# Communication Systems E303: Comments 2017-2018

## Introductory Concepts

1. Important:
    - pages 18,19,20,23-26
    - FT table page 27: scaling (2), time shift (3), rect (14) sinc (15), $\Lambda$ (21),rep (22), comb (23)
    - Pages 26-33
    - Tail function Graph - page 36
2. Corrections*:
    - Pages: 21

## Information Sources

3. Important: Everything
4. Corrections*:
    - Pages: 12, 13, 22

## Communication Channels & Criteria and Limits

5. Important - Everything Except:
    - Equs 14, 15,16,17, 29 & 30
    - Pages 40 & 41

   Note: please remember one of the following two equations: Equ 11 or Equ 12
6. Corrections*:
    - Pages: 7, 23, 36

## Wireless Comms

7. For reference**:
    - pages 17, 41-42, 47, 50
8. Important:
    - pages 21, 28-34, 45-46
9. Corrections*:
    - Pages: 34

## Digital Modulators & Line Codes

10. For reference**:
    - Pages 34-40, 52-53, 57
11. Important:
    - AMI & HDB3 (page 55)
    - BER (Equ 13) pages 32 and 33
12. Corrections*:
    - Page: 64

## PN-codes, PN-signals and Principles of Spread Spectrum (Part-A)

13. For reference**:
    - Pages 5, 6, 7, 47 and Appendices
14. Corrections*:
    - Pages: 16

## PN-codes, PN-signals and Principles of Spread Spectrum (Part-B)

15. For reference**:
    - Pages 8, 16, 39-42, Appendices (54-56)
16. Important
    - Page 23: remember Fig below Equ 20 and be able to write equations 20 and 21 and plot Fig below Equ 21
    - Page 32: the power of the noise at o/p (i.e. at T0) and the power of the jammer at o/p (i.e. at T0)
    - Equations 28 and 30
    - pages 35-38, 43-44
17. Corrections*:
    - Pages: 23, 35, 53

## PCM

18. For reference**:
    - Pages 22-24, 35-36, 47-49
19. Important:
    - pages 7, 8, 18, 19, 29 (6dB Law)
20. Page 28: For A-Law or mu-law questions the equation on page 28 will be provided in the exam.
21. Corrections*:
    - Pages: 25

## 3G, 4G and 5G

22. For reference**:
    - Pages 14, 44, 47, 66, 68
23. Corrections*:
    - Pages: 46, 57, 71

## NB:

** the term “for reference” refers to non-examinable material

* these corrections are shown in the following pages

## "Accumulators" and "Averaging" Devices

- Definitions

| | | | |
|---|---|---|---|
| Accumulators: | $\int_{t_1}^{t_2}$ | $\sum_{i=1}^{M}$ | |
| Averaging: | $\frac{1}{t_2-t_1}\int_{t_1}^{t_2}$ | $\frac{1}{M}\sum_{i=1}^{M}$ | $\mathcal{E}\{.\}$ |

- Examples

| | |
|---|---|
| Accumulators corresponding to Energy | $R_{gg}(\tau) \triangleq \int_{t_1}^{t_2} g(t).g(t-\tau)dt$ <br> $\text{ESD}_g(f) = \text{FT}\{R_{gg}(\tau)\}$ |
| Averaging corresponding to Power | $R_{gg}(\tau) \triangleq \frac{1}{t_2-t_1}\int_{t_1}^{t_2=t_1+\tau} g(t).g(t-\tau)dt$ <br> $R_{gg}(\tau) \triangleq \mathcal{E}\{g(t).g(t-\tau)\}$ <br> $\text{PSD}_g(f) = \text{FT}\{R_{gg}(\tau)\}$ |





### Example 1

- If $X = \{0, 1\}$ with probablities

$$\underline{p} = \begin{bmatrix} \Pr(0) \\ \Pr(1) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \quad (12)$$

then

**data rate** : $r_b = r_X = 10\frac{bits}{\text{sec}}$

entropy : $H_X = 1\frac{bits}{\text{symbol}} = 1\frac{info\ bit}{data\ bit}$

**info rate** : $r_{\text{inf}} = r_X.H_X = 10\frac{bits}{\text{sec}}$

i.e.

