SOLUTIONS

EE4-01E
EE9-SC2

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE/EIE PART IV: MEng and ACGI

ADVANCED COMMUNICATION THEORY

Friday, 2 May 2014, 10:00 am

Time allowed: 3:00 hours

There are 20 questions on this paper.

Answer ALL questions.

The multiple choice questions together account for 40% of the marks.

Answers to multiple choice questions 1-14 should be given on the paper itself.

Students are not permitted to use more than one answer book.

Students are not permitted to take the question paper away.

The following are provided:
A table of Fourier transforms
A Gaussian Tail Function graph

Examiners responsible:

First Marker(s): A. Manikas
Second Marker(s): D. Mandic

PART-I

1. Consider a binary communication system which uses the following two equiprobable signals $s_0(t)$ and $s_1(t)$ of equal energy E and cross correlation $\rho_{01} = -1$. The signals are transmitted over a communication channel which adds white Gaussian noise having a double-sided power spectral density of 10^{-6} W/Hz. If the forward transition matrix $\mathbb{F}$ of the equivalent discrete channel is $\mathbb{F}=\begin{bmatrix} 0.994, & 0.006 \\ 0.006, & 0.994 \end{bmatrix}$ then the energy E is [3 marks]

   (a) 2.25×10^{-6}.
   (b) 4.26×10^{-6}.
   (c) 6.25×10^{-6}. [x]
   (d) 8.25×10^{-6}.
   (e) None of the above.

2. With reference to 'multi-user (MU) CDMA receivers', which of the following statements is correct? [2 marks]

   (a) A RAKE receiver is a multi-user receiver.
   (b) A multi-user receiver is used to resolve paths (in a multipath environment), delayed by more than the chip period T_c.
   (c) A minimum-mse MU receiver requires no knowledge of the cross-correlation matrix of the PN-signals.
   (d) A decorrelating MU receiver is a sub-optimum multi-user receiver. [x]
   (e) None of the above.

3. If the path-loss is 23.0103 dB and the power of the received signal is 6.9897 dBm, then the power of the transmitter is [3 marks]

   (a) 20 W;
   (b) 10 W;
   (c) 1 W; [x]
   (d) 10 mW;
   (e) 20 mW.

4. Consider an antenna array systems of 5 elements operating in the presence of one desired and two uncorrelated co-channel interfering signals all of power equal to $P_s = 1$. The power of the noise is equal to $\sigma_n^2 = 10$. If $\mathbb{R}_{xx}$ is the theoretical covariance matrix of the received signal vector $x(t)$ then which of the following statements is correct? [2 marks]

   (a) The minimum eigenvalue of $\mathbb{R}_{xx}$ is equal to 1.
   (b) The principal eigenvalue of $\mathbb{R}_{xx}$ is equal to 10.
   (c) The rank of $\mathbb{R}_{xx}$ is equal to 2.
   (d) The rank of $\mathbb{R}_{xx}$ is equal to 3.
   (e) The rank of $\mathbb{R}_{xx}$ is equal to 5. [x]

5. Consider an antenna array system of N elements operating in the presence M co-channel sources ($M < N$). If $\underline{S}_i$ is the manifold vector associated with the i^{th} source and $\mathbb{E}_s$ and $\mathbb{E}_n$ denote the matrices with columns the signal eigenvectors and the noise eigenvectors respectively of data covariance matrix $\mathbb{R}_{xx}$ then which of the following expresssions is correct? [2 marks]

   (a) $\mathbb{E}_n.\mathbb{E}_n^H.\underline{S}_i = \underline{S}_i$.

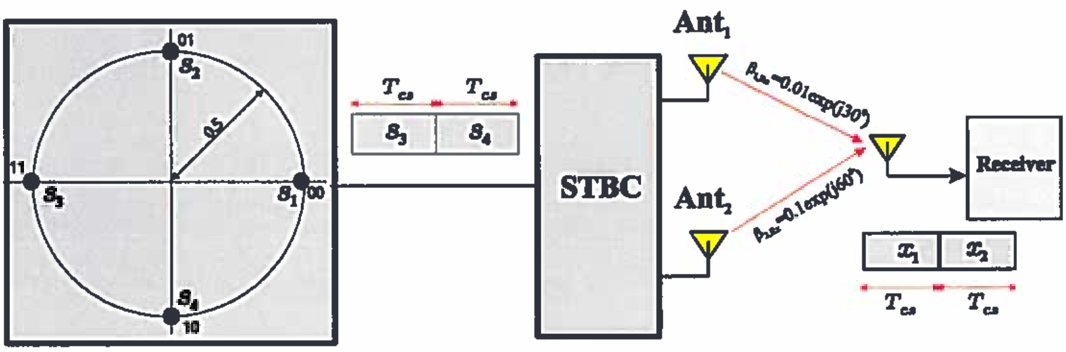
   (b) $\mathbb{E}_s.\mathbb{E}_s^H.\underline{S}_i = \underline{0}$.

   (c) $\mathbb{E}_n.\mathbb{E}_n^H.\underline{S}_i = \underline{0}$. [x]

   (d) $\left(\mathbb{I}_N - \mathbb{E}_s.\mathbb{E}_s^H\right).\underline{S}_i = \underline{S}_i$.

   (e) None of the above.

6. Consider the QPSK MISO system of 2 Tx antennas operating in a frequency flat wireless channel as shown the following figure:



   If the QPSK symbols $[s_3, s_4]$ are transmitted using the above "Space-Time Block Coder" (STBC) then the receiver's input $[x_1, x_2]$, ignoring the noise, is [4 marks]

   (a) $\left[+0.0173 + j0.0458, \quad -0.0298 + j0.42\right]$;

   (b) $\left[-0.0173 + j0.0458, \quad -0.0298 - j0.42\right]$;....[x]

   (c) $\left[-0.0173 - j0.0458, \quad -0.0298 + j0.42\right]$;

   (d) $\left[+0.0173 - j0.0458, \quad -0.0298 - j0.42\right]$;

   (e) none of the above.

