Unsupervised learning ⟷ Supervised learning

Generative model ⟷ Discriminative model

PCA

FLD/LDA

Maximum margin classifier

Algorithm-independent learning

Ensemble learning

Randomisation

Committee machine

Online learning

Probabilistic model

Regression

→ Feature extraction → Classification →

PCA

NN classification

FLD/LDA

Discriminant functions

Maximum margin classifier

Subspace learning

PCA

FLD/LDA

Projection/pooling

Orthonormal basis vectors

Dimension reduction

Data visualisation

Reconstruction

Optimisation ↔ Randomisation

Gradient-based

Lagrange multipliers

Eigenvalue problem

Generalized eigenvalue problem

Gaussian distributions

Underfitting → Generalization → Overfitting

Training loss

Testing (real) loss

Linear model ↔ Nonlinear model

Eigenface

Fisherface

Kernel trick

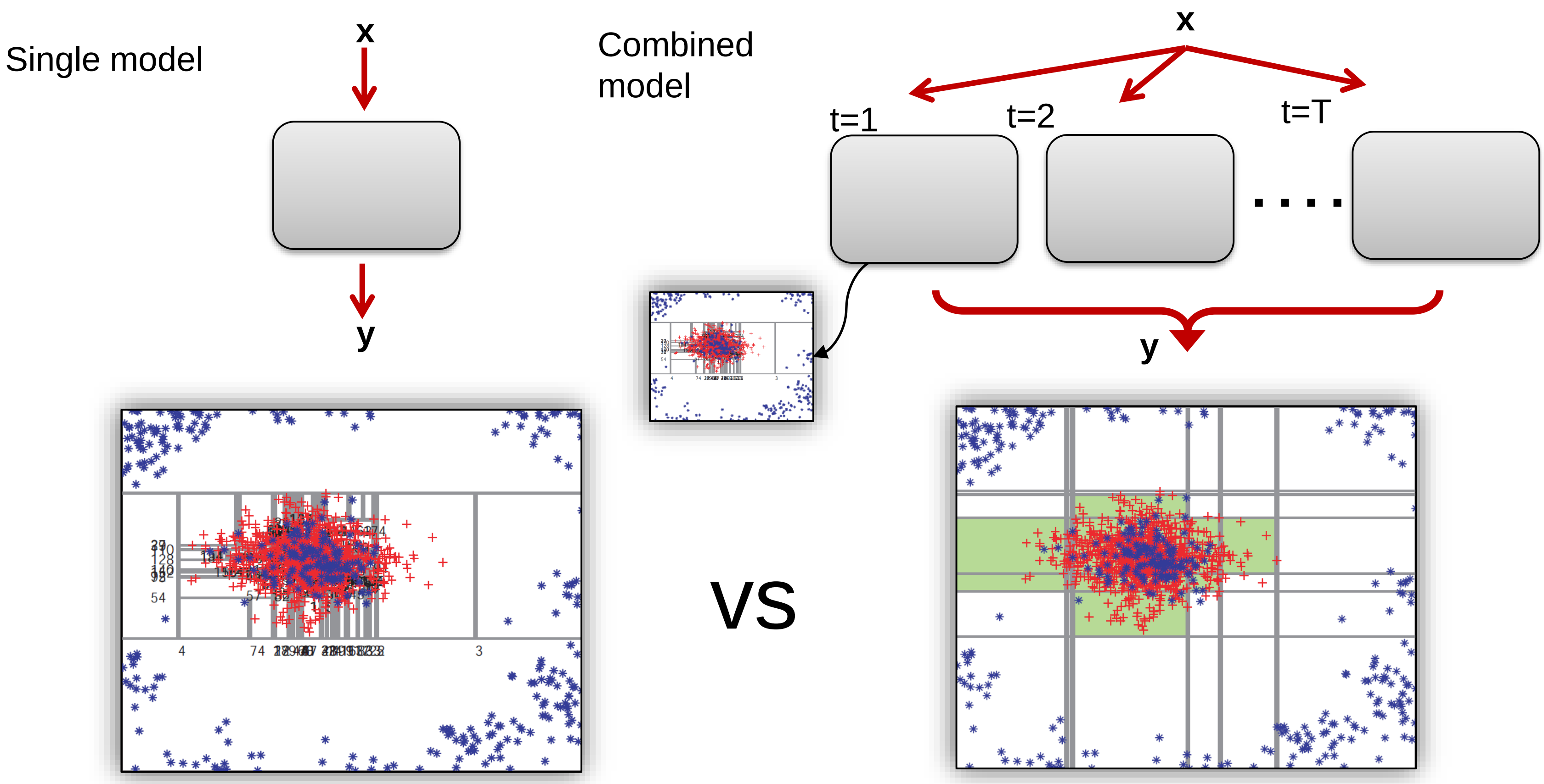
Feature space

Committee Machine, Ensemble Learning

Random Sampling LDA for Face Recognition

Tae-Kyun Kim
Senior Lecturer
https://labicvl.github.io/

Overfitting



Overfit (axis-aligned weaklearners, 2 class problem)

Generalised, smooth decision regions (axis-aligned weaklearners, 2 class problem)

Ensemble of models

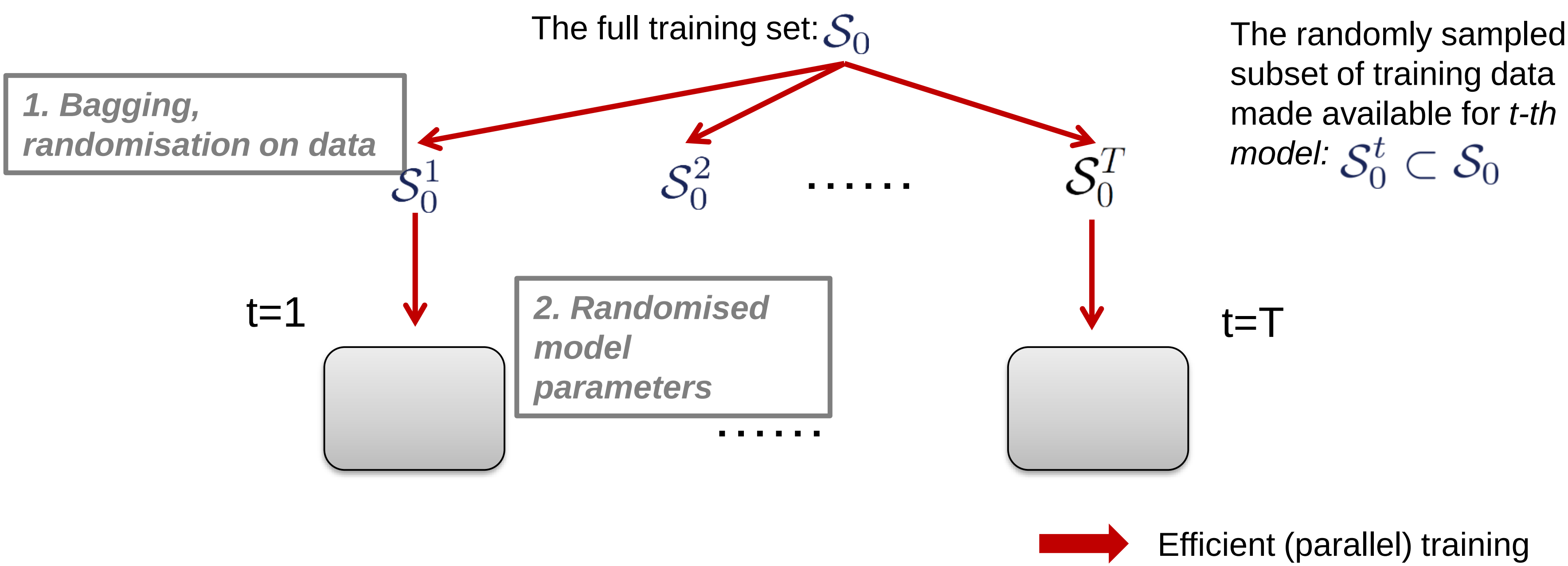
- The key aspect of the ensemble model is the fact that its component models are all randomly different from one another.
- This leads to decorrelation between the individual model predictions and, in turn, results in improved generalization and robustness.
- The combined model is characterized by the same components as the individual models.
- The amount of randomness influence the prediction/estimation properties of the models.

