Study Group

Professor A. Manikas

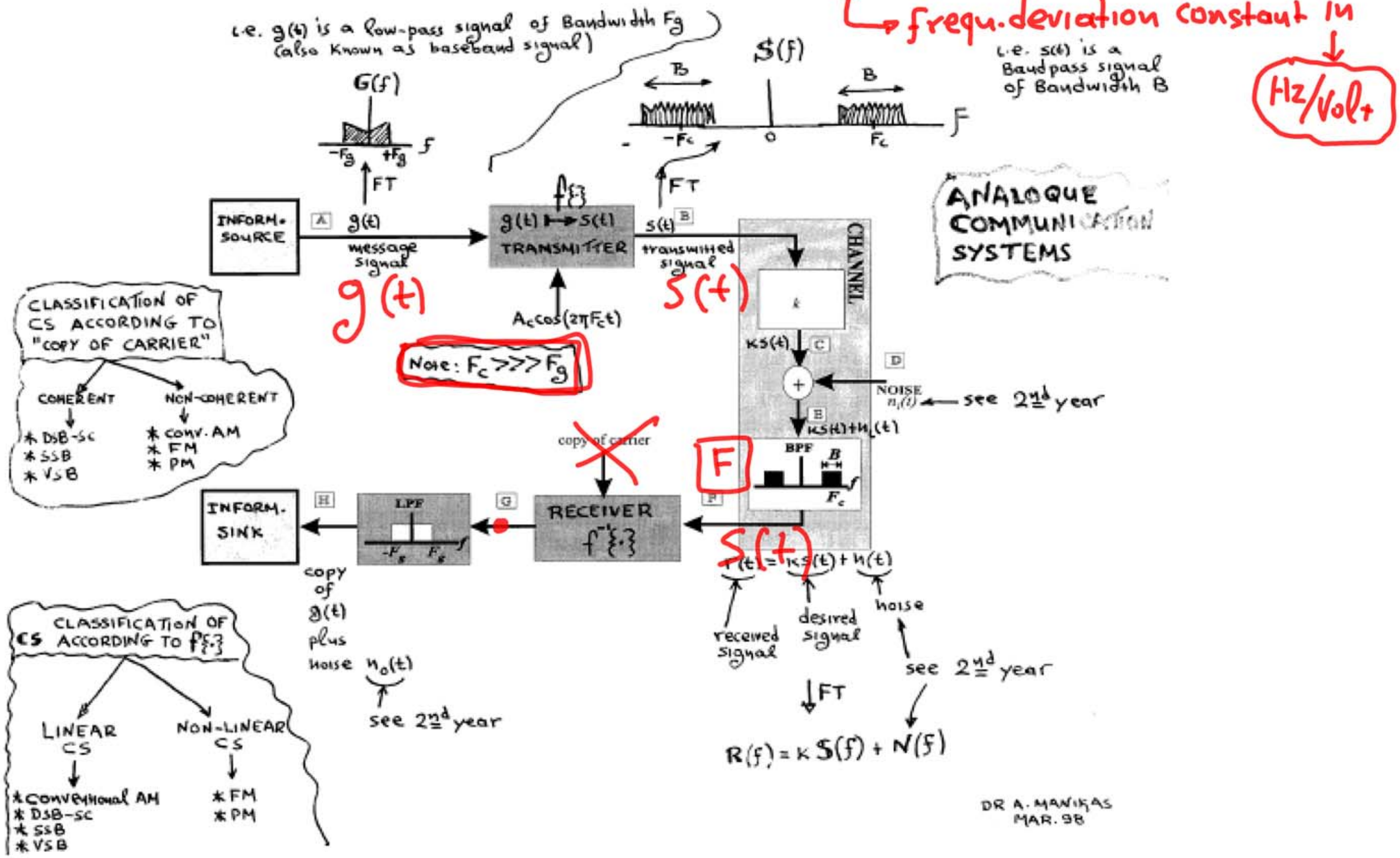
Imperial College London

Comms-1

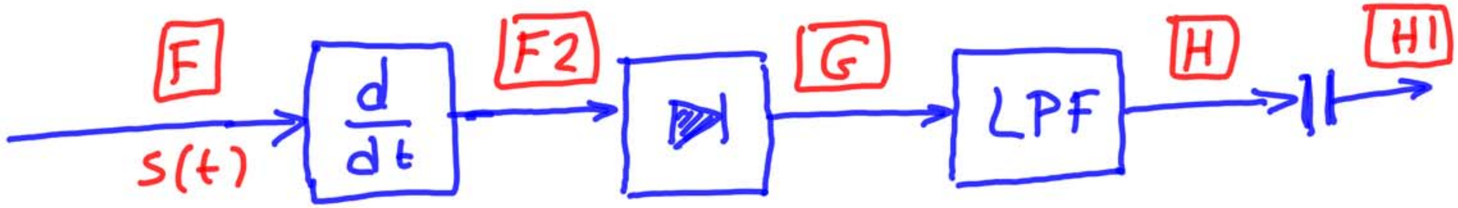
FM + PM

$$\text{FM}: s(t) = A_c \cos\left(2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} g(u)\,du\right)$$

k_f → frequ. deviation constant in Hz/Volt



$$\boxed{F} = s(t) = A_c \cdot \cos\left(2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} \underbrace{g(u)}_{\text{message}} du\right)$$



$$\boxed{F2} = A_c \cdot \left[2\pi F_c + 2\pi k_f g(t)\right] \cdot \sin\left(2\pi F_c t + \cdots\right)$$

$$\boxed{G} = 2\pi A_c F_c + 2\pi A_c k_f g(t)$$

$$\boxed{H} = \boxed{G}$$

$$\boxed{H1} = \underbrace{2\pi A_c k_f}_{\text{gain factor}} g(t)$$

$$\therefore P_{out} = \overline{4\pi^2 A_c^2 k_f^2 g^2(t)} = 4\pi^2 A_c^2 k_f^2 \overline{g^2(t)} = 4\pi^2 A_c^2 k_f^2 \overbrace{P_g}^{\text{power of } g(t)}$$

* PM : $s(t) = A_c \cdot \cos\big(\underbrace{2\pi F_c t + k_p \cdot g(t)}_{\triangleq\, \Theta(t)}\big)$