7. With reference to a MIMO wireless communication system where the Cartesian coordinates of the Tx and Rx antenna array elements are given by the columns of the following matrices

$$\text{Tx} : [\underline{\overline{r}}_1, \underline{\overline{r}}_2] = \begin{bmatrix} -1, & +2 \\ 0, & 0 \\ 0, & 0 \end{bmatrix} \text{ in units of half-wavelength.}$$

$$\text{Rx} : [\underline{r}_1, \underline{r}_2] = \begin{bmatrix} 0, & 0 \\ 0, & 0 \\ -0.5, & 0.5 \end{bmatrix} \text{ in units of half-wavelength.}$$

which of the following statements, associated with its virtual Rx antenna array of an equivalent SIMO wireless communication system, is correct? [3 marks]

(a) $\begin{bmatrix} -1, & -1, & 2, & 2 \\ 0, & 0, & 0, & 0 \\ -0.5, & 0.5, & -0.5, & 0.5 \end{bmatrix}$ [x]

(b) $\begin{bmatrix} -1, & 2, & -1, & 2 \\ -0.5, & -0.5, & 0.5, & 0.5 \\ 0, & 0, & 0, & 0 \end{bmatrix}$.

(c) $\begin{bmatrix} -0.5, & -0.5, & 0.5, & 0.5 \\ -1, & 2, & -1, & 2 \\ 0, & 0, & 0, & 0 \end{bmatrix}$.

(d) $\begin{bmatrix} -0.5, & -0.5, & 0.5, & 0.5 \\ 0, & 0, & 0, & 0 \\ -1, & 2, & -1, & 2 \end{bmatrix}$.

(e) None of the above.

8. Consider a linear array of 5 Rx-antennas having the following Cartesian coordinates:

$$[\underline{r}_1, \underline{r}_2, \underline{r}_3, \underline{r}_4, \underline{r}_5] = \begin{bmatrix} -5, & -1, & +1, & +2, & +3 \\ 0, & 0, & 0, & 0, & 0, \\ 0, & 0, & 0, & 0, & 0, \end{bmatrix} \text{ in units of half-wavelength.}$$

The rate of change of the arclength $\dot{s}(\theta)$ of the array manifold for a source with Direction-of-Arrival (azimuth) $\theta = 30°$ is [3 marks]

(a) $\dot{s}(30°) = 19.631$;
(b) $\dot{s}(30°) = 9.9346$; [x]
(c) $\dot{s}(30°) = 5.4414$;
(d) $\dot{s}(30°) = 3.1623$;
(e) none of the above.

9. Consider a beamformer which employes a uniform array of N antennas and operates in the presence of a single signal with direction ($\theta = 30^0, \phi = 0^0$). The carrier frequency is 2.4 GHz and the manifold vector for the Direction-of-Arrival ($\theta = 30^0, \phi = 0^0$) is

$$[-0.1125 + 0.9936i, 0.6661 + 0.7458i, 1.0000, 0.6661 - 0.7458i, -0.1125 - 0.9936i]^T$$

Consider that the array steers its main lobe towards the direction ($\theta = 30^0, \phi = 0^0$), the power of the received signal is 1 and the channel noise is additive white Gaussian noise of power 0.01. If at the output of the beamformer P_{out} is the power of the desired signal and SNR_{out} denotes the signal-to-noise ratio, which of the following statements is correct? [3 marks]

(a) P_{out}=5 and SNR_{out}=100.
(b) P_{out}=25 and SNR_{out}=100.
(c) P_{out}=5 and SNR_{out}=500.
(d) P_{out}=25 and SNR_{out}=500. [x]
(e) None of the above.

10. For a uniform linear array of 5 sensors operating at 2.4GHz frequency with an inter-antenna spacing 6.25cm the beamwidth is [3 marks]

(a) 47.156°; [x]
(b) 45.537°;
(c) 23.074°;
(d) 11.537°;
(e) none of the above.

11. With reference to a Wiener-Hopf beamformer, which of the following statements is correct? [2 marks]

(a) It is a superresolution beamformer.
(b) It is robust to errors associated with the direction of the desired signal.
(c) It provides, asymptotically, complete interference cancellation.
(d) It is optimum with respect to SNIR criterion. [x]
(e) None of the above.

12. Consider a beamformer which employes a uniform linear array of N antennas and uses the following weight vector:

$$[-0.1125 + 0.9936i, 0.6661 + 0.7458i, 1.0000, 0.6661 - 0.7458i, -0.1125 - 0.9936i]^T.$$

If the channel noise is additive white Gaussian noise with power $\sigma_n^2 = 0.001$ then the noise power at the beamformer's output is: [3 marks]

(a) 0.00025;
(b) 0.0005;
(c) 0.005;[x]
(d) 0.025;
(e) none of the above.

13. Consider a beamformer which employes a uniform array of N antennas and operates in the presence of a single signal with direction ($\theta = 30^0, \phi = 0^0$). The carrier frequency is 2.4 GHz and the manifold vector for the Direction-of-Arrival ($\theta = 30^0, \phi = 0^0$) is

$$[-0.1125 + 0.9936i, 0.6661 + 0.7458i, 1.0000, 0.6661 - 0.7458i, -0.1125 - 0.9936i]^T$$

Consider that the array steers its main lobe towards the direction ($\theta = 30^0, \phi = 0^0$), the power of the received signal is 1 and the channel noise is additive white Gaussian noise of power 0.01. If at the output of the beamformer P_{out} is the power of the desired signal and SNR_{out} denotes the signal-to-noise ratio, which of the following statements is correct? [4 marks]

(a) P_{out}=5 and SNR_{out}=100.
(b) P_{out}=25 and SNR_{out}=100.
(c) P_{out}=5 and SNR_{out}=500.
(d) P_{out}=25 and SNR_{out}=500. [x]
(e) None of the above.

