

Ex.

large-scale fading
(shadowing)

$$S_{dB} \sim N(0, \sigma_s^2)$$
$$= 10 \log_{10} S$$

$$M: \pi = 10 \log_{10} S \sim N(0, \sigma_s^2)$$

calculate $S = 10^{\frac{\pi}{10}}$ \rightarrow shadowing
realisation

multiple for $i = 1:1000$
realisation $x = \text{randn}(0, \sigma_s^2)$;
of $S[i] = 10^{\frac{x}{10}}$;
shadowing
end

Plot(S)

$$\sigma_s^2 = 8 \text{ dB} \quad \sigma_s^2 = 10^{\frac{8}{10}}$$

Ex 2:

$$h_i \sim \mathcal{CN}(0, 1)$$

↓ ↘
real imag

$$h_i = h_{\text{real}} + i h_{\text{imag}}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \mathcal{CN}(0, 1) & \mathcal{N}(0, \frac{1}{2}) & \mathcal{N}(0, \frac{1}{2}) \\ & = \frac{\mathcal{N}(0, 1)}{\sqrt{2}} & \end{array}$$

M:

for i = 1 : 1000

$$h_R = \frac{\mathcal{N}(0, 1)}{\sqrt{2}} \sim \frac{1}{\sqrt{2}} \times \text{randn}$$

$$h_I = \frac{\mathcal{N}(0, 1)}{\sqrt{2}}$$

$$h = h_R + i h_I$$

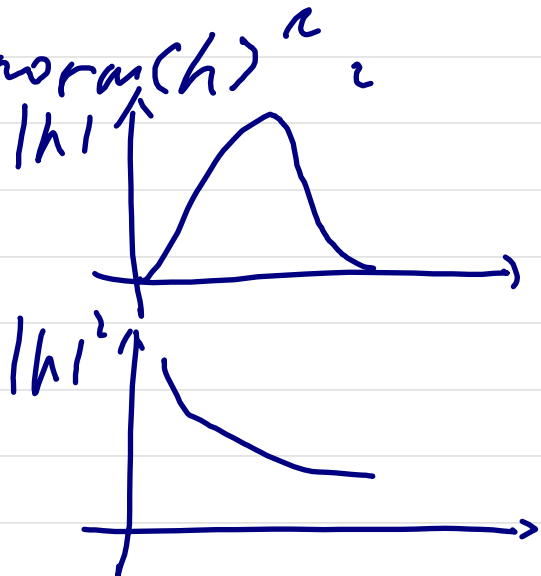
$$\text{norm}[i] = \text{abs}(h)$$

$$\text{square abs.}[i] = \text{norm}(h)^2$$

end

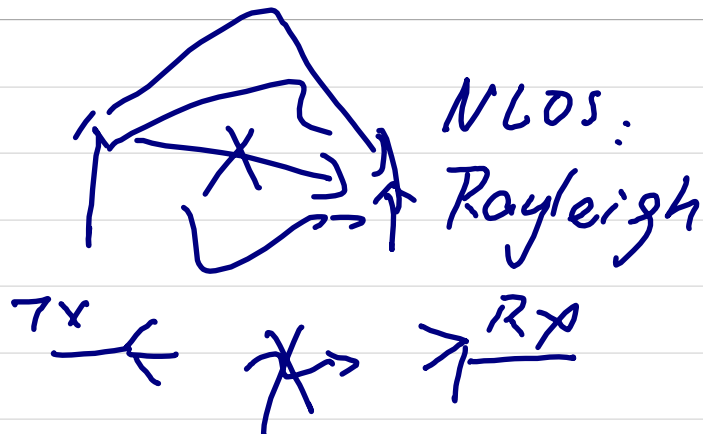
plot(norm)

plot(square abs)



