EE2 Mathematics – Probability & Statistics

Exercise 6

1. (a) An aircraft component has a lifetime which is exponentially distributed with mean 2000 hours. Find the probability that it lasts more than 1000 hours.

   (b) Two independent components of this type are arranged in series to form a new device; this will operate if and only if both components function. What is the probability that this device will operate after 1000 hours of use?

   (c) Now suppose that the two independent components are arranged in parallel to form a new device; this will operate if and only if at least one of the two components functions. What is the probability that this device will operate after 1000 hours of use?

   Pick your answers from:
   (i) 0.845 (ii) 0.368 (iii) 0.431 (iv) 0.135 (v) 0.001 (vi) 0.607

2. Find the hazard rate and cumulative hazard function for the lifespan of the two devices described in Question 1 (series and parallel).

   Pick your answers from:
   (i) $(2 - e^{t/2000})/[1000(1 - e^{t/2000})]$ (ii) $t^2/1000$ (iii) $1/1000$ (iv) $t/1000$
   (v) $(1-e^{t/2000})/[1000(2-e^{t/2000})]$ (vi) $t/2000-\log(2-e^{t/1000})$ (vii) $1/2000$
   (viii) $1/100$ (ix) $t/1000 - \log(2 - e^{t/2000})$

3. Let T be a non-negative random variable with hazard rate

$$z_T(t) = \frac{\alpha}{t+1}$$

   for some parameter $\alpha > 0$.

   Find the cumulative hazard function, the cumulative distribution function (CDF) and the probability density function (PDF) of T.

   Pick your answers from:
   (i) $\alpha/(t+1)^2$ (ii) $1 - (t+1)^{-\alpha}$ (iii) $\alpha(t+1)^{-\alpha+1}$ (iv) $\alpha(t+1)^{-(\alpha+1)}$
   (v) $(t+1)^{-\alpha}$ (vi) $\alpha \log(t+1)$