14. Consider an array of 4 antennas with Cartesian coordinates given by the following matrix

$$\begin{bmatrix} -2, & 2, & 2, & -2 \\ -0.5, & -0.5, & 1, & 0.5 \\ 0, & 0, & 0, & 0 \end{bmatrix}$$ in units for halfwavelength

The array aperture is [3 marks]

(a) 4.272; [x]
(b) 4.0311;
(c) 4;
(d) 1.5;
(e) none of the above.

PART-I: SOLUTIONS

Question	Answer
1	c
2	d
3	c
4	e
5	e
6	b
7	a
8	b
9	d
10	a
11	d
12	c
13	d
14	a

PART-II

15. (a) The two signals $s_0(t)$ and $s_1(t)$ of a binary communication system each have energy equal to 93.3 and cross correlation coefficient 0.866.

- Draw the constellation diagram of the system properly labeled. [2 marks]
- What is the distance of these two signals? [2 marks]

(b) Prove that the maximum signal-to-noise ratio $\mathrm{SNR}_{\substack{out \\ max}}$ at the output of a matched filter is given by:

$$\mathrm{SNR}_{\substack{out \\ max}} = \int_0^T h_o(z).s(T-z).dz$$

where $h_o(t)$ is the impulse response of the filter matched to the signal $s(t)$. [3 marks]

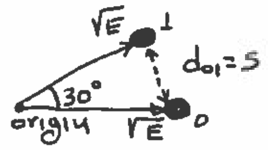
(c) What is the Fredholm integral equation of the first kind which provides the general equation for a matched filter? [2 marks]

(d) What are the Fredholm and $\mathrm{SNR}_{\substack{out \\ max}}$ equations when the noise is additive white Gaussian? [3 marks]

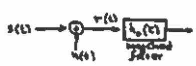
<u>Answer</u>

$$\underset{\substack{\uparrow \\ 0.866}}{\rho_{01}} = \cos\varphi \Rightarrow \varphi = 30^\circ$$

$$d_{01}^2 = 2E - 2\times 0.866 \times \sqrt{EE} \Rightarrow d_{01}^2 = 25 \Rightarrow d_{01} = 5$$



(a)



$$\underbrace{\int_0^T h_o(z).s(T-z)dz}_{\tilde{s}(T)} + \underbrace{\int_0^T h_o(z)n(T-z)dz}_{\tilde{n}(T)}$$

$$\therefore \mathrm{SNR}_{\substack{out \\ max}} = \frac{E\{\tilde{s}^2(T)\}}{E\{\tilde{n}(T)^2\}} = \frac{\tilde{s}^2(T)}{E\{\tilde{n}(T)\}} = \frac{\int_0^T\int_0^T h_o(z)h_o(u)s(T-z)s(T-u)dz\,du}{E\{\int_0^T\int_0^T h_o(z)h_o(u)n(T-z)n(T-u)dz\,du\}}$$

$$= \frac{\int_0^T\int_0^T \ldots}{\int\int h_o(z)h_o(u)\underbrace{E\{n(T-z)n(T-u)\}}_{R_{nn}(z-u)}dz\,du}$$

$$= \frac{\int_0^T h_o(z)s(T-z)dz \cdot \int_0^T h_o(u)\,s(T-u)\,du}{\int_0^T h_o(z)\underbrace{\left\{\int_0^T h_o(u)\,R_{nn}(z-u)\,du\right\}}_{s(T-z)}dz}$$

$$= \frac{\cancel{\int_0^T h_o(z)s(T-z)dz} \cdot \int_0^T h_o(u)s(T-u)du}{\cancel{\int_0^T h_o(z)s(T-z)dz}}$$

$$= \int_0^T h_o(u)\,s(T-u)du.$$

(b)

(c)

$$\boxed{\int_0^{T_{cs}} h_{opt}(z)\cdot R_{nn}(\tau - z)\cdot dz = s(T_{cs}-\tau);\ 0 \le \tau \le T_{cs}}$$

GENERAL EXPRESSION FOR MATCHED FILTERS

(d)

$$\int_0^{T_{cs}} h_{opt}(z) \cdot \underbrace{\frac{N_0}{2} \cdot \delta(\tau - z)}_{=R_{nn}(\tau - z)} \cdot dz = s(T_{cs} - \tau)$$

$$\Longrightarrow \quad h_{opt}(\tau) \cdot \frac{N_0}{2} = s(T_{cs} - \tau) \Longrightarrow \boxed{h_{opt} = \frac{2}{N_0} \cdot s(T_{cs} - \tau)}$$

In this case the SNR becomes:

$$\begin{aligned}
\mathrm{SNR}_{\substack{out\\ \max}} &= \int_0^{T_{cs}} \underbrace{\frac{2}{N_0} \cdot s(T_{cs} - z)}_{=h_{opt}} \cdot s(T_{cs} - z) \cdot dz \\
&= \frac{2}{N_0} \int_0^{T_{cs}} s^2(T_{cs} - z) \cdot dz = \\
\Longrightarrow \mathrm{SNR}_{\substack{out\\ \max}} &= 2\frac{E}{N_0} \text{ for white noise}
\end{aligned}$$

