Study Group

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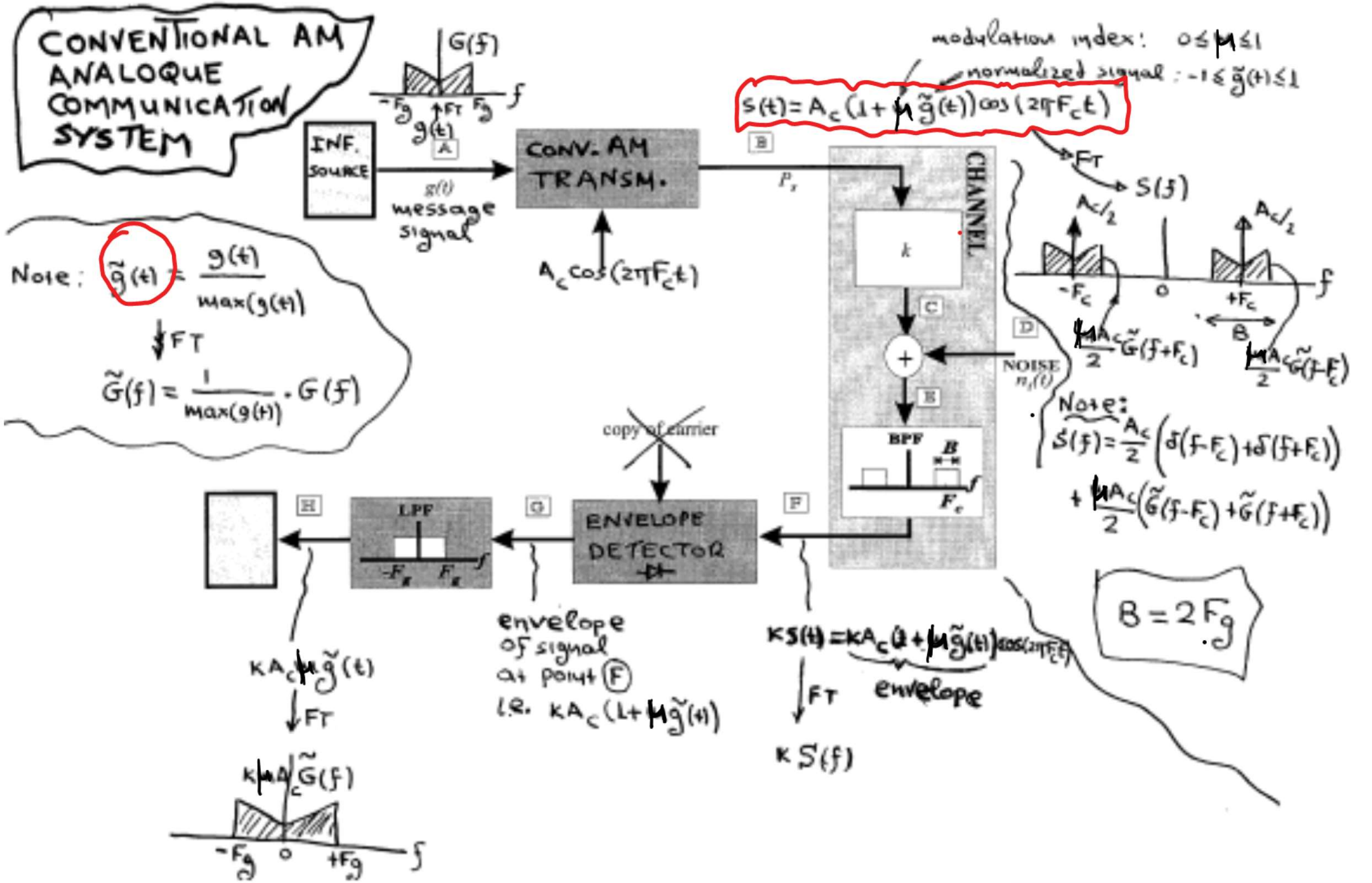
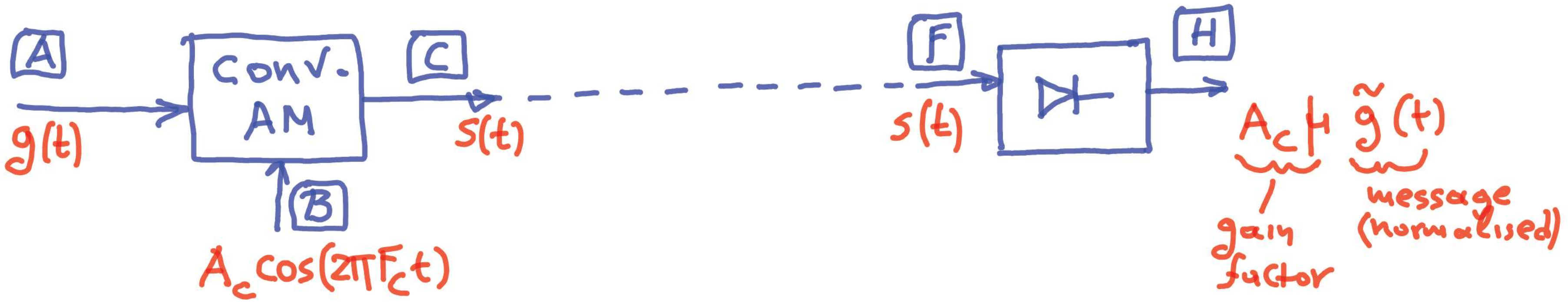
Comms-1

