

SOLUTIONS: Communication Systems

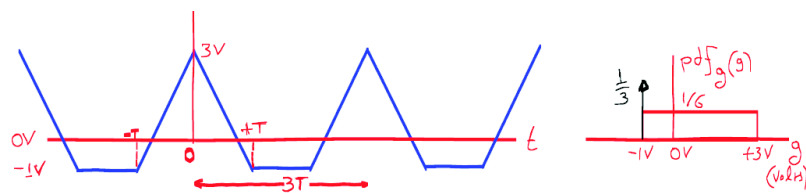
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2011

1 Topic: Introductory Concepts

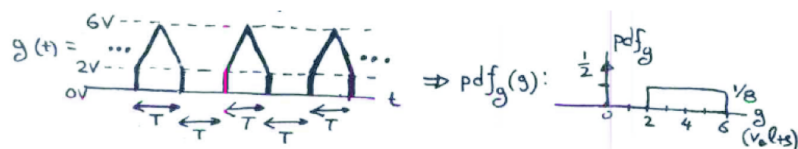
1. Solution

(a)



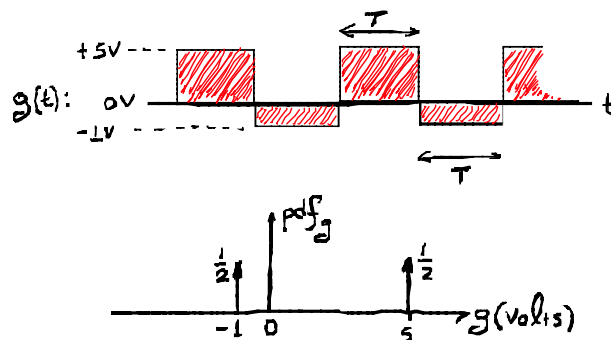
$$\text{i.e. pdf}_g(g) = \frac{1}{3}\delta(g+1) + \frac{1}{6}\text{rect}\left\{\frac{g-1}{4}\right\}$$

(b)



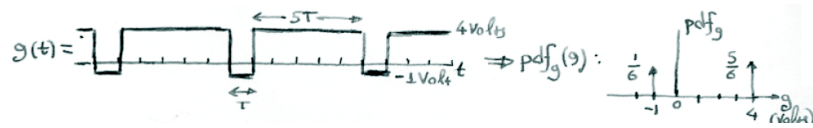
$$\text{i.e. pdf}_g(g) = \frac{1}{2}\delta(g) + \frac{1}{8}\text{rect}\left\{\frac{g-4}{4}\right\}$$

(c)



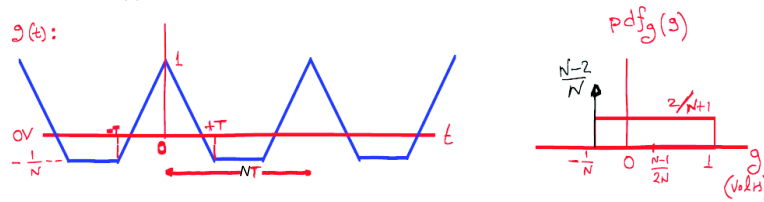
$$\text{pdf}_g(g) = \frac{1}{2}\delta(g+1) + \frac{1}{2}\delta(g-5)$$

(d)



$$\text{pdf}_g(g) = \frac{1}{6}\delta(g+1) + \frac{5}{6}\delta(g-4)$$

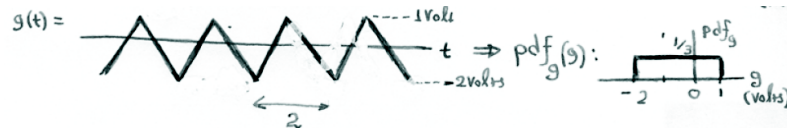
(e)



i.e.

$$\begin{aligned} \text{pdf}_g(g) &= \frac{N-2}{N}\delta\left(g + \frac{1}{N}\right) + \frac{2}{N+1}\text{rect}\left\{\frac{g - \frac{N-1}{2N}}{\frac{N+1}{N}}\right\} \\ &= \frac{N-2}{N}\delta\left(g + \frac{1}{N}\right) + \frac{2}{N+1}\text{rect}\left\{\frac{2Ng - N + 1}{2(N+1)}\right\} \end{aligned}$$

(f)



$$\text{pdf}_g(g) = \frac{1}{3}\text{rect}\left\{\frac{g - 0.5}{3}\right\}$$

2. Solution

(a)

$$\int_{-\infty}^{\infty} (t^4 - 3t + 1) \cdot \delta(t - 2) \cdot dt = (t^4 - 3t + 1)|_{t=2} = 2^4 - 3 \times 2 + 1 = 11$$

$$\begin{aligned} \text{(b)} \quad \int_{-\infty}^{\infty} (\cos(4\pi t) * \delta(t + \frac{1}{4})) \cdot \delta(t - \frac{1}{8}) \cdot dt &= \int_{-\infty}^{\infty} \cos(4\pi(t + \frac{1}{4})) \cdot \delta(t - \frac{1}{8}) \cdot dt = \\ &= \cos(4\pi(t + \frac{1}{4}))|_{t=\frac{1}{8}} = \cos(4\pi(\frac{1}{8} + \frac{1}{4})) = \cos(\frac{3}{2}\pi) = 0 \end{aligned}$$

$$\text{(c)} \quad \int_{-\infty}^{\infty} (t^3 - 3t^2 - 11) \cdot \delta(t - 1) \cdot dt = (t^3 - 3t^2 - 11)|_{t=1} = 1^3 - 3 \times 1^2 - 11 = -13$$

$$\begin{aligned} \text{(d)} \quad \int_{-\infty}^{\infty} \{(\sin(4\pi t) * \delta(t + \frac{1}{4}))\} \cdot \delta(t - \frac{1}{4}) \cdot dt &= \int_{-\infty}^{\infty} (\sin(4\pi(t + \frac{1}{4}))) \cdot \delta(t - \frac{1}{4}) \cdot dt = \\ \sin(4\pi(t + \frac{1}{4}))|_{t=\frac{1}{4}} &= \sin(4\pi(\frac{1}{4} + \frac{1}{4})) = \sin(2\pi) = 0 \end{aligned}$$

$$\text{(e)} \quad \int_{-\infty}^{\infty} (t^3 - 2t^2 + 1) \cdot \delta(t - 2) \cdot dt = (t^3 - 2t^2 + 1)|_{t=2} = 2^3 - 2 \times 2^2 + 1 = 1$$

$$(f) \int_{-\infty}^{\infty} (\cos(2\pi t) * \delta(t - \frac{1}{4})) \cdot \delta(t - \frac{1}{12}) \cdot dt = \int_{-\infty}^{\infty} \cos(2\pi(t - \frac{1}{4})) \cdot \delta(t - \frac{1}{12}) \cdot dt =$$

$$= \cos(2\pi(t - \frac{1}{4}))|_{t=\frac{1}{12}} = \cos(2\pi(\frac{1}{12} - \frac{1}{4})) = \cos(-\frac{1}{3}\pi) = \frac{1}{2}$$

$$(g) h(3) \text{ where } h(t) = (t \cdot \text{rect}\{\frac{t}{8}\}) * \delta(t+3) \quad (5\%)$$

$$h(t) = (t \cdot \text{rect}\{\frac{t}{8}\}) * \delta(t+3) = (t+3) \cdot \text{rect}\{\frac{t+3}{8}\}$$

$$\Rightarrow h(3) = 0$$

$$(h) h(3) \text{ where } h(t) = (t \cdot \text{rect}\{\frac{1}{8T}\}) * \delta(t-2) \quad (10\%)$$

$$h(t) = (t \cdot \text{rect}\{\frac{t}{8T}\}) * \delta(t-2) = (t-2) \cdot \text{rect}\{\frac{t-2}{8T}\}$$

$$h(3) = (3-2) \cdot \text{rect}\{\frac{3-2}{8T}\}$$

$$\Rightarrow h(t) = \begin{cases} t-2 & \text{if } -0.5 < \frac{t-2}{8T} < 0.5 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow h(3) = \begin{cases} 1 & \text{if } -0.5 < \frac{3-2}{8T} < 0.5 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 1 & \text{if } T > \frac{1}{4} \\ 0 & \text{if } T \leq \frac{1}{4} \end{cases}$$

$$(i) h(3.5) \text{ where } h(t) = (t \cdot \text{rect}\{\frac{1}{8T}\}) * \delta(t-3) \quad (10\%)$$

$\text{rect}\{\frac{t}{8T}\}$
 $t \cdot \text{rect}\{\frac{t}{8T}\}$
 $h(t)$
 $\text{L.e. } h(t) = (t-3) \text{rect}\{\frac{t-3}{8T}\} \Rightarrow h(3.5) = (3.5-3) \text{rect}\{\frac{3.5-3}{8T}\} \Rightarrow$
 $h(3.5) = 0.5 \times \text{rect}\{\frac{0.5}{8T}\}$
 $\frac{0.5}{8T} < \frac{1}{2} \Rightarrow T > \frac{1}{8}$
 $\text{L.e. } h(3.5) = \begin{cases} 0.5 & \text{if } T > \frac{1}{8} \\ 0 & \text{if } T \leq \frac{1}{8} \end{cases}$

3. Solution

$$(a) R_{bb}(\tau) = \frac{N+1}{N} \text{rep}_{NT_c} \left\{ \Lambda\left(\frac{\tau}{T_c}\right) \right\} - \frac{1}{N}$$

$$(b) \text{PSD}(f) = \text{FT}\{R_{bb}(\tau)\} = \frac{N+1}{N^2} \text{comb}_{\frac{1}{NT_c}} \{ \text{sinc}^2(fT_c) \} - \frac{1}{N} \delta(f)$$

4. Solution

$n_c(t) \rightarrow [H(f)] \rightarrow n_o(t)$
 $\text{PSD}_{n_c}(f)$
 $\text{PSD}_{n_o}(f) = \text{PSD}_{n_c}(f) \cdot |H(f)|^2$
 $P_{n_o} = \int_{-\infty}^{\infty} \text{PSD}_{n_c}(f) \cdot |H(f)|^2 \cdot df$
 $= \frac{N_0}{2} \int_{-B}^{+B} |H(f)|^2 df = \frac{N_0}{2} 2 \int_{-B}^{+B} |H(f)|^2 df$
 $= N_0 \int_{-B}^{+B} \frac{(f+B)^2}{B^2} df = \frac{N_0}{B^2} \left[\frac{(f+B)^3}{3} \right]_{-B}^{+B}$
 $\Rightarrow P_{n_o} = \frac{N_0 B}{3} = \frac{3 \times 10^{-6} \times 10^6}{3} = 1$