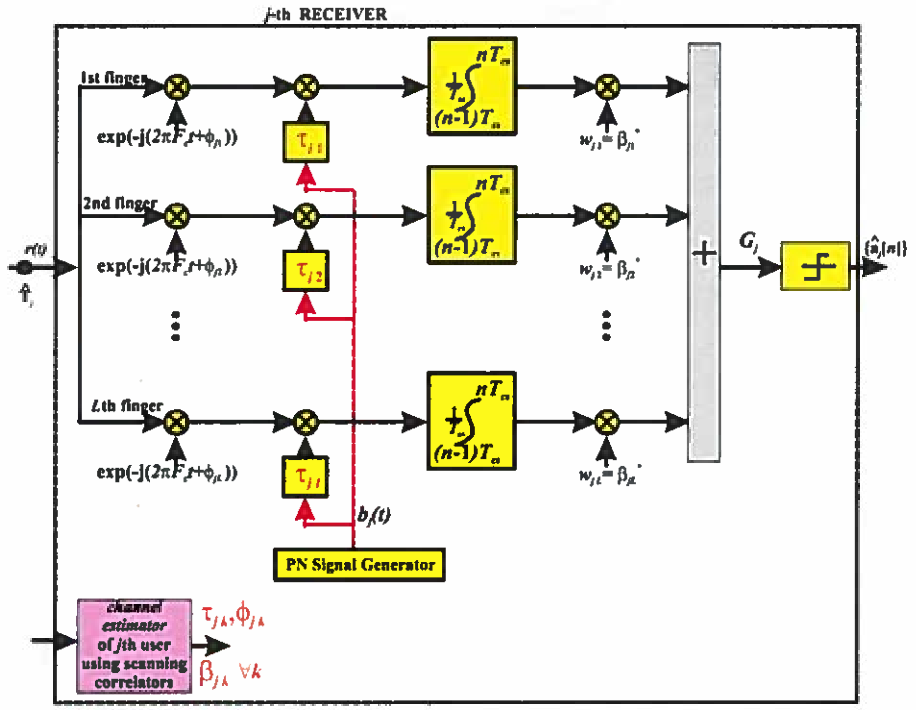
16. (a) Define the concept of "Diversity".Draw a block diagram of a RAKE receiver in a CDMA mobile system and describe briefly its operation. ~~[3 marks]~~ dM [3 marks]

(b) Define the concept of "Diversity" [3 marks]

(c) Define the main four diversity combining rules [4 marks]

Answer

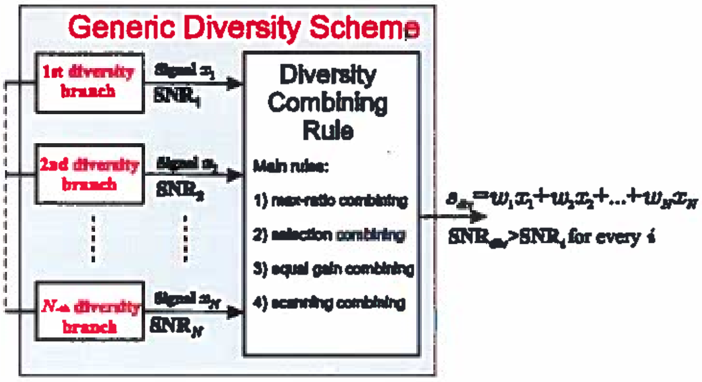
(a) The generic structure of a RAKE receiver is below.Hence, if one path (or more)



is "lost" (because its associated path gain coefficient becomes very small) the output SNR remains in an optimum level.

A finger is, by itself, an optimum coherent receiver (correlator or matched filter receiver) while, according to Brennan's original paper [Brennan 1959] there are four main diversity combining rules to combine the outputs of the fingers.

(b) Diversity: the utilization of two or more copies of a signal with varying degrees of noise/interference effects to achieve, by selection or a combination scheme, higher degree of message-recovery performance than that achievable by any one of the individual copies separately.



(c) max ratio combining (MRC) or, equivalently, maximum signal-to-noise-power ratio combining:

$$\underline{w}_{MRC} = \arg\left\{\max_{\underline{w}}(\mathrm{SNR}_{out,\mathrm{div}})\right\}$$

selection combining (SC) diversity rule:

$$w_k = \begin{cases} 1 & \text{if } \mathrm{SNR}_k > \mathrm{SNR}_i \forall i \\ 0 & \text{otherwise} \end{cases}$$

equal gain combining (EGC):

$$w_1 = w_2 = ... = w_N$$

$$\text{scanning combining (SCC):} \quad \left.\begin{array}{l} \text{if } \mathrm{SNR}_k > \text{threshold then } \begin{cases} w_k = 1 \\ w_j = 0; \forall j \neq k \end{cases} \\ \text{if } \mathrm{SNR}_k \text{ fall below threshold then } k = k+1 \\ \text{repeat} \end{array}\right\}$$

- That is, a branch (say the k-th branch) is selected that has SNR above a predetermined threshold (SNR_k >threshold) and is used until its SNR drops below that threshold.
- Then the next branch is selected if its SNR is above the threshold, etc.

17. Consider that one of the paths from the transmitter of a CDMA user arrives at the reference point of an antenna array CDMA receiver from direction (azimuth, elevation)= $(60°, 0°)$. The corresponding PN-sequence, of period N_c, is generated by the polynomial D^2+D+1 in GF(2) while the discrete path delay (mod-N_c) is equal to two. For this path, if the Cartesian coordinates of the antenna array elements are given by the columns of the following matrix

$$[r_1, r_2, r_3] = \begin{bmatrix} -2, & 0, & +2 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \end{bmatrix} \text{ in units of half-wavelength.}$$

find

(a) the manifold vector [5 marks]

(b) the spatio-temporal array manifold vector. [5 marks]

Answer

(a) manifold vector: $\underline{S}(\theta) = \exp(-j\pi \underline{r} \cos\theta) = \exp(-j\pi \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \cos(\pi/6)) =$

$$= \begin{bmatrix} \exp(-i\pi(-2)\cos(60\pi/180)) \\ \exp(-i\pi(0)\cos(60\pi/180)) \\ \exp(-i\pi(2)\cos(60\pi/180)) \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \\ -1.0 \end{bmatrix}$$