* FM : $s(t) = A_c \cdot \cos\Big(\underbrace{2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} g(u)\, du}_{\triangleq\, \Theta(t)}\Big)$

* Angle modulation : $s(t) = A_c \cdot \cos\big(\Theta(t)\big)$

$\Downarrow$

$$\text{FM}: \Theta(t) \triangleq 2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} g(u) \cdot du$$

$$\therefore \frac{d\Theta(t)}{dt} = \Theta'(t) = 2\pi F_c + 2\pi k_f\, g(t) = 2\pi \underbrace{\big(F_c + k_f \cdot g(t)\big)}_{\triangleq\, f_i = \text{instanteneous frequency}}$$

* $g(t) \longrightarrow \boxed{\int} \longrightarrow \boxed{\begin{matrix}\text{PM}\\ \text{Tx}\end{matrix}} \longrightarrow \begin{matrix} s(t) \\ \text{FM} \end{matrix}$

* $g(t) \longrightarrow \boxed{\dfrac{d}{dt}} \longrightarrow \boxed{\begin{matrix}\text{FM}\\ \text{Tx}\end{matrix}} \longrightarrow \begin{matrix} s(t) \\ \text{PM} \end{matrix}$

* modulation index: $\longrightarrow$ $\boxed{\text{it is defined for sinewave messages } A_g \cdot \cos(2\pi F_g t)}$