5. Solution

$$(a) \quad h(t) = \delta(t) - \delta(t - T)$$

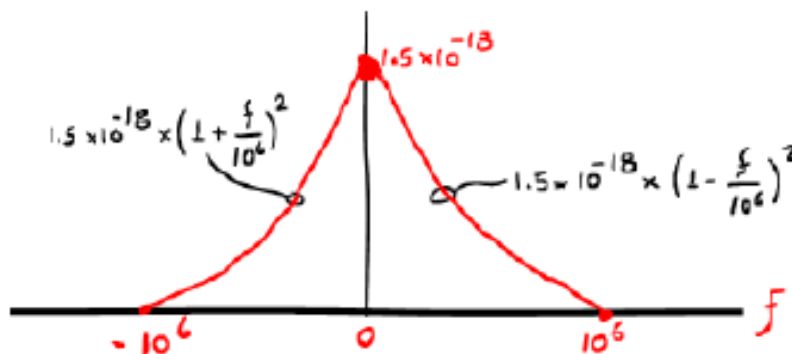


$$\begin{aligned} (b) \quad H(f) &= \text{FT}\{h(t)\} = 1 - \exp(-j2\pi fT) \\ &= \{\exp(j\pi fT) - \exp(-j\pi fT)\} \exp(-j\pi fT) \\ &= 2j \sin(\pi fT) \exp(-j\pi fT) \end{aligned}$$

$$\Rightarrow |H(f)| = 2 |\sin(\pi fT)| =$$

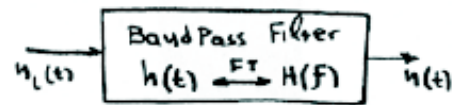
6. Solution

$$\begin{aligned} (a) \quad H(f) &= \text{FT}\{h(t)\} = \frac{1}{10^6} \Lambda\left(\frac{f}{10^6}\right) \exp(-j2\pi f \times 3) \\ \Rightarrow \text{PSD}_n(f) &= \text{PSD}_{n_i}(f) \cdot |H(f)|^2 = \\ 1.5 \times 10^{-6} \left(\frac{1}{10^6}\right)^2 \Lambda^2\left(\frac{f}{10^6}\right) &= 1.5 \times 10^{-18} \Lambda^2\left(\frac{f}{10^6}\right) \end{aligned}$$



$$\begin{aligned} (b) \quad P_n &= \int_{-10^6}^{10^6} \text{PSD}_n(f) \cdot df = 2 \int_0^{10^6} \text{PSD}_n(f) \cdot df = \\ &= 2 \int_0^{10^6} \left(1.5 \times 10^{-18} \times \left(1 - \frac{f}{10^6} \right)^2 \right) \cdot df = \\ &= 2 \times 1.5 \times 10^{-18} \times \int_0^{10^6} \left(1 - \frac{f}{10^6} \right)^2 \cdot df = \\ &= 3 \times 10^{-18} \times \int_0^{10^6} \left(1 - 2\frac{f}{10^6} + \frac{f^2}{10^{12}} \right) \cdot df \\ &= 3 \times 10^{-18} \times \left(10^6 - 2\frac{10^{12}}{2 \times 10^6} + \frac{10^{18}}{3 \times 10^{12}} \right) \\ &= 3 \times 10^{-18} \times \left(10^6 - 10^6 + \frac{1}{3} 10^6 \right) \\ &= 10^{-12} \end{aligned}$$

7. Solution



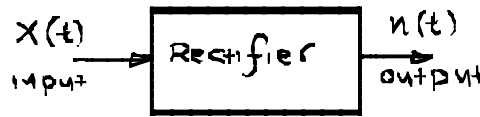
$$\begin{aligned}
 H(f) &= \text{FT}\{h(t)\} = 8 \times 10^3 \text{ FT}\{\text{sinc}(4 \times 10^3 t) \cos(2\pi 10^4 t)\} \\
 &= 8 \times 10^3 \underbrace{\text{FT}\{\text{sinc}(4 \times 10^3 t)\}}_{\frac{1}{4 \times 10^3} \text{rect} \frac{f}{4 \times 10^3}} \otimes \left[\frac{1}{2} \delta(f - 10^4) + \frac{1}{2} \delta(f + 10^4) \right] \\
 &\quad \quad \quad \uparrow \text{convolution} \\
 &= \text{rect} \frac{f - 10^4}{4 \times 10^3} + \text{rect} \frac{f + 10^4}{4 \times 10^3}
 \end{aligned}$$

$$\text{PSD}_y(f) = \text{PSD}_{x_L}(f) \cdot |H(f)|^2 = 10^{-6} \text{rect} \frac{f - 10^4}{4 \times 10^3} + 10^{-6} \text{rect} \frac{f + 10^4}{4 \times 10^3}$$

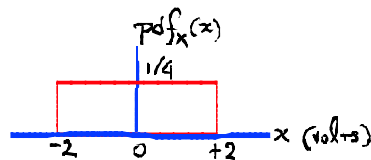
$$P_y = 4 \times 10^3 \times 10^{-6} \times 2 = 8 \times 10^{-3} \text{ W} = \underline{\underline{8 \text{ mW}}}$$

2 Topic: Information Sources

8. Solution



(a) $\text{pdf}_x(x) = \frac{1}{4} \text{rect}\left\{\frac{x}{4}\right\}$

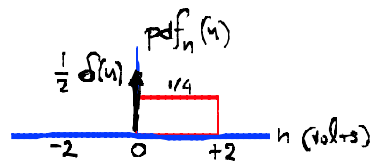


mean: $\mathcal{E}\{x(t)\} = \int_{-\infty}^{+\infty} x \cdot \text{pdf}_x(x) \cdot dx = \int_{-2}^{+2} x \cdot \frac{1}{4} \cdot dx = 0$ Volts

power: $P_x = \mathcal{E}\{x^2(t)\} = \int_{-\infty}^{+\infty} x^2 \cdot \text{pdf}_x(x) \cdot dx = \int_{-2}^{+2} x^2 \cdot \frac{1}{4} \cdot dx = \frac{4}{3}$

rms = $\sqrt{P_x} = \frac{2}{\sqrt{3}} = 1.1547$

(b) $\text{pdf}_n(n) = \frac{1}{2} \delta(n) + \frac{1}{4} \text{rect}\left\{\frac{n-1}{2}\right\}$

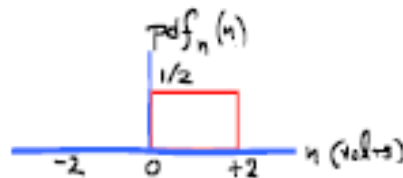


mean: $\mathcal{E}\{n(t)\} = \int_{-\infty}^{+\infty} n \cdot \text{pdf}_n(n) \cdot dn = \int_0^{+2} n \cdot \frac{1}{4} \cdot dn = \frac{1}{2}$ Volts

power: $P_n = \mathcal{E}\{n^2(t)\} = \int_{-\infty}^{+\infty} n^2 \cdot \text{pdf}_n(n) \cdot dn = \int_0^{+2} n^2 \cdot \frac{1}{4} \cdot dn = \frac{2}{3}$

rms = $\sqrt{P_n} = \sqrt{\frac{2}{3}} = 0.81650$

(c) $\text{pdf}_n(n) = \frac{1}{2} \text{rect}\left\{\frac{n-1}{2}\right\}$



mean: $\mathcal{E}\{n(t)\} = \int_{-\infty}^{+\infty} n \cdot \text{pdf}_n(n) \cdot dn = \int_0^{+2} n \cdot \frac{1}{2} \cdot dn = 1$ Volts

power: $P_n = \mathcal{E}\{n^2(t)\} = \int_{-\infty}^{+\infty} n^2 \cdot \text{pdf}_n(n) \cdot dn = \int_0^{+2} n^2 \cdot \frac{1}{2} \cdot dn = \frac{4}{3}$

rms = $\sqrt{P_n} = \frac{2}{\sqrt{3}} = 1.1547$

9. Solution

(a) average power (source with uniform pdf):

$$P_x = \mathcal{E}\{x^2(t)\} = \int_{-\infty}^{+\infty} x^2 \cdot \text{pdf}_x(x) \cdot dx = \int_{-3}^{+3} x^2 \cdot \frac{1}{6} \cdot dx = \frac{9}{3} = 3$$

(b) $H_x = - \int_{-\infty}^{+\infty} \text{pdf}_x(x) \cdot \log_2(\text{pdf}_x(x)) \cdot dx = - \int_{-3}^{+3} \frac{1}{6} \cdot \log_2\left(\frac{1}{6}\right) \cdot dx =$
 $= -\frac{1}{6} \cdot \log_2\left(\frac{1}{6}\right) \int_{-3}^{+3} dx = \log_2 6 = 2.585$

(c) If $y(t)$ = Gaussian with mean = μ and rms = $\sqrt{P_x}$ then

$$H_y = \log_2 \sqrt{2\pi e P_x} = \log_2 \sqrt{2\pi e 3} = 2.8396$$

$$H_y - H_x = 2.8396 - 2.585 = 0.2546$$

$$(d) N_x = \frac{1}{2\pi e} 2^{2H_x} = \frac{1}{2\pi e} 2^{2 \times 2.585} = 2.1079$$

10. Solution

$$(a) r_x = 2 \times F_g = 8k \frac{\text{levels}}{\text{sec}}$$

$$(b) \Pr(-2V) = \frac{3}{4}; \Pr(+2V) = \frac{1}{4} \Rightarrow \underline{p} = [\frac{3}{4}, \frac{1}{4}]^T$$

pdf:



$$(c) \text{rms} = \sqrt{(-2)^2 \Pr(-2V) + 2^2 \Pr(+2V)} = \sqrt{(-2)^2 \frac{3}{4} + 2^2 \frac{1}{4}} = 2V$$

$$(d) H_X = -\frac{3}{4} \log_2 \left(\frac{3}{4} \right) - \frac{1}{4} \log_2 \left(\frac{1}{4} \right) = 0.81128 \frac{\text{bits}}{\text{level}}$$

$$(e) (X \times X, \mathbb{J}) = \left\{ \begin{array}{cc} (x_1 x_1, \frac{9}{16}) & (x_2 x_1, \frac{3}{16}) \\ (x_1 x_2, \frac{3}{16}) & (x_2 x_2, \frac{1}{16}) \end{array} \right\}$$

$$H_{X \times X} = -\frac{9}{16} \log_2 \left(\frac{9}{16} \right) - \frac{3}{16} \log_2 \left(\frac{3}{16} \right) - \frac{3}{16} \log_2 \left(\frac{3}{16} \right) - \frac{1}{16} \log_2 \left(\frac{1}{16} \right) = 1.6226 \frac{\text{bits}}{\text{double level}}$$

11. Solution

(a) Quantiser:

"end"-points Quantisation levels

$$b_0 = -2V \text{ (or } -\infty)$$

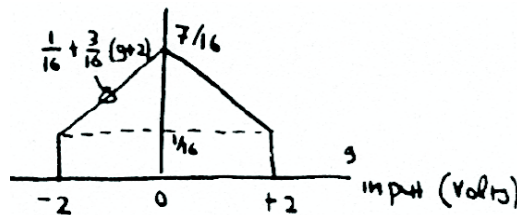
$$b_1 = -1V \quad x_1 = -1.5V$$

$$b_2 = 0V \quad x_2 = -0.5V$$

$$b_3 = 1V \quad x_3 = 0.5V$$

$$b_4 = 2V \text{ (or } +\infty) \quad x_4 = 1.5V$$

input pdf:

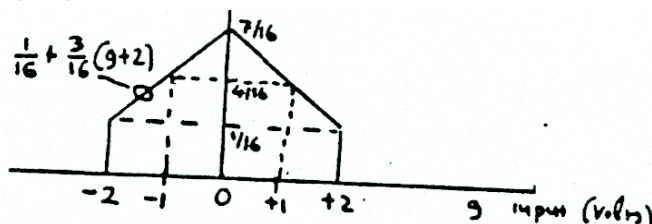


$$\text{Therefore, power of } g(t) = P_g = \int_{-\infty}^{+\infty} g^2 \cdot \text{pdf}_g(g) \cdot dg$$

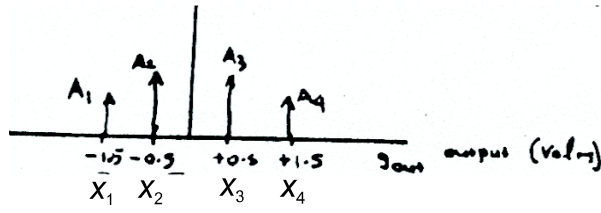
$$= 2 \int_{-2}^0 g^2 \cdot \left(\frac{1}{16} + \frac{3}{16}(g+2) \right) \cdot dg = 2 \int_{-2}^0 \left(\frac{1}{16}g^2 + \frac{3}{16}g^3 + \frac{6}{16}g^2 \right) \cdot dg = 2 \int_{-2}^0 \left(\frac{7}{16}g^2 + \frac{3}{16}g^3 \right) \cdot dg$$

$$= \left[\frac{2 \times 7}{16} \frac{g^3}{3} \right]_{-2}^0 + \left[\frac{2 \times 3}{16} \frac{g^4}{4} \right]_{-2}^0 = \frac{5}{6} = 0.8333$$

input pdf:



output pdf:



(b) rms:

Handwritten calculations for the RMS value of a signal. The calculations are divided into two parts, A1 and A2, separated by a vertical line.

