EE401: Advanced Communication Theory

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Principles of Diversity Theory

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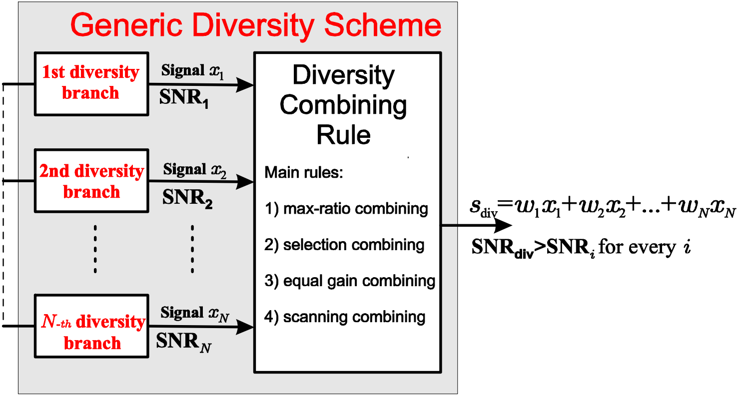
Introduction

Diversity : ≥ 2 closely similar copies (noise/interference vary)
⇓ selection/combination
better recovered signal.

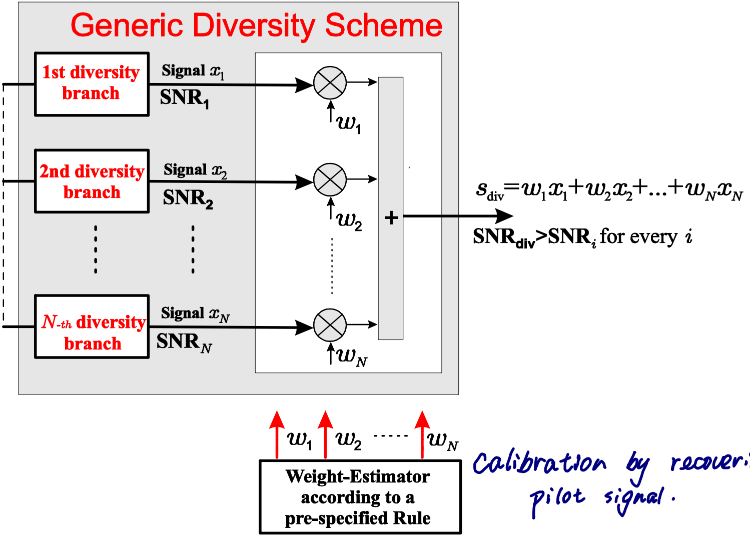
- In general, the term "diversity system" refers to a system in which there are available two or more closely similar copies of some desired signal.
- The 'Diversity Combining' (or briefly "diversity") Concept: **Diversity** is defined as a general technique that utilizes **two or more copies** of a signal with varying degrees of noise/interference effects to achieve, by selection or a combination scheme, higher degree of message-recovery performance than that achievable by any one of the individual copies separately.
- The concept of diversity was introduced in 1959 by Brennan [D. G. Brennan, "LinearDiversityCombiningTechniques", *Proceedings of the IRE*, vol.47, no.6, pp.1075-1102, 1959]

Generic Diversity Diagram



- N.B.: The terms "**diversity branch** " and "diversity channel" are equivalent

Equivalent Generic Diversity Diagram



Calibration by recovering pilot signal.

- If s represents the desired signal and x_i denotes its i^{th}distorted copy, then the input (i/p) and output (o/p) may be represented as follows:
  - i/p

$$\left.\begin{array}{l} x_1 = \beta_1 s_d + n_1 \\ x_2 = \beta_2 s_d + n_2 \\ \cdots\cdot \\ x_N = \beta_N s_d + n_N \end{array}\right\} \Rightarrow \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_N \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \ldots \\ \beta_N \end{bmatrix} s_d + \begin{bmatrix} n_1 \\ n_2 \\ \ldots \\ n_N \end{bmatrix}}_{\underline{x}=\underline{\beta} s_d+\underline{n}} \qquad (1)$$