* Dropout in deep neural networks ≈ randomisation

Randomness model

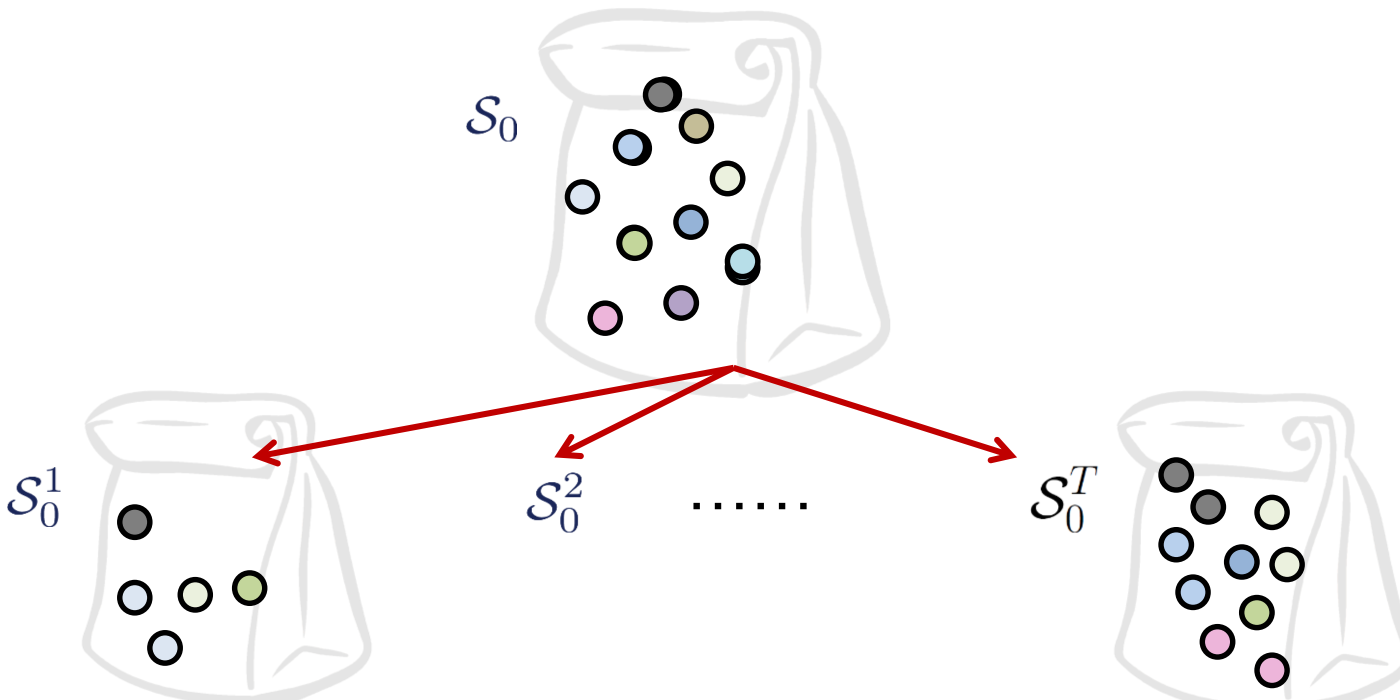
Randomness is injected into the models during the two phases. Two techniques used together are:

- random training set sampling (i.e. bagging), and
- randomized model parameters.



Bagging (Bootstrap AGGregatING) - randomizing the training set

- Given a data set $\mathcal{S}_0$ of size n, it generates T data subsets $\mathcal{S}_0^t$, t=1,…,T.
- Each subset has e.g. n_t=n, by sampling data from $\mathcal{S}_0$ uniformly and <u>with replacement.</u>
- Some data are repeated in $\mathcal{S}_0^t$. If n_t=n and n is large, $\mathcal{S}_0^t$ is likely to have 63.2% of unique data.