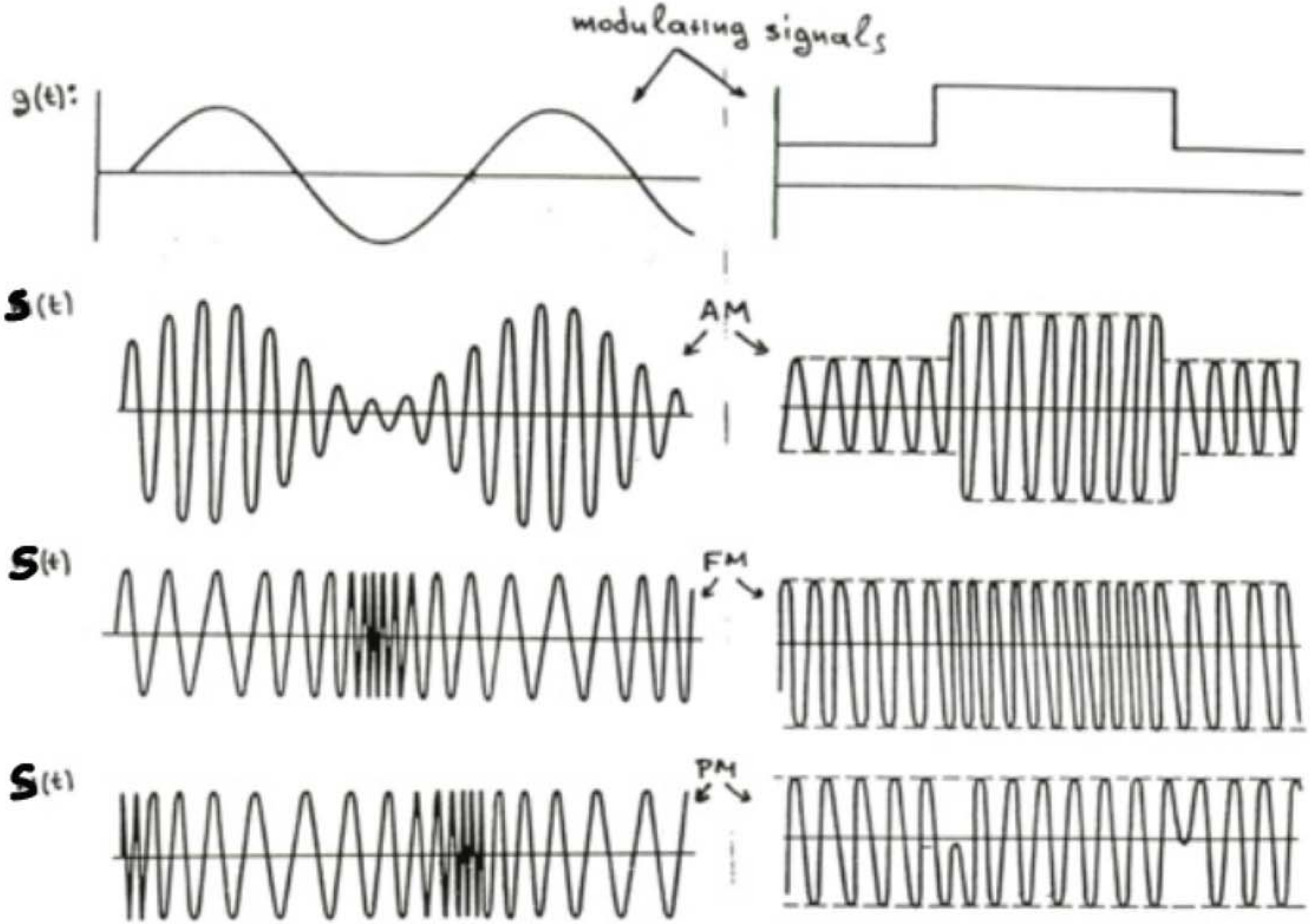
FM: $\quad \mathcal{B}_{FM} \triangleq \dfrac{k_f \cdot A_g}{F_g}$

PM: $\quad \mathcal{B}_{PM} \triangleq k_p \cdot A_g$

* Bandwidth-FM $= \infty$

$\therefore$ CARLSON'S RULE (98%) $= B_{FM} = 2(\beta_{FM}+1)F_g$

$$= 2(\underbrace{\beta_{FM}F_g}_{\triangleq F_D} + F_g)$$


modulating signals
g(t):
S(t)
AM
S(t)
FM
S(t)
PM

PS7

1) Over an interval $0 \le t \le 1$, an angle modulated signal is given by

$$s(t) = 10\cos 13000t$$

The carrier frequency is $\omega_c = 10000$.

(a) If this were a PM signal with $k_p = 1000$, determine $g(t)$ over $0 \le t \le 1$.