(b) $D^2 + D + 1$ provides the a PN-sequence with period 3: $-1, -1, +1$

$$\underline{h}(60°, 2T_c) = \underline{S}(60°) \otimes \mathbb{J}^2 \underline{c} = \overbrace{\begin{bmatrix} -1.0 \\ 1.0 \\ -1.0 \end{bmatrix}}^{\triangleq \underline{S}(30°)} \otimes \mathbb{J}^2 \overbrace{\begin{bmatrix} -1 \\ -1 \\ +1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{\triangleq \underline{c}} = \begin{bmatrix} -\mathbb{J}^2\underline{c} \\ \mathbb{J}^2\underline{c} \\ -\mathbb{J}^2\underline{c} \end{bmatrix}$$

$$\underline{h}(60°, 2T_c) = \begin{bmatrix} 0, & 0, & +1, & +1, & -1, & 0, & 0, & 0, & -1, & -1, & +1, & 0, & 0, & 0, & +1, & +1, & -1, & 0 \end{bmatrix}^T$$

18. Draw a block structure and write a mathematical equation of the impulse response of the following multipath frequency selective channels:

(a) SISO, [2 marks]

(b) SIMO, [2 marks]

(c) MISO, and [2 marks]

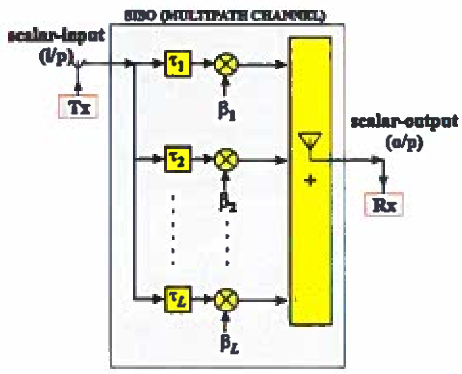
(d) MIMO [2 marks]

Answer

The vector $\overline{\underline{S}}_\ell = \overline{\underline{S}}(\overline{\theta}_\ell, \overline{\phi}_\ell) \in C^{\overline{N}}$ is the Tx array manifold vector of the ℓ-th path . and The vector $\underline{S}_\ell = \underline{S}(\theta_\ell, \phi_\ell) \in C^N$ is the Rx array manifold vector of the ℓ-th path .

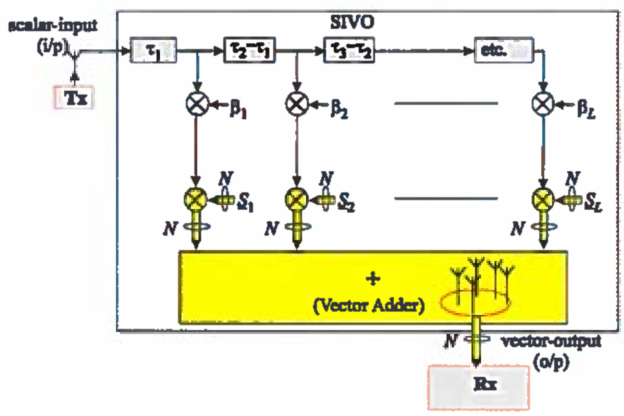
(a) The impulse response of the SISO multipath channel is

$$\boxed{\text{SISO: } h(t) = \sum_{\ell=1}^{L} \beta_\ell . \delta(t - \tau_\ell)} \tag{1}$$



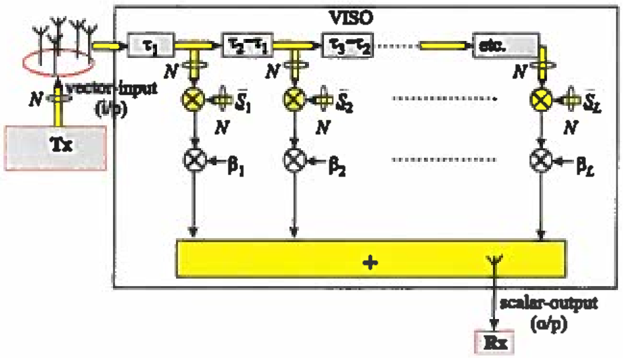
(b) The impulse response (vector) of the SIVO multipath channel is

$$\boxed{\text{SIVO: } \underline{h}(t) = \sum_{\ell=1}^{L} \underline{S}(\theta_\ell, \phi_\ell) . \beta_\ell . \delta(t - \tau_\ell)} \tag{2}$$



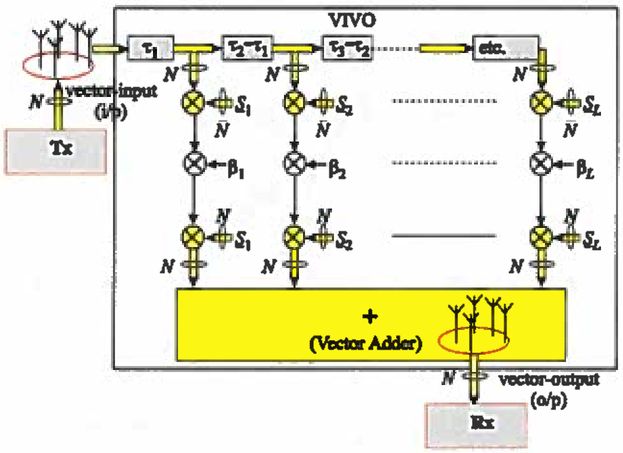
(c) The impulse response of the MISO (VISO) channel is

$$\boxed{\text{MISO (VISO): } h(t) = \sum_{\ell=1}^{L} \beta_\ell . \overline{\underline{S}}_\ell^H \underline{\delta}(t - \tau_\ell)} \tag{3}$$



(d) The impulse response of the MIMO (VIVO) channel is

$$\boxed{\text{MIMO (VIVO)}: \underline{h}(t) = \sum_{\ell=1}^{L} \beta_\ell . \underline{S}_\ell . \underline{\bar{S}}_\ell^H \, \underline{\delta}(t - \tau_\ell)} \tag{4}$$



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19. Consider a binary pulse-code-modulation (binary-PCM) system where the input to the digital modulator is a binary sequence of 1's and 0's with the number of 1's being twice the number of zeros. The binary sequence is transmitted as a pulse signal $s(t)$ with a *one* being sent as $\text{rect}(\frac{t}{T_b})$ $+4\Lambda\left\{\frac{t}{T_b/2}\right\}$ and *zero* being sent as $2\text{rect}(\frac{t}{T_b})$ $-4\Lambda\left\{\frac{t}{T_b/2}\right\}$.