  - o/p

$$\begin{aligned} s_{\text{div}} &= w_1 x_1 + w_2 x_2 + \ldots + w_N x_N \\ &= \underline{w}^T \underline{x} \\ &= \underbrace{\underline{w}^T \underline{\beta} s_d}_{desired} + \underbrace{\underline{w}^T \underline{n}}_{noise} \end{aligned} \qquad (2)$$

where

$$\underline{w} = \begin{bmatrix} w_1, & w_2, & \ldots, & w_N \end{bmatrix}^T$$

- output SNR: The first term of Equ 2 is the desired term and the second term is due to the noise. The powers of these two terms are:

$$P_{out,desired} = \mathcal{E}\left\{\left(\underline{w}^T \underline{\beta} s_d\right)^2\right\} = P_d \underline{w}^T \mathbb{R}_{\beta\beta} \underline{w} \tag{3}$$

$$P_{out,noise} = \mathcal{E}\left\{\left(\underline{w}^T \underline{n}\right)^2\right\} = \underline{w}^T \mathbb{R}_{nn} \underline{w} \tag{4}$$

$$\text{SNR}_{out,\text{div}} = \frac{P_{out,desired}}{P_{out,noise}} = \frac{P_d \underline{w}^T \mathbb{R}_{\beta\beta} \underline{w}}{\underline{w}^T \mathbb{R}_{nn} \underline{w}} = \frac{P_d}{\sigma_n^2} \frac{\underline{w}^T \mathbb{R}_{\beta\beta} \underline{w}}{\underline{w}^T \underline{w}} \tag{5}$$

where

$$\text{cov}(\underline{\beta}) = \mathbb{R}_{\beta\beta} = \mathcal{E}\left\{\underline{\beta}\, \underline{\beta}^T\right\} \tag{6}$$

$$\text{cov}(\underline{n}) = \mathbb{R}_{nn} = \mathcal{E}\{\underline{n}\, \underline{n}^T\} \overset{(a)}{=} \sigma_n^2 \mathbb{I}_N \tag{7}$$

Note: the equality (a) is valid if the noise is AWGN

Diversity Combining Rules

- max ratio combining (MRC) or, equivalently, maximum signal-to-noise-power ratio combining:

$$\underline{w}_{MRC} = \arg\left\{\max_{\underline{w}}(\mathrm{SNR}_{out,\mathrm{div}})\right\} \tag{8}$$

- selection combining (SC) diversity rule:

$$w_k = \begin{cases} 1 & \text{if } \mathrm{SNR}_k > \mathrm{SNR}_i \forall i \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

- equal gain combining (EGC):

$$w_1 = w_2 = \ldots = w_N \tag{10}$$

- scanning combining (SCC):

$$\left.\begin{array}{l} -\text{if } \mathrm{SNR}_k > \text{threshold then } \left\{\begin{array}{l} w_k = 1 \\ w_j = 0; \forall j \neq k \end{array}\right. \\ -\text{if } \mathrm{SNR}_k \text{ fall below threshold then } k = k+1 \\ -\text{repeat} \end{array}\right\} \tag{11}$$

  - That is, a branch (say the k-th branch) is selected that has SNR above a predetermined threshold ($\mathrm{SNR}_k >$threshold) and is used until its SNR drops below that threshold.
  - Then the next branch is selected if its SNR is above the threshold, etc.

SNRout,div as a function of Diversity Channels/Branches

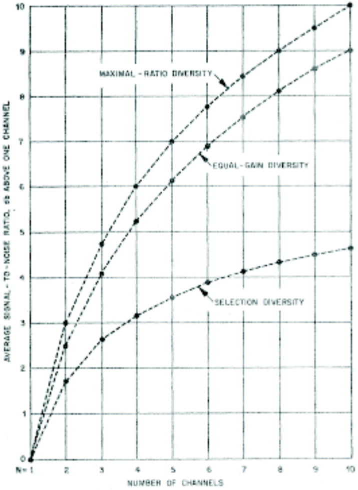


Fig. 8—Diversity improvement (in decibels) in average SNR, for independently fading Rayleigh-distributed locally coherent signals in locally incoherent noises with constant local rms values.

TABLE I
COMPARATIVE AVERAGE SNR (SAME CONDITIONS AS IN FIG. 8)

Number of Channels N	Number of DB by which Maximal-Ratio Exceeds		
	Equal-Gain	Selection	One Channel
2	0.49	1.25	3.01
3	0.67	2.14	4.77
4	0.76	2.83	6.02
6	0.85	3.89	7.78
8	0.90	4.69	9.03
.	.	.	.
.	.	.	.
.	.	.	.
∞	1.05	∞	∞

Classification of Diversity Techniques/Systems

- In diversity systems, there are several known methods of obtaining two or more signals s_i, in order to be combined using one of the diversity combining rules, to obtain an improved signal.