For A1: $A_1 = \Pr(g_{out} \geq 1.5V) = \Pr(-2 < g < -1)$. The diagram shows a trapezoid with vertices at (-2, 0), (-1, 1/16), (-1, 3/16), and (-2, 0). The area is labeled A_1 . The calculation is:

$$= \int_{-2}^{-1} \left(\frac{1}{16} + \frac{3}{16}(g+2) \right) dg =$$

$$= \frac{1}{16}g + \frac{3}{16} \frac{(g+2)^2}{2} \Big|_{-2}^{-1} =$$

$$-\frac{1}{16} + \frac{3}{32} + \frac{2}{16} - 0 = \frac{5}{32} = 0.1563$$

For A2: $A_2 = \Pr(g_{out} = -0.5V) = \Pr(-1 < g < 0)$. The diagram shows a trapezoid with vertices at (-1, 0), (0, 1/16), (0, 3/16), and (-1, 0). The area is labeled A_2 . The calculation is:

$$= \int_{-1}^0 \left(\frac{1}{16} + \frac{3}{16}(g+2) \right) dg =$$

$$= \frac{1}{16}g + \frac{3}{16} \frac{(g+2)^2}{2} \Big|_{-1}^0 =$$

$$\frac{6}{16} + \frac{1}{16} - \frac{3}{32} = \frac{11}{32} = 0.3438$$

i.e. $A_3 = A_2 = \frac{11}{32}$ and $A_4 = A_1 = \frac{5}{32}$

$$P_{g_{out}} = 2 \times ((-1.5)^2 \times 0.1563 + (-0.5)^2 \times 0.3438) = 0.8752 \Rightarrow rms = \sqrt{0.8752} = 0.9355$$

(c) $(X, \underline{p}) = \left\{ (-1.5V, \frac{5}{32}), (-0.5V, \frac{11}{32}), (+0.5V, \frac{11}{32}), (+1.5V, \frac{5}{32}) \right\}$

$$(X \times X, \mathbb{J}) = \left\{ \begin{array}{cccc} (x_1x_1, \frac{25}{1024}) & (x_2x_1, \frac{55}{1024}) & (x_3x_1, \frac{55}{1024}) & (x_4x_1, \frac{25}{1024}) \\ (x_1x_2, \frac{55}{1024}) & (x_2x_2, \frac{121}{1024}) & (x_3x_2, \frac{121}{1024}) & (x_4x_2, \frac{55}{1024}) \\ (x_1x_3, \frac{55}{1024}) & (x_2x_3, \frac{121}{1024}) & (x_3x_3, \frac{121}{1024}) & (x_4x_3, \frac{55}{1024}) \\ (x_1x_4, \frac{25}{1024}) & (x_2x_4, \frac{55}{1024}) & (x_3x_4, \frac{55}{1024}) & (x_4x_4, \frac{25}{1024}) \end{array} \right\}$$

(d) Entropy:

$$H_{X \times X} = \frac{1}{4}^T (\mathbb{J} \odot \log_2 \mathbb{J}) \mathbb{1}_4 = 3.7921 \frac{bits}{double\ level}$$

3 Topic: Communication Channels

12. Solution

- The matrix \mathbb{F} is:

$$\mathbb{F} = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

- \therefore the overall matrix \mathbb{F}_{csc} is as follows:

$$\begin{aligned} \mathbb{F}_{casc} &= \mathbb{F} \cdot \mathbb{F} = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix} \\ &= \begin{bmatrix} 0.7 \times 0.7 + 0.3 \times 0.3, & 0.7 \times 0.3 + 0.3 \times 0.7 \\ 0.3 \times 0.7 + 0.7 \times 0.3, & 0.7 \times 0.7 + 0.3 \times 0.3 \end{bmatrix} \\ &= \begin{bmatrix} 0.58 & 0.42 \\ 0.42 & 0.58 \end{bmatrix} = \begin{bmatrix} \Pr(z_0|x_0), & \Pr(z_0|x_1) \\ \Pr(z_1|x_0) & \Pr(z_1|x_1) \end{bmatrix} \end{aligned}$$

\therefore

$$\begin{aligned} p_e &= \Pr(z_1|x_0) \Pr(x_0) + \Pr(z_0|x_1) \Pr(x_1) \\ &= 0.42 \times 0.4 + 0.42 \times 0.6 = 0.42 \end{aligned}$$

13. Solution

$$C = B \log_2(1 + \underbrace{SNR_{in}}_{=30/2}) = B \log_2(1 + 15) = 4B \quad (1)$$

$$\text{However, } BUE = \frac{B}{r_b} \Rightarrow B = \underbrace{BUE}_{=2} \times \underbrace{r_b}_{=100} = 200 \quad (2)$$

$$\text{Therefore: } (1) \wedge (2) \Rightarrow C = 4 \times 200 = 800 \frac{\text{bits}}{\text{sec}}$$

14. Solution

(a)

Handwritten solution for part (a):

$$\begin{aligned} &\left. \begin{array}{l} EUE = 30 \\ N_0/2 = 0.5 \times 10^{-6} \Rightarrow N_0 = 10^{-6} \\ C = 16 \frac{\text{kbits}}{\text{sec}} \\ B = 4 \text{ KHz} \end{array} \right\} \Rightarrow C = B \log_2(1 + SNR_{in}) \quad (1) \\ &\quad \quad \quad BUE = \frac{B}{r_b} \quad (2) \\ &\textcircled{1} \Rightarrow SNR_{in} = 2^{C/B} - 1 \Rightarrow \frac{EUE}{BUE} = 2^{C/B} - 1 \Rightarrow \\ &\quad \Rightarrow BUE = \frac{EUE}{2^{C/B} - 1} \Rightarrow \frac{B}{r_b} = \frac{EUE}{2^{C/B} - 1} \Rightarrow \\ &\quad \Rightarrow r_b = \frac{(2^{C/B} - 1) B}{EUE} = 2 \text{ k} \frac{\text{bits}}{\text{sec}} \end{aligned}$$

$$(b) P_n = N_0 B = 10^{-6} \times 4 \text{ k} = 4 \text{ mW}$$

15. Solution

$$\begin{aligned} (a) p_e &= \Pr(H_2, D_1) + \Pr(H_1, D_2) = \Pr(D_1|H_2) \Pr(H_2) + \Pr(D_2|H_1) \Pr(H_1) \\ &= 0.04 \times \frac{2}{3} + 0.018 \times \frac{1}{3} = \boxed{0.032667} \end{aligned}$$

(b) any expression of mutual information H_{mut} can be used.

$$\text{For instance: } H_{mut} = \sum_{m=1}^2 \sum_{k=1}^2 J_{km} \log_2 \left(\frac{p_m q_m}{J_{km}} \right)$$

where

$$\underline{q} = \mathbb{F} \cdot \underline{p} \Rightarrow \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.982 & 0.04 \\ 0.018 & 0.96 \end{bmatrix}}_{\mathbb{F}} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$p_1 = \Pr(H_1) = \frac{1}{3},$$

$$p_2 = \Pr(H_2) = \frac{2}{3}$$

$$J_{11} = \Pr(H_1, D_1) = \Pr(D_1|H_1) \Pr(H_1) = 0.982 \times \frac{1}{3} = -0.3273$$

$$J_{12} = \Pr(H_2, D_1) = \Pr(D_1|H_2) \Pr(H_2) = 0.04 \times \frac{2}{3} = -0.0267$$

$$J_{21} = \Pr(H_1, D_2) = \Pr(D_2|H_1) \Pr(H_1) = 0.018 \times \frac{1}{3} = -0.006$$

$$J_{22} = \Pr(H_2, D_2) = \Pr(D_2|H_2) \Pr(H_2) = 0.96 \times \frac{2}{3} = -0.64$$

$$\text{or } \mathbb{J} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \mathbb{F} \text{diag}(\underline{p}) = \underbrace{\begin{bmatrix} 0.982 & 0.04 \\ 0.018 & 0.96 \end{bmatrix}}_{\mathbb{F}} \cdot \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{2}{3} \end{bmatrix}$$

$$\Rightarrow \boxed{H_{mut} = 0.7326}$$

16. Solution

$$(a) B = \frac{r_{cs}}{2} = \frac{1}{2T_{cs}} = \frac{1}{2 \times 20 \times 10^{-6}} = \boxed{25 kHz}$$

$$(b) r_{cs} = \frac{1}{T_{cs}} = r_b = \boxed{50 kbits/sec}$$

$$(c) EUE = \frac{E_b}{N_0},$$

$$E_1 = \int_{-10\mu}^{10\mu} 4\Lambda^2 \left\{ \frac{t}{10\mu} \right\} dt = 2 \int_{-10\mu}^0 4 \left(\frac{t+10\mu}{10\mu} \right)^2 dt$$

$$= \frac{8}{(10\mu)^2} \int_{-10\mu}^0 \left(t^2 + 20\mu t + (10\mu)^2 \right) dt = \frac{1}{3} 80\mu = E_2 = E_b$$

$$\text{i.e. } E_b = \frac{80}{3} \times 10^{-6} = 26.667 \times 10^{-6}$$

$$\frac{N_0}{2} = 10^{-6} \Rightarrow N_0 = 2 \times 10^{-6}$$

$$\text{Thus } \boxed{EUE = \frac{E_b}{N_0} = \frac{26.667 \times 10^{-6}}{2 \times 10^{-6}} = 13.334}$$

$$(d) C = B \log_2(1 + SNR_{in}) = B \log_2 \left(1 + \frac{EUE}{BUE} \right) =$$

$$25 \times 10^3 \times \log_2 \left(1 + \frac{13.334}{\frac{25 \times 10^3}{50 \times 10^3}} \right) = 25 \times 10^3 \times \log_2(1 + 26.668)$$

$$\Rightarrow C = \boxed{119.75 \times 10^3}$$

17. Solution

$$(a) T_{cs} = 10\mu s \Rightarrow B = \frac{1}{2T_{cs}} = \frac{1}{2 \times 10\mu s} = \frac{10^5}{2} = 50 kHz$$

$$(b) r_{cs} = \frac{1}{T_{cs}} = 10^5 \text{ symbols/sec (=bit rate } r_b)$$

$$(c) N_0 = 2 \times 0.5 \times 10^{-6} \Rightarrow EUE = \frac{E_b}{N_0} = \frac{E_b}{10^{-6}}$$