① Ex2

$$h \sim (\mathcal{N}(0,1))$$



② Ex3

$$h = h_{\text{LOS}} + h_{\text{Rayleigh}}$$



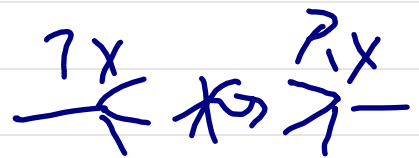
$$h = \sqrt{\frac{k}{k+1}} \bar{h} + \sqrt{\frac{1}{1+k}} \tilde{h}$$

energy

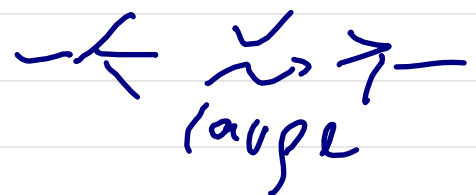
k : how much is in LOS

Ricean factor

$$k \rightarrow 0 \Rightarrow h = \tilde{h} \text{ (Rayleigh)}$$



$$k \rightarrow \infty \Rightarrow h \approx \bar{h} = e^{j\phi}$$



fix k

for i = 1:10000

$$h_R = \frac{N(0,1)}{\sigma_R}$$

$$h_I = \frac{N(0,1)}{\sigma_I}$$

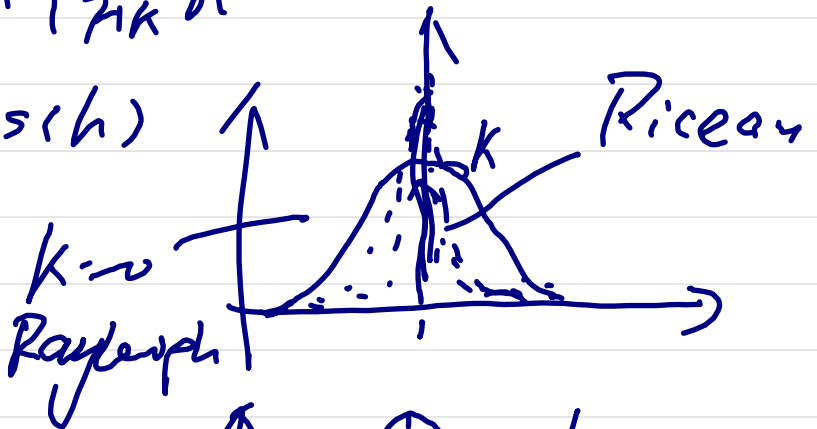
$$\tilde{h} = h_R + i h_I \rightarrow \text{Rayleigh}$$

$$\bar{h} = e^{j\phi} = 1$$

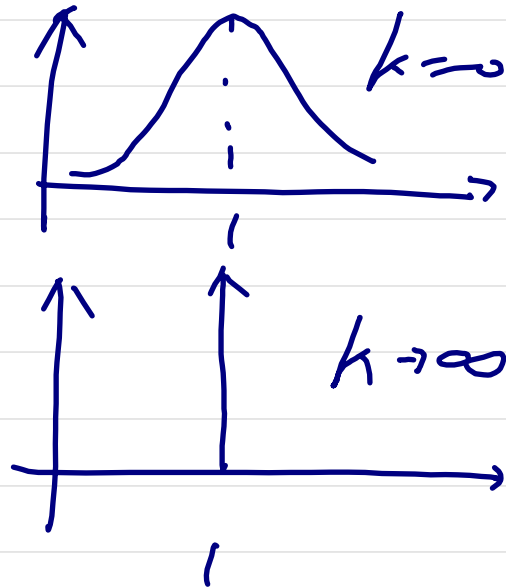
$$h = \sqrt{\frac{k}{1+k}} \bar{h} + \sqrt{\frac{1}{1+k}} \tilde{h}$$

$$\text{norm}[i] = \text{abs}(h)$$

end
plot(norm)



Correlated
uncorrelated

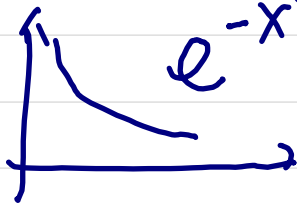


Ex 4:

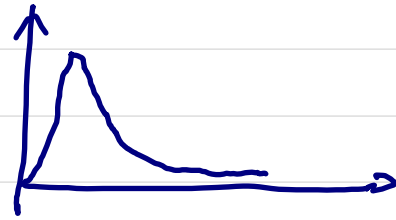
n complex normal RV

$$X_1, X_2, \dots, X_n \Rightarrow Y = |X_1|^2 + |X_2|^2 + \dots + |X_n|^2$$

$n=1 \Rightarrow |X_1|^2$ (orig. RV)



$n=3$



$n=3$

for $i = 1:100$

$$h_R = \frac{N(0,1)}{\sqrt{2}}$$

$$h_I = \frac{N(0,1)}{\sqrt{2}}$$

$$h_1 = h_R + i h_I$$

$$\vdots$$

$$h_2 = h_R + i h_I$$

$$\text{sum_norm_h}[i] = \text{abs}(h_1)^2 + \text{abs}(h_2)^2$$

plot (sum_norm_h)

for $i = 1 : 100$

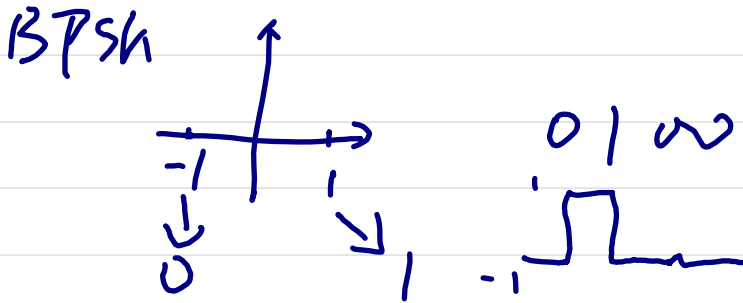
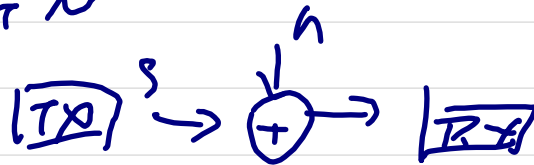
for $j = 1 : n$

$h = h_R + j h_z$

$\text{sum_norm} \cdot h[i] = \text{sum_norm}[i-1]$
 $+ \text{abs}(h)^2$

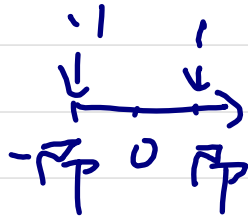
Ex 5:

BPSK $\boxed{TX} \rightarrow \text{modulator} \rightarrow \boxed{RX}$
 ① Analog

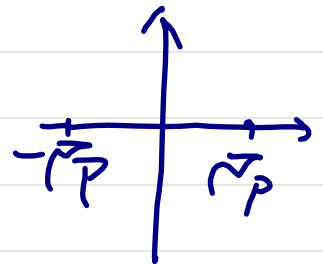


$$\text{Power} = |s|^2$$

$$110100 \rightarrow \{1, 1, -1, 1, -1, -1\}$$

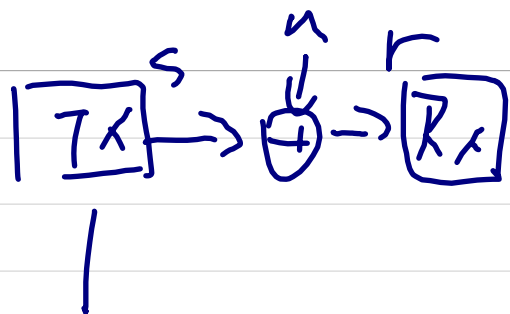


BPSK with power $P \Rightarrow$ symbols



$$P = 10 \text{ dB} \Rightarrow 10 \Rightarrow 3 \dots$$

$$P = 20 \text{ dB} \Rightarrow 100 \Rightarrow 10$$



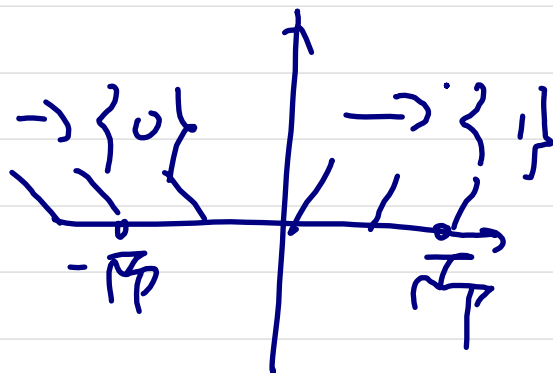
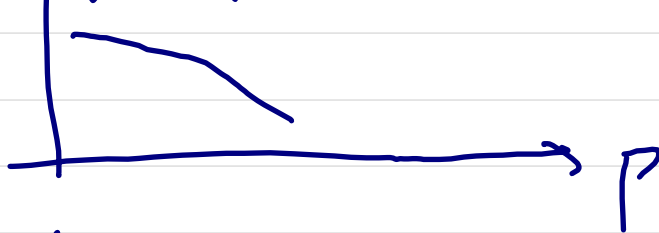
generate 1000 random bits

$$0 \rightarrow (-1) \rightarrow -\sqrt{P}$$

$$1 \rightarrow \sqrt{P}$$

$$r = s + n \sim \mathcal{N}(0, 1)$$

BER



$$b_i \in \{0, 1\} \mapsto s_i \in \{-\sqrt{P}, \sqrt{P}\}$$

$$\oplus \leftarrow n$$

$$\begin{matrix} 70 \rightarrow 1 \\ 00 \rightarrow 0 \end{matrix} \leftarrow \text{Re}\{y_i\} \leftarrow y_i = s_i + n$$

Ex 5. 1)

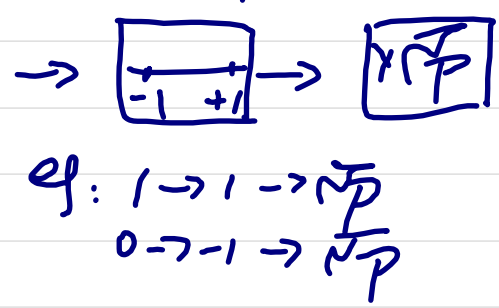
BPSK/QPSK on AWGN channel

① BPSK

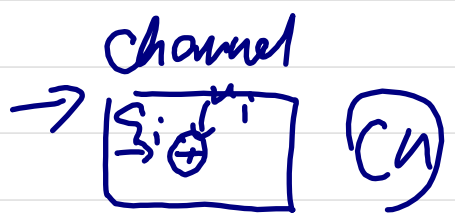
bit/data generation

101...011
1e4

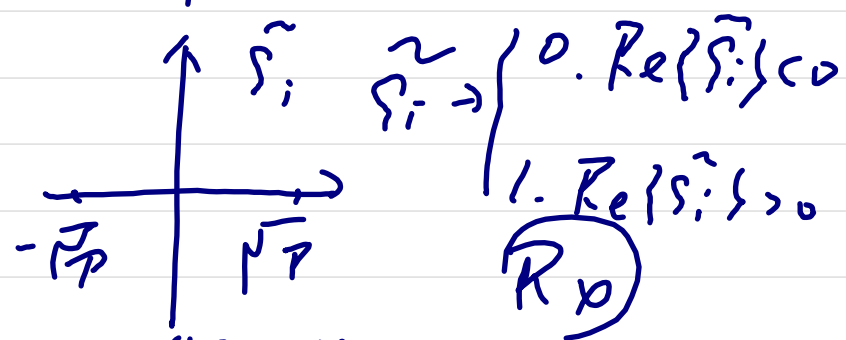
mapping power=SNR



(7v)



$s_i \sim \mathcal{CN}(0,1)$



$$P_e = \frac{\# \text{error}}{\# \text{bits}}$$

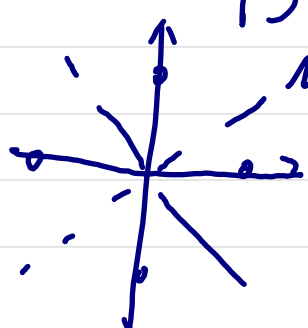
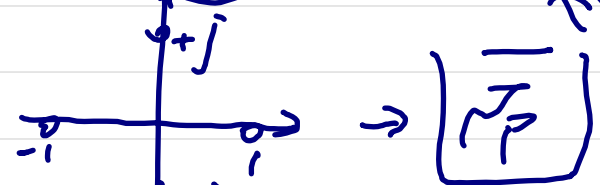
② QPSK