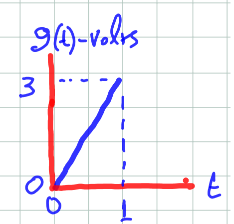
(b) If this were an FM signal with $k_f = 1000$, determine $g(t)$ over $0 \le t \le 1$.

PM:

$$s(t) = \underbrace{A_c}_{10} \cdot \cos\Big(\underbrace{\overbrace{2\pi F_c t}^{10K} + \overbrace{K_p}^{1000} g(t)}_{13000t}\Big)$$

$$\Rightarrow 13Kt = 10Kt + 1000\,g(t) \Rightarrow$$

$$\Rightarrow g(t) = \frac{3000t}{1000} = 3t$$



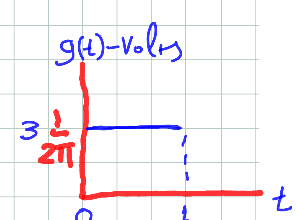
FM:

$$s(t) = \underbrace{A_c}_{10} \cdot \cos\Big(\underbrace{\overbrace{2\pi F_c t}^{10K} + 2\pi \overbrace{K_f}^{1000} \int_{-\infty}^{t} g(u)\,du}_{13000t}\Big)$$

$$\Rightarrow 13Kt = 10Kt + 2\pi 1000 \int_{-\infty}^{t} g(u)\,du$$

$$\Rightarrow 13K = 10K + 2\pi\, 1000\, g(t)$$

$$\Rightarrow g(t) = \frac{3000}{1000 \times 2\pi} = 3\,\frac{1}{2\pi}$$



2. An angle modulated signal has the form $s(t) = \underbrace{100}_{A_c}\cos[2\pi f_c t + 4\sin(2000\pi t)]$ where $f_c = 10$ MHz.

(a) Determine the average transmitted power.

(b) Is this an FM or PM signal? Expain.

(c) Determine the frequency deviation Δf. [4000Hz]

(d) Using Carson's rule, find the bandwidth of the modulated signal. [10kHz]

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a> $P_s = \frac{A_c^2}{2} = \frac{100^2}{2} = 5000$

b> PM

$$s(t) = A_c \cdot \cos\left(2\pi F_c t + \underbrace{K_p}_{4} \overbrace{g(t)}^{\sin(2000\pi t)}\right)$$

FM

$$s(t) = A_c \cdot \cos\left(2\pi F_c t + \underbrace{2\pi K_f \int_{-\infty}^{t} g(u)\,du}_{4\sin(2000\pi t)}\right)$$

$$\Rightarrow 4\sin(2000\pi t) = \underbrace{2\pi K_f}_{} \int_{-\infty}^{t} \underbrace{g(u)}_{g(t)=\cos(2000\pi t)}\,du$$