    The channel noise is assumed to be additive and uniformly distributed between -2 Volts and $+2$ Volts. Find:

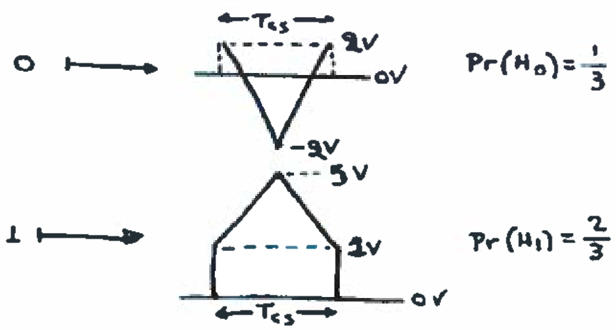
    (a) the probability density function $\text{pdf}_s(s)$, of the transmitted signal $s(t)$; [3 marks]

    (b) the probability density function, $\text{pdf}_r(r)$, of the received signal $r(t)$; [3 marks]

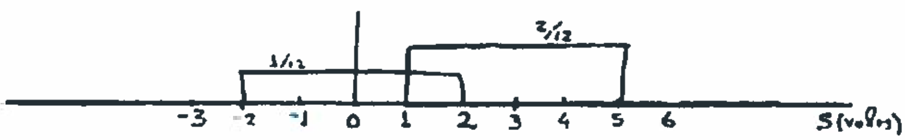
    (c) the likelihood functions of the above system. [4 marks]

    <u>Answer</u>

    (a) Probability density function $\text{pdf}_s(s)$

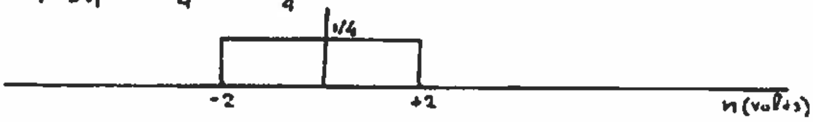


$$\text{pdf}_s(s) = \frac{1}{3} \times \frac{1}{4}\,\text{rect}\,\frac{s}{4} + \frac{2}{3} \times \frac{1}{4}\,\text{rect}\,\frac{s-3}{4}$$



(b) Probability density function, $\mathrm{pdf}_r(r)$

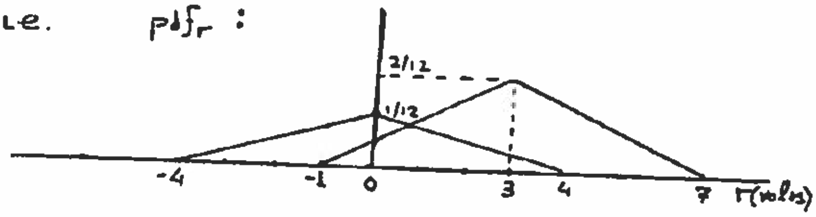
$$\mathrm{pdf}_n(n) = \frac{1}{4}\,\mathrm{rect}\,\frac{n}{4}$$



$$r(t) = s(t) + n(t) \Rightarrow \mathrm{pdf}_r(r) = \mathrm{pdf}_s \underset{\substack{\uparrow \\ \text{convolution}}}{*} \mathrm{pdf}_n$$

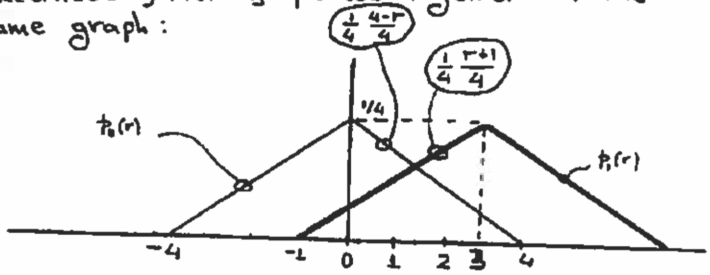
$$\Rightarrow \mathrm{pdf}_r(r) = \frac{1}{3}\cdot\underbrace{\frac{1}{4}\cdot\frac{4}{4}\,\Lambda\left\{\frac{r}{4}\right\}}_{\mathrm{pdf}_{r|H_0} \equiv p_0(r)} + \frac{2}{3}\cdot\underbrace{\frac{1}{4}\cdot\frac{4}{4}\,\Lambda\left\{\frac{r-3}{4}\right\}}_{\mathrm{pdf}_{r|H_1} \equiv p_1(r)}$$

i.e. pdf_r :



(c) Likelihood Functions

likelihood functions placed together on the same graph:



$$\left.\begin{aligned} \text{i.e.}\quad p_0(r) &= \frac{1}{4}\,\Lambda\left\{\frac{r}{4}\right\} \\ p_1(r) &= \frac{1}{4}\,\Lambda\left\{\frac{r-3}{4}\right\} \end{aligned}\right\} \Rightarrow \text{likelihood ratio } \ell(r) = \frac{\Lambda\left\{\frac{r-3}{4}\right\}}{\Lambda\left\{\frac{r}{4}\right\}}$$

20. Consider an M-ary communication system with its signal set described as follows: $s_i(t) = A_i\cos(2\pi F_c t)$, $i = 1, 2, .., M$, $0 < t < 2$ sec

$$\text{with} \begin{cases} M = 4 \\ A_i = (2i - 1 - M) \times 10^{-3}\text{Volts} \\ \Pr(H_1) = \Pr(H_4) = 0.2 \ \text{ and } \ \Pr(H_2) = \Pr(H_3) = 0.3 \end{cases}$$

The signals are transmitted over a communication channel which adds white Gaussian noise having a double-sided power spectral density of 10^{-6} W/Hz.