$$r_b = r_{\text{inf}} = 10\frac{bits}{\text{sec}} \quad (13)$$

## Example 2

- If $X = \{0, 1\}$ with probablities

$$\underline{p} = \begin{bmatrix} \Pr(0) \\ \Pr(1) \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} \quad (14)$$

then

$$\begin{aligned} \textbf{data rate} &: r_b = r_X = 10 \frac{bits}{sec} \\ \text{entropy} &: H_X = 0.8813 \frac{bits}{\text{symbol}} = 0.8813 \frac{info\ bit}{data\ bit} \\ \textbf{info rate} &: r_{\inf} = r_X . H_X = 8.813 \frac{bits}{sec} \end{aligned}$$

i.e.

$$r_b > r_{\inf} \quad (15)$$





## Important Relationships

- At the output of an information source $g(t)$ the following are very important:

$$\begin{aligned} \text{Entropy} &: H_g && (28) \\ \max(Entropy) &: \text{Gaussian Entropy} \underbrace{=}_{= H_g} \log_2 \sqrt{2\pi e \sigma_g^2} && (29) \\ \text{Entropy Power} &: N_g \triangleq \frac{1}{2\pi e} 2^{2H_g} && (30) \\ \text{Average Power} &: P_g \triangleq \mathcal{E}\left\{g(t)^2\right\} && (31) \\ \text{In general} &: P_g \geqq N_g && (32) \end{aligned}$$

$$\begin{aligned} \text{if pdf}_g &= \text{Gaussian} \Rightarrow P_g = N_g && (33) \\ \text{if pdf}_g &\neq \text{Gaussian} \Rightarrow P_g > N_g && (34) \end{aligned}$$

- In many situations the input and output alphabets $X$ and $Y$ are identical but in the general case these are different. Instead of using $X$ and $Y$, it is common practice to use the symbols $H$ and $D$ and thus define the two alphabets and the associated probabilities as

$$\text{input:} \quad H = \{H_1, H_2, ..., H_M\} \quad \underline{p} = [\overbrace{\Pr(H_1)}^{\triangleq p_1}, \overbrace{\Pr(H_2)}^{\triangleq p_2}, ..., \overbrace{\Pr(H_M)}^{\triangleq p_M}]^T$$

$$\text{output:} \quad D = \{D_1, D_2, ..., D_K\} \quad \underline{q} = [\overbrace{\Pr(D_1)}^{\triangleq q_1}, \overbrace{\Pr(D_2)}^{\triangleq q_2}, ..., \overbrace{\Pr(D_K)}^{\triangleq q_K}]^T$$

where $p_m$ abbreviates the probability $\Pr(H_m)$ that the symbol $H_m$ may appear at the input while $q_k$ abbreviates the probability $\Pr(D_k)$ that the symbol $D_k$ may appear at the output of the channel.





# Capacity of AWGN Channels

- In the case of a continuous channel corrupted by additive white Gaussian noise the capacity is given by

$$C = \frac{1}{2}\log_2(1+\mathrm{SNR}_{in}) \quad \frac{\text{bits}}{\text{symbol}} \qquad (25)$$

or $\cancel{(26)}$

$$C = B\log_2(1+\mathrm{SNR}_{in}) \quad \frac{\text{bits}}{\text{sec}} \qquad (27)$$

where

$$\begin{aligned} B &= \text{baseband band width of channel} \\ \mathrm{SNR}_{in} &= \frac{P_s}{P_n} \\ P_s &= \text{Power of the signal at point } \widehat{\mathrm{T}} \\ P_n &= \text{Power of the noise at point } \widehat{\mathrm{T}} = N_0 B \end{aligned}$$

