Ex1

large-scale fading (shadowing) $\quad S_{dB} = 10\log_{10} S \sim N(0, \sigma_s^2)$

M: $x = 10\log_{10} S \sim N(0, \sigma_s^2)$

calculate $S = 10^{\frac{x}{10}}$ → shadowing realisation

multiple realisations of shadowing

```
for i=1:1000
    x = randn(0, σs²);
    s[i] = 10^(x/10);
end
plot(s)
```

$\sigma_s^2 = 8\,dB \qquad \sigma_s^2 = 10^{\frac{8}{10}}$

Ex2:

$h_i \sim CN(0,1)$

real　　imag

$h_i = h_{real} + i h_{imag}$

$CN(0,1)$　　$N(0,\frac{1}{2}) = \frac{N(0,1)}{\sqrt{2}}$　　$N(0,\frac{1}{2})$

M:

for $i = 1:10000$

$h_R = \frac{N(0,1)}{\sqrt{2}} \sim \frac{1}{\sqrt{2}} * \text{randn}$

$h_I = \frac{N(0,1)}{\sqrt{2}}$

$h = h_R + i h_I$

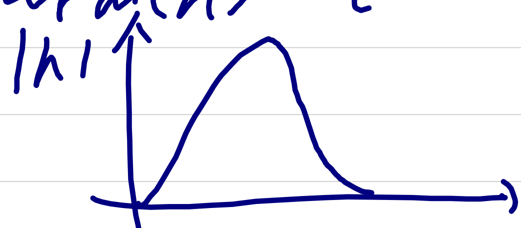
norm[i] = abs(h)

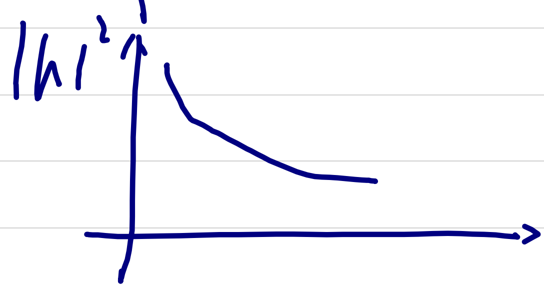
square abs.[i] = norm(h)^2

end

plot(norm)

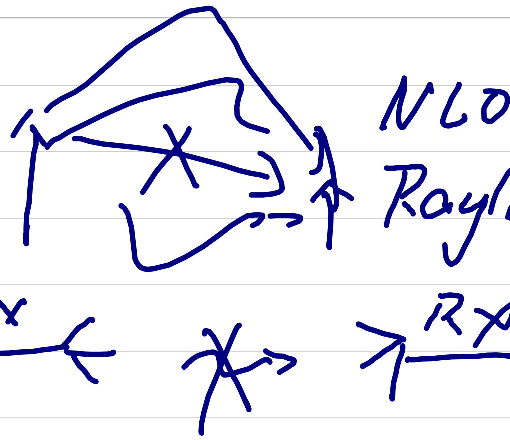
plot(square abs)





① Ex2

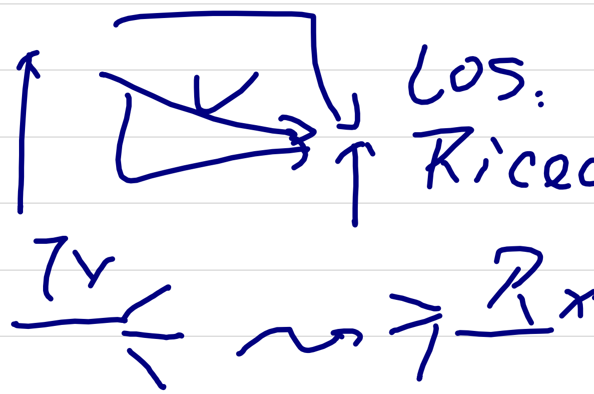
$h \sim \mathcal{CN}(0,1)$



NLOS: Rayleigh

② Ex3

$h = h_{LOS} + h_{Rayleigh}$



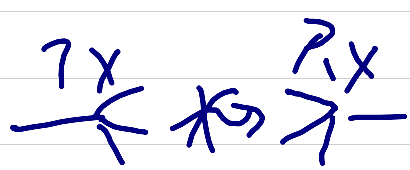
LOS: Ricean

$$h = \sqrt{\frac{K}{K+1}}\,\bar{h} + \sqrt{\frac{1}{1+K}}\,\tilde{h}$$

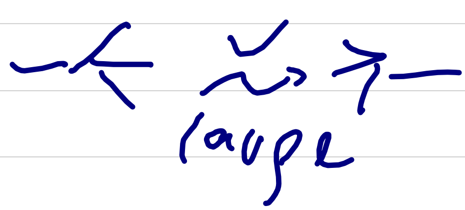
K : how much energy is in LOS

Ricean factor

$K = 0 \Rightarrow h = \tilde{h}$ (Rayleigh)



