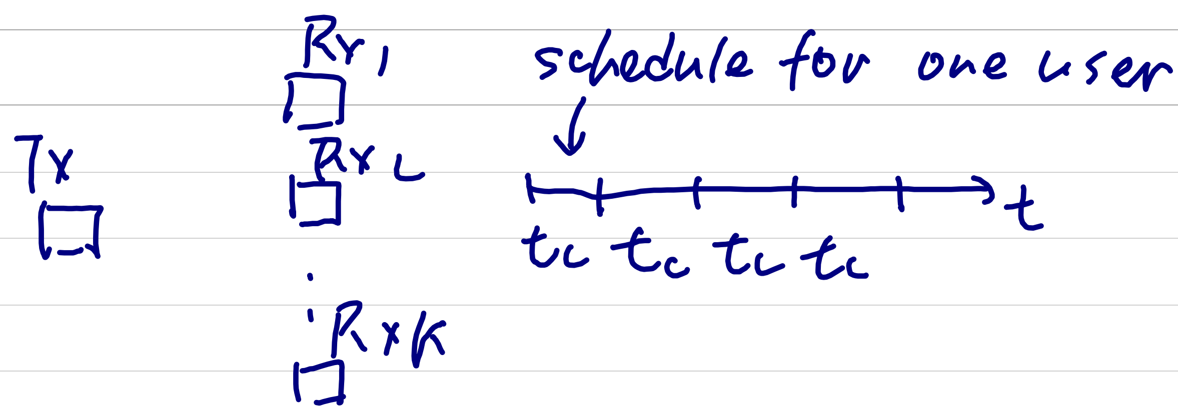
$q^* = \arg\max_{q \in \{1..K\}} U_q$ target func.

Ex) $q^* = \arg\max_q R_q$ rate ~ CSI

$q^* = \arg\max_q r_q \frac{R_q}{\bar{R}_q}$ long-term avg.

QoS

proportional fair scheduling



For each user in each slot

3 params: $\begin{cases} r_q : \text{QoS} \\ R(t) : \text{current rate} \\ \overline{R(t)} : \text{long-term rate} \end{cases}$

r_q : given

$$R(t) = \log_2(1 + |h_q|^2 \rho)$$

$$\overline{R(t)} \begin{cases} (1 - \frac{1}{t_c})\overline{R_q(t-1)} + \frac{1}{t_c} R_q(t) & \text{if } q \text{ scheduled} \\ (1 - \frac{1}{t_c})\overline{R_q(t-1)} & \text{if } q \text{ not scheduled} \end{cases}$$

$$q^* = \arg\max_q r_q \frac{R_q}{\overline{R_q}}$$

$$h_k(t) = \varepsilon h_k(t-1) + \sqrt{1-\varepsilon^2}\, n_k$$ noise $\mathcal{CN}(0,1)$

tradeoff by slot length.
$t_c \to T$: max rate
$t_c \to 0$: round robin (max fairness)

for $t = 1 : 1e^5$

① generate the channel of each user $q \in \{1..N\}$

$$h_q(t) = \varepsilon h_q(t-1) + \sqrt{1-\varepsilon^2}\, n$$

② calculate the instantaneous rate and long-term rate of each user

$$R_q(t) = \log_2(1 + |h_q(t)|^2 P)$$

select $q^* = \arg\max_q \gamma_q \frac{R_q(t)}{\bar{R}_q(t-1)}$

update the long-term rate $\bar{R}_q(t)$,

in each slot t

① generate channel for each user

② calculate the inst. rate of user k

$$R_q(t) = \log_2(1 + |h_q(t)|^2 P)$$

③ choose user q that maximise

$$q^* = \arg\max_q \gamma_q \frac{R_q(t)}{\bar{R}_q(t-1)}$$

⑤ iterate.

④ update long-term rate $\bar{R}_q(t)$