00 10 01 00 10
1 -j +j 1 -j

$\mathcal{CN}(0,1)$



$\{R_x \text{ knows } h_i\}$



ML decoding

$$\hat{s}_i = s_i + h_i$$

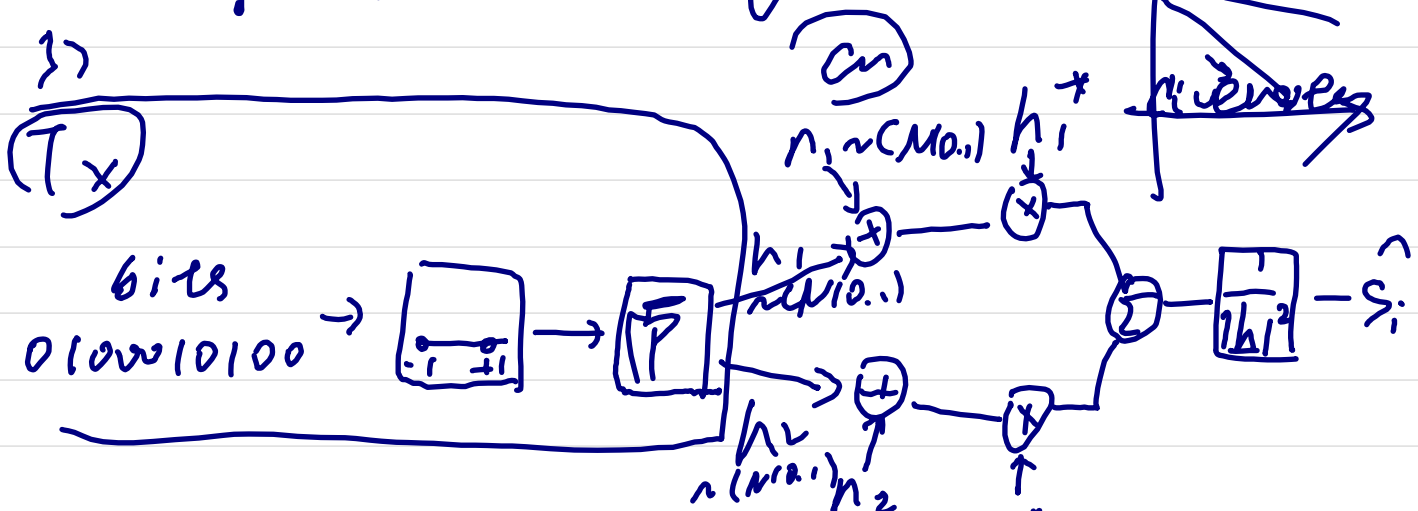
$h_i \in \{j\}$


$$\hat{s}_i = \frac{s_i}{h_i}$$

$$P_e(\text{symbol}) = \frac{\# \text{ wrong symbols}}{\# \text{ Tx symbols}}$$

2) channel: $h_i s_i + \text{CN}(0,1)$. $h_i \sim \text{CN}(0,1)$

change [→] after several symbols.



6) MRT $\triangle R$ not to do anything h^* $\#$ noise
 h^* \rightarrow  x_k $\#$ receiver



$\begin{bmatrix} h_1^* s_i \\ h_2^* s_i \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$|\vec{p} h_1^* s_1|^2 + |\vec{p} h_2^* s_2|^2$$

$$= P(E_2 | h_1) + (h_2)$$

MRT
≡ MRC
(Tx needs
to know c/v)

5)

$$\begin{array}{c} \underline{000110111010} \\ s_6 \ s_5 \ s_4 \ s_3 \ s_2 \ s_1 \end{array} \left| \begin{array}{l} 2 \times \text{symbols} \\ \text{each time} \\ (s_2, s_1) \end{array} \right.$$