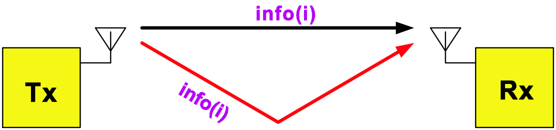
- According to the method of obtaining copies of the same signal the diversity systems and their associated techniques are classified as follows:

  - Multi-Path Diversity
  - Time Diversity
  - Frequency Diversity
  - Space Diversity
  - Polarization Diversity
  - Code Diversity

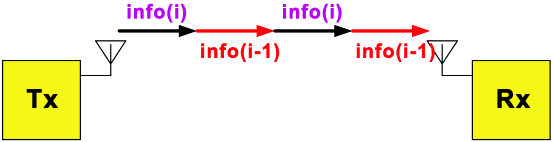
Examples

Multipath: remove interference? utilise diversity?

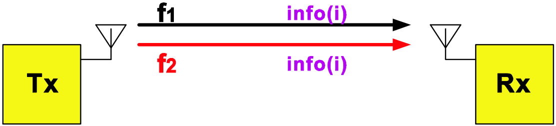
- Multi-Path Diversity:



- Time Diversity:



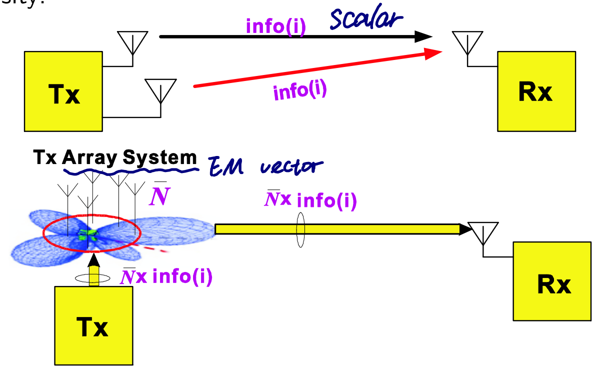
- Frequency Diversity:



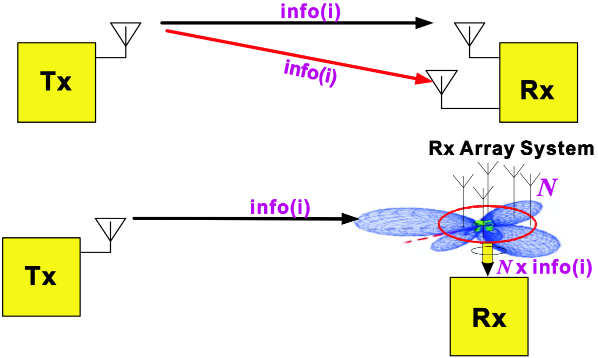
More Examples

- Why MIMO ?
- Create space diversity !

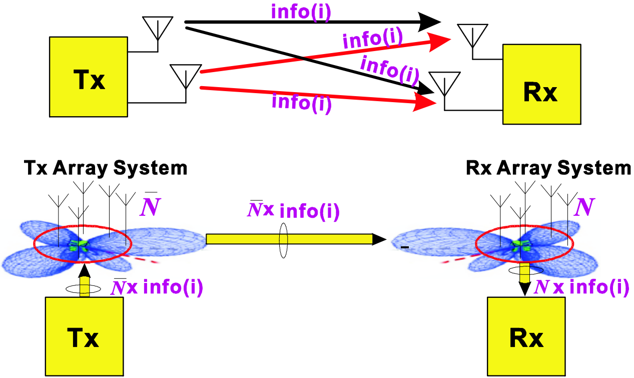
- Space Diversity (or Antenna Diversity):
  1) Tx Diversity:



2) Rx Diversity:



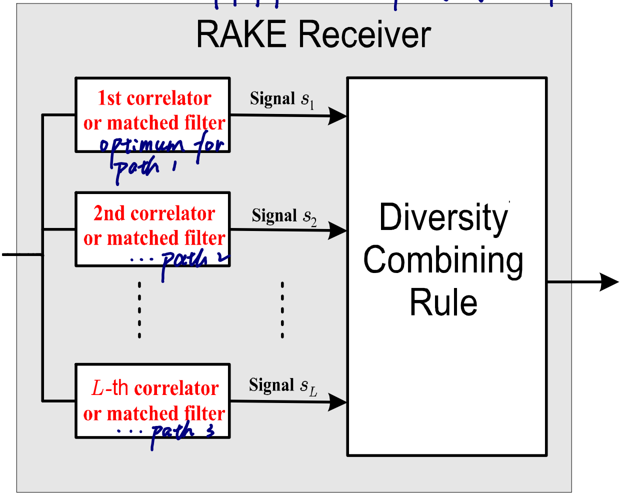
3) Tx-Rx Diversity:



- Array systems/techniques can be seen as the most sophisticated and advanced *space diversity* systems/techniques.