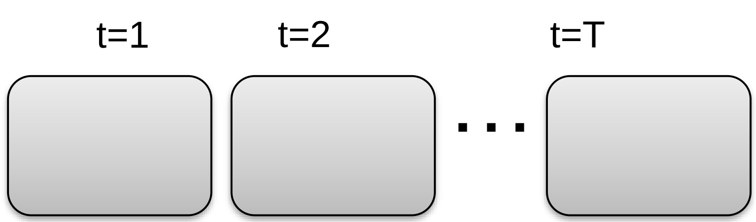


Randomizing model parameters

- The full set of all possible parameters (or their values) is denoted by $\mathcal{T}$
- A small random subset $\mathcal{T}_j \subset \mathcal{T}$ of parameters is considered.
- The randomness parameter $\rho = |\mathcal{T}_j|$ controls not only the amount of randomness within each model but also the amount of correlation between different models in the ensemble.
- As illustrated, when $\rho = |\mathcal{T}|$ all the models will be identical and as ρ decreases the models become more decorrelated (different from one another).

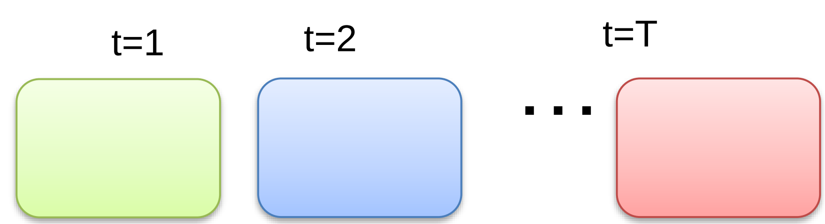
The effect of ρ

$\rho = |\mathcal{T}|$



Low randomness, high model correlation

$\rho = 1$



High randomness, low model correlation

https://goo.gl/m6Mnov for questions

Model correlation vs strength

- Randomisation on data and model parameters increases diversity among component models.
- For the fixed data, the randomised model parameters decreases strength of each model.
- This compromising issue is further explained in the perspective of a generic committee machine.

Committee machine

- We consider multiple models or experts, $y_t(x),\ t = 1, \ldots, T$.

- Output of each model is

$$y_t(x) = h(x) + \epsilon_t(x)$$

  where $h(x), \epsilon_t(x)$ are the true value and error of each model.

- The average sum-of-squares error is

$$\mathrm{E}[\{y_t(x) - h(x)\}^2] = \mathrm{E}[\epsilon_t(x)^2]$$

- The average error by acting individually is

$$\mathrm{E}_{av} = \frac{1}{T}\sum_{t=1}^{T} \mathrm{E}[\epsilon_t(x)^2]$$

Committee machine

– The committee machine is

$$y_{com}(x) = \frac{1}{T}\sum_{t=1}^{T} y_t(x)$$

– The expected error of the committee machine is

$$\mathrm{E}_{com} = E\left[\left\{\frac{1}{T}\sum_{t=1}^{T} y_t(x) - h(x)\right\}^2\right]$$

$$= E\left[\left\{\frac{1}{T}\sum_{t=1}^{T} \epsilon_t(x)\right\}^2\right] = E[\frac{1}{T^2}(\epsilon_1^2 + \epsilon_1\epsilon_2 + \epsilon_2^2 + \cdots)]$$

Committee machine

– If we assume

$$E\left[\epsilon_i(x)\epsilon_j(x)\right] = 0,$$

$$\text{for any } i, j \in \{1, \dots, T\}\ and\ i \neq j$$

then we obtain

$$E_{com} = \frac{1}{T} E_{av}$$

– In practice, the errors are typically highly correlated, but we can still expect that