$K \to \infty \Rightarrow h \approx \bar{h} = e^{j\phi}$



fix K

for i=1:10000

$$h_R = \frac{N(0,1)}{\sqrt{2}}$$

$$h_I = \frac{N(0,1)}{\sqrt{2}}$$

$$\tilde{h} = h_R + i h_I \rightarrow \text{Rayleigh}$$

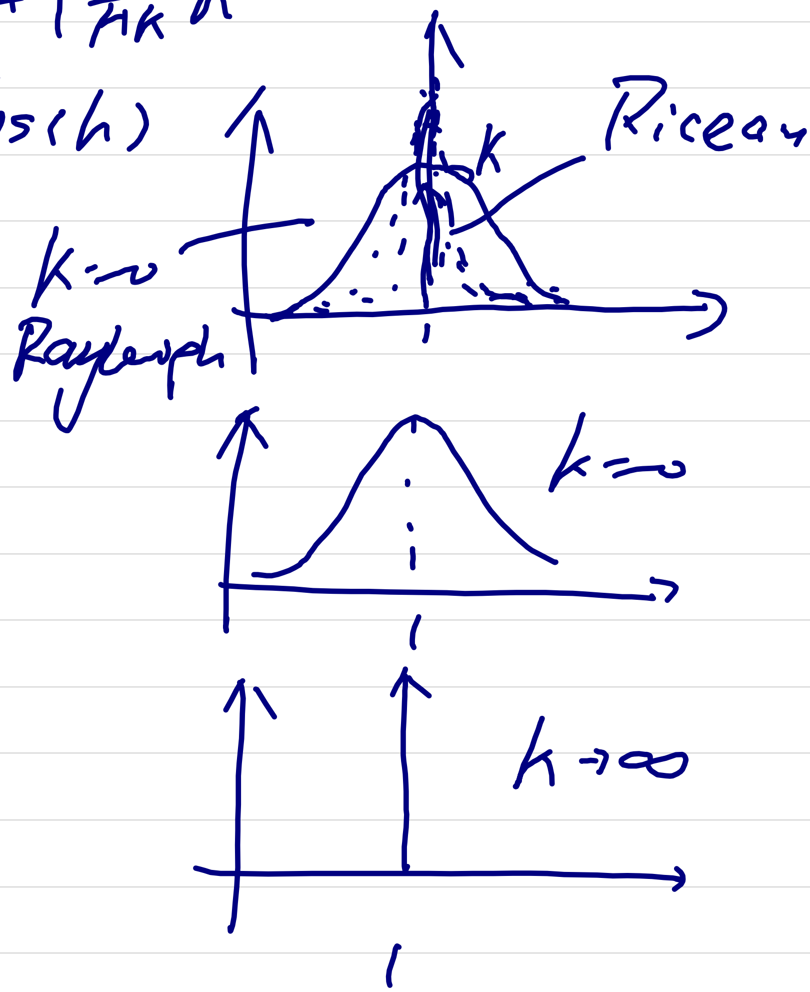
$$\bar{h} = e^{j\phi} = 1$$

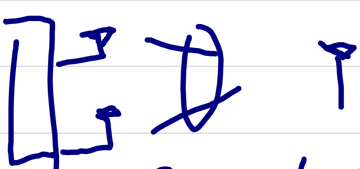
$$h = \sqrt{\frac{K}{1+K}}\,\bar{h} + \sqrt{\frac{1}{1+K}}\,\tilde{h}$$

norm[i] = abs(h)

end

plot(norm)





correlated

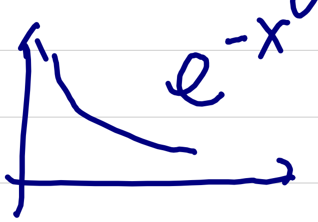
uncorrelated

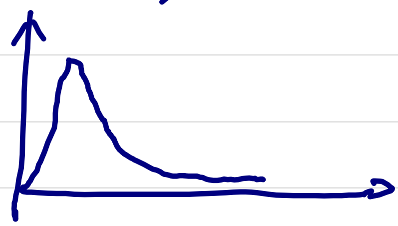
Ex4:

n complex normal RV

$X_1 . X_2 .. X_n \Rightarrow Y = |X_1|^2 + |X_2|^2 + .. + |X_n|^2$

$n=1 \Rightarrow |X_1|^2$ conj (RV) n=3





n=3

for i = 1:100

$h_R = \frac{N(0.1)}{\sqrt{2}}$

$h_I = \frac{N(0.1)}{\sqrt{2}}$

$h_1 = h_R + i h_I$

⋮

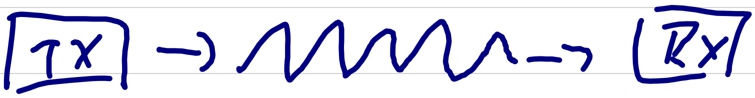
$h_2 = h_R' + i h_2'$

sum_nom_h[i] = abs(h_1)^2 + abs(h_2)^2

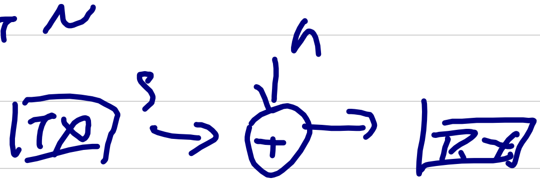
plot(sum_nom_h)

```
for i:1:100
  for j=1:n
    h=hR+jhI
    sum_norm.h[i] = sum_norm[i-1]
                        + abs(h)^2
```