Alamouti: get diversity gain w/o CSI

$$s_1, s_2 \rightarrow \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{array}{l} \xrightarrow{h_1} \\ \xrightarrow{h_2} \end{array} \begin{array}{l} y^{\textcircled{1}} = \frac{1}{\sqrt{2}} (h_1 s_1 + h_2 s_2) + n^{\textcircled{1}} \\ y^{\textcircled{2}} = \frac{1}{\sqrt{2}} (h_1 (-s_2^*) + h_2 s_1^*) + n^{\textcircled{2}} \end{array}$$

$$\begin{bmatrix} s_1 & s_2 \end{bmatrix} \begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix}^* = 0 \Rightarrow \begin{array}{l} y^{\textcircled{1}} = \frac{1}{\sqrt{2}} (h_1 s_1 + h_2 s_2) + n^{\textcircled{1}} \\ y^{\textcircled{2}*} = \frac{1}{\sqrt{2}} (-h_1^* s_2 + h_2^* s_1) + n^{\textcircled{2}} \end{array}$$

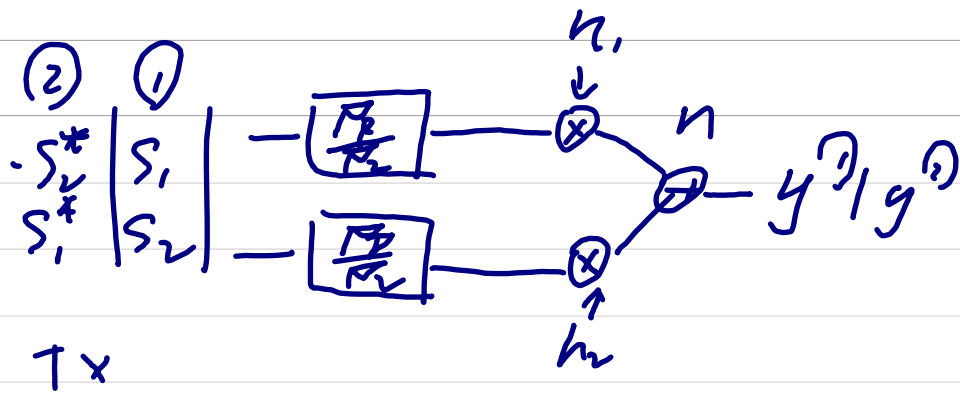
orthogonal

$$\begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}*} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n^{\textcircled{1}} \\ n^{\textcircled{2}} \end{bmatrix}$$

$$s_1 = \begin{bmatrix} h_1^* & h_2 \end{bmatrix} \begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} (|h_1|^2 + |h_2|^2) s_1$$

$$s_2 = \begin{bmatrix} h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} (|h_1|^2 + |h_2|^2) s_2$$

diversity gain
 $\frac{1}{\sqrt{2}} \rightarrow 3 \text{ dB}$



$$\Rightarrow \begin{pmatrix} \hat{s}_1^{pre} = (h_1^* \ h_2) \\ \hat{s}_2^{pre} = (h_2^* \ -h_1) \end{pmatrix} (y^D, y^D)$$

$$\Rightarrow \hat{s}_1 = \frac{\hat{s}_1^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

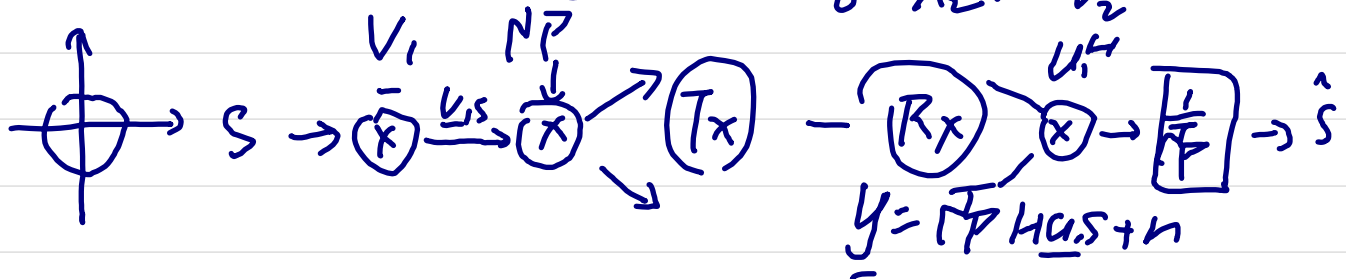
$$\hat{s}_2 = \frac{\hat{s}_2^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