- **LIMIT-1** : **limit on bit rate**
  - when binary information is transmitted in the channel, $r_b$ should be limited as follows:
$$r_b \leq C \tag{42}$$
  - ideal case:
$$r_b = C \tag{43}$$
- **LIMIT-2** : **limit on EUE**
  - the best Energy Efficiency is EUE=0.693. This is the ultimate limit below which no physical channel can transmit without errors i.e
$$\text{EUE} \geq 0.693$$
- **LIMIT-3** : **Shannon's threshold channel capacity curve**
  - This is the curve EUE=f{BUE} for a bit rate $r_b$ equal to its maximum value, i.e.
$$r_b = C \Rightarrow \text{EUE} = \frac{2^{\text{BUE}^{-1}} - 1}{\text{BUE}^{-1}} \tag{44}$$





# Channel Selectivity and Channel Coherence

- Channel *Selectivity*: A channel has selectivity if **it varies** as a function of either time, or frequency, ~~or space~~
- Channel Coherence: (opposite of Channel Selectivity)
  - A channel has coherence if **it does not vary** as a function of either time, frequency, or space over a specified 'window' of interest.
  - This is the **most important** concept in describing wireless channels
  - coherence: $\begin{cases} \textit{temporal}\text{ coherence} & \text{-coherence time } T_{\text{coh}} \\ \textit{frequency}\text{ coherence} & \text{-coherence bandwidth } B_{\text{coh}} \\ \cancel{\textit{spatial}\text{ coherence}} & \cancel{\text{-coherence distance } D_{\text{coh}}} \end{cases}$

# Solution: PSD(f)

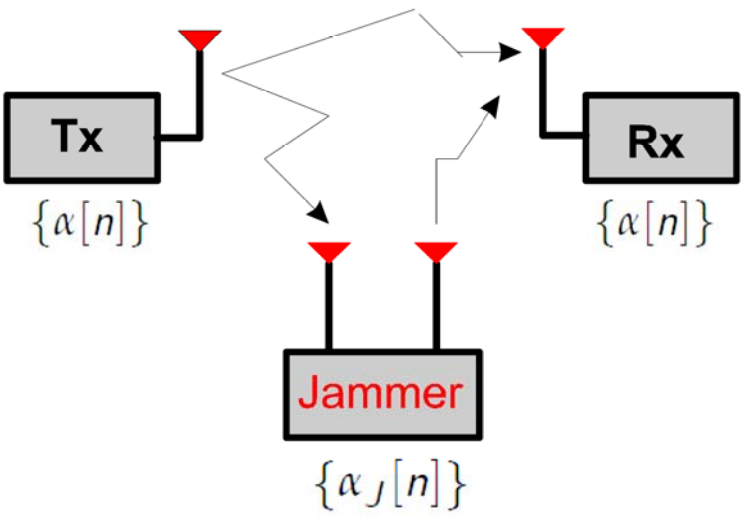
$$
\begin{aligned}
\text{PSD}(f) &= \frac{\left|\text{FT}\left(\text{rect}\left\{\frac{t}{T_{cs}}\right\}\right)\right|^2}{T_{cs}}\left\{R[0]+2R[k]\cos(2\pi f k T_{cs})\right\} \\
&= \frac{T_{cs}^2\text{sinc}^2(fT_{cs})}{T_{cs}}\left\{\frac{1}{2}-2\times\frac{1}{4}\cos(2\pi f T_{cs})\right\} \\
&= T_{cs}\text{sinc}^2(fT_{cs})\left\{\frac{1}{2}-\frac{1}{2}\cos(2\pi f T_{cs})\right\} \\
&= T_{cs}\text{sinc}^2(fT_{cs})\cdot\sin^2\left(\frac{2\pi f T_{cs}}{2}\right) \qquad (25)
\end{aligned}
$$





- Jamming source, or, simply Jammer is defined as follows:

$$\boxed{\text{Jammer} \triangleq \text{intentional (hostile) interference}}$$

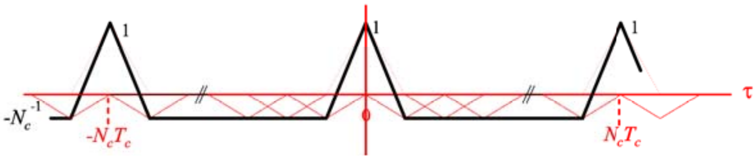



- ★ the jammer has full knowledge of SSS design except the jammer does not have the key to the PN-sequence generator,
- ★ i.e. the jammer may have full knowledge of the SSSystem but it does not know the PN sequence used.