However,

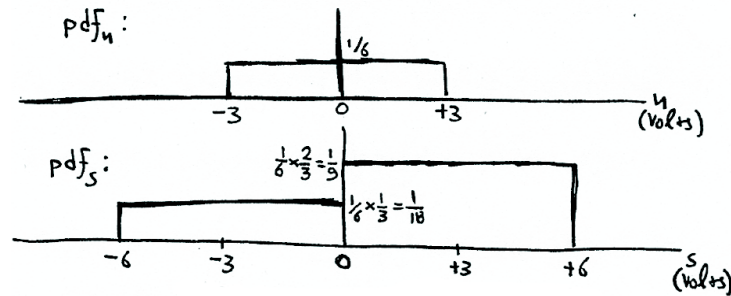
$$\begin{aligned}
 E_b &= \int_{-5\mu}^{5\mu} 9 \left(\Lambda \left\{ \frac{t}{5\mu} \right\} + \text{rect} \left\{ \frac{t}{10\mu} \right\} \right)^2 dt \\
 &= 2 \times 9 \int_0^{5\mu} \left(\Lambda \left\{ \frac{t}{5\mu} \right\} + \text{rect} \left\{ \frac{t}{10\mu} \right\} \right)^2 dt \\
 &\quad \text{note that } \Lambda \left\{ \frac{t}{5\mu} \right\} \Big|_0^{5\mu} \text{ is equal to } \frac{-t+5\mu}{5\mu} \\
 &\quad \text{and } \Lambda \left\{ \frac{t}{5\mu} \right\} + \text{rect} \left\{ \frac{t}{10\mu} \right\} = 1 + \frac{-t+5\mu}{5\mu} = 2 - 2 \times 10^5 t \\
 &\quad \text{and } \left(\Lambda \left\{ \frac{2t}{T_{cs}} \right\} + \text{rect} \left\{ \frac{t}{T_{cs}} \right\} \right)^2 = 4 + 4 \times 10^{10} t^2 - 8 \times 10^5 t \\
 &= 2 \times 9 \int_0^{5 \times 10^{-6}} (4 + 4 \times 10^{10} t^2 - 8 \times 10^5 t) dt \\
 &= 18 \left(\int_0^{5\mu} 4 dt + \int_0^{5\mu} 4 \times 10^{10} t^2 dt - \int_0^{5\mu} 8 \times 10^5 t dt \right) \\
 &= 18 \times \left(20 + \frac{4}{3} \times 10^{-2} \times 125 - 4 \times 10^{-1} \times 25 \right) \times 10^{-6} \\
 &= 210 \times 10^{-6}
 \end{aligned}$$

$$\text{EUE} = \frac{210 \times 10^{-6}}{10^{-6}} = 210$$

$$(d) \text{ EUE}=210; \text{ BUE} = \frac{B}{r_b} = \frac{50 \times 10^3}{10^5} = 0.5$$

$$C = 50 \times 10^3 \log_2(1 + \frac{210}{0.5}) = 435.88 \text{ kbits/s}$$

18. Solution



$$P_s = \mathcal{E} \{s(t)^2\} = \int_{-\infty}^{+\infty} s^2 \text{pdf}_s(s) ds = \int_{-6}^0 s^2 \frac{1}{18} ds + \int_0^6 s^2 \frac{1}{9} ds = \frac{1}{18} \frac{s^3}{3} \Big|_{-6}^0 + \frac{1}{9} \frac{s^3}{3} \Big|_0^6 = 4 + 8 = 12W$$

$$\text{Since noise} \neq \text{white Gaussian} \Rightarrow B \log_2 \left(\frac{P_s + N_n}{N_n} \right) \leq C \leq B \log_2 \left(\frac{P_s + P_n}{N_n} \right)$$

$$\text{However, } P_n = \int_{-3}^{+3} n^2 \frac{1}{6} dn = \frac{1}{6} \frac{n^3}{3} \Big|_{-3}^{+3} = \frac{1}{18} (27 + 27) = 3W$$

$$N_n = \text{entropy power} = \frac{1}{2\pi e} 2^{2H}$$

$$\text{where } H = - \int_{-\infty}^{+\infty} \text{pdf}_n(n) \log_2(\text{pdf}_n(n)) \, dn = -\frac{1}{6} \log_2 \frac{1}{6} \int_{-3}^{+3} dn = -\log_2 \frac{1}{6} = \log_2 6 = 2.5850$$

$$\text{Thus, } N_n = \frac{1}{2\pi e} 2^{2 \times 2.5850} = \frac{36.002}{2\pi e} = 2.1079$$

$$\log_2 \left(\frac{12+2.1079}{2.1079} \right) \leq \frac{C}{B} \leq \log_2 \left(\frac{12+3}{2.1079} \right)$$

$$2.7426 \leq \frac{C}{B} \leq 2.8311$$

19. Solution

(a)

$$\begin{aligned} P_n &= \mathcal{E}\{n(t)^2\} = \int_{-\infty}^{\infty} n \cdot \text{pdf}_n(n) \cdot dn \\ &= \int_{-3}^3 n \cdot \frac{1}{6} \cdot dn = 3 \\ N_n &= (\text{entropy power}) = \frac{1}{2\pi e} 2^{2H} \\ \text{where } H &= - \int_{-\infty}^{\infty} \text{pdf}_n(n) \log_2(\text{pdf}_n(n)) \cdot dn \\ &= -\frac{1}{6} \log_2 \left(\frac{1}{6} \right) \int_{-3}^3 dn \\ &= -\log_2 \left(\frac{1}{6} \right) = \log_2(6) = 2.585 \end{aligned}$$

$$\begin{aligned} \therefore N_n = (\text{entropy power}) &= \frac{1}{2\pi e} 2^{2H} \\ &= \frac{36}{2\pi e} = 2.1078 \end{aligned}$$

(b)

$$\begin{aligned} B \log_2 \left(\frac{P_s + N_n}{N_n} \right) &\leq C \leq B \log_2 \left(\frac{P_s + P_n}{N_n} \right) \frac{\text{bits}}{\text{sec}} \\ \log_2 \left(\frac{P_s + N_n}{N_n} \right) &\leq \frac{C}{B} \leq \log_2 \left(\frac{P_s + P_n}{N_n} \right) \\ \log_2 \left(\frac{12 + 2.1078}{2.1078} \right) &\leq \frac{C}{B} \leq \log_2 \left(\frac{12 + 3}{2.1078} \right) \\ 2.7427 &\leq \frac{C}{B} \leq 2.8312 \end{aligned}$$

20. Solution

$$(a) \, p_e = \underbrace{\Pr(y_2|x_1) \cdot \Pr(x_1)}_{\Pr(y_2, x_1)} + \underbrace{\Pr(y_1|x_2) \cdot \Pr(x_2)}_{\Pr(y_1, x_2)} = 0.1 \times 0.25 + 0.2 \times 0.75 = 0.175$$

$$(b) \, \mathbb{F} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$$

$$\underline{p} = \begin{bmatrix} 0.25 & 0.75 \end{bmatrix}^T$$

$$\mathbb{J} = \mathbb{F} \cdot \text{diag}(\underline{p}) = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \text{diag} \left(\begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} \right) = \begin{bmatrix} 0.225 & 0.15 \\ 0.025 & 0.6 \end{bmatrix}$$

$$\underline{q} = \mathbb{F} \underline{p} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$

$$\begin{aligned}
H_R &= \underline{q}^T \cdot \log_2(\underline{q}) = 0.9544 \\
H_{R|M} &= -1_2^T (\mathbb{J} \odot \log_2(\mathbb{F})) 1_2 = 0.658695 \\
H_{mut} &= H_R - H_{R|M} = 0.9544 - 0.658695 = 0.295705 \text{ bits/symbol}
\end{aligned}$$

21. Solution

(a)

$$\begin{aligned}
p_e &= \Pr(y_1, x_2) + \Pr(y_2, x_1) \\
&= \Pr(y_1|x_2) \cdot \Pr(x_2) + \Pr(y_2|x_1) \cdot \Pr(x_1) \\
&= 0.04 \times \frac{2}{3} + 0.018 \times \frac{1}{3} = 0.032667
\end{aligned}$$

(b)

$$\mathbb{J} = \begin{bmatrix} J_{11} = \Pr(x_1, y_1) = 0.3267 & J_{12} = \Pr(x_1, y_2) = 0.006 \\ J_{21} = \Pr(x_2, y_1) = 0.02667 & J_{22} = \Pr(x_2, y_2) = 0.64 \end{bmatrix}^T$$

$$\underline{p} = \begin{bmatrix} p_1 = \Pr(x_1) = \frac{1}{3} \\ p_2 = \Pr(x_2) = \frac{2}{3} \end{bmatrix}, \mathbb{F} = \begin{bmatrix} F_{11} = 0.982 & F_{12} = 0.04 \\ F_{21} = 0.018 & F_{22} = 0.96 \end{bmatrix}$$

$$\underline{q} = \mathbb{F} \underline{p} = \begin{bmatrix} q_1 = \Pr(y_1) = 0.982 \times \frac{1}{3} + 0.04 \times \frac{2}{3} = 0.354 \\ q_2 = \Pr(y_2) = 0.018 \times \frac{1}{3} + 0.96 \times \frac{2}{3} = 0.646 \end{bmatrix}$$

$$\begin{aligned}
H_{mut} &\triangleq H_{mut}(\underline{p}, \mathbb{F}) = \text{(using any of the following expression)} \\
&= - \sum_{m=1}^M \sum_{k=1}^K F_{km} \cdot p_m \log_2 \left(\frac{q_k}{F_{km}} \right) \\
&= - \sum_{m=1}^M \sum_{k=1}^K J_{km} \log_2 \left(\frac{p_m \cdot q_k}{J_{km}} \right) \\
&= -1_K^T \left(\underbrace{\mathbb{J} \odot \log_2 \left[\left(\widehat{\mathbb{F} \cdot \underline{p} \cdot \underline{p}^T} \right) \oslash \mathbb{J} \right]}_{K \times M \text{ matrix}} \right) 1_M \frac{\text{bits}}{\text{symbol}} \\
&= 0.7327
\end{aligned}$$

22. Solution

$$a> F_g = 4 \times 10^3 \text{ Hz}$$

$$F_s = 2 \times F_g = 8 \times 10^3 \text{ Hz}$$

$$\bar{\ell}_3 = 1 \times \frac{2^2}{64} + \left(3 \times \frac{9}{64}\right) \times 3 + \left(5 \times \frac{3}{64}\right) \times 3 + 5 \times \frac{1}{64} = 2.4685 \frac{\text{bits}}{\text{trip.}}$$

$$= 2.4685 \frac{\text{bits}}{\text{triple-level}}$$

$$\text{Alphabet: } \underline{x} = \begin{pmatrix} x_1 = 1 \\ x_2 = 0 \end{pmatrix}$$