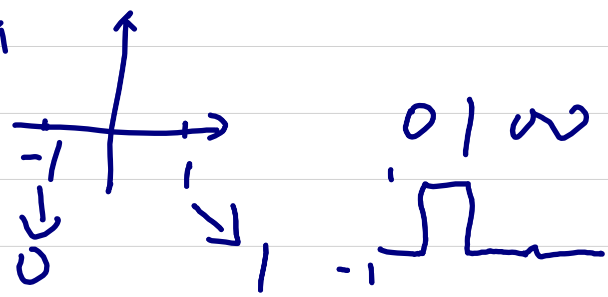
Ex 5:

BPSK 

① AWGN

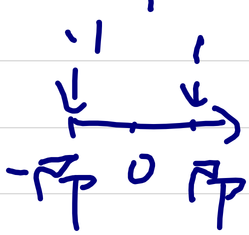


BPSK

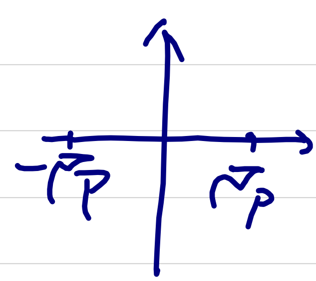


Power $= |s|^2$

$110100 \rightarrow \{1, 1, -1, 1, -1, -1\}$



BPSK with power $P \Rightarrow$ symbols



$P = 10dB \Rightarrow 10 \Rightarrow 3..$

$P = 20dB \Rightarrow 100 \Rightarrow 10$

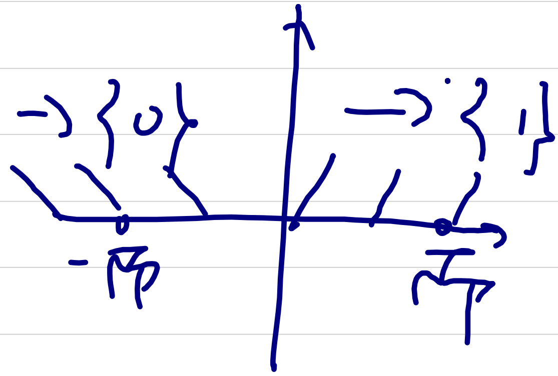
$$\text{Tx} \xrightarrow{s} \oplus \xrightarrow{r} \text{Rx}, \quad n \to \oplus$$

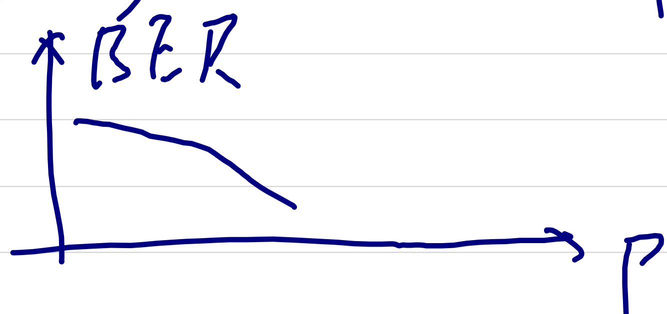
generate 10000 random bits

$0 \to (-1) \to -\sqrt{p}$

$1 \to \sqrt{p}$

$r = s + n \quad n \sim \mathcal{N}(0,1)$





$b_i \in \{0,1\} \mapsto s_i \in \{-\sqrt{p}, \sqrt{p}\} \to \oplus \leftarrow n$

$$\left.\begin{matrix} >0 \to 1 \\ <0 \to 0 \end{matrix}\right\} \leftarrow \mathrm{Re}\{y_i\} \leftarrow y_i = s_i + n$$

Ex5. 1)

BPSK/QPSK on AWGN channel

① BPSK

bit/data generation   mapping   power = SNR

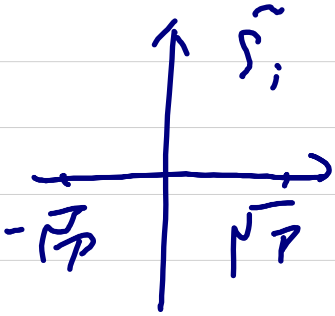
$\underbrace{101\ldots011}_{1e^4} \rightarrow \boxed{-1 \quad +1} \rightarrow \boxed{\times\sqrt{P}}$ Tx

eg: $1 \rightarrow 1 \rightarrow \sqrt{P}$

$0 \rightarrow -1 \rightarrow -\sqrt{P}$

channel

$\rightarrow \boxed{s_i + n_i}$ Cn $\rightarrow$



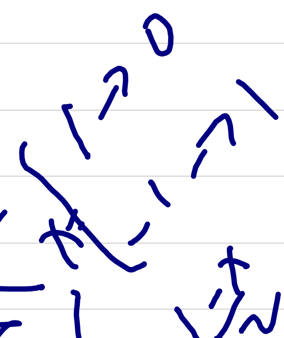
$\tilde{s}_i \rightarrow \begin{cases} 0. & Re\{\tilde{s}_i\} < 0 \\ 1. & Re\{\tilde{s}_i\} > 0 \end{cases}$ Rx