# PSD(f) of a PN-Signal b(t) in DS-SSSs

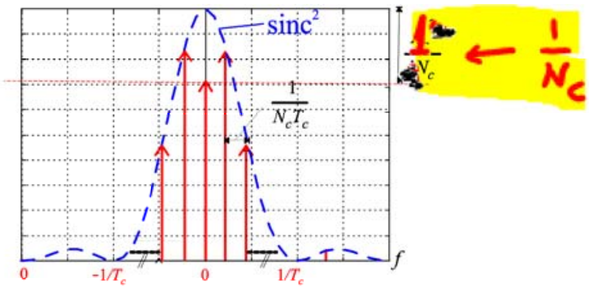
- Autocorrelation function: $R_{bb}(\tau)$

$$\boxed{R_{bb}(\tau) = \frac{N_c+1}{N_c}\mathrm{rep}_{N_c T_c}\left\{\Lambda\left(\frac{\tau}{T_c}\right)\right\} - \frac{1}{N_c}} \qquad (20)$$




- Using the FT tables the $\mathrm{PSD}_b(f) = \mathrm{FT}\{R_{bb}(\tau)\}$ of the signal $b(t)$ is:

$$\boxed{\mathrm{PSD}_b(f) = \frac{N_c+1}{N_c^2}\mathrm{comb}_{\frac{1}{N_c T_c}}\left\{\mathrm{sinc}^2\left\{f\cdot T_c\right\}\right\} - \frac{1}{N_c}\delta(f)} \qquad (21)$$








# Bit Error Probability with Jamming

## A. CONSTANT POWER BROADBAND JAMMER:

- From the "Detection Theory" topic we know that the bit-error-probability $p_e$ for a **Binary Phase-Shift Key (BPSK)** communication system is given by:

$$p_e = \mathrm{T}\left\{\sqrt{\underbrace{2\cdot \mathrm{EUE}}_{\mathrm{SNR}_{\text{out, matched filter}}}}\right\} \quad \text{where } \mathrm{EUE} = \frac{E_b}{N_0} \qquad (31)$$

- Consider a DS/BPSK SSS which operates in the presence of a constant amplitude broadband jammer with double sided power spectral density

$$\mathrm{PSD}_j(f) = \frac{N_j}{2} \qquad (32)$$

- A serious problem is the $\overbrace{\text{“near-far” problem}}^{\substack{\text{very strong signals at receiver} \\ \text{swapping out the effects} \\ \text{of weaker signal} \\ \Uparrow}}$ :
  - DS: severe problem
  - FH: much ~~more~~ less susceptible
- acquisition: much faster in FH than in DS
- PG $= \frac{B_{ss}}{B} =$ it is not very good criterion for FH