$M=2$ channel symbols

$$\text{Probabilities: } \underline{p} = \begin{bmatrix} p_1 = \Pr(H_1) = 0.6344 \\ p_2 = \Pr(H_0) = 0.3656 \end{bmatrix} \leftarrow \text{to be proven}$$

$$H_x = - \sum_{m=1}^2 p_m \log_2(p_m) = -\underline{p}^T \cdot \log_2(\underline{p}) = 0.9473 \frac{\text{bits}}{\text{symbol}}$$

$$r_{inf} = H_x \cdot r_{cs}$$

$$\text{symbol rate} = r_b = F_s \frac{1}{3} \bar{\ell}_3 = 6583.3 \frac{\text{symbols}}{\text{sec}}$$

$$\text{i.e. } r_{inf} = 0.9473 \times 6583.3 = 6236.4 \frac{\text{bits}}{\text{symbol}}$$

$$r_d = \underset{\substack{\uparrow \\ 1 \text{ bit}}}{\ell} \cdot \underset{\substack{\uparrow \\ 6583.3}}{r_{cs}} = 6583.3 \frac{\text{bits}}{\text{sec}}$$

$$b> p_e = 0.6344 \times 0.05 + 0.3656 \times 0.2 = 0.1048$$

$$c> 1 = x_1 \mapsto A_1 \sqrt{\frac{1}{8}} \Delta\left(\frac{t}{0.5T_{cs}}\right) \text{ of Energy} = E_1 = ?$$

$$0 = x_2 \mapsto 0 \text{ Volts i.e. Energy} = E_2 = 0$$

$$E_1 = 2 \int_0^{T_{cs}/2} A_1^2 \Delta^2\left(\frac{t}{0.5T_{cs}}\right) dt \quad (\text{where } A_1 = \sqrt{\frac{3}{8}})$$

$$= 2 \int_0^{0.5T_{cs}} A_1^2 \left(\frac{-t + 0.5T_{cs}}{0.5T_{cs}}\right)^2 dt$$

$$= \dots = \frac{1}{3} A_1^2 T_{cs} = \frac{1}{3} \frac{3}{8} T_{cs} = \frac{1}{8} T_{cs}$$

$$E_b = E_1 \cdot P_1 + E_2 \cdot P_2 = \frac{1}{8} T_{cs} P_1 + 0 = 1.2046 \times 10^{-5}$$

$$EVE = \frac{E_b}{N_0} = 6.0228 \times 10^{-3} \quad (\text{data EVE})$$

\downarrow
 2×10^{-3}

$$BVE = \frac{B}{r_b} = \frac{B}{r_{cs}} = \frac{B}{2B} = \frac{1}{2}$$

Note: $B = \frac{r_{cs}}{2} \Rightarrow r_{cs} = 2B = r_b$

$$\text{data point} = (EVE, BVE) = (6.0228 \times 10^{-3}, \frac{1}{2})$$

$$CS = \text{inf. point} = (EVE_{\text{inf}}, BVE_{\text{inf}}) = (\text{data point}) \times \frac{\log_2 M}{H_{\text{mut}}}$$

therefore the mutual entropy H_{mut} of the channel should be estimated

$$\text{i.e. } H_{\text{mut}} = H_Y - H_{Y|X} \quad (\text{or } H_{\text{mut}} = H_X - H_{X|Y})$$

as follows:

$$* \underline{P} = \begin{bmatrix} 0.6344 \\ 0.3656 \end{bmatrix}; \quad \underline{F} = \begin{bmatrix} 0.95 & 0.2 \\ 0.05 & 0.8 \end{bmatrix}; \quad \underline{q} = \underline{F} \cdot \underline{P} = \begin{bmatrix} 0.6758 \\ 0.3242 \end{bmatrix}$$

$$\underline{J} = \underline{F} \cdot \text{diag}(\underline{P}) = \begin{bmatrix} 0.6027 & 0.0731 \\ 0.0317 & 0.2925 \end{bmatrix}$$

$$* H_Y = -\underline{q}^T \cdot \log_2(\underline{q}) = 0.9089 \frac{\text{bits}}{\text{symbol}}$$

$$* H_{Y|X} = -\sum_{m=1}^2 \sum_{k=1}^2 J_{km} \log_2 \left(\frac{J_{km}}{P_m} \right) = -\| \underline{J} \odot \log_2(\underline{F}) \|_{1*}$$

$$= 0.4456 \frac{\text{bits}}{\text{symbol}}$$

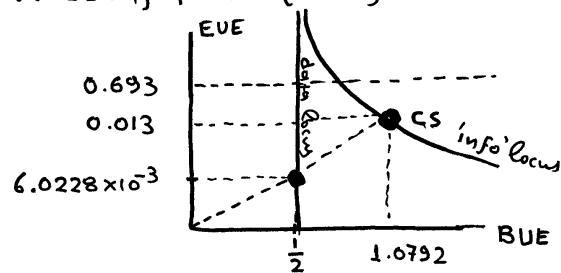
$$* H_{\text{mut}} = H_Y - H_{Y|X} = 0.4633 \frac{\text{bits}}{\text{symbol}}$$

Note: a different approach is to use the following expression

$$H_{\text{mut}} = -\| \underline{J} \odot \log_2 \left(\frac{\underline{F} \cdot \underline{P} \cdot \underline{P}^T}{\underline{J}} \right) \|_{1*} \frac{\text{bits}}{\text{symbol}} = 0.4633$$

where $\| \text{matrix} \|_{1*} = \text{sum of the elements of the matrix-argument}$.

d> $\therefore CS = \text{inf. point} = (0.013, 1.0792)$



e> $\therefore CS = \text{inf. point} = (0.013, 1.0792) \Rightarrow$ ^{+ theoretical limit}
 CS is not realizable (since $EUE_{\text{inf}} = 0.013 < 0.693$)

f>
$$SNR_{\text{inf}} = \frac{EUE_{\text{inf}}}{BUE_{\text{inf}}} = \frac{EUE_d}{BUE_d} = 0.012 \Rightarrow SNR_{\text{inf}} = -19.2082 \text{ dB}$$

23. Solution

a) $F_g = 4 \times 10^3 \text{ Hz}; \quad F_s = 2 \times F_g = 8 \times 10^3 \text{ Hz}; \quad Q = 2$

From Figures 1 and 2 we get:

$$\Pr(-2V) = 3/4$$

$$\Pr(+2V) = 1/4$$

$$N_0 = 2 \times 10^{-3}$$

symbols	probabilities	Huffman	l_i (bits)
$x_1x_1x_1$	$27/64$	1	1
$x_1x_1x_2$	$9/64$	001	3
$x_1x_2x_1$	$9/64$	010	3
$x_2x_1x_1$	$9/64$	011	3
$x_1x_2x_2$	$3/64$	00000	5
$x_2x_1x_2$	$3/64$	00001	5
$x_2x_2x_1$	$3/64$	00010	5
$x_2x_2x_2$	$1/64$	00011	5

$$\bar{l} = 1 \times 27/64 + 3 \times 9/64 + 3 \times 9/64 + 3 \times 9/64 + 5 \times 3/64 + 5 \times 3/64 + 5 \times 3/64 + 5 \times 1/64 = 2.46875 \text{ bits/triple-level}$$

(or 3-samples)

Alphabet: $\underline{X} = \begin{cases} x_1 = 1 \\ x_2 = 0 \end{cases}$ (since $\Pr(x_1) > \Pr(x_2)$)

with probabilities: $\underline{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \Pr(x_1) \\ \Pr(x_2) \end{bmatrix} = \begin{bmatrix} 0.6344 \\ 0.3656 \end{bmatrix}$

Note:

$$\Pr(x_2) = \frac{2}{3} \times \frac{9}{64} + \frac{2}{3} \times \frac{9}{64} + \frac{1}{3} \times \frac{9}{64} + \frac{5}{5} \times \frac{3}{64} + \frac{4}{5} \times \frac{3}{64} + \frac{4}{5} \times \frac{3}{64} + \frac{3}{5} \times \frac{1}{64} = 0.3656$$

$$\begin{aligned} p_e &= \Pr(y_2, x_1) + \Pr(y_1, x_2) \\ &= \Pr(y_2|x_1)\Pr(x_1) + \Pr(y_1|x_2)\Pr(x_2) \\ &= 0.05 \times 0.6344 + 0.2 \times 0.3656 \\ &= 0.1048 \end{aligned}$$

b) $H_x = -\sum_{m=1}^2 p_m \cdot \log_2 p_m = -\underline{p}^T \cdot \log_2 \underline{p} = 0.9473 \frac{\text{bits}}{\text{symbol}}$

data rate:

$$r_{\text{data}} = r_b = F_s \frac{1}{3} \bar{l} = 6583.3 \text{ bits/sec}$$

information rate:

$$r_{\text{inf}} = r_b \times H_x = r_b \times 0.9473 = 6236.4 \text{ bits/sec}$$

c) $M = 2$ i.e. binary CS

Therefore: $T_{cs} = \frac{1}{r_{cs}} = 1.5190 \times 10^{-4} \text{ sec}$

$$E_b = \frac{0.5^2}{2} T_{cs} \times \Pr(x_1) = 1.2046 \times 10^{-5}$$

$$\text{EUE} = \frac{E_b}{N_0} = 6.0228 \times 10^{-3} \text{ (data EUE)}$$

$$\text{BUE} = \frac{B}{r_{cs}} = \frac{B}{2B \times \log_2(M)} = \frac{1}{2} \text{ (data BUE with } B \text{ denoting the baseband bandwidth)}$$

$$\text{data point} = (\text{EUE}, \text{BUE}) = (6.0228 \times 10^{-3}, \frac{1}{2})$$

$$\mathbf{d)} \quad \text{CS} = \text{inf.point} = (\text{EUE}_{inf}, \text{BUE}_{inf}) = (\text{data point}) \times \frac{\log_2(M)}{\mathbf{H}_{\text{mut}}}$$

Therefore we have to estimate the mutual information \mathbf{H}_{mut}

$$\mathbf{H}_{\text{mut}} = \mathbf{H}_Y - \mathbf{H}_{Y|X} \quad \text{or} \quad (\mathbf{H}_{\text{mut}} = \mathbf{H}_X - \mathbf{H}_{X|Y})$$

i.e.