$s_i + CN(0,1)$

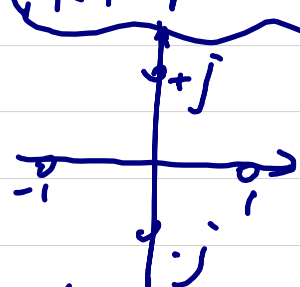
$P_e = \frac{\#error}{\#bits}$

② QPSK

Rx knows h_i



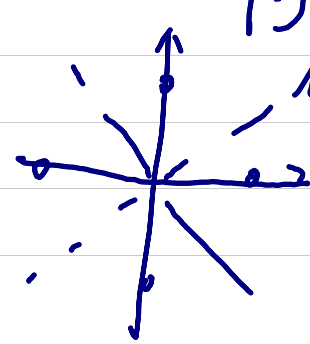
$\underbrace{00}_{1} \ \underbrace{10}_{-j} \ \underbrace{01}_{+j} \ \underbrace{00}_{1} \ \underbrace{10}_{-j} \rightarrow$



$\rightarrow \boxed{\sqrt{P}}$

$CN(0,1)$

$\rightarrow \oplus \rightarrow$



ML decoding

$\tilde{s}_i = s_i * h_i$

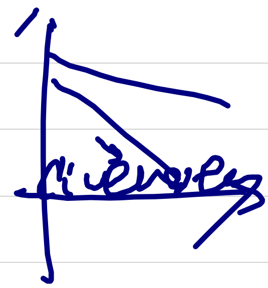
$\min D_i$

$\hat{s}_i = \frac{\tilde{s}_i}{h_i}$

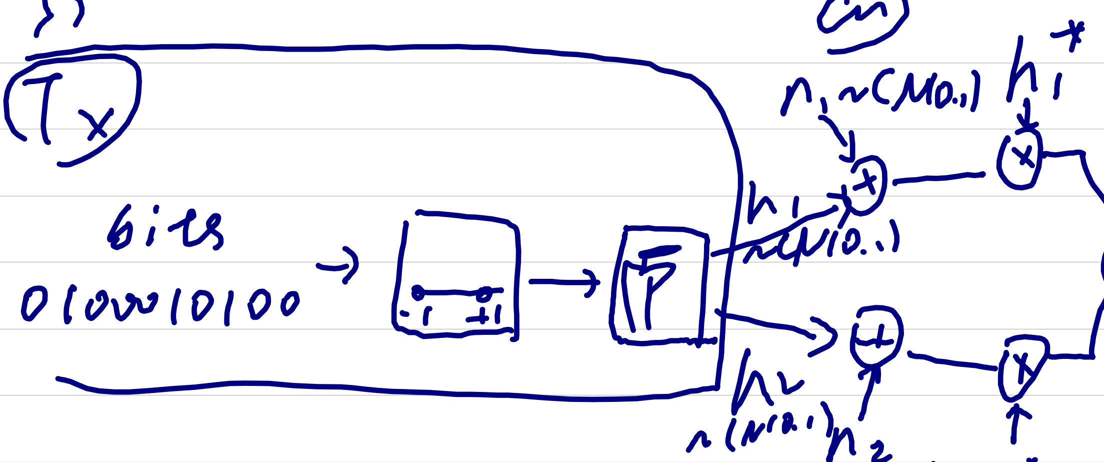
$P_e(\text{symbol}) = \dfrac{\#\text{ wrong symbol}}{\#\text{ Tx symbol}}$

2) channel: $h_i s_i + CN(0,1)$. $h_i \sim CN(0,1)$

change after several symbols.

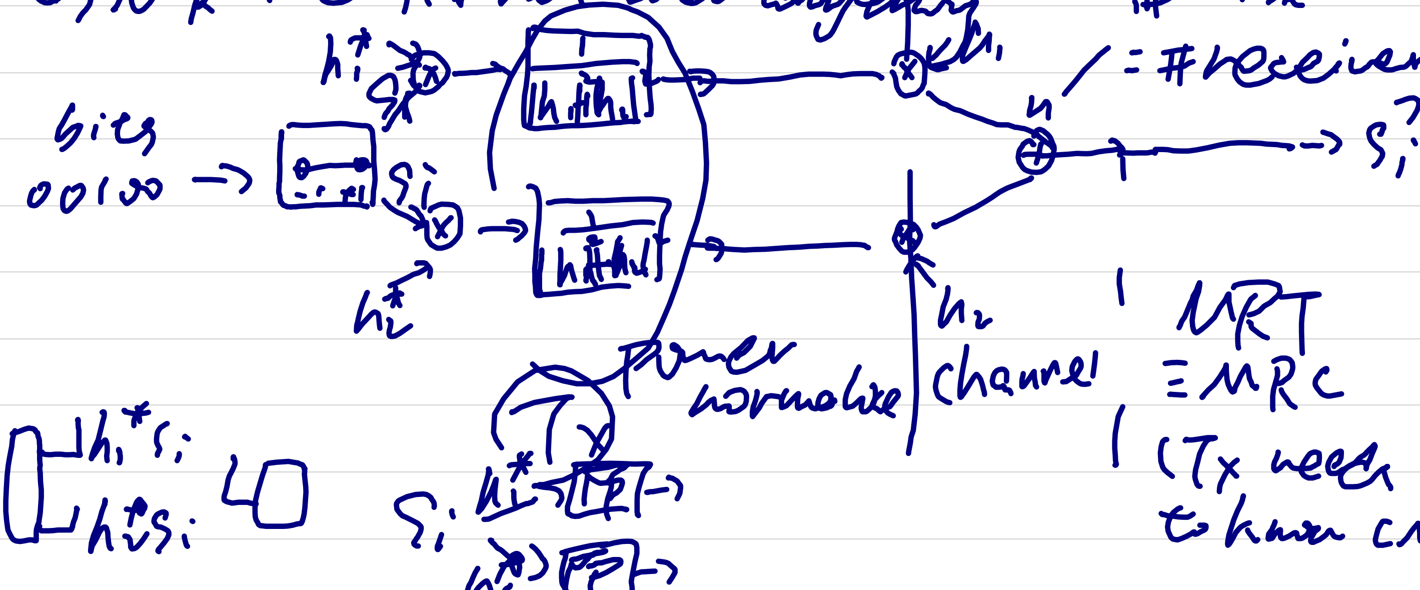


3)