Rake Rx - Price & Green 1958

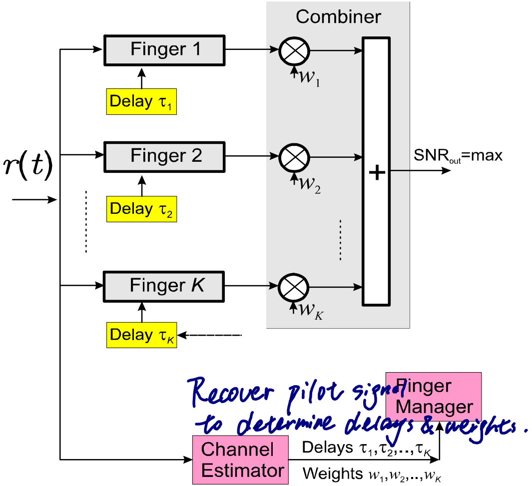
- Employs the "diversity" concept which is shown in generic terms in the figure below. More capacity by multipath.



Generate diversity branches by transmitting a single signal once (over diverse paths and match), rather than transmit repeatly.

- In this case the diversity branches are replaced by correlation receivers (or matched filters). This is known as multipath diversity, the overall structure as RAKE receiver and the associated diversity branches (i.e. correlators) as RAKE **fingers** .
- Thus, in this way, a plurality of parallel correlation receivers (fingers) is used with each of them synchronised to a different path. Then the outputs of these fingers are combined according to a diversity combining criterion/rule (followed by a decision device).
- RAKE receiver is an optimum coherent receiver for multipath channels and it was introduced in 1958 by Price and Green [R. Price and P. E. Green, "A Communication Technique for Multipath Channels", Proceedings of IRE, pp.555-570, March 1958]

- The generic structure of a RAKE receiver is below.



Hence, if one path (or more) is “lost” (because its associated path gain coefficient becomes very small) the output SNR remains in an optimum level.

Types of Rake Rx

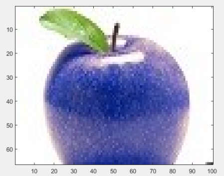
- A finger is, by itself, an optimum coherent receiver (correlator or matched filter receiver) while, according to Brennan's original paper [Brennan 1959] there are four main diversity combining rules to combine the outputs of the fingers.
- Thus, there are four classes of RAKE receivers - based on "diversity combining" rules:

1. RAKE with maximum-ratio combining (this is the best of these four types and provides the highest output SNR - but is more complex);
2. RAKE with selection combining (RAKE-SC);
3. RAKE with equal gain combining (RAKE-EGC);
4. RAKE with scanning combining (RAKE SCC)

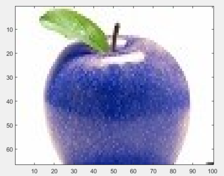
Summary of a Rake Receiver

- First implementation: 1950s.
- RAKE receiver is a matched filter receiver for wide-band signals.
- It is used to resolve paths delayed by more than T_c.
- Main assumption: the channel is time invariant over the decision interval $[0, T_{cs}]$.
- Rake-Receiver = optimum Single-user Receiver
- it has a number of ‘fingers’ (One Rake-‘finger’ per path)

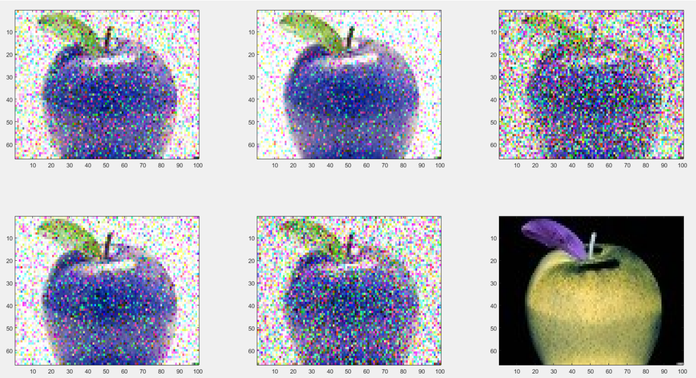
Diversity Example: Apple image



< TX Image | Rake's o/p >



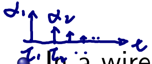
Images at each branch of a Rake Rx of 6 fingers



Multipath Diversity in CDMA

(not absolute!)

delay spread: the time difference between the arrival of earliest and latest multipath components, i.e. 'echo' duration.