$$\underline{p} = \begin{bmatrix} 0.6344 \\ 0.3656 \end{bmatrix} \quad \mathbb{F} = \begin{bmatrix} 0.95, & 0.2 \\ 0.05, & 0.8 \end{bmatrix} \quad \underline{q} = \mathbb{F} \cdot \underline{p} = \begin{bmatrix} 0.6758 \\ 0.3242 \end{bmatrix}$$

$$\mathbb{B} = \text{diag}(\underline{q})^{-1} \cdot \mathbb{F} \cdot \text{diag}(\underline{p}) = \begin{bmatrix} 0.8918, & 0.1082 \\ 0.0978, & 0.9022 \end{bmatrix}$$

$$\mathbb{J} = \mathbb{F} \cdot \text{diag}(\underline{p}) = \text{diag}(\underline{q}) \cdot \mathbb{B} = \begin{bmatrix} 0.6027, & 0.0731 \\ 0.0317 & 0.2925 \end{bmatrix}$$

$$\mathbf{H}_X = - \sum_{m=1}^2 p_m \cdot \log_2(p_m) = - \underline{p}^T \log_2(\underline{p}) = 0.9473 \frac{\text{bits}}{\text{symbol}}$$

$$\mathbf{H}_Y = - \sum_{k=1}^2 p_k \cdot \log_2(p_k) = - \underline{q}^T \log_2(\underline{q}) = 0.9089 \frac{\text{bits}}{\text{symbol}}$$

$$\mathbf{H}_{X \times Y} = - \sum_{m=1}^2 \sum_{k=1}^2 J_{km} \cdot \log_2(J_{km}) = - \left\| \mathbb{J} \odot \log_2(\mathbb{J}) \right\|_{1*} = 1.3929 \frac{\text{bits}}{\text{symbol}}$$

$$\mathbf{H}_{Y|X} = \mathbf{H}_{Y|X}(\mathbb{J}) \equiv - \sum_{m=1}^2 \sum_{k=1}^2 J_{km} \cdot \log_2\left(\frac{J_{km}}{p_m}\right)$$

$$= - \left\| \mathbb{J} \odot \log_2\left(\underbrace{\mathbb{J} \cdot \text{diag}(\underline{p})^{-1}}_{\mathbb{F}}\right) \right\|_{1*} = 0.4456 \frac{\text{bits}}{\text{symbol}}$$

$$\Rightarrow \boxed{\mathbf{H}_{\text{mut}} = \mathbf{H}_Y - \mathbf{H}_{Y|X} = 0.4633}$$

$$\text{Therefore } \boxed{\text{CS} = \text{inf.point} = (0.013, 1.0792)}$$

e)

$$\Rightarrow \boxed{\text{CS is not realizable}} \quad (\text{since } \text{EUE}_{inf} = 0.013 < 0.693)$$

f)

$$\text{SNR}_{\text{in}} = \frac{\text{EUE}}{\text{BUE}} = 0.012 \Rightarrow \text{SNR}_{\text{in}} = -19.2082 \text{dB}$$

4 Topic: Wireless Channels

24. Solution

$$T_{spread} = \frac{30}{3 \times 10^8} = 10^{-7} \text{ sec} = T_{c, \max}$$

$$\text{In this case } L = \left\lfloor \frac{T_{spread}}{T_c} \right\rfloor + 1 = 1 + 1 = 2$$

$$\text{chip rate} = \frac{1}{T_c} = \frac{1}{10^{-7}} = 10 \text{ Mchips/sec}$$

25. Solution

$$\begin{aligned} \text{(a) } T_c &= 61 \text{ ns}; \quad B_{coh} = 3 \text{ MHz} \quad T_c = 61 \text{ ns} < T_{spread} = \frac{1}{B_{coh}} = \frac{1}{3 \times 10^6} = 333 \text{ ns} \\ \text{and } T_c &= 61 \text{ ns} < T_{coh} = \frac{1}{B_D} = \frac{1}{8 \times 10^6} = 125 \text{ ns} \end{aligned}$$

$$\begin{aligned} \text{(b) } T_c &= 61 \text{ ns}; \quad B_{coh} = 100 \text{ MHz} \quad T_c = 61 \text{ ns} > T_{spread} = \frac{1}{B_{coh}} = \frac{1}{100 \times 10^6} = 10 \text{ ns} \\ \text{and } T_c &= 61 \text{ ns} < T_{coh} = \frac{1}{B_D} = \frac{1}{8 \times 10^6} = 125 \text{ ns} \end{aligned}$$

$$\begin{aligned} \text{(c) } T_c &= 244 \text{ ns} \Rightarrow B_{coh} = 3 \text{ MHz} \quad T_c = 244 \text{ ns} < T_{spread} = \frac{1}{B_{coh}} = \frac{1}{3 \times 10^6} = 333 \text{ ns} \quad \text{and} \\ T_c &= 244 \text{ ns} > T_{coh} = \frac{1}{B_D} = \frac{1}{8 \times 10^6} = 125 \text{ ns} \end{aligned}$$

$$\begin{aligned} \text{(d) } T_c &= 244 \text{ ns} \Rightarrow B_{coh} = 100 \text{ MHz} \quad T_c = 244 \text{ ns} > T_{spread} = \frac{1}{B_{coh}} = \frac{1}{100 \times 10^6} = 10 \text{ ns} \\ \text{and } T_c &= 244 \text{ ns} > T_{coh} = \frac{1}{B_D} = \frac{1}{8 \times 10^6} = 125 \text{ ns} \end{aligned}$$

(e) None of the above.

Thus, the answer is (c)

26. Solution

$$T_{spread} = \frac{30}{3 \times 10^8} = 10^{-7} \text{ sec} = T_{c, \max}$$

$$\text{In this case } L = \left\lfloor \frac{T_{spread}}{T_c} \right\rfloor + 1 = 1 + 1 = 2$$

$$\text{chip rate} = \frac{1}{T_c} = \frac{1}{10^{-7}} = 10 \text{ Mchips/sec}$$

That is, the answer is (a)

5 Topic: Digital Modulators & Line Codes

27. Solution

$$(a) d_{ij}^2 = E_{s_i} + E_{s_j} - 2\rho_{ij}\sqrt{E_{s_i}E_{s_j}} \text{ with } E_{s_i} = E_{s_j} = E$$

$$\rho_{ij} = -1$$

$$10^2 = 2E + 2E = 4E \Rightarrow E = 25$$

That is, the correct answer is (a)

28. Solution

$$A = 3mV$$

$$T_{cs} = 1ms$$

single pulse = $\alpha \cdot \text{rect} \frac{t}{T_{cs}}$ with $\alpha = \text{random}$ $\begin{cases} \alpha = +A \text{ with prob. } 0.5 \\ \alpha = -A \text{ with prob. } 0.5 \end{cases}$

$$\begin{aligned} \text{PSD}(f) &= \frac{1}{T_{cs}} \mathbb{E} \left\{ \left| \text{FT}(\text{single pulse}) \right|^2 \right\} = \\ &= \frac{1}{T_{cs}} \mathbb{E} \left\{ \left| \text{FT} \left(\alpha \text{rect} \frac{t}{T_{cs}} \right) \right|^2 \right\} = \\ &= \frac{1}{T_{cs}} \mathbb{E} \left\{ \left| \alpha T_{cs} \text{sinc}(f T_{cs}) \right|^2 \right\} = \\ &= \frac{1}{T_{cs}} T_{cs}^2 \text{sinc}^2(f T_{cs}) \cdot \underbrace{\mathbb{E} \left\{ \alpha^2 \right\}}_{(-A)^2 \times 0.5 + (A)^2 \times 0.5 = A^2} \\ &= T_{cs} A^2 \text{sinc}^2(f T_{cs}) \end{aligned}$$

$$= 9 \times 10^{-9} \text{sinc}^2(f 10^{-3})$$

29. Solution

$$\begin{aligned} \frac{N_0}{2} &= 0.5 \times 10^{-6} \Rightarrow N_0 = 10^{-6} \\ r_b &= 220 \text{ kbits/sec} \\ 10^{-5} &= \Gamma \left(\sqrt{2 \frac{E_b}{N_0}} \right) \Rightarrow \sqrt{2 \frac{E_b}{N_0}} = 4.2 \quad \left\{ \begin{array}{l} \text{from 'Tail' graph} \end{array} \right\} \\ P_s &= E_b \cdot r_b \Rightarrow E_b = \frac{P_s}{r_b} \\ \sqrt{2 \frac{P_s}{r_b N_0}} &= 4.2 \\ \Downarrow \\ P_s &\approx 2W \quad (1.9869) \end{aligned}$$

30. Solution

$$(a) \text{EUE} = \frac{E_b}{N_0} = \frac{\frac{3^2}{2} \times 10^{-9}}{10^{-9}} = 18$$

$$(b) \text{Initially you have to prove that } p_e = \text{T} \left\{ \sqrt{2 \text{EUE} \sin^2(30^\circ)} \right\}$$

$$p_e = \text{T} \left\{ \sqrt{2 \times 18 \sin^2(30^\circ)} \right\} = \text{T} \left\{ \sqrt{2 \times 18 \times \frac{1}{4}} \right\} = \text{T} \{3\} \approx 10^{-3}$$

31. Solution

The correct answer is (c)

6 Topic: SSS and PN-Codes

32. Solution

$$(a) N_c = 31 \Rightarrow N_c = 2^m - 1$$

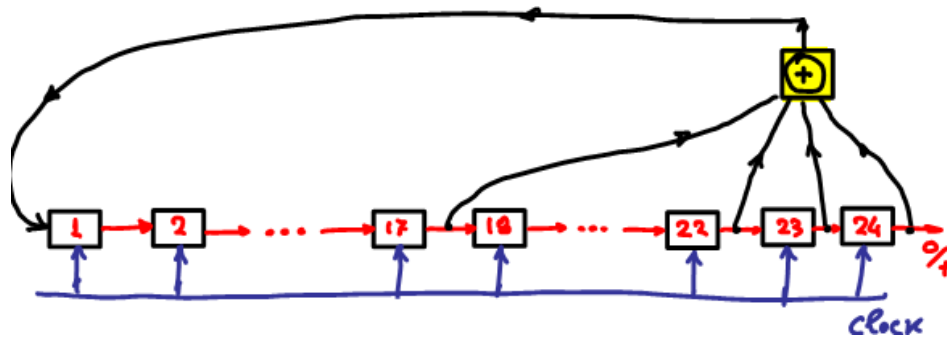
$$\Rightarrow m = \log_2(N_c + 1)$$

$$\Rightarrow m = \log_2(31 + 1) = 5$$

$$(b) T_c = 10^{-8}$$

$$\Delta f = \frac{1}{N_c T_c} = \frac{1}{31 \times 10^{-8}} = 3.2258 \times 10^6$$

33. Solution



$$N = 2^m - 1$$

$$m = 24$$

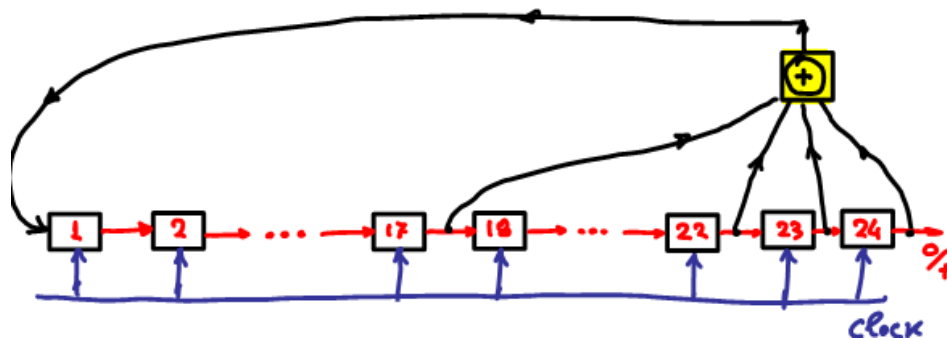
$$\text{Thus } N = 2^{24} - 1 = 16777215$$

$$T_c = \frac{1}{2.7 \text{ chip}} = 0.37037 \frac{s}{\text{chip}}$$

$$NT_c = (2^{24} - 1) \times 0.37037 = 6.21377712 \times 10^6 s$$

$$= \frac{6.21377712 \times 10^6}{60} \text{ min} = 1.0356 \times 10^5 \text{ min}$$