6. ^a Dominant eigenmode transmission (DET)

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

$$\stackrel{\text{SVD}}{=} U \Lambda V^H$$

$$R_x \quad T_x = (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \underline{v}_1^H \\ \underline{v}_2^H \end{pmatrix}$$



$$\underline{y} = \underline{P}^H H \underline{u}_1 s + n$$

2x2 DET:

$$R_{x1} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underline{P}^H U \Lambda V^H \underline{v}_1 s + n$$

array gain = 4

dir. gain = 4

$$= \frac{\underline{v}_1^H}{\underline{v}_1^H \underline{v}_1} \underline{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad 2 \times 2 \text{ Alamouti} \dots$$

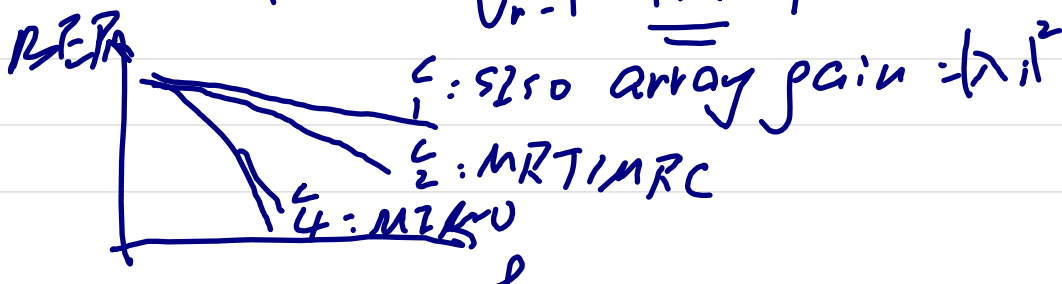
$$= \underline{P}^H (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} s \quad \text{array gain} = 2 \quad (\text{MRT/MSI: 3 dB loss})$$

$$= \underline{P}^H (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 s \\ 0 \end{pmatrix} = \underline{P}^H \underline{u}_1 \lambda_1 s \quad \text{d.v. gain} = 4$$

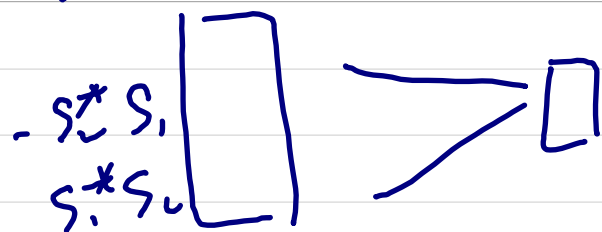
$$R_x: y = \underline{P}^H H \underline{u}_1 s + n$$

$$\underline{u}_1^H y = \underline{P}^H H \underline{u}_1^H \underline{u}_1 \lambda_1 s + \underline{u}_1^H n = \underline{P}^H \lambda_1 s + n$$

$$SNR = |\lambda_1|^2 \frac{P}{\sigma_n^2} = |\lambda_1|^2 \rho$$



b)

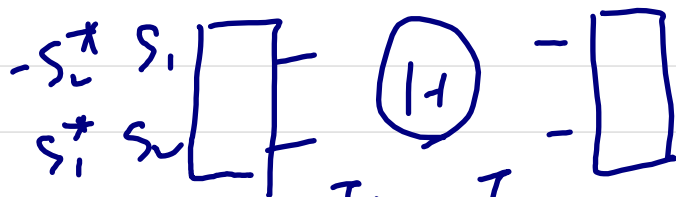


$$\begin{aligned} y(1) &= h_1 s_1 + h_2 s_2 + n \\ y(2) &= -h_1 s_2^* + h_2 s_1^* + n \end{aligned} \Rightarrow \begin{bmatrix} y(1) \\ y(2)^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