$\bar{\tau} = E(\tau) = \sum_i \frac{|\alpha_i|^2}{\sum_k |\alpha_k|^2} \tau_i$

$\sigma_\tau = \sqrt{E[(\tau - \bar{\tau})^2]} = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$

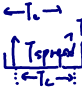
RMS delay spread (std. deviation) ~ cell size, path difference

⇕

Coherence bandwidth B_{coh}

- In a wireless channel, in addition to the noise, the transmitted signal is also affected by fading effects (due to multipath propagation and Doppler effects).
- If the paths in a multipath channel can be separated (i.e. $T_{spread} > T_c$ (symbol period)) **then** there are two main ways to exploit the multipath propagation channel for enhancing the performance of the receiver, using the auto-correlation properties of the PN-codes:

  $B_{coh} < B_m \Rightarrow$ frequency-selective fading.

  T_c : affect later.

  1. to use a correlation receiver (CORRELATION Rx) that is perfectly synchronised to one of the paths (i.e. correct delay).
  2. to employ the "diversity" concept (RAKE Rx).

1. perfect correlator
2. RAKE Rx

$T_{spread} > T_c \Rightarrow$ f-selective fading $\Rightarrow$ multipath $\Rightarrow$

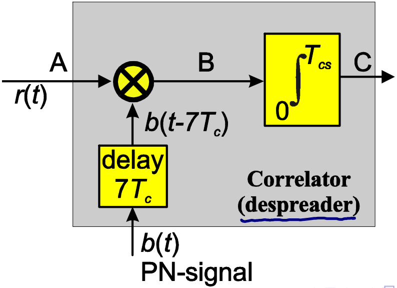
perfect correlator: too ideal to implement. low SNR if β small.

RAKE Rx: require suitable algorithm. complex but always better!

Correlation Rx

- A correlation receiver is **perfectly synchronised to one** of the paths (i.e. correct delay)
  - This will **suppress all the other received paths** based on the correlation properties of the PN-code (with m-sequences having the best auto-correlation properties).
- The figure below shows **a representative example** of the correlation function at the input and at the output of a correlator operating in the presence of 3 paths.

Despreador:
- delayer
- convolator
- integrator.



- The PN-signal $b(t)$ is an m-sequence of length 31 and is synchronised to the 3rd path which has a delay equal to $7T_c$ with T_c denoting the chip period.
- Thus, at the o/p of the correlator only the contribution of the 3rd path will appear (i.e. the evaluation of the correlation function at $7T_c$) while the 1st and 2nd paths will be rejected as "unwanted" self-interference. Indeed:
  - at point-A:

$$r(t) = \beta_1 b(t-2T_c)\mathrm{a}[n] + \beta_2 b(t-4T_c)\mathrm{a}[n] + \beta_3 b(t-7T_c)\mathrm{a}[n]$$

  - at point-B:

$$r(t)b(t-7T_c)$$

  - at point-C:

$$\int_{nT_{cs}}^{(n+1)T_{cs}} r(t)b(t-7T_c)dt =$$

$$
\begin{aligned}
\int_{nT_{cs}}^{(n+1)T_{cs}} r(t)b(t-7T_c)dt &= \overbrace{\int_{nT_{cs}}^{(n+1)T_{cs}} \beta_1 b(t-2T_c)b(t-7T_c)\mathrm{a}[n]dt}^{\simeq 0} \\
&+ \overbrace{\int_{nT_{cs}}^{(n+1)T_{cs}} \beta_2 b(t-4T_c)b(t-7T_c)\mathrm{a}[n]dt}^{\simeq 0} \\
&+ \overbrace{\int_{nT_{cs}}^{(n+1)T_{cs}} \beta_3 b(t-7T_c)b(t-7T_c)\mathrm{a}[n]dt}^{=\beta_3 N_c T_c \mathrm{a}[n]} \\
&= 0 + 0 + \beta_3 N_c T_c \mathrm{a}[n]
\end{aligned}
$$

- However, although the correlation receiver handles the multipath problem, if the associated path gain (i.e. β_3, which changes with time) is reduced significantly (e.g. shadowing effects) this may result in the loss of the signal at the output of the correlator (i.e. output SNR=0 or very small).

CDMA Rake Receivers

RAKE single-user Rx: PN sequence, single user timing

RAKE multi-user Rx: PN sequence, timing of all users, amplitude of all users. noise level.