$$E_{com} \leq E_{av}$$

Prediction models and testing

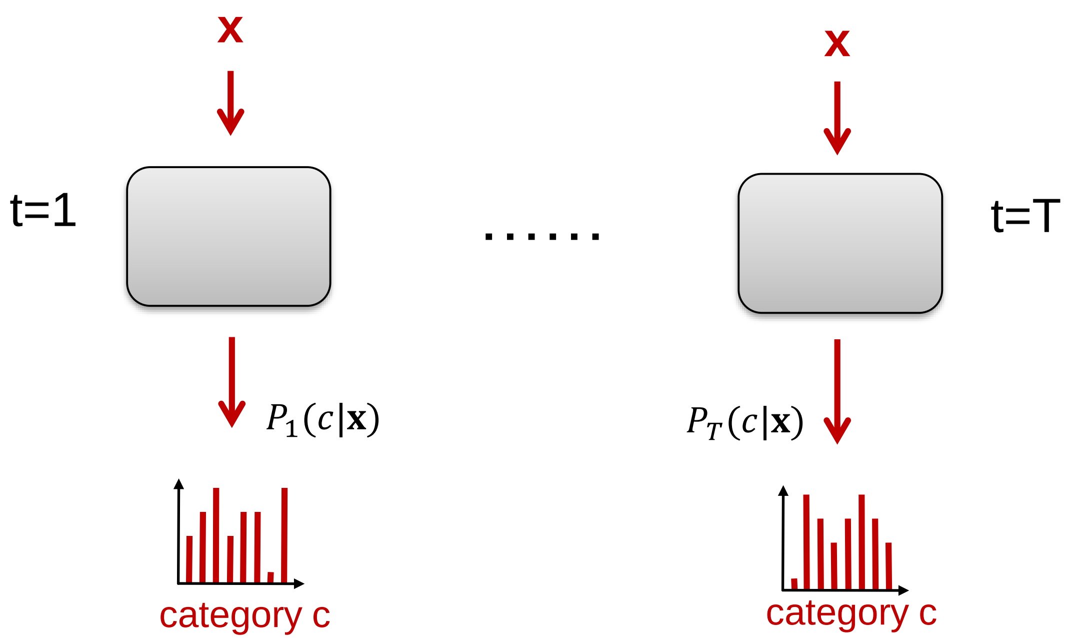
- In an ensemble with T models we use the variable $t \in \{1, . . . , T\}$ to index each component model.
- All models are trained independently (and possibly in parallel).
- During testing, each test point $\mathbf{x}$ is simultaneously pushed through all models.
- Testing can also often be done in parallel, thus achieving high computational efficiency on modern parallel CPU or GPU hardware.
- Combining all model predictions into a single prediction is done by a simple averaging operation. E.g. in classification

$$P(c|\mathbf{x}) = \frac{1}{T}\sum_{t=1}^{T} P_t(c|\mathbf{x})$$

where $P_t(c|\mathbf{x})$ denotes the class posterior distribution obtained by the t-th model.

Ensemble of models: evaluation

– A data point is passed down all models, and the respective posterior distributions are collected.



– Classification is done by $P(c|\mathbf{x}) = \frac{1}{T}\sum_{t=1}^{T} P_t(c|\mathbf{x})$

Prediction models and testing

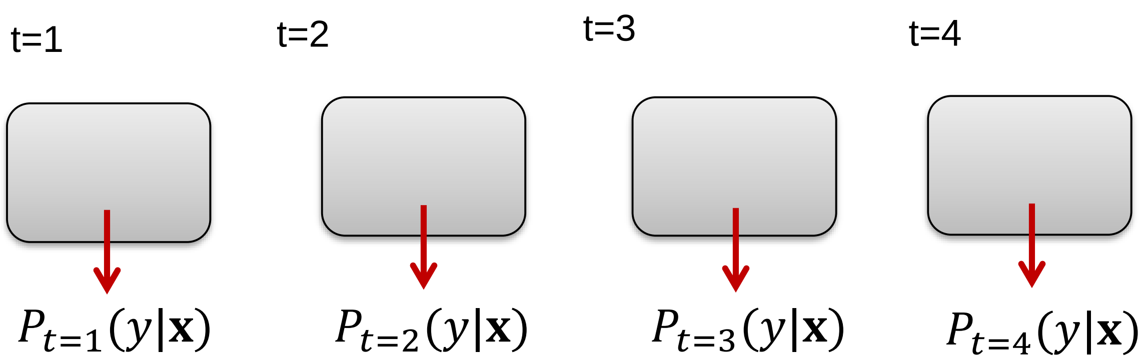
– Alternatively one could also multiply the model outputs together (though the models are not statistically independent)

$$P(c|\mathbf{x}) = \frac{1}{Z}\prod_{t=1}^{T} P_t(c|\mathbf{x})$$

with Z ensuring probabilistic normalization.