y

$$\hat{s}_1 = [h_1^* \ h_2^T] y \quad \text{div. gain} = 2$$

$$\hat{s}_2 = [h_2^* \ -h_1^T] y$$



$$H = \begin{bmatrix} \overline{h_{11}} & \overline{h_{12}} \\ \overline{h_{21}} & \overline{h_{22}} \end{bmatrix} = (\underline{h}_1 | \underline{h}_2)$$

$$\underline{y}(1) = \underline{h}_1 s_1 + \underline{h}_2 s_2 + \underline{n}(1)$$

$$\underline{y}(2) = -\underline{h}_1 s_2^* + \underline{h}_2 s_1^* + \underline{n}_2$$

$$\Rightarrow \begin{bmatrix} \underline{y}(1) \\ \underline{y}(2)^* \end{bmatrix} = \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \\ \underline{h}_2^* & -\underline{h}_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \underline{n}(1) \\ \underline{n}_2 \end{bmatrix}$$

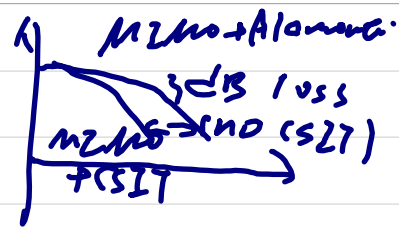
$$\hat{s}_1 = [\underline{h}_1^* \ \underline{h}_2^T] \underline{y}$$

$$\hat{s}_2 = [\underline{h}_2^* \ -\underline{h}_1^T] \underline{y}$$

$$\Rightarrow \text{SNR} = \frac{|\underline{h}_1|^2 + |\underline{h}_2|^2}{2}$$

$$\text{array gain} = 1$$

$$\text{diversity gain} = 4$$



$$s_1^* s_1 = \left[\frac{1}{\sqrt{2}} \right] \left[\frac{1}{\sqrt{2}} \right] = 1$$

$$s_2^* s_2 = \left[\frac{1}{\sqrt{2}} \right] \left[\frac{1}{\sqrt{2}} \right] = 1$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix}$$

$$\begin{matrix} \text{---} & y(1) & | & y(2) \\ \text{---} & \underline{2 \times 1} & & \underline{2 \times 1} \end{matrix}$$

$$P_{\text{out}} = \left(\frac{\sqrt{P}}{2} \right)^2 \cdot 2 = P$$

$$\underline{y} = \begin{pmatrix} y(1) \\ y(2) \end{pmatrix} \quad 4 \times 1$$

$$\hat{s}_1 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_1^H \ \underline{h}_2^T] \underline{y}$$

$$\hat{s}_2 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_2^H \ -\underline{h}_1^T] \underline{y}$$

7.8

$$\begin{array}{c} \downarrow \\ \text{---} \\ \downarrow \end{array} \quad H = \begin{array}{c} \downarrow \\ \text{---} \\ \downarrow \end{array} \quad H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

spacing $> \frac{\lambda}{2}$: h uncorrelated.

$< \frac{\lambda}{2}$: h correlated.

How correlation impact the performance?

Kronecker model correlation at R_x, T_x (1.2)

$$R_r = \begin{bmatrix} 1 & r^* \\ r & 1 \end{bmatrix} \quad R_t = \begin{bmatrix} 1 & t^* \\ t & 1 \end{bmatrix}$$

$$H = R_r^H H_w R_t^H$$

no R_x correlation. \Rightarrow $R_r = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R_r^H$
 only T_x correlation $\Rightarrow R_t = \begin{bmatrix} 1 & t \\ t^* & 1 \end{bmatrix}$

$$H = H_w R_t^H$$

task: repeat the previous exercise with channel $H = H_w R_t^H$ ($t = 0, 0.9$)

$$E\{H H^H\} = R_r \otimes R_t$$

$$R_t^H = \begin{pmatrix} 1 & t^* \\ t & 1 \end{pmatrix}$$