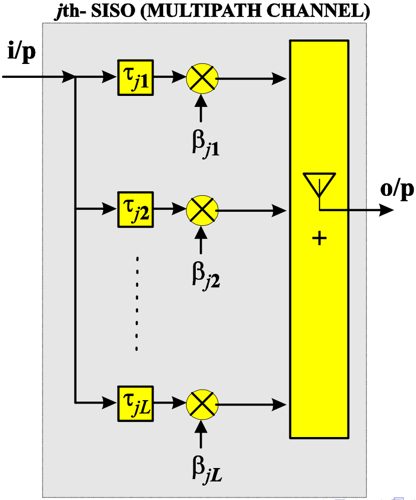
- Two classes:
  1. Single-user Receivers
  2. Multi-user Receivers

- Single user Receiver: requires no knowledge beyond the PN-sequence and the timing of the user it wants to demodulate/receive ("desired" user)

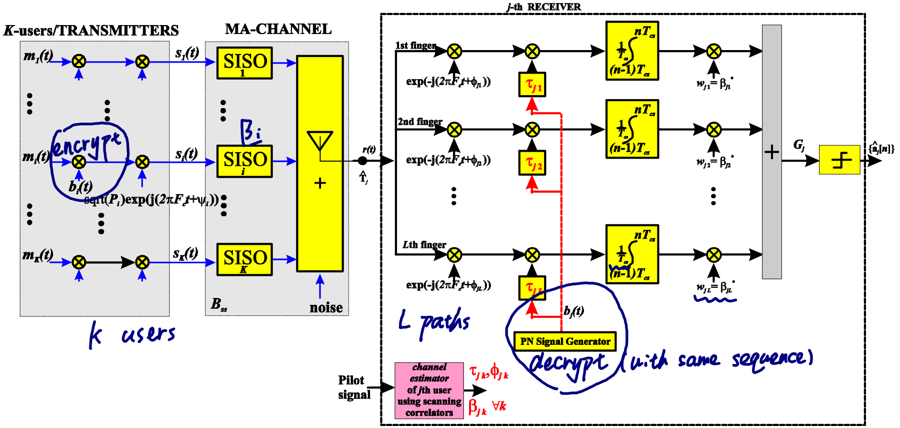
- Multi-user Receiver: not only knows the PN-sequence and the timing of every active user but also knows (or can estimate) the received amplitudes of all users and the noise level

Optimum Single-user Rx (employs Multipath Diversity)

- Optimum Single-user (SU)Receiver = RAKE Rx
- Multipath Channel (j-th user):



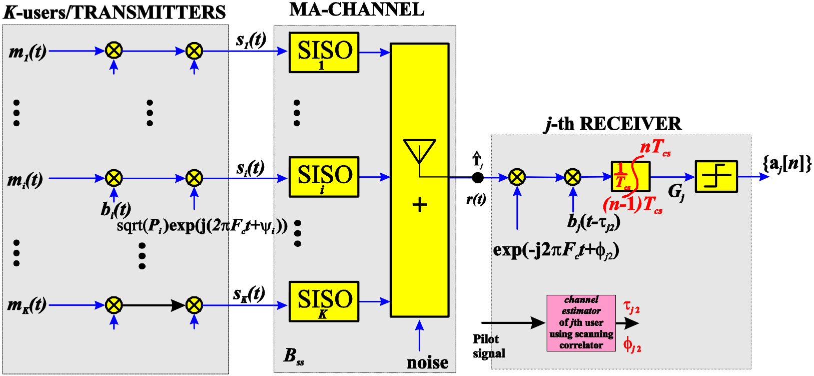
- BPSK-CDMA RAKE Rx (multipath-diversity)



★ The delay τ_{jk}, fading coeffiient β_{jk},and ϕ_{jk} of the k-th path of the j-th user is estimated $\forall k$ using 'scanning' correlators monitoring the pilot channel.

L only 1 activated correlator index change over time

- BPSK-CDMA Correlation Rx (without diversity): if there is a single path i.e. no multipath then optimum



The delay τ_{j2} of the 2nd path of the j-th user is estimated using a 'scanning' correlator monitoring the pilot channel.