$$4 = \frac{2\pi k_f}{2000\pi} \Rightarrow k_f = \frac{8000\pi}{2\pi} = 4000$$

Hz/Volt

c) $f_i = F_c + k_f \cdot g(t)$

$= 10\,MHz + 4000 \cdot \cos(2000\pi t)$

$\Delta f = k_f \cdot A_g = 4000 \cdot 1 = 4000$

d) $B_{FM} = 2(\underbrace{\Delta f}_{F_D} + F_g) = 2(4k + 1k) = 10\,kHz$

SOME THEORY

Bandwidth Expansion Factor

$$BEF = \frac{\text{Tx Bandwidth}}{\text{message Bandwidth}}$$

remember: message Bandwidth = F_g

$$BEF_{FM} = \frac{\text{Carson's Rule}}{F_g} = 2(\beta_{FM} + 1)$$

$$BEF_{DSB} = \frac{2F_g}{F_g} = 2 = BEF_{conv\ AF}$$

$$BEF_{SSB} = \frac{F_g}{F_g} = 1$$

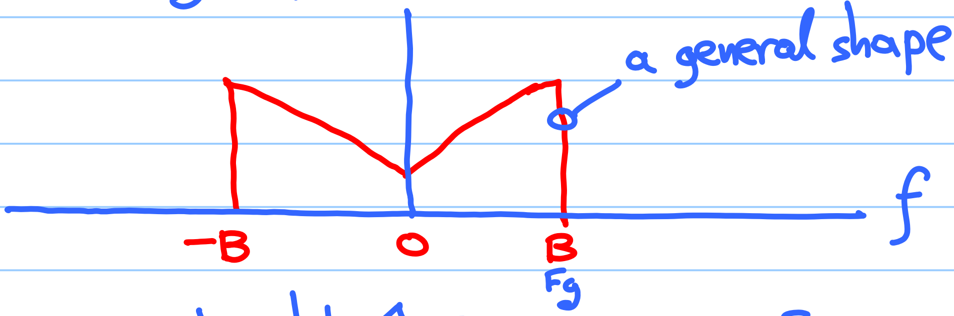
$$BEF_{VSB} = \frac{k F_g}{F_g} = k \quad \text{with} \quad 1 < k < 2$$

VSB: vestigial

eg $k = 1.5$

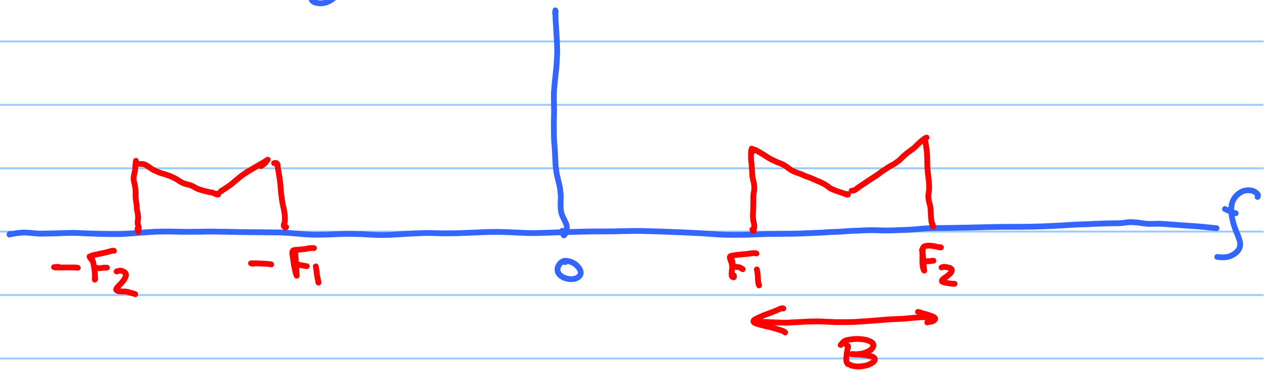
More on Bandwidth

1. Baseband signal
(or Low Pass signal)



$$\text{Bandwidth} \triangleq B = F_g - 0 = F_g$$

2. Bandpass Signal:

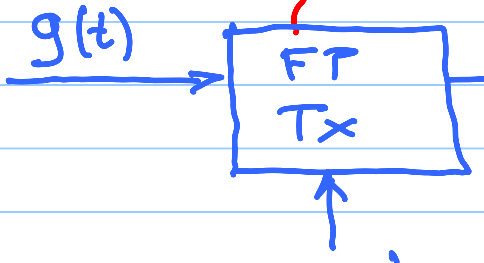


$$\text{Bandwidth} = B \triangleq F_2 - F_1$$

PS7 [Q3]

Note Title

$\beta \triangleq$ modulation index ?



$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} g(u)\,du\right)$$

carrier $A_c \cos(2\pi f_c t)$

max $(g(t))$

$$\beta \triangleq \frac{k_f \cdot A_g}{F_g}$$

← Note: the modulation index β is only defined for sinewave messages
eg: $g(t) = A_g \cos(2\pi F_g t)$
or $g(t) = A_g \sin(2\pi F_g t)$

$$\boxed{F_D \text{ or } \Delta f = k_f \cdot A_g}$$

remember:

$k_f \rightarrow \frac{Hz}{Volt}$ (For FM)

$k_p \rightarrow \frac{rads}{Volt}$ (for PM)