6) MRT @ Rx not to do anything

#noise = #receiver



MRT = MRC (Tx need to know CN)

$|\sqrt{P} h_1^* s_1|^2 + |\sqrt{P} h_2^* s_2|^2$

$= P(t)(|h_1|^2 + |h_2|^2)$

5)

$\underline{00}\,\underline{01}\,\underline{10}\,\underline{11}\,\underline{10}\,\underline{10}$

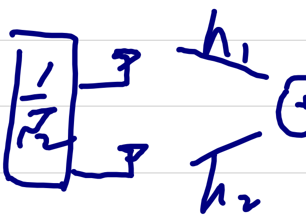
$s_6\ s_5\ s_4\ s_3\ s_2\ s_1$

2x symbols each time

(s_2, s_1)

Alamouti: get diversity gain w/o CSI

$$s_1, s_2 \Rightarrow \begin{array}{cc} -s_2^* & s_1 \\ s_1^* & s_2 \\ (2) & (1) \end{array}$$



$$y^{(1)} = \frac{1}{\sqrt{2}}(h_1 s_1 + h_2 s_2) + n^{(1)}$$

$$y^{(2)} = \frac{1}{\sqrt{2}}(h_1(-s_2^*) + h_2 s_1^*) + n^{(2)}$$

$$[s_1\ s_2]\begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix}^* = 0$$

orthogonal

$$\Rightarrow y^{(1)} = \frac{1}{\sqrt{2}}(h_1 s_1 + h_2 s_2) + n^{(1)}$$

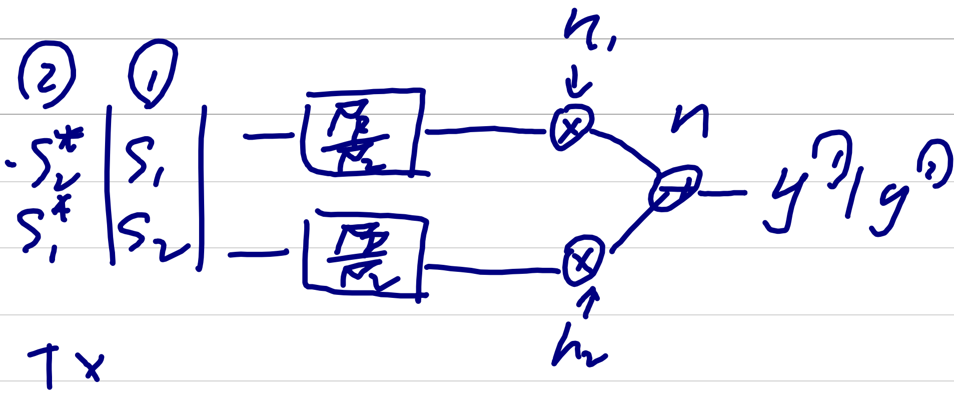
$$y^{(2)*} = \frac{1}{\sqrt{2}}(-h_1^* s_2 + h_2^* s_1) + n^{(2)}$$

$$\begin{bmatrix} y^{(1)} \\ y^{(2)*} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n^{(1)} \\ n^{(2)} \end{bmatrix}$$

$$\hat{s}_1 = [h_1^*\ h_2]\begin{bmatrix} y^{(1)} \\ y^{(2)} \end{bmatrix} = \frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)s_1$$

$$\hat{s}_2 = [h_2^*\ -h_1]\begin{bmatrix} y^{(1)} \\ y^{(2)} \end{bmatrix} = \underbrace{\frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)}_{\text{diversity gain}} s_2$$

$\frac{1}{\sqrt{2}} \to 3\text{dB}$

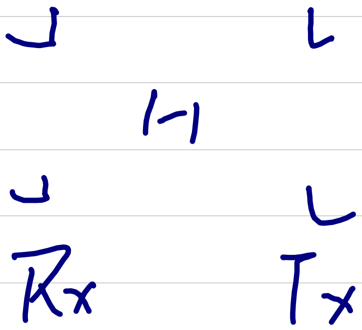


$$\Rightarrow \begin{cases} \hat{s}_1^{pre} = (h_1^* \;\; h_2)\,(y^{(1)}, y^{(2)}) \\ \hat{s}_2^{pre} = [h_2^* \;\; -h_1]\,(y^{(1)}, y^{(2)}) \end{cases}$$

$$\Rightarrow \hat{s}_1 = \frac{\hat{s}_1^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

$$\hat{s}_2 = \frac{\hat{s}_2^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