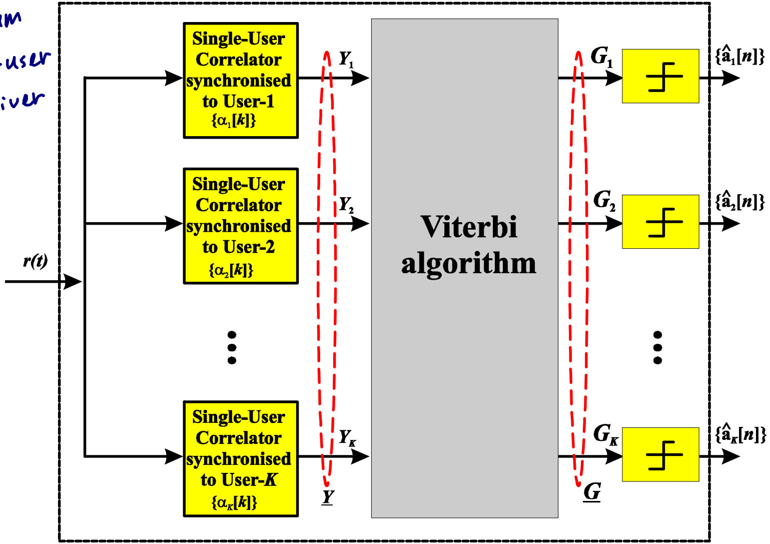
Optimum Multi-User Receiver

- Optimum Detection:
  A detector which employs correlators has to take into account the PN-codes (or equivalent the PN signals) of all other CDMA users in the system in order to be optimum in the Maximum Likelihood sense, unless the PN-codes are perfectly orthogonal.

- The optimum detector, however, is non-linear and computationally far too complex.

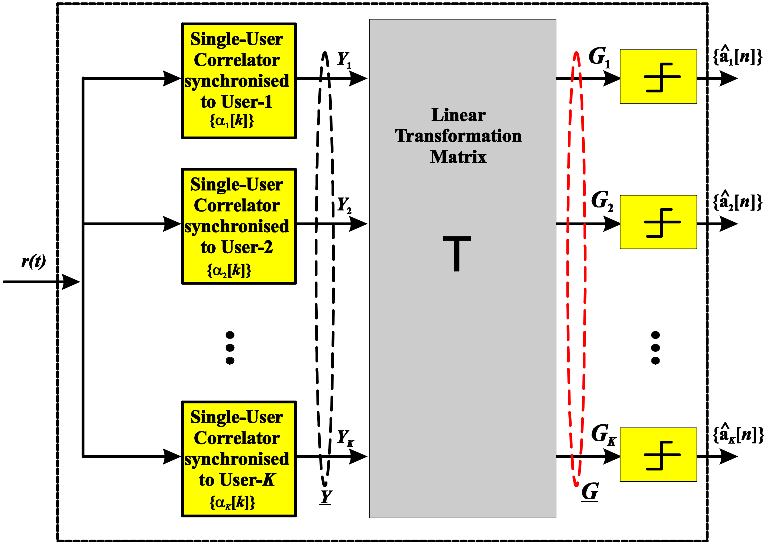
- Optimum MU Receiver Architecture

Optimum multi-user receiver



- Note that there is a huge gap in performance and complexity between the optimum single-user and optimum multi-user receivers

Suboptimal Linear Multi-user Receiver Architecture



- If $\mathbb{T} = \mathbb{R}^{-1}$
  then the Rx is known as decorrelating MU receiver
- If $\mathbb{T} = \left(\mathbb{R} + \sigma_n^2 \mathbb{A}^{-2}\right)^{-1}$
  mean square error
  then the Rx is known as minimum mse (mmse) MU receiver
- where

$$\begin{cases} \mathbb{R} \text{ denoting the } (K \times K) \text{ cross-correlation matrix of PN-signals} \\ \mathbb{A} = \begin{bmatrix} \sqrt{P_1}, & 0, & ..., & 0 \\ 0, & \sqrt{P_2}, & ..., & 0 \\ ..., & ..., & ..., & ... \\ 0, & 0, & ..., & \sqrt{P_K} \end{bmatrix}, \ (K \times K) \text{ matrix} \\ \sigma_n^2 = \text{noise power} \end{cases}$$

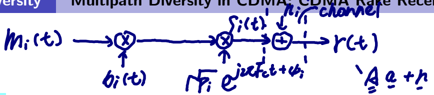
Suboptimal Multi-User (MU) Receiver: Decorrelating Rx

- $\mathbb{T} = \mathbb{R}^{-1}$: decorrelating MU receiver

- This is a typical MU receiver which is suboptimal but much simpler than the optimum MU receiver. Also it is optimum according to the following three different criteria when the received amplitudes are unknown
  - least squares
  - near-far resistance
  - maximum-likelihood

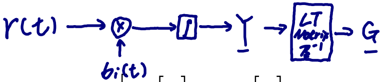
- The performance of this MU receiver is used as a comparative upper bound for the performace of different single-user receivers.