FM Bandwidth : B_{FM} $(= \infty)$

CARLSON'S RULE (98%) : $B_{FM} = 2(\beta + 1) F_g$

FM Bandwidth — modulation index — max frequ. of the message

$$= 2(\underbrace{\beta F_g}_{\triangleq F_D} + F_g)$$

max $(g(t))$

Remember: $\boxed{F_D = k_f \cdot A_g = \beta \cdot F_g}$

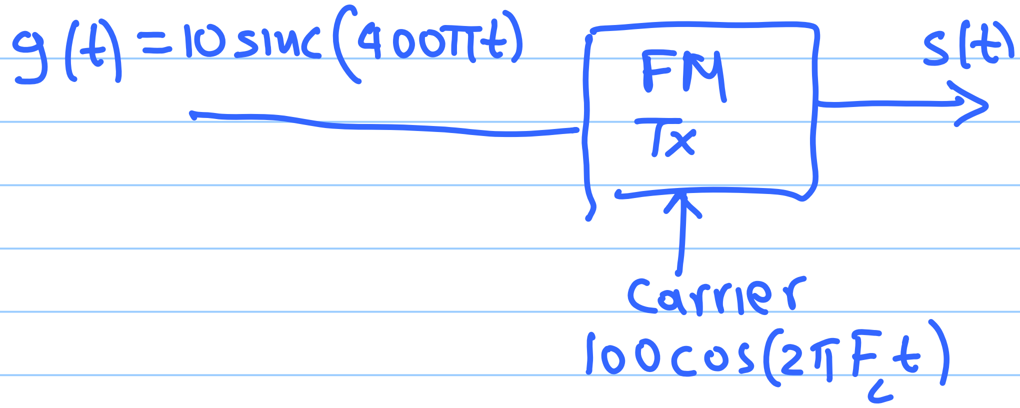
3. The message signal $m(t) = 10sinc(400\pi t)$ frequency modulates the carrier $c(t) = 100\cos(2\pi f_c t)$. The modulation index is $\beta = 6$.

(a) Write an expression for the modulated signal $s(t)$. [Hint: you need to find the value of k_f]

(b) What is the maximum frequency deviation of the modulation signal? [1200Hz]

(c) Using Carson's rule, find the bandwidth of the modulated signal. [2800Hz]

$\beta = 6$



a) $s(t) = ?$

$$= \underbrace{A_c}_{100} \cos\left(2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} \underbrace{g(u)}_{10sinc(400\pi t)} du\right)$$

$$\underbrace{\beta}_{6} = \frac{k_f A_g}{F_g}$$

$A_g = max(g(t))$

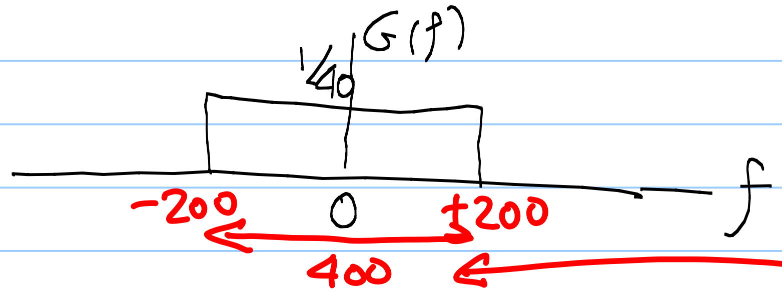
F_g → Bandwidth of message

Bandwidth of message = max frequ of message (F_g)

$g(t) \xrightarrow{FT} G(f)$

$$10\,\mathrm{sinc}(400\pi t) = 10\,\mathrm{sinc}\,\frac{\pi t}{1/400} \rightarrow 10\,\frac{1}{400}\,\mathrm{rect}\left(f\,\frac{1}{400}\right)$$

$$G(f) = \frac{10}{400}\,\mathrm{rect}\left(\frac{f}{400}\right) = \frac{1}{40}\,\mathrm{rect}\left(\frac{f}{400}\right)$$



a) Bandwidth = 200 Hz

$$K_f = \frac{6\times 200}{10} = 120\ \frac{Hz}{Volts}$$

(240π)

b) $F_D = K_f \underbrace{A_g}_{\max(g(t))} = 120\times 10 = 1200$

$$= \beta \cdot F_g = 6\times 200 = 1200$$

c) $B_{FM} = 2(\overset{6}{\beta} + \overset{1}{1})\overset{200}{F_g} = 2800$ Hz.