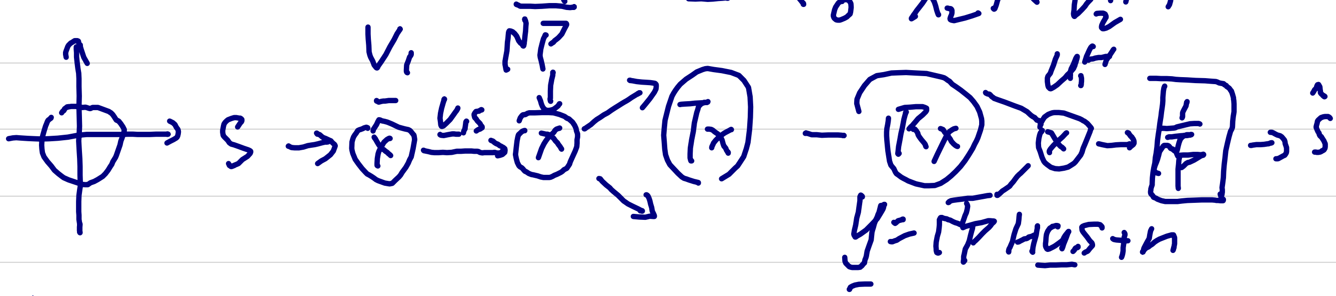
6. a) Dominant eigenmode transmission (DET)



$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$

$\overset{SVD}{=} U \Lambda V^H$

$= (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \underline{v}_1^H \\ \underline{v}_2^H \end{pmatrix}$



$\underline{y} = \sqrt{P} H \underline{v}_1 s + \underline{n}$

2x2 DET:
array gain = 4
div. gain = 4

$\begin{matrix} Rx_1 \\ Rx_2 \end{matrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \sqrt{P}\, U \Lambda \underline{V^H v_1} s + n$

$= \begin{pmatrix} \underline{v}_1^H \\ \underline{v}_2^H \end{pmatrix} v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

2x2 Alamouti:
array gain = 2 (no CSI T: 3 dB loss)
d.v. gain = 4

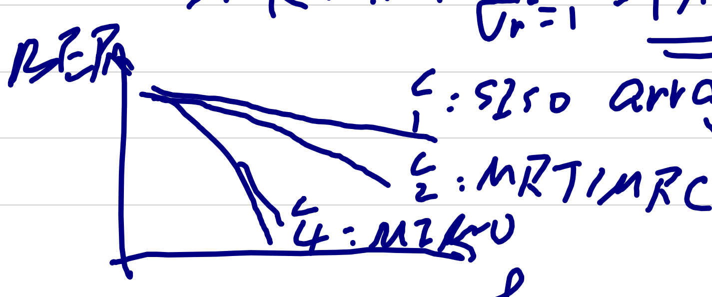
$= \sqrt{P} (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} s$

$= \sqrt{P} (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 s \\ 0 \end{pmatrix} = \sqrt{P}\, \underline{u}_1 \lambda_1 s$

Rx: $y = \sqrt{P} H \underline{u}_1 s + n$

$u_1^H y = \sqrt{P} H \underline{u}_1^H u_1 \lambda_1 s + u_1^H n = \sqrt{P} \lambda_1 s + \underline{n}$

$SNR = |\lambda_1|^2 \frac{P}{\sigma_n^2 = 1} = \underline{\underline{|\lambda_1|^2 P}}$



b)

$-s_2^* \quad s_1$

$s_1^* \quad s_2$

$$y(1) = h_1 s_1 + h_2 s_2 + n$$
$$y(2) = -h_1 s_2^* + h_2 s_1^* + n$$

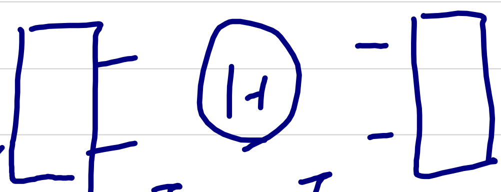
$$\Rightarrow \underbrace{\begin{bmatrix} y(1) \\ y^*(2) \end{bmatrix}}_{y} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\hat{s}_1 = [h_1^* \quad h_2]^T y \qquad \text{div. gain} = 2$$

$$\hat{s}_2 = [h_2^* \quad -h_1]^T y$$

$-s_2^* \quad s_1$

$s_1^* \quad s_2$



$$H = \begin{array}{c} \\ Rx_1 \\ Rx_2 \end{array} \begin{array}{cc} Tx_1 & Tx_2 \\ \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \end{array} = (\underline{h}_1 | \underline{h}_2)$$

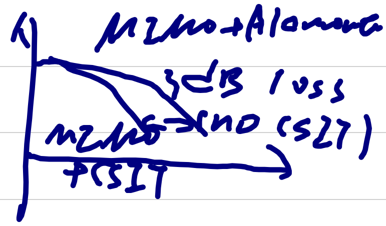
$$\underline{y}(1) = \underline{h}_1 s_1 + \underline{h}_2 s_2 + \underline{n}(1)$$
$$\underline{y}(2) = -\underline{h}_1 s_2^* + \underline{h}_2 s_1^* + \underline{n}(2)$$

$$\Rightarrow \begin{bmatrix} \underline{y}(1) \\ \underline{y}^*(2) \end{bmatrix} = \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \\ \underline{h}_2^* & -\underline{h}_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \underline{n}(1) \\ \underline{n}(2) \end{bmatrix}$$