Conventional AM + SSB

(or "full" AM) → Q1, Q2

SSB → Q3

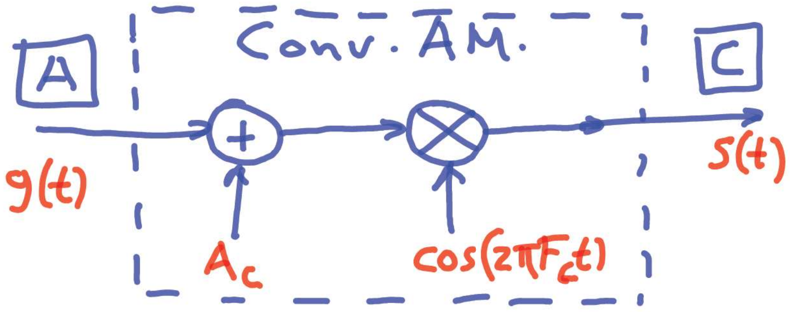

CONVENTIONAL AM ANALOQUE COMMUNICATION SYSTEM
G(f)
g(t)
INF. SOURCE
g(t) message signal
CONV. AM TRANSM.
A_c cos(2πF_c t)
modulation index: 0 ≤ μ ≤ 1
normalized signal: -1 ≤ g̃(t) ≤ 1
s(t) = A_c(1 + μ g̃(t)) cos(2πF_c t)
FT
S(f)
A_c/2
-F_c
0
+F_c
B
μA_c/2 G̃(f+F_c)
μA_c/2 G̃(f-F_c)
Note:
S(f) = A_c/2 (δ(f-F_c) + δ(f+F_c)) + μA_c/2 (G̃(f-F_c) + G̃(f+F_c))
B = 2F_g
Note: g̃(t) = g(t) / max(g(t))
FT
G̃(f) = 1/max(g(t)) · G(f)
CHANNEL
k
NOISE n_i(t)
BPF
copy of carrier
ENVELOPE DETECTOR
LPF
envelope of signal at point F i.e. KA_c(1+μg̃(t))
KS(t) = KA_c(1+μg̃(t))cos(2πF_c t)
envelope
FT
KS(f)
KA_c μ g̃(t)
FT
KμA_c G̃(f)
-F_g
0
+F_g



$$\boxed{C}: \quad s(t) = A_c\left(1+\mu\cdot\tilde{g}(t)\right)\cdot\cos(2\pi F_c t)$$

$$\tilde{g}(t) \triangleq \frac{g(t)}{\max(g(t))} \xrightarrow{FT} \tilde{G}(f) = \frac{G(f)}{\max(g(t))}$$

Note:

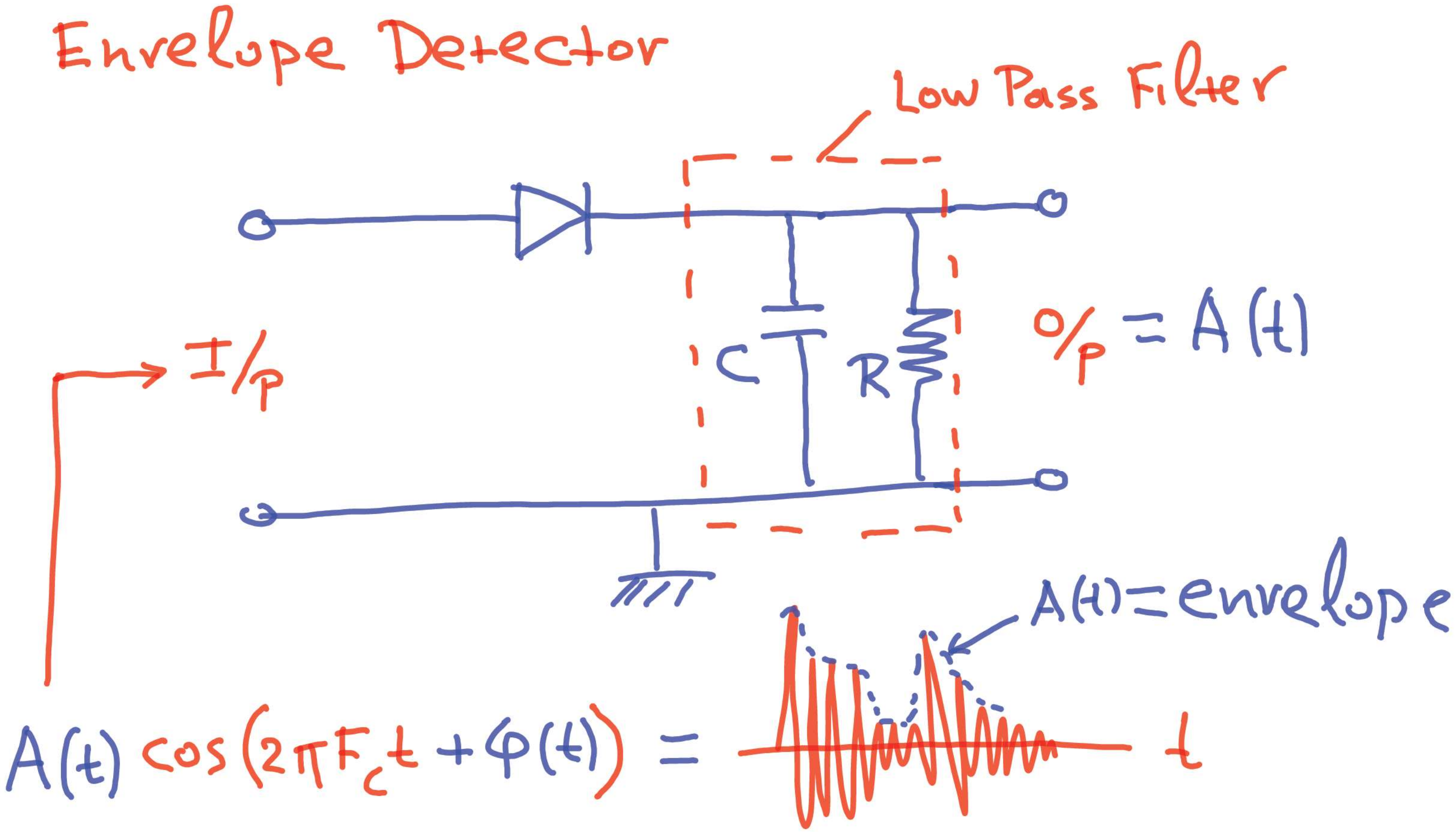


$$s(t) = \left(A_c + g(t)\right)\cos(2\pi F_c t)$$

$$= A_c\left(1 + \frac{1}{A_c}\cdot\frac{\max(g(t))}{\max(g(t))}\cdot g(t)\right)\cdot\cos(2\pi F_c t)$$

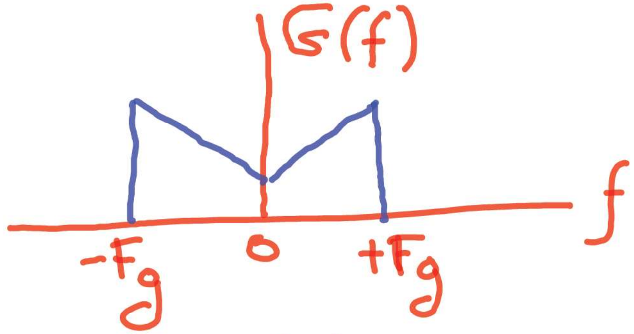
$$= A_c\left(1+\mu\cdot\tilde{g}(t)\right)\cdot\cos(2\pi F_c t)$$

