C477: Computing for Optimal Decisions
Quiz 0

January 10, 2020

Note: This quiz assesses your mathematics background, but it does not count towards your final mark. Understanding the background material in this quiz is required for taking C477. If this material is unfamiliar, please revise ASAP.

Exercise 1 (Differentiating a 1D function)**.** Let $x \in \mathbb{R}$; $f : \mathbb{R} \mapsto \mathbb{R}$ and define:

$$f(x) = x\ e^x + \frac{1}{6}\ (2\ x + 1)^3$$

(a) What is the first derivative, $\frac{df}{dx}$?

(b) What is the second derivative, $\frac{d^2 f}{dx^2}$?

Exercise 2 (Differentiating a 2D quadratic function)**.** Let: $\boldsymbol{x}$, $\boldsymbol{b} \in \mathbb{R}^2$; $Q \in \mathbb{R}^{\{2\times 2\}}$ be a symmetric matrix; $c \in \mathbb{R}$; $f : \mathbb{R}^2 \mapsto \mathbb{R}$ and define:

$$\begin{aligned} f(\boldsymbol{x}) &= \frac{1}{2}\ \boldsymbol{x}^\top\ Q\ \boldsymbol{x} - \boldsymbol{x}^\top\ \boldsymbol{b} + c \\ &= \frac{1}{2}\ \boldsymbol{x}^\top \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \boldsymbol{x} - \boldsymbol{x}^\top \begin{bmatrix} 1 \\ 4 \end{bmatrix} + c \end{aligned}$$

(a) What is the gradient, $\nabla f(\boldsymbol{x})$?

(b) What is the Hessian, $\nabla^2 f(\boldsymbol{x})$?

Exercise 3 (Operating with vectors and matrices).

(a) Multiply:

$$\begin{bmatrix} 3 & -1 \end{bmatrix} \times \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(b) Multiply:

$$\begin{bmatrix} 2 & 5 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(c) Find the transpose of matrix M_1:

$$\begin{bmatrix} 3 & -1 \\ -1 & 2 \\ 2 & 7 \end{bmatrix}^\top = \ ?$$

(d) Find the eigenvalues (λ_1, λ_2) of matrix M_2:

$$M_2 = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$$

(e) Is matrix M_2 positive definite? Is M_2 negative definite?

(f) What is the rank of matrix M_1? What is the rank of matrix M_2?

(g) What is the 1-norm, $\|\boldsymbol{v}\|_1$ of vector $\boldsymbol{v}^\top = [3, 4]$?

(h) What is the 2-norm, $\|\boldsymbol{v}\|_2$ of vector $\boldsymbol{v}^\top = [3, 4]$?

(i) What is the ∞-norm, $\|\boldsymbol{v}\|_\infty$ of vector $\boldsymbol{v}^\top = [3, 4]$?