$\hat{s}_1 = [\underline{h}_1^* \; \underline{h}_2^T] \, y$

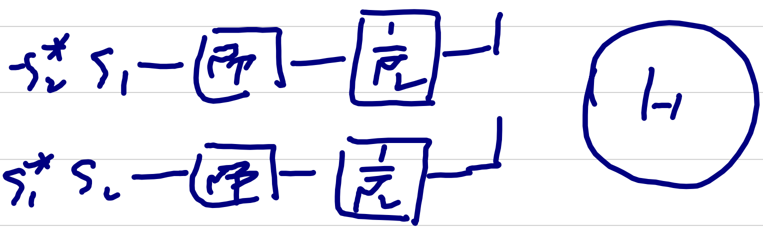
$\hat{s}_2 = [\underline{h}_2^* \; -\underline{h}_1^T] \, y$

$\Rightarrow SNR = \frac{|h_1|^2 + |h_2|^2}{2} \rightarrow$



array gain = 1

diversity gain = 4



$y(1) \mid y(2)$

$2\times1 \quad 2\times1$

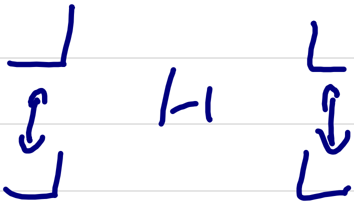
$\underline{y} = \begin{pmatrix} y(1) \\ y(2) \end{pmatrix} \quad 4\times1$

$P_{tot} = \left(\frac{\sqrt{P}}{\sqrt{2}}\right)^2 \cdot 2 = P$

$\hat{s}_1 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_1^H \; \underline{h}_2^T] \, \underline{y}$

$\hat{s}_2 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_2^H \; -\underline{h}_1^T] \, \underline{y}$

7.8



$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$

spacing $> \frac{\lambda}{2}$: h uncorrelated.

$< \frac{\lambda}{2}$: h correlated.

How correlation impact the performance?

Kronecker model — correlation at Rx. Tx (1.2)

$R_r = \begin{bmatrix} 1 & r^* \\ r & 1 \end{bmatrix}$ $R_t = \begin{bmatrix} 1 & t^* \\ t & 1 \end{bmatrix}$

$H = R_r^{\frac{1}{2}} H_w R_t^{\frac{1}{2}}$

no Rx correlation. only Tx correlation $\Rightarrow$ $R_r = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R_r^{\frac{1}{2}}$, $R_t = \begin{bmatrix} 1 & t \\ t & 1 \end{bmatrix}$

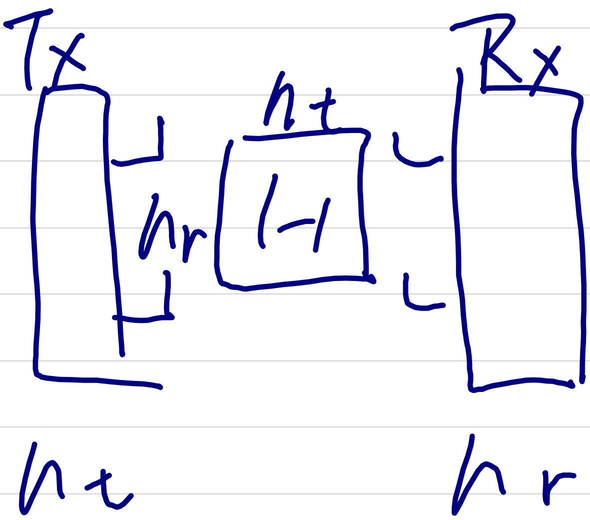
$H = H_w R_t^{\frac{1}{2}}$

task: repeat the previous exercise with channel $H = H_w R_t^{\frac{1}{2}}$ ($t = 0, 0.9$)

$E\{HH^H\} = R_r \otimes R_t$

$R_t^{\frac{1}{2}} \neq \begin{pmatrix} 1 & \sqrt{t} \\ \sqrt{t} & 1 \end{pmatrix}$

ensure at least 100 errors



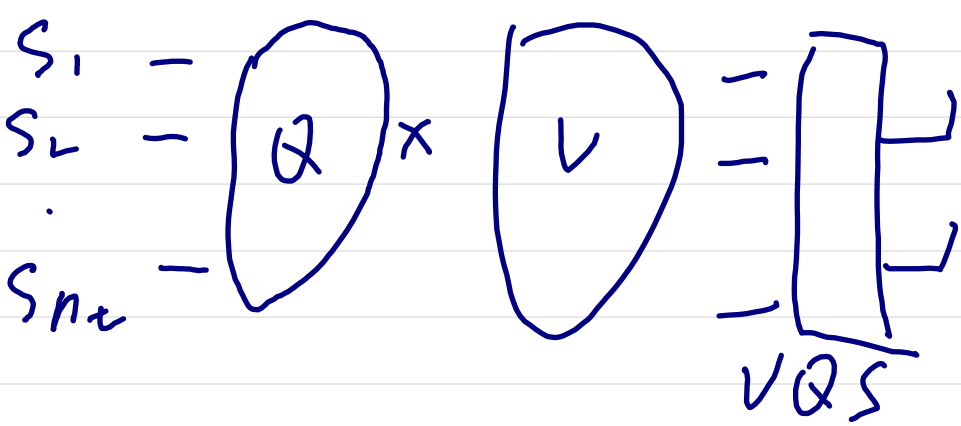
$$I(H,Q) = \log_2 \det(I + \rho H Q H^H)$$