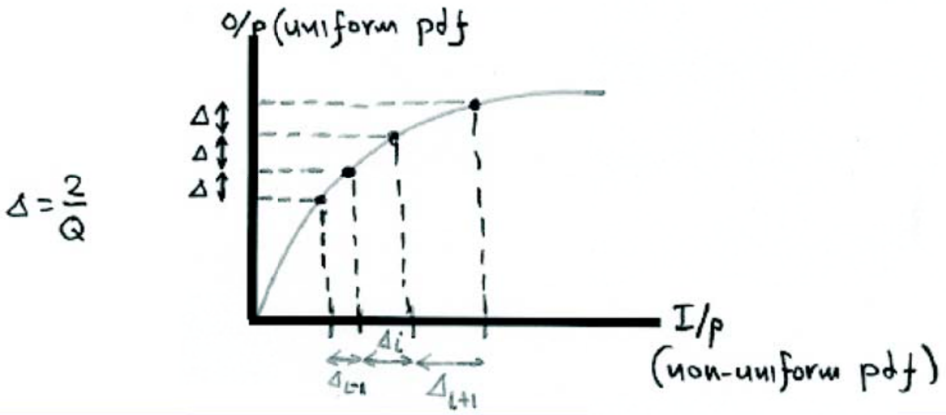




# Companders (non-Uniform Quantizers)

- Their performance independent of CF
- Non-unif. Quant $=$ SAMPLE COMPRESSION $+$ UNIFORM QUANTIZER $+$ SAMPLE EXPANDER
- Compressor $+$ Expander $\equiv$ **Compander**

$$g \xmapsto{f} g_c \;\; i.e. \;\; \boxed{g_c = \text{f}\{g\} \underset{\substack{\uparrow \\ \text{means} \\ \text{"such that"}}}{:} \text{pdf}_{g_c} = \text{uniform}} \xmapsto{f^{-1}} g_c$$



# Sectorization

- **Sectorization** is achieved by using directional antennas instead of omnidirectional antennas.
- Each cell is divided to three sectors using **three directional antennas** each having 120° beamwidth.
- Using sectorization the performance can be improved even more. (The expected value of the total interference is reduced by a factor of 3 wrt single omnidirectional antenna case)

$$\textbf{BPSK} \quad : \quad \text{SNIR}_{\text{out}} = 2.\text{EUE}_{equ} = 2\frac{E_b}{N_j + N_0} \tag{20}$$

$$\text{where } N_j = \frac{(K-1).P_s.s}{B_{ss}} \tag{21}$$

In practice: 3 dB $<\text{SNIR}_{\text{out}} <$ 15dB





| | TS1 (iFFT output) | | | | TS2 (iFFT output) | | | |
|---|---|---|---|---|---|---|---|---|
| 1st | +1 | 0.5+0.0j | 0.5 | **0** ° | +1 | 0.5+0.0j | 0.5 | **0** ° |
| 2nd | +1 | 0.5+0.0j | 0.5 | **0** ° | +1 | 0.0+0.5j | 0.5 | **90** ° |
| 3rd | −1 | -0.5+0.0j | 0.5 | **180** ° | +1 | 0.5+0.0j | 0.5 | **0** ° |
| 4th | +1 | 0.5+0.0j | 0.5 | **0** ° | −1 | 0.0+0.5j | 0.5 | **-90** ° |
| | TS3 (iFFT output) | | | | TS4 (iFFT output) | | | |
| 1st | +1 | -0.5+0.0j | 0.5 | **180** ° | −1 | -0.5+0.0j | 0.5 | **180** ° |
| 2nd | −1 | 0.5+0.0j | 0.5 | **0** ° | +1 | 0.0+0.5j | 0.5 | **90** ° |
| 3rd | −1 | 0.5+0.0j | 0.5 | **0** ° | −1 | -0.5+0.0j | 0.5 | **180** ° |
| 4th | −1 | 0.5+0.0j | 0.5 | **0** ° | −1 | 0.0+0.5j | 0.5 | **-90** ° |
| | TS5 (iFFT output) | | | | TS6 (iFFT output) | | | |
| 1st | −1 | 0.0+0.0j | 0 | **0** ° | −1 | 0.0+0.0j | 0 | **0** ° |
| 2nd | +1 | -0.5+0.5j | $\frac{\sqrt{2}}{2}$ | **135** ° | −1 | -0.5-0.5j | $\frac{\sqrt{2}}{2}$ | **-135** ° |
| 3rd | +1 | 0.0+0.0j | 0 | **-90** ° | +1 | 0.0+0.0j | 0 | **-90** ° |
| 4th | −1 | -0.5+0.5j | $\frac{\sqrt{2}}{2}$ | **-135** ° | +1 | -0.5+0.5j | $\frac{\sqrt{2}}{2}$ | **135** ° |

# Enhanced Mobile Broadband