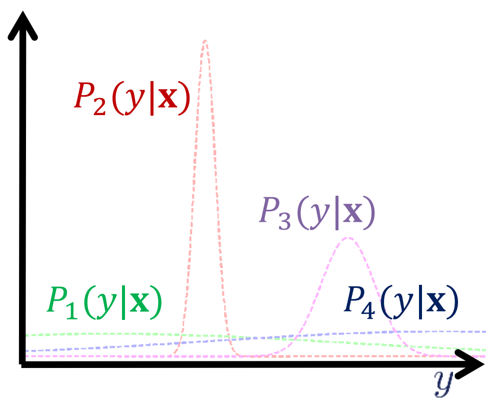
Prediction models and testing

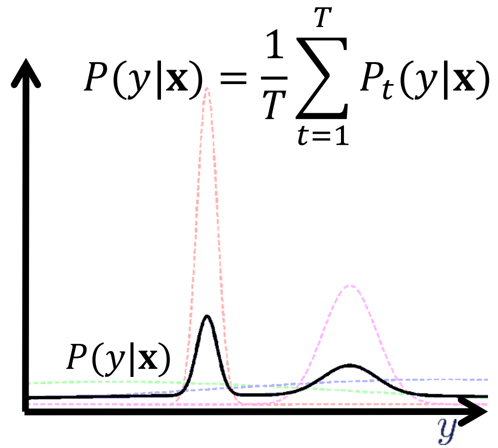
- Model output fusion is illustrated in the next slide, for a simple example where the attribute we want to predict is a continuous variable *y*.
- Imagine that we have trained an ensemble with $T = 4$ models.
- For a test data point **x**, we get the corresponding posteriors $p_t(y|\mathbf{x})$, with $t = \{1, . . . , 4\}$.

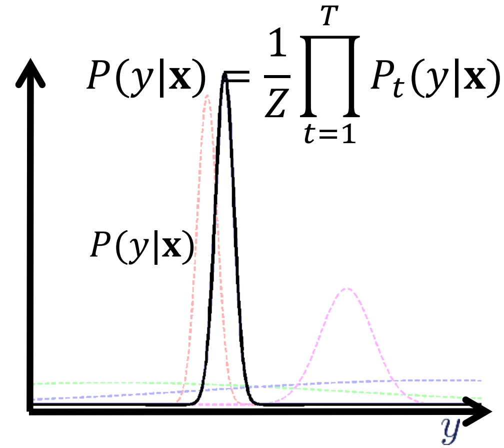


Prediction models and testing

- Some models produce peakier (more confident) predictions than others.
- Both the averaging and the product operations produce combined distributions (shown in black) which are heavily influenced by the most confident i.e. most informative models.
- Therefore, such simple operations have the effect of selecting (softly) the more confident models out of the ensemble.
- Averaging many posteriors also has the advantage of reducing the effect of possibly noisy model contributions.
- In general, the product based ensemble model may be less robust to noise.