* Envelope Detector



$A(t)\cos\left(2\pi F_c t + \varphi(t)\right) =$

$\boxed{A}$: $g(t) \xrightarrow{FT} G(f) =$



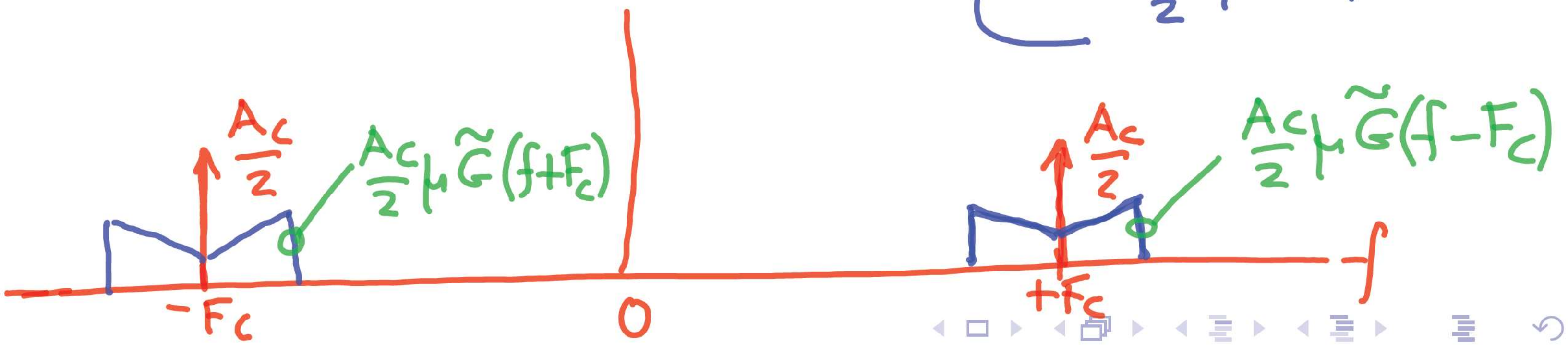
Bandwidth $= F_g =$ max frequency

$\boxed{C}$: $s(t) = A_c\left(1+\mu\,\tilde{g}(t)\right)\cdot\cos(2\pi F_c t)$

$$= A_c\cos(2\pi F_c t) + A_c\mu\,\tilde{g}(t)\cos(2\pi F_c t)$$

$$S(f) = FT\{s(t)\} = \frac{A_c}{2}\delta(f+F_c) + \frac{A_c}{2}\delta(f-F_c) + \frac{A_c}{2}\mu\,\tilde{G}(f+F_c) + \frac{A_c}{2}\mu\,\tilde{G}(f-F_c)$$

ie



$\boxed{Q1}$

$$g(t) = \cos(100t) + \sin(100t)$$

$$A_c = 2$$

$$\tilde{g}(t) = ?$$

$$\mu = ?$$

carrier frequ $= F_c = \frac{10000}{2\pi} = \frac{5000}{\pi}$

$$g(t) = \sqrt{2}\cos\left(100t - \frac{\pi}{4}\right) \longrightarrow \tilde{g}(t) = \cos(\underbrace{100}_{\downarrow}t - \frac{\pi}{4})$$

$$F_g = \frac{100}{2\pi} = \frac{50}{\pi}$$

$$\tilde{G}(f) = \frac{1}{2}\delta(f+F_g) + \frac{1}{2}\delta(f-F_g)$$

$$\therefore \max(g(t)) = \sqrt{2}$$

$$\therefore \mu \triangleq \frac{\max(g(t))}{A_c} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\therefore s(t) = \underset{\substack{\uparrow\\2}}{A_c}\Big(1 + \underset{\substack{\uparrow\\1/\sqrt{2}}}{\mu}\,\underbrace{\tilde{g}(t)}_{\to \cos(100t - \frac{\pi}{4})}\Big)\cdot\cos(2\pi \underbrace{F_c t}_{10000})$$

$$n = \frac{P_s}{P_c + P_s} = ?$$

$$\text{carrier} = \underset{\substack{\uparrow \\ 2}}{A_c} \cos(2\pi \underbrace{F_c}_{10000} t)$$

