1. a) $E(Y) = \sum k \, P(Y=k) = \frac{1}{2}$

   b) $P\{j \text{ heads} / n \text{ trials}\} = \binom{n}{j} P^j (1-P)^{n-j}$
   $$= \binom{n}{j}\left(\frac{1}{2}\right)^n$$

   c) $P\{n=k\} = (1-P)^{k-1} P = \left(\frac{1}{2}\right)^k$

2. a) $P\{1 \times 6\} = \binom{2}{1} P(1-P) = 2 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{18}$

   b) $P\{2 \times \text{odd}\} = P\{\text{odd}\}^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

   c) $P\{\text{sum}=4\} = P\{1,3\} + P\{2,2\} + P\{3,1\} = \frac{1}{12}$

   d) $P\{\text{mod}(\text{sum},3)=0\} = P\{\text{sum}=3\} + P\{\text{sum}=6\}$
   $$\qquad\qquad + P\{\text{sum}=9\} + P\{\text{sum}=12\}$$
   $$= \frac{2}{36} + \frac{5}{36} + \frac{4}{36} + \frac{1}{36} = \frac{1}{3}$$

3. a) $P\{1\text{st head}\} = (1-P)^{h-1} P = \left(\frac{1}{2}\right)^n$

   b) $P\{\text{heads} = \text{tails}\} = \begin{cases} \binom{n}{\frac{n}{2}}(1-P)^{\frac{n}{2}} P^{\frac{n}{2}} = \binom{n}{\frac{n}{2}}\left(\frac{1}{2}\right)^n, & n \text{ even} \\[2mm] 0, & n \text{ odd} \end{cases}$

3. c) $P\{2 \times heads\} = \binom{n}{2} P^2 (1-p)^{n-2} = \binom{n}{2}(\frac{1}{2})^n$

d) $P\{\geq 2 \times heads\} = 1 - P\{1 \times heads\} - P\{0 \times heads\}$
$$= 1 - \binom{n}{1} P(1-p)^{n-1} - (1-p)^n$$
$$= 1 - \binom{n}{1}(\frac{1}{2})^n - (\frac{1}{2})^n$$

4. $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$$- P(AB) - P(BC) - P(CA) + P(ABC)$$
$$= \frac{1}{2} + \frac{3}{10} + \frac{1}{2} - \frac{2}{10} - \frac{1}{10} - \frac{2}{10} + \frac{1}{10} = \frac{9}{10}$$

5. $P\{error\} = P\{2 \times e\} + P\{3 \times e\}$
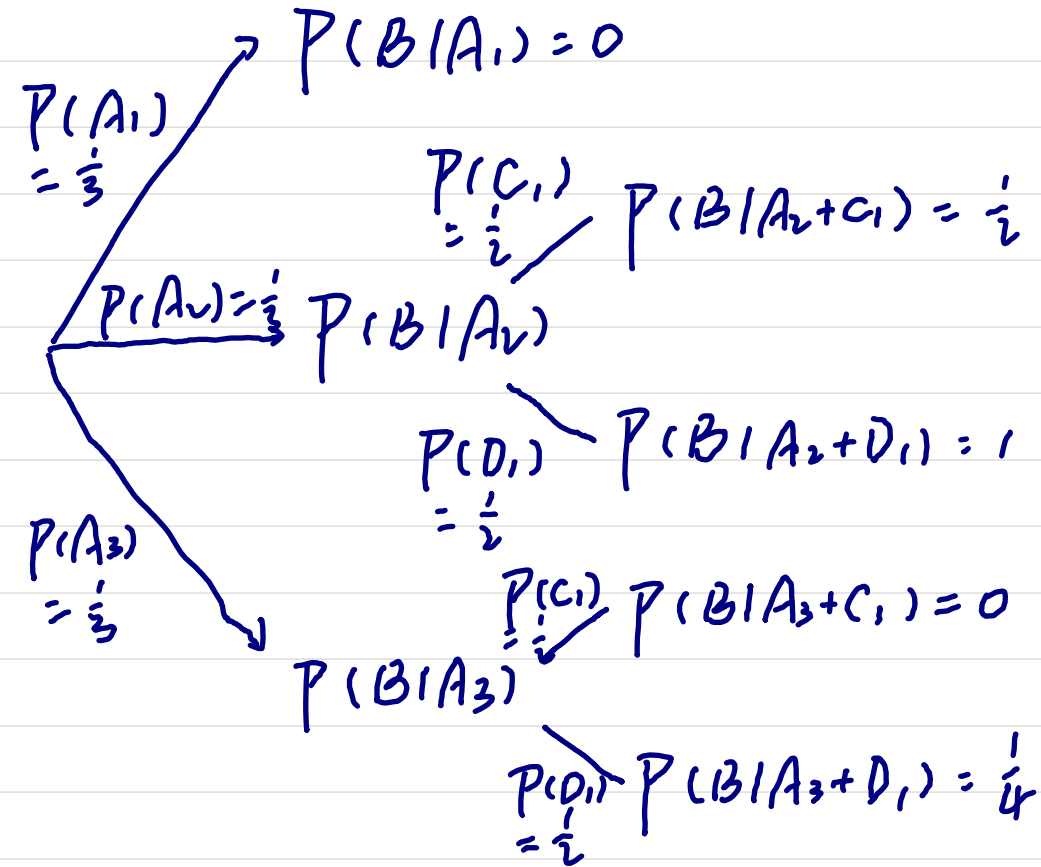$$= \binom{3}{2} P_e^2 (1-P_e) + P_e^3 = 2.898 \times 10^{-6}$$

6. $P(A) = \frac{1}{2}$

$P(B) = \frac{1}{2}$

$P(C) = \frac{1}{2}$

$P(D) = 0$

7. $A_i \to i$ works. $B \to$ the event. $C_i \to$ works $i$.
$D_i \to$ fails.

$P(A_1)$
$= \frac{1}{3}$ $\nearrow$ $P(B|A_1) = 0$

$P(A_2) = \frac{1}{3}$ $P(B|A_2)$

$\quad\quad\quad P(C_1)$
$\quad\quad\quad = \frac{1}{2}$ $\nearrow$ $P(B|A_2 + C_1) = \frac{1}{2}$

$\quad\quad\quad P(D_1)$ $\searrow$ $P(B|A_2 + D_1) = 1$
$\quad\quad\quad = \frac{1}{2}$

$P(A_3)$
$= \frac{1}{3}$ $\searrow$ $P(B|A_3)$

$\quad\quad\quad P(C_1)$ $P(B|A_3 + C_1) = 0$
$\quad\quad\quad = \frac{1}{2}$ $\nearrow$

$\quad\quad\quad P(D_1)$ $P(B|A_3 + D_1) = \frac{1}{4}$
$\quad\quad\quad = \frac{1}{2}$

$P(A_2|B) = P(B|A_2) \frac{P(A_2)}{P(B)}$

$= (\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 1) \times \dfrac{\frac{1}{3}}{(\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 1) \times \frac{1}{3} + (\frac{1}{4} \times \frac{1}{2}) \times \frac{1}{3}}$

$= \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}.$