Problem Sheet: Introductory Concepts
Communication Systems

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1. Sketch and mathematicaly represent the pdfs of the following signals:

   (a) $4\ \text{rep}_{3T}\left\{\Lambda\left(\frac{t}{T}\right)\right\} -1$ (10%)

   (b) $\text{rep}_{2T}\left\{2\text{rect}\left(\frac{t}{T}\right)+4\Lambda\left(\frac{2t}{T}\right)\right\}$ (10%)

   (c) $\text{rep}_{2T}\left\{5\ \text{rect}\left(\frac{t}{T}\right)-\ \text{rect}\left(\frac{t-T}{T}\right)\right\}$ (10%)

   (d) $\text{rep}_{6T}\left\{4\ \text{rect}\left(\frac{t}{5T}\right)-\ \text{rect}\left(\frac{t-3T}{T}\right)\right\}$ (5%)

   (e) $\frac{N+1}{N}\ \text{rep}_{NT}\left\{\Lambda\left(\frac{t}{T}\right)\right\}-\frac{1}{N}\quad \text{with}\quad N\in Z^{+}>2$ (15%)

   (f) $3\text{rep}_{2}\left\{\Lambda\left(t\right)\right\}-2$ (5%)

2. Evaluate:

   (a) ${}_{\text{-}\infty}\int^{\infty}(t^4-3t+1).\delta(t-2)\ .dt$ (10%)

   (b) ${}_{\text{-}\infty}\int^{\infty}\left\{(\cos(4\pi t)*\delta(t+\frac{1}{4})\right\}.\delta(t-\frac{1}{8}).dt$ (10%)

   (c) $\int_{-\infty}^{\infty}(t^3-3t^2-11).\delta(t-1)\ .dt$ (5%)

   (d) $\int_{-\infty}^{\infty}\left\{(\sin(4\pi t)*\delta(t+\frac{1}{4})\right\}.\delta(t-\frac{1}{4}).dt$ (5%)

   (e) $\int_{-\infty}^{\infty}(t^3-2t^2+1).\delta(t-2)\ .dt$ (5%)

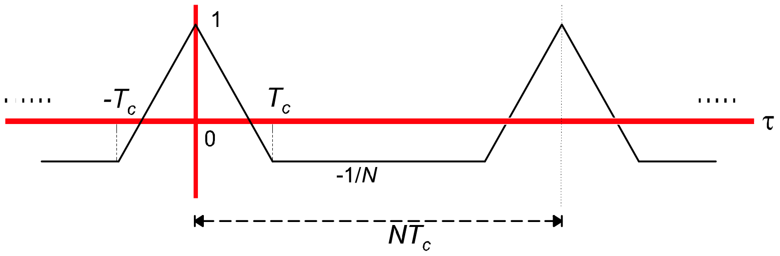
   (f) $\int_{-\infty}^{\infty}\left\{(\cos(2\pi t)*\delta(t-\frac{1}{4})\right\}.\delta(t-\frac{1}{12}).dt$ (5%)

   (g) $h(3)$ where $h(t)=\left(t.\text{rect}\left\{\frac{t}{8}\right\}\right)*\delta(t+3)$ (5%)

   (h) $h(3)$ where $h(t)=\left(t.\text{rect}\left\{\frac{1}{8T}\right\}\right)*\delta(t-2)$ (10%)

   (i) $h(3.5)$ where $h(t)=\left(t.\text{rect}\left\{\frac{1}{8T}\right\}\right)*\delta(t-3)$ (10%)

3. The waveform below shows the autocorrelation function $R_{bb}(\tau)$ of what is called in communications a pseudo-random (PN) signal $b(t)$.



(a) Write a mathematical expression, using Woodward's notation, to describe the above autocorrelation function. (15%)

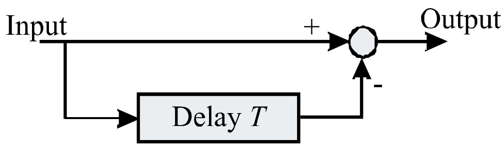
(b) Find the power spectral density $\text{PSD}_b(f)$ of $b(t)$. (20%)

4. At the input of a filter there is white Gaussian noise of power spectral density $\text{PSD}_{n_i}(f) = \frac{3}{2}10^{-6}$. If the transfer function of the filter is

$$H(f) = \Lambda\left\{\frac{f}{10^6}\right\}\exp(-j\phi(f))$$

calculate the power of the signal at the output of the filter. (10%)

5. For the following differential circuitfind:



(a) the impulse response and (5%)

(b) frequency response (5%)

6. Consider the filter with impulse response

$$h(t) = \text{sinc}^2\left\{10^6(t-3)\right\}$$

and assume that the input signal $n_i(t)$ is white Gaussian noise with double-sided power spectral density $\text{PSD}_{n_i}(f) = 1.5 \times 10^{-6}$ W/Hz.

For the signal $n(t)$ at the output of the filter

(a) find and plot its power spectral density $\text{PSD}_n(f)$; (10%)

(b) calculate its power P_n (5%)

7. Consider a bandpass filter with impulse response

$$h(t) = 8 \times 10^3 \text{sinc}\left\{4 \times 10^3 t\right\} . \cos(2\pi 10^4 t)$$

and assume that at the input of this filter there is white Gaussian noise $n_i(t)$ of power spectral density $\text{PSD}_{n_i}(f) = 10^{-6}$.

For the signal $n(t)$ at the output of the filter

(a) find and plot its power spectral density $\text{PSD}_n(f)$; (10%)

(b) calculate its power P_n (5%)

END