(a) Draw a labelled block diagram of the MAP receiver. [5 marks]

(b) Plot the constellation diagram and label the decision regions. [5 marks]

Answer

(a)

$N_0 = 2\times10^{-6}$

$T_{cs} = 2$

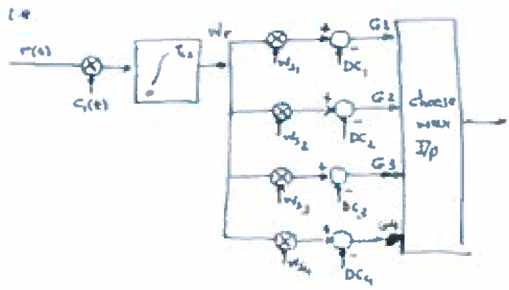
$D = 1$ (dim) $\longrightarrow c_1(t) = \frac{2}{T_{cs}}\cos(2\pi F_c t) = \cos(2\pi F_c t)$

$A_i = (2i-1-4)\times10^{-3} \Rightarrow \begin{cases} A_1 = -3\text{mV} \\ A_2 = -1\text{mV} \\ A_3 = 1\text{mV} \\ A_4 = 3\text{mV} \end{cases}$

$w_{s_i} = -\sqrt{E_i} = -\sqrt{\frac{A_i^2}{2}T_{cs}} \Rightarrow \begin{cases} w_{s_1} = -3\times10^{-3} \\ w_{s_2} = -10^{-3} \\ w_{s_3} = +10^{-3} \\ w_{s_4} = 3\times10^{-3} \end{cases}$

$DC_i = \frac{N_0}{2}\ln(P_i) - \frac{1}{2}E_i \Rightarrow \begin{cases} DC_1 = -6.109\times10^{-6} \\ DC_2 = -1.706\times10^{-6} \\ DC_3 = -1.704\times10^{-6} \\ DC_4 = -6.109\times10^{-6} \end{cases}$

$\boxed{\text{Note: } G_i = w_r \cdot w_{s_i} + DC_i}$



(b)

1st threshold $w_{r_{thr1}}$: $G_1 = G_2 \Rightarrow$

$\Rightarrow w_{r_{thr1}}\cdot w_{s_1} + DC_1 = w_{r_{thr1}}\cdot w_{s_2} + DC_2$

$\Rightarrow w_{r_{thr1}} = \frac{DC_2 - DC_1}{w_{s_1} - w_{s_2}} = -2.2\times10^{-3}$

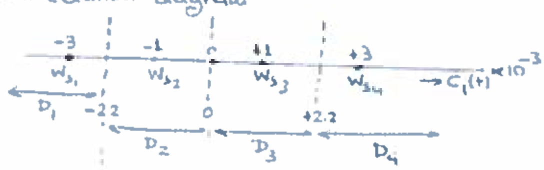
similarly $\rightarrow$ 2nd threshold $w_{r_{thr2}}$: $G_2 = G_3$

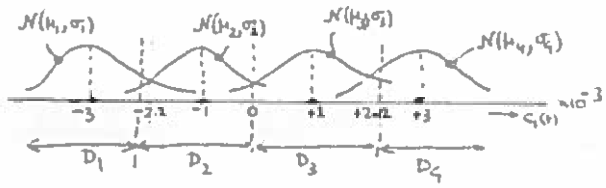
$\Rightarrow w_{r_{thr2}} = \frac{DC_3 - DC_2}{w_{s_2} - w_{s_3}} = 0$

and 3rd threshold $w_{r_{thr3}}$: $G_3 = G_4$

$\Rightarrow w_{r_{thr3}} = \frac{DC_4 - DC_3}{w_{s_3} - w_{s_4}} = +2.2\times10^{-3}$

$\therefore$ constellation diagram





$$\left.\begin{aligned} \mu_1 &= -3\times10^{-3} \\ \mu_2 &= -1\times10^{-3} \\ \mu_3 &= 1\times10^{-3} \\ \mu_4 &= 3\times10^{-3} \end{aligned}\right\} \quad \begin{aligned} &\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma \\ &\text{where } \sigma^2 = \frac{N_0}{2} \times 2\underbrace{B}_{\frac{1}{2T_{cs}}} \times T_{cs} = \frac{N_0}{2} = 10^{-6} \end{aligned}$$