- Note that:



$$\underline{Y} = \mathbb{R}\,\mathbb{A}\,\underline{a} + \underline{n} \quad (12)$$

$$\Rightarrow \underline{G} = \mathbb{R}^{-1}\underline{Y} = \mathbb{R}^{-1}\mathbb{R}\,\mathbb{A}\underline{a} + \mathbb{R}^{-1}\underline{n} = \mathbb{A}\underline{a} + \mathbb{R}^{-1}\underline{n} \quad (13)$$



where

$$\underline{a} = \left[a_1[n],\quad a_2[n],\quad \ldots,\quad a_K[n]\right]^{\mathbf{T}} \quad (14)$$

$\underline{\mathbb{R}}$ (PN signals cross-correlation)

- e.g. Two users ($K = 2$) with cross-correlation $\rho_{12} = \rho_{21} = \rho$

$$\Rightarrow \mathbb{R}^{-1} = \begin{bmatrix} 1, & \rho \\ \rho, & 1 \end{bmatrix}^{-1} = \frac{1}{1-\rho^2}.\begin{bmatrix} 1, & -\rho \\ -\rho, & 1 \end{bmatrix} \quad (15)$$

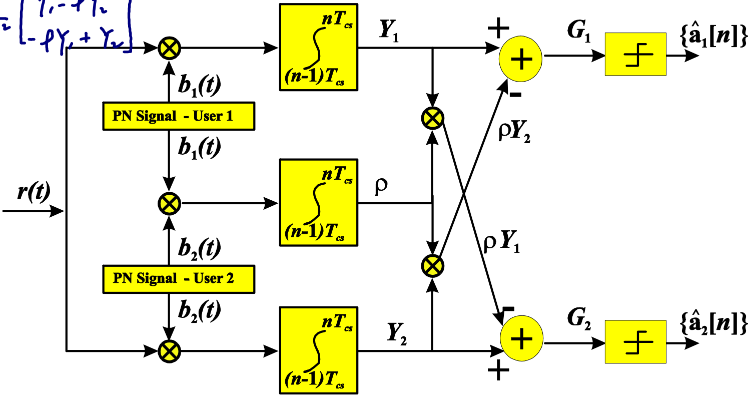
- This implies that:

$$\underline{G} = \mathbb{R}^{-1}\underline{Y} \Rightarrow \quad \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \frac{1}{1-\rho^2}.\begin{bmatrix} 1, & -\rho \\ -\rho, & 1 \end{bmatrix}.\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad (16)$$

$$\Rightarrow \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho^2}.(Y_1 - \rho.Y_2) \\ \frac{1}{1-\rho^2}.(-\rho Y_1 + Y_2) \end{bmatrix} \quad (17)$$

- Therefore, in this case, the decorrelating receiver, for $K = 2$, can be implemented as follows:

$$\begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \frac{1}{1-\rho^2} \begin{bmatrix} Y_1 - \rho Y_2 \\ -\rho Y_1 + Y_2 \end{bmatrix}$$



- Note: The normalisation with $\frac{1}{1-\rho^2}$ has been ignored (it is not necessary since both 'desired' and 'noise' terms are weighted)

Some Comments on 3G: Main Specs

	IS-95	UMTS
Generation	2G (USA)	3G (EU)
Type	CDMA	W-CDMA
Frame duration	20msec	10msec
Uplink	$\overbrace{\underbrace{824 - 849\text{MHz}}_{\text{mod:QPSK}}}^{25\text{MHz}}$	$\overbrace{1920 - 1980\text{MHz}}^{60\text{MHz}}$ mod:balanced QPSK, i.e. **QPSK2**
Downlink	$\overbrace{\underbrace{869 - 894\text{MHz}}_{\text{mod:oQPSK}}}^{25\text{MHz}}$	$\overbrace{2110 - 2170\text{MHz}}^{60\text{MHz}}$ mod:dual QPSK, i.e. **QPSK1**
B_{ss}	1.23MHz	5MHz,10MHz,20MHz
T_c symbol duration	813.8ns	244ns,122ns, 61ns
$r_c = \frac{1}{T_c}$ symbol rate	$1.2288 \frac{\text{Mchips}}{\text{sec}}$	$4.096 \frac{\text{Mchips}}{\text{sec}}$ (reduced to $3.084 \frac{\text{Mchips}}{\text{sec}}$)
roll-off-factor		0.22;Note: $B_{ss} = \frac{1}{T_c}$(1+roll-off-factor)=4.99712MHz$\simeq$5MHz
Receiver's type	RAKE	RAKE