Prediction models and testing

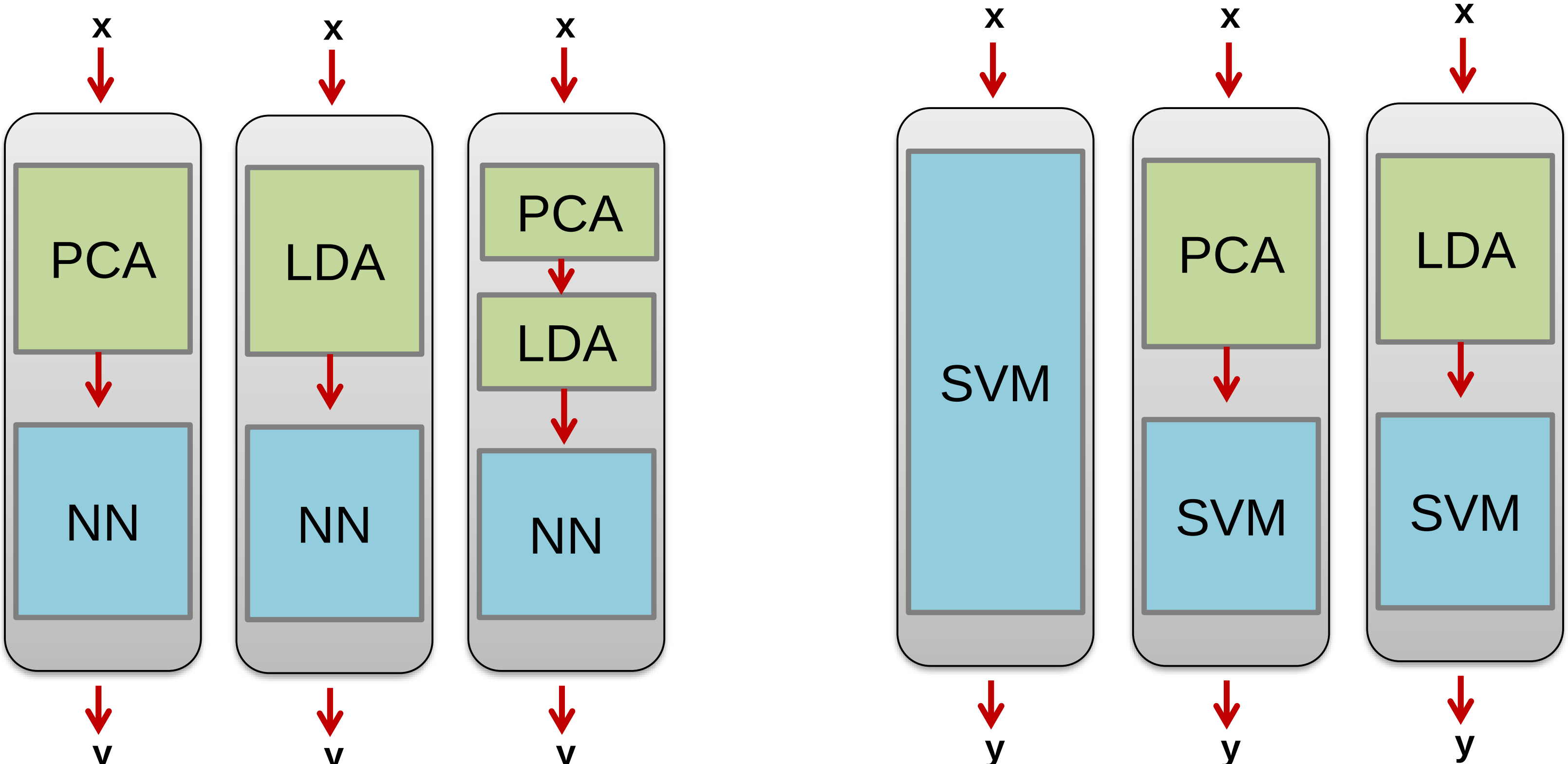
- Alternative ensemble models are possible, where for instance one may choose to select individual models in a hard way, or may do majority voting.

  - Min: $P(y|\mathbf{x}) = \min_t P_t(y|\mathbf{x})$

  - Max: $P(y|\mathbf{x}) = \max_t P_t(y|\mathbf{x})$

  - Majority voting (in classification):
    - each learned model votes for a class to assign to a query image.
    - Classification of the query image is by assigning the class has the highest number of 'votes'.

In our case, each single model can be



More discriminative than PCA

When $\mathbf{S}_W^{-1}$ is not attainable

SVM is a global optimiser?

PCA helps the computational complexity of SVM, but accuracy?

Two discriminative parts in a sequence?