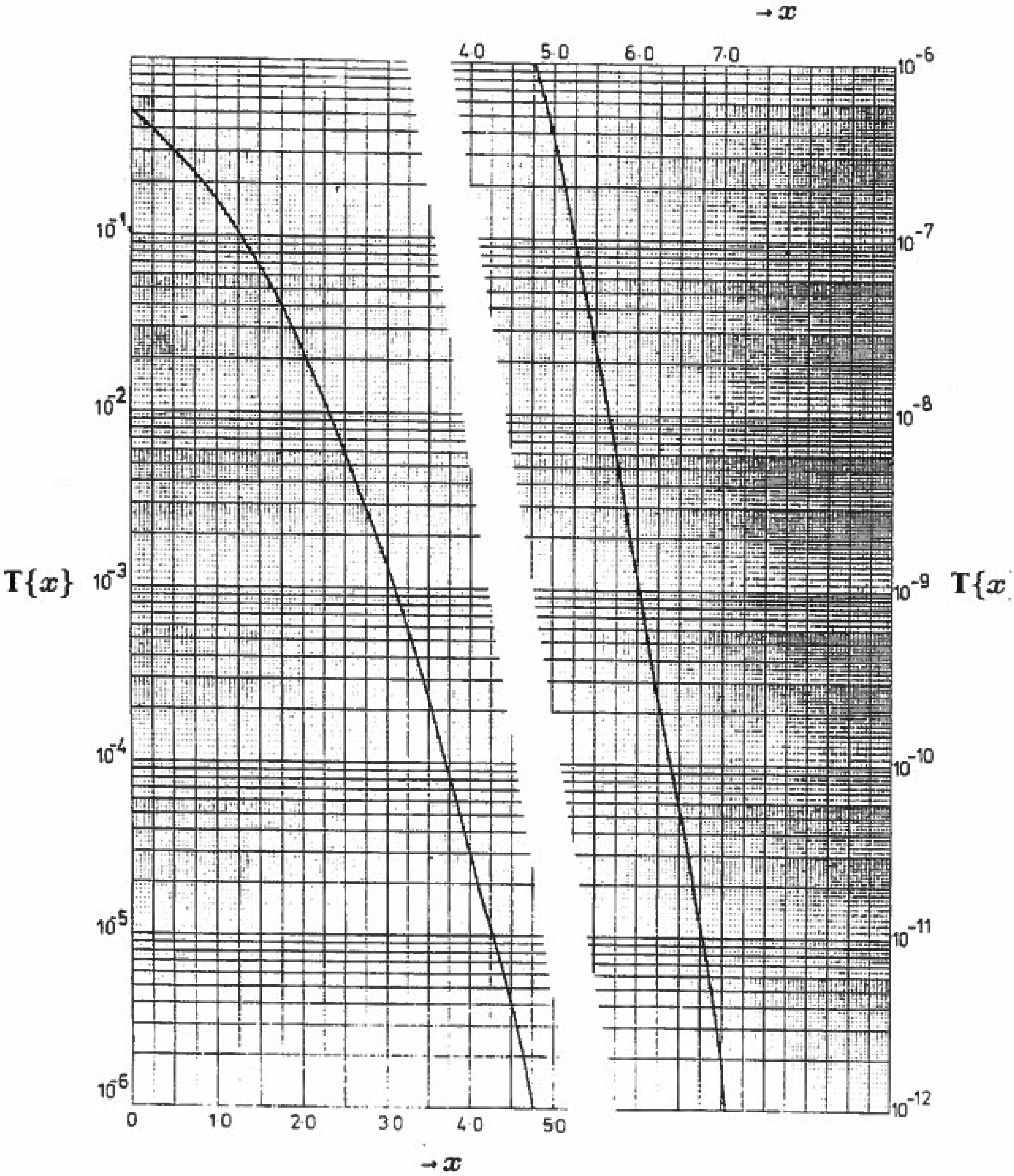
noise energy over T_{cs}

$$\therefore \boxed{\sigma = 10^{-3}}$$

Fourier Transform Tables			
	Description	Function	Transformation
1	Definition	$g(t)$	$G(f) = \int_{-\infty}^{\infty} g(t).e^{-j2\pi ft}dt$
2	Scaling	$g\left(\frac{t}{T}\right)$	$\lvert T\rvert .G(fT)$
3	Time shift	$g(t-T)$	$G(f).e^{-j2\pi fT}$
4	Frequency shift	$g(t).e^{j2\pi Ft}$	$G(f-F)$
5	Complex conjugate	$g^*(t)$	$G^*(-f)$
6	Temporal derivative	$\frac{d^n}{dt^n}g(t)$	$(j2\pi f)^n.G(f)$
7	Spectral derivative	$(-j2\pi t)^n.g(t)$	$\frac{d^n}{df^n}G(f)$
8	Reciprocity	$G(t)$	$g(-f)$
9	Linearity	$A.g(t)+B.h(t)$	$A.G(f)+B.H(f)$
10	Multiplication	$g(t).h(t)$	$G(f)*H(f)$
11	Convolution	$g(t)*h(t)$	$G(f).H(f)$
12	Delta function	$\delta(t)$	1
13	Constant	1	$\delta(f)$
14	Rectangular function	$\mathbf{rect}\{t\} \triangleq \begin{cases} 1 & \text{if } \lvert t\rvert < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$	$\mathbf{sinc}\{f\} \triangleq \frac{\sin(\pi f)}{\pi f}$
15	Sinc function	$\mathbf{sinc}(t)$	$\mathbf{rect}\{f\}$
16	Unit step function	$u(t) \triangleq \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$	$\frac{1}{2}\delta(f) - \frac{j}{2\pi f}$
17	Signum function	$\mathrm{sgn}(t) \triangleq \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$	$-\frac{j}{\pi f}$
18	decaying exp (two-sided)	$e^{-\lvert t\rvert}$	$\frac{2}{1+(2\pi f)^2}$
19	decaying exp (one-sided)	$e^{-\lvert t\rvert}.u(t)$	$\frac{1-j2\pi f}{1+(2\pi f)^2}$
20	Gaussian function	$e^{-\pi t^2}$	$e^{-\pi f^2}$
21	Lambda function	$\Lambda\{t\} \triangleq \begin{cases} 1-t & \text{if } \quad 0 \le t \le 1 \\ 1+t & \text{if } -1 \le t \le 0 \end{cases}$	$\mathbf{sinc}^2\{f\}$
22	Repeated function	$\mathbf{rep}_T\{g(t)\} = g(t)*\mathbf{rep}_T\{\delta(t)\}$	$\left\lvert\frac{1}{T}\right\rvert \mathbf{comb}_{\frac{1}{T}}\{G(f)\}$
23	Sampled function	$\mathbf{comb}_T\{g(t)\} = g(t).\mathbf{rep}_T\{\delta(t)\}$	$\left\lvert\frac{1}{T}\right\rvert \mathbf{rep}_{\frac{1}{T}}\{G(f)\}$

The graph below shows the Tail function $\mathbf{T}\{x\}$ which represents the area from x to ∞ of the Gaussian probability density function N(0,1), i.e.

$$\mathbf{T}\{x\} = \int_x^{\infty} \frac{1}{\sqrt{2\pi}}.\exp\left(-\frac{y^2}{2}\right)dy$$



Note that if $x > 6.5$ then $\mathbf{T}\{x\}$ may be approximated by $\mathbf{T}\{x\} \approx \frac{1}{\sqrt{2\pi}.x}.\exp\left\{-\frac{x^2}{2}\right\}$