34. Solution



$$\left. \begin{array}{l} N = 2^m - 1 \\ m = 24 \end{array} \right\} \Rightarrow N = 2^{24} - 1 = 16.777 \times 10^6 (16777215)$$

$$T_c = \frac{1}{10^6} s/\text{bit} = 10^{-6} s/\text{bit}$$

$$NT_c = (2^{24} - 1) 10^{-6} / 60 = 0.27962 \text{ minutes}$$

35. Solution

1 1 1 1

0 1 1 1

0 0 1 1

0 0 0 1

1 0 0 0

0 1 0 0

0 0 1 0

1 0 0 1

1 1 0 0

0 1 1 0

1 0 1 1

0 1 0 1

1 0 1 0

1 1 0 1

1 1 1 0

1 1 1 1

o/p = 1 1 1 1 0 0 0 1 0 0 1 1 0 1 0

7 Topic: Direct Sequence and Frequency Hopping

36. Solution

$$r_b = 9.6 \text{ kbits} \Rightarrow T_{cs} = \frac{1}{9.6 \times 10^3}$$

$$PG = N = \frac{T_{cs}}{T_c}$$

$$B_{ss} \leq 25 \text{ MHz} \Rightarrow \frac{1}{T_c} \leq 25 \times 10^6 \Rightarrow T_c \geq \frac{1}{25 \times 10^6}$$

$$\Rightarrow \frac{T_{cs}}{N} \geq \frac{1}{25 \times 10^6} \Rightarrow N \leq 25 \times 10^6 T_{cs}$$

$$\Rightarrow N \leq \frac{25 \times 10^6}{9.6 \times 10^3} = 2604.2$$

$$\text{However, } N = 2^m - 1 \Rightarrow N = 2^{11} - 1 = 2047 \leq 2604.2$$

That is, the correct answer is (d)

37. Solution

(a) $r_{cs} = 8 \times 10^3 \text{ bits/sec}$

$$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2 \times 10^{-12}$$

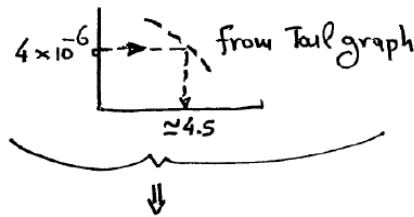
$$P_j = 1 \text{ W}; PG = 10^5; p_e = 4 \times 10^{-6}; p_{e,PR} = 4 \times 10^{-2}$$

$$PG = \frac{T_{cs}}{T_c} \Rightarrow B_{ss} = \frac{1}{T_c} = PG \times \underbrace{r_{cs}}_{= \frac{1}{T_{cs}}} = 10^5 \times 8 \times 10^3 = 800 \times 10^6 \text{ Hz}$$

Baseline Performance: $B_j = B_{ss}$

$$\Rightarrow p_e = \mathbf{T} \left\{ \sqrt{(1-\rho) \text{EUE}_{equ}} \right\}; \rho = -1$$

$$\Rightarrow 4 \times 10^{-6} = \mathbf{T} \left\{ \sqrt{2 \text{EUE}_{equ}} \right\}$$



$$\Rightarrow \sqrt{2 \text{EUE}_{equ}} = 4.5 \Rightarrow \text{EUE}_{equ} = \frac{4.5^2}{2} = 10.125$$

$$\Rightarrow \frac{E_b}{N_0 + \frac{P_j}{B_{ss}}} = 10.125 \Rightarrow E_b = 10.125 \times \left(N_0 + \frac{P_j}{B_{ss}} \right) = 10.125 \times \left(2 \times 10^{-12} + \frac{1}{800 \times 10^6} \right) = 12.677 \times 10^{-9}$$

$$Ps = \frac{E_b}{T_{cs}} = E_b r_{cs} = 12.677 \times 10^{-9} \times 8 \times 10^3 = 101.42 \times 10^{-6}$$

$$A = \sqrt{2Ps} = \sqrt{2 \times 101.42 \times 10^{-6}} \Rightarrow \boxed{A = 14.242 \text{ mV}}$$

(b) pulse jammer with $q = 0.4$ "on"

$$p_e = (1-q) \underbrace{\mathbf{T} \left\{ \sqrt{2 \frac{E_b}{N_0}} \right\}}_{\text{jammer="off"}} + q \underbrace{\mathbf{T} \left\{ \sqrt{2 \frac{E_b}{N_0 + \frac{P_L}{qB_{ss}}}} \right\}}_{\text{jammer="on"}}$$

$$p_e = 0 + 0.4 \times \mathbf{T} \left\{ \sqrt{2 \times 4.05} \right\} = 0.4 \times \mathbf{T} \left\{ 2.8474 \right\} = 0.4 \times 2.2 \times 10^{-3}$$

i.e. $\boxed{p_e = 8.8 \times 10^{-4}}$

$\rightarrow 4.05 = q \cdot \text{EUE}_{equ} \Rightarrow \text{EUE}_{equ} = \frac{4.05}{0.4} = 10.125$

(c) $p_{e,PR} = q \mathbf{T} \left\{ \sqrt{2q \text{EUE}_{PR}} \right\} \Rightarrow 4 \times 10^{-2} = 0.4 \mathbf{T} \left\{ \sqrt{2 \times 0.4 \times \text{EUE}_{PR}} \right\}$

$$\Rightarrow 10^{-1} = \mathbf{T} \left\{ \sqrt{0.8 \times \text{EUE}_{PR}} \right\}$$

$$\Rightarrow \sqrt{0.8 \times \text{EUE}_{PR}} = 1.25 \Rightarrow \text{EUE}_{PR} = \frac{1.25^2}{0.8} = 1.9531$$

$$\text{AJM} = 10 \log_{10} \text{EUE}_{equ} - 10 \log_{10} \text{EUE}_{PR} = 10 \log_{10} 10.125 - 10 \log_{10} 1.9531 = 7.1467 \text{ dB}$$

(or ~~AJM = 10 log₁₀ (10.125) - 10 log₁₀ (0.7812) = 11.126~~)

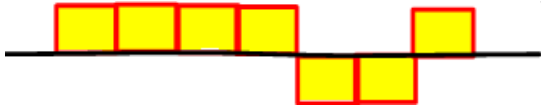
Note that the same result is produced if the EUE_{equ} & EUE_{PR} include in their definition the $q=0.4$. In this case

$$\begin{cases} \text{EUE}_{equ} = 10.125 \times 0.4 \\ \text{EUE}_{PR} = 1.9531 \times 0.4 \\ \text{AJM} = 7.1467 \end{cases}$$

38. Solution

- (a) fully synchronised system \Rightarrow code noise = zero
- (b) $F_s = 2 \times 2 \times 4k = 16kHz \Rightarrow \text{bit-rate} = r_b = r_{cs} = 8 \times 16k = 128k$
 $\Rightarrow T_{cs} = \frac{1}{r_{cs}} = \frac{1}{128 \times 10^3} = 7.8125 \times 10^{-6}$
 $N_0 = 2 \times 0.5 \times 10^{-12} = 10^{-12}$
 $P_{n,out} = \frac{N_0}{T_{cs}} = \frac{10^{-12}}{7.8125 \times 10^{-6}} = \underline{1.28 \times 10^{-7}}$
- (c) $T_c = \frac{1}{B_{ss}} = \frac{1}{32 \times 10^6} = 3.125 \times 10^{-8} \text{ sec}$
 $PG = \frac{T_{cs}}{T_c} = \frac{7.8125 \times 10^{-6}}{3.125 \times 10^{-8}} = 250$
 $P_{j,out} = \frac{P_j}{PG} = \frac{1.6}{25} = \underline{0.0064}$

39. Solution



(or, other valid codes - for various delays)

40. Solution

$$r_b = 28 \text{ kbits/sec} \Rightarrow T_{cs} = \frac{1}{28 \times 10^3}$$

Note: $PG = N = \frac{T_{cs}}{T_c}$

$$B_{ss} \leq 25 \text{ MHz} \Rightarrow \frac{1}{T_c} \leq 25 \times 10^6 \Rightarrow T_c \geq \frac{1}{25 \times 10^6} = 40 \times 10^{-9} \text{ sec}$$

$$\Rightarrow \frac{T_{cs}}{N} \geq 40 \times 10^{-9} \text{ sec} \Rightarrow N \leq 25 \times 10^6 T_{cs} \Rightarrow N \leq \frac{25 \times 10^6}{28 \times 10^3} = 892$$

However,

$$\underline{N = 2^m - 1 \leq 892} \quad (1)$$

This implies that $m = 9 \Rightarrow N = 2^9 - 1 = 511$ that satisfies Equation 1 given above.

41. Solution

$$\left. \begin{aligned} \frac{1}{T_c} &= 10M \Rightarrow T_c = 10^{-7} \text{ sec} \\ \frac{1}{NT_c} &= 39.2k \Rightarrow NT_c = 25.5 \mu \text{ sec} \end{aligned} \right\} \Rightarrow N = 255$$

$$N = 2^m - 1 \Rightarrow m = \log_2(N + 1) = 8$$

42. Solution

$\gamma = 2$
 $F_g = 4 \text{ kHz}$
 $PG = 10^8$
 $\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2 \times 10^{-12}$
 $EUE = 40$
 $q = 50\% = 0.5$

$\Rightarrow P_e = 3 \times 10^{-5} \Rightarrow$
 $T \left(\sqrt{\frac{2E_b}{N_0 + N_J}} \right) = 3 \times 10^{-5} \Rightarrow$
 $\sqrt{\frac{2E_b}{N_0 + N_J}} = 4 \Rightarrow P_J = q B_{ss} N_0 \left(\frac{2EUE}{16} - 1 \right)$ (1)

However, $r_{cs} = \frac{1}{T_{cs}} = \gamma \frac{2F_g}{2} \Rightarrow r_{cs} = 16 \text{ k} \frac{\text{bits}}{\text{sec}}$
 $B_{ss} = \frac{1}{T_c}$
 $PG = \frac{T_{cs}}{T_c} = \frac{B_{ss}}{r_{cs}} = 10^8 \Rightarrow B_{ss} = 10^8 \times 16 \text{ k}$ (2)

$\textcircled{1} \wedge \textcircled{2} \Rightarrow P_J = 6.4 \text{ W}$

N.B.: $EUE_{equ} = 8; T_c = \frac{1}{16} \times 10^{-11}; T_{cs} = \frac{1}{16} \times 10^{-3}$

43. Solution

$$r_b = \frac{1}{T_{cs}} = \frac{1}{MT_c} = \frac{1}{100 \times 4 \times 10^{-6}} = 2.5 \text{ kbits}$$

$$\text{bandwidth} = B_{ss} = c \times L \times F_1 = \begin{pmatrix} c = 8 \\ L = 2^{10} \\ F_1 = 250 \text{ k} \end{pmatrix} = 8 \times 2^{10} \times 250 \times 10^3 = 2.048 \times 10^9 \text{ Hz}$$

$$\frac{\text{bandwidth}}{r_b} = \frac{c \times L \times F_1}{r_b} = \frac{2.048 \times 10^9}{2.5 \times 10^3} = 819200 = 59.13 \text{ dB}$$

8 Topic: DS-CDMA

44. Solution

$$\begin{aligned}
\text{(a)} \quad & F_s = 2 \times 4K = 8KHz \\
& \gamma = \log_2 Q = \log_2 128 = 7 \\
& r_b = \gamma F_s = 7 \times 8K = 56Kbits/s \\
& T_{cs} = 2T_b = 2 \frac{1}{r_b} = 2 \frac{1}{56K} = 3.5714 \times 10^{-5} \\
& PG = 20dB \Rightarrow 10 \log_{10} \frac{T_{cs}}{T_c} = 20 \Rightarrow 100 = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{100} = 0.35714 \times 10^{-6} \\
& \text{i.e. chip rate} = r_c = \frac{1}{T_c} = \frac{1}{0.35714 \times 10^{-6}} = 2.8 \text{Mchips/s} \\
\text{(b)} \quad & N_c T_c = 5 \text{hours} \Rightarrow N_c \geq \frac{5 \text{hours}}{T_c} = \frac{5 \times 3600}{T_c} = \frac{5 \times 3600}{0.35714 \times 10^{-6}} = 0.0504 \times 10^{12} \\
& \Rightarrow N_c = 2^m - 1 \Rightarrow 2^m = N_c + 1 \Rightarrow m = \log_2(N_c + 1) = \log_2(0.0504 \times 10^{12} + 1) = 35.553 \\
& \text{i.e. } m = 36
\end{aligned}$$