$$C = \max_{Q} I(H,Q)$$

Q (obtained by WF)

$$\underset{n_r \times n_t}{H} = \underset{n_r \times n_r}{U}\ \overset{n_r \times n_t}{\Lambda}\ \underset{n_t \times n_t}{V^H}$$

$$Q = \begin{bmatrix} Q_1 & & \\ & \ddots & \\ & & Q_{n_t} \end{bmatrix} \quad Tr(Q) = P$$



$$\underline{J} = U\Lambda V^H V Q S = U\Lambda Q S$$

$$\underbrace{U^H U}_{\text{Tx} \quad I}\Lambda \underbrace{V^H V}_{I} Q S + n = \Lambda Q S + n$$

$$\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{n_t} \end{pmatrix}\begin{pmatrix} q_1 & & \\ & \ddots & \\ & & q_{n_t} \end{pmatrix}\begin{pmatrix} s_1 \\ \vdots \\ s_{n_t} \end{pmatrix}$$

$$C = \sum_{i=1}^{n_t} \log_2 (1 + \lambda_i^2 q_i P) \qquad \begin{pmatrix} \lambda_1 q_1 s_1 \\ \lambda_2 q_2 s_2 \\ \vdots \\ \lambda_{n_t} q_{n_t} s_{n_t} \end{pmatrix}$$

$$H = U\Lambda U^H$$

$$HH^H = U\Lambda \underbrace{V^H V}_{I} \Lambda^H U^H = U\Lambda^2 U^H$$

Tx $\to V$

Rx $\to U^H$

$\boxed{2\times 2}$

$\boxed{2\times 4}$

$\boxed{4\times 2}$

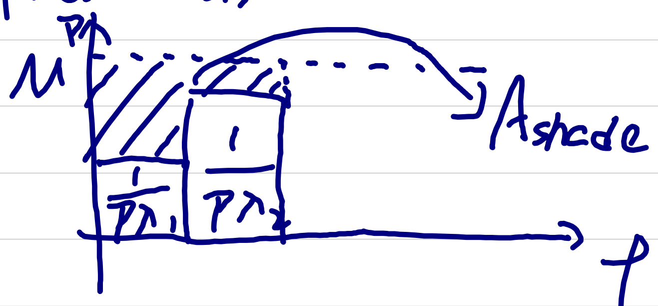
$$C = \overbrace{\log_2(1+\lambda_1^2 q_1) + \log(1+\lambda_2^2 q_2)}^{WF}$$

$$H = U\Lambda V^H \quad \downarrow \quad (\lambda_1, \lambda_2)$$

$$q_1 + q_2 = P \Rightarrow \begin{cases} q_1 = x_1 P \\ q_2 = x_2 P \\ x_1 + x_2 = 1 \end{cases}$$

$N_s = \min(n_t, n_r) = 2$

#streams



$A_{shade} = 1$

SVD(H) : 2

SVD($H^H H$) : no square

$$\left(\mu_i - \frac{1}{P\lambda_1^2}\right)^+ + \left(\mu_i - \frac{1}{P\lambda_2^2}\right)^+ = 1$$

μ^* such that

$$\max\left\{\mu^* - \frac{1}{P\lambda_1^2}, 0\right\} + \max\left\{\mu^* - \frac{1}{P\lambda_2^2}, 0\right\} = 1$$

$$H = U \Sigma V^H$$

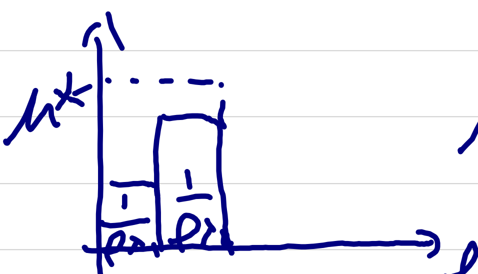
$$\Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

$$\downarrow$$

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \qquad q_1 + q_2 = 1$$

$$\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} \to Q \to V \to \overset{Tx}{\boxed{\vec{P}}}$$

$$C = \log_2(1 + P q_1 \underbrace{\sigma_1^2}_{=\lambda_1}) + \log_2(1 + P q_2 \underbrace{\sigma_2^2}_{=\lambda_2})$$



μ^* such that

$$\left\{\mu^* - \frac{1}{\rho\lambda_1}\right\}^+ + \left\{\mu^* - \frac{1}{\rho\lambda_2}\right\}^+ = 1$$

① generate matrix H

② SVD $H = U\Sigma V^H$ ③ $\Sigma = \begin{pmatrix}\sigma_1 & \\ & \sigma_2\end{pmatrix} \to \lambda = \begin{pmatrix}\lambda_1 & \\ & \lambda_2\end{pmatrix}$

④ 

$$\left(\mu^* - \frac{1}{\rho\lambda_1}\right)^+ + \left(\mu^* - \frac{1}{\rho\lambda_2}\right)^+ = 1$$

⑤ calculate $C = \log_2(1+\rho\lambda_1 S_1) + \log_2(1+\rho\lambda_2 S_2)$

$$C = E\left(\log_2\det\left(I + \frac{\rho}{n_t} H H^H\right)\right)$$