* From the spectrum on page 5

$$S(f) = FT\{s(t)\} = \underbrace{\frac{A_c}{2}\delta(f+F_c) + \frac{A_c}{2}\delta(f-F_c)}_{\text{carrier}} + \underbrace{\frac{A_c}{2}\mu\tilde{G}(f+F_c) + \frac{A_c}{2}\mu\tilde{G}(f-F_c)}_{\text{sidebands}}$$

$$= \overbrace{\frac{A_c}{2}\delta(f+F_c) + \frac{A_c}{2}\delta(f-F_c)} + \frac{A_c}{2}\mu\frac{1}{2}\delta(f+F_g+F_c) + \frac{A_c}{2}\mu\frac{1}{2}\delta(f-F_g+F_c) + \frac{A_c}{2}\mu\frac{1}{2}\delta(f+F_g-F_c) + \frac{A_c}{2}\mu\frac{1}{2}\delta(f-F_g-F_c)$$

$\therefore$ P_c = power of carrier

$$= \frac{A_c^2}{4} + \frac{A_c^2}{4} = \frac{A_c^2}{2} = \frac{2^2}{2} = 2$$

$$P_s = \frac{A_c^2}{4}\mu^2\frac{1}{4} \times 4 = \frac{2^2}{4} \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{4} \times 4 = \frac{1}{2}$$

$$\therefore n = \frac{1/2}{2 + 1/2} = \frac{1}{5} = 0.2$$

(the same solution can be found in the time domain)

$\boxed{Q2}$

$$s(t) = 5\cos(\underbrace{1800\pi t}_{\text{Lower sideband}}) + 20\cos(\underbrace{2000\pi t}_{\text{carrier frequency}}) + 5\cos(\underbrace{2200\pi t}_{\text{upper sideband}})$$

$$F_c = \frac{2000\pi}{2\pi} = 1000\,Hz$$

$$\therefore \frac{2000\pi - 1800\pi}{2\pi} = \frac{200\pi}{2\pi} = 100\,Hz$$

$$\therefore \text{Bandwidth} = 2 \times 100\,Hz = 200\,Hz$$

$$s(t) = \left(20 + 10\cdot\cos(200\pi t)\right)\cdot\cos(2000\pi t)$$

$$= \underbrace{20}_{A_c}\Big(1 + \underbrace{\frac{10}{20}}_{\mu}\underbrace{\cos(200\pi t)}_{\tilde{g}(t)}\Big)\cdot\underbrace{\cos(2000\pi t)}_{\text{carrier}}$$

Q2

$$P_c = \text{power of carrier} = \frac{A_c^2}{2} = \frac{20^2}{2} = 200$$

$$P_s = \text{power of sidebands} = \frac{1}{2} \cdot P_g = \frac{1}{2}\overline{g^2(t)} = \frac{1}{2}\overline{10^2\cos^2(200\pi t)}$$

$$= \frac{1}{2} 100 \overline{\cos^2(200\pi t)}$$

$$= \frac{1}{2} 100 \frac{1}{2} = 25$$

$$\therefore \frac{P_s}{P_c} = \frac{25}{200} = \frac{1}{8}$$

$\boxed{Q3}$ $g(t) = \cos(100t) \longrightarrow G(f) = \frac{1}{2}\delta\left(f + \frac{100}{2\pi}\right) + \frac{1}{2}\delta\left(f - \frac{100}{2\pi}\right)$

carrier $= 2\cos(1000t)$

DSB-SC : $s(t) = 2g(t)\cos(1000t)$

spectrum: $S(f) = FT\{s(t)\} = 2\,G(f) * \left\{\frac{1}{2}\delta\left(f + \frac{1k}{2\pi}\right) + \frac{1}{2}\delta\left(f - \frac{1k}{2\pi}\right)\right\}$

$= G\left(f + \frac{1k}{2\pi}\right) + G\left(f - \frac{1k}{2\pi}\right)$

$= \frac{1}{2}\delta\left(f + \frac{100}{2\pi} + \frac{1k}{2\pi}\right) + \frac{1}{2}\delta\left(f - \frac{100}{2\pi} + \frac{1k}{2\pi}\right) + \frac{1}{2}\delta\left(f + \frac{100}{2\pi} - \frac{1k}{2\pi}\right)$

$+ \frac{1}{2}\delta\left(f - \frac{100}{2\pi} - \frac{1k}{2\pi}\right)$