45. Solution

$$\begin{aligned}
& P = 10^{-2} \\
& SNR_{out} = 14 \\
& r_{cs} = 25 \Rightarrow T_{cs} = \frac{1}{25k} \\
& PG = 400 \Rightarrow PG = \frac{B_{ss}}{B} = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{PG} = 10^{-7} \\
& B_{ss} = \frac{1}{T_c} = 10 \text{MHz} \\
& SNR_{out} = 2EUE_{equ} = 2 \frac{E_b}{N_0 + N_j} = 2 \frac{PT_{cs}}{N_0 + (K-1) \frac{P}{B_{ss}}} \\
& \Rightarrow N_0 + (K-1) \frac{P}{B_{ss}} = \frac{2PT_{cs}}{SNR_{out}} \\
& \Rightarrow K = \left(\frac{2PT_{cs}}{SNR_{out}} - N_0 \right) \frac{B_{ss}}{P} + 1 \simeq 58 \text{ users}
\end{aligned}$$

46. Solution

$$\begin{aligned}
\text{(a)} \quad & P = 10mW \\
& r_b = 500 \text{kbits/sec} \Rightarrow T_{cs} = \frac{1}{500} \text{ msec} \\
\text{(b)} \quad & K = 201 \text{ users} \\
& N_0 = 2 \times 10^{-9} \\
& p_e = 3 \times 10^{-5} \\
& a = 0.375 \\
& s = 1/3 \\
& E_b = PT_{cs} = 10 \times 10^{-3} \times \frac{1}{500} \times 10^{-3} = 2 \times 10^{-8} \\
& p_e = T \{ \sqrt{2EUE_{equ}} \} \Rightarrow 3 \times 10^{-5} = T \{ \sqrt{2EUE_{equ}} \} \\
& \Rightarrow (\text{using "tail graph" supplied}) \\
& 4 = \sqrt{2EUE_{equ}} \Rightarrow EUE_{equ} = 8 \\
\text{(c)} \quad & \text{However, } EUE_{equ} = \frac{E_b}{N_0 + N_j} \\
& \text{where } E_b = PT_{cs} \text{ and } N_j = \frac{(K-1).P.a.s}{B_{ss}} = \frac{(K-1).P.a.s}{PG/T_{cs}} \\
& \text{Therefore, } EUE_{equ} = \frac{PT_{cs}}{N_0 + \frac{(K-1).P.a.s}{PG/T_{cs}}} \Rightarrow \dots \Rightarrow PG = \frac{(K-1).P.a.s.T_{cs}}{\frac{PT_{cs}}{EUE_{equ}} - N_0} \\
& \Rightarrow \dots \Rightarrow PG = 1000
\end{aligned}$$

47. Solution

(a) $K = 256$

$$AJM=30dB \log_{10} EUE_{equ} - 10 \log_{10} EUE_{PR} = 30$$

$$\Rightarrow \frac{EUE_{equ}}{EUE_{PR}} = 10^3 \quad (1)$$

$$p_{e,PR} = 10^{-2}$$

$$m = 21 \Rightarrow N_c = 2^m - 1 \Rightarrow N_c = 2^{21} - 1 = 2.0972 \times 10^6$$

$$P = 0.1915$$

$$N_0 = 10^{-6}$$

$$p_{e,PR} = T\{\sqrt{2 EUE_{PR}}\} \Rightarrow 10^{-2} = T\{\sqrt{2 EUE_{PR}}\}$$

using tail function graph we have

$$\sqrt{2 EUE_{PR}} = 2.3 \text{ (or } 2.3263) \Rightarrow EUE_{PR} = 2.645 \text{ (or } 2.7058) \quad (2)$$

$$(1) \wedge (2) \Rightarrow EUE_{equ} = 2645 \text{ (or } 2705.8)$$

$$EUE_{equ} = \frac{E_b}{N_0 + N_j} = 2645 \text{ (or } 2705.8)$$

$$\Rightarrow E_b = EUE_{equ} (N_0 + \frac{(K-1) \cdot E_b}{N_c})$$

$$E_b \left(1 - \frac{EUE_{equ} (K-1)}{N_c}\right) = EUE_{equ} N_0$$

$$E_b = \frac{EUE_{equ} N_0}{1 - \frac{EUE_{equ} (K-1)}{N_c}} = \frac{2705.8 \times 10^{-6}}{1 - \frac{2705.8 \times (256-1)}{2.0972 \times 10^6}} = 4.0325 \times 10^{-3}$$

$$(b) T_{cs} = \frac{E_b}{P} = \frac{4.0325 \times 10^{-3}}{0.1915} = 21.057 \times 10^{-3} = 21.057 ms$$

$$N_c = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{N_c} = \frac{21.056 \times 10^{-3}}{2.0972 \times 10^6} = 10.04 \times 10^{-9} = 10.04 ns$$

$$PN\text{-code-rate} = \frac{1}{T_c} = \frac{1}{10.04 \times 10^{-9}} = 99.602 \times 10^6 = \boxed{99.602 Mchips/s}$$

48. Solution

$$P = 5mW$$

$$r_b = 25 kbits/sec \Rightarrow T_{cs} = 2T_b = 2 \frac{1}{r_b} = 2 \frac{1}{25 \times 10^3} = \frac{1}{12500} = 8 \times 10^{-5}$$

$$\frac{N_0}{2} = 10^{-9} \Rightarrow N_0 = 2 \times 10^{-9}$$

$$PG = N_c = 400 \Rightarrow 400 = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{8 \times 10^{-5}}{400} = 2 \times 10^{-7}$$

$$p_e = 3 \times 10^{-5}$$

$$a = 0.375$$

$$s = 3$$

$$p_e = T\{\sqrt{2 EUE_{equ}}\} \Rightarrow 3 \times 10^{-5} = T\{\sqrt{2 EUE_{equ}}\} \xrightarrow{\text{(using the tail function graph)}} 4 = \sqrt{2 EUE_{equ}}$$

$$\Rightarrow EUE_{equ} = \frac{16}{2} = 8 \Rightarrow$$

$$E_b = \frac{PT_{cs}}{2} = \frac{\frac{E_b}{N_0 + N_j}}{2} = \frac{5 \times 10^{-3} \times 8 \times 10^{-5}}{2} = 2 \times 10^{-7} \quad \left. \begin{array}{l} \frac{E_b}{N_0 + N_j} = 8 \\ N_j = \frac{(K-1)Pas}{B_{ss}} \end{array} \right\} \Rightarrow \frac{E_b}{N_0 + (K-1)PasT_c} = 8$$

$$\Rightarrow K = \left(\frac{PT_{cs}/2}{8} - N_0\right) \cdot \frac{1}{PasT_c} + 1$$

$$\Rightarrow K = \left(\frac{5 \times 10^{-3} \times 8 \times 10^{-5}}{2 \times 8} - 2 \times 10^{-9}\right) \frac{1}{5 \times 10^{-3} \times 0.375 \times 1/3 \times 2 \times 10^{-7}} + 1 = 185$$

9 Topic: PCM & PSTN

49. Solution

$$Q = 2^\gamma \Rightarrow r_b = 2F_g\gamma = 4 \times 10^3 \times 2 \times \log_2(256) = 64k$$

That is, the correct answer is (d)

50. Solution

point 'D' data sequence: 23V, 46V, 40V, 41V, 42V

51. Solution

PCM using a 256-level uniform quantizer:

$$Q = 256 \Rightarrow 2^\gamma = 256 \Rightarrow \gamma = 8 \frac{\text{bits}}{\text{level}}$$

$$\text{CF} = \text{crest factor of the signal} = \frac{\text{peak}}{\text{rms}}$$

$$\text{peak} = \hat{g} = 2V$$

$$\text{rms} = \sigma_g = \sqrt{P_g} \Rightarrow P_g = 2 \int_0^2 g^2 \text{pdf}_g(g) dg = 2 \int_0^2 g^2 \frac{1}{2} \Lambda\left(\frac{g}{2}\right) dg$$

$$= 2 \int_0^2 g^2 \frac{1}{2} \frac{2-g}{2} dg = \int_0^2 \left(g^2 - \frac{1}{2}g^3\right) dg$$

$$= \left(\frac{g^3}{3} - \frac{1}{2} \frac{g^4}{4}\right) \Big|_0^2 = \frac{2}{3}$$

$$\text{i.e. rms} = \sqrt{\frac{2}{3}}$$

$$\text{SNR}_q = 4.77 + 6 \times \gamma - \underbrace{20 \log_{10} \frac{2}{\sqrt{\frac{2}{3}}}}_{20 \log_{10}(\sqrt{6}) = 7.7815} = 4.77 + 6 \times 8 - 7.7815 = 44.989 \text{dB}$$

52. Solution

$$\text{SNR}_q \geq 50 \text{ dB}; \text{CR} = \frac{\hat{V}}{\sigma} = 4.4668$$

$$\text{SNR}_q = 4.77 + 6\gamma - \underbrace{20 \log_{10} \frac{\hat{V}}{\sigma}}_{=13 \text{dB}} \text{ dB}$$

$$\text{SNR}_q = 4.77 + 6\gamma - 13$$

i.e.

$$\Rightarrow 4.77 + 6\gamma - 13 \geq 50$$

$$\Rightarrow \gamma \geq \frac{50 + 13 - 4.77}{6}$$

$$\Rightarrow \gamma \geq 10 \Rightarrow \log_2 Q \geq 10 \Rightarrow Q \geq 2^{10} = 1024$$

Therefore,

$$r_b = \gamma F_s = \gamma 2F_g = 10 \times 2 \times 18k = 360 \text{ kbits/sec}$$

53. Solution

$$B = \frac{\text{channel symbol rate}}{2} = \frac{\text{bit rate}}{2} = \frac{\gamma F_s}{2} = \gamma F_g \Rightarrow B = \gamma F_g \Rightarrow \frac{B}{F_g} = \gamma \Rightarrow \beta = \gamma$$

54. Solution

$$(a) \frac{g}{g_{\max}} = \frac{2.4}{10} = 0.24$$

$$g_c = \frac{\ln(1+100 \times 0.24)}{\ln(1+100)} \times g_{\max} = \frac{3.218}{4.615} \times 10 = 6.974$$

$$\Rightarrow b_{13} < g_c < b_{14} \Rightarrow g_c = 6.875V = m_{14}$$

$$(b) \ g_{q,out} = \frac{1}{\mu} \left(\exp \left(\frac{m_{14}}{g_{maz}} \times \ln(1 + \mu) \right) - 1 \right) \times 10 = 2.287$$

$$(c) \ n_q = 2.4 - 2.28 = 0.12 \text{ V (or } -0.12 \text{ V)}$$

55. Solution

The correct answer is (a)

56. Solution

The correct answer is (d)

END