QUESTION 1.1

1) MULTI-RATE SIGNAL PROCESSING

(a)

① $X(z)H(z^{-1})$

② $\frac{1}{2}\left(X(z^{1/2})H(z^{-1/2}) + X(-z^{1/2})H(-z^{-1/2})\right)$

③ $\frac{1}{2}\left(X(z)H(z^{-1}) + X(-z)H(-z)\right)$

④ $\frac{1}{2}H(z)\left[X(z)H(z^{-1}) + X(-z)H(-z^{-1})\right]$

IN FOURIER DOMAIN

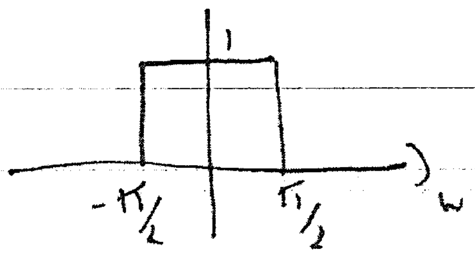
① $X(e^{j\omega})H(e^{-j\omega})$

② $\frac{1}{2}\left(X(e^{j\omega/2})H(e^{-j\omega/2}) + X\left(e^{j\left(\frac{\omega}{2}+\pi\right)}\right)H\left(e^{-j\left(\frac{\omega}{2}+\pi\right)}\right)\right)$

③ $\frac{1}{2}\left[X(e^{j\omega})H(e^{-j\omega}) + X(e^{j\omega+\pi})H(e^{-j\omega+\pi})\right]$

④ $\frac{1}{2}H(e^{j\omega})\left[X(e^{j\omega})H(e^{-j\omega}) + X(e^{j(\omega+\pi)})H(e^{-j(\omega+\pi)})\right]$

$H(e^{j\omega}) = H(e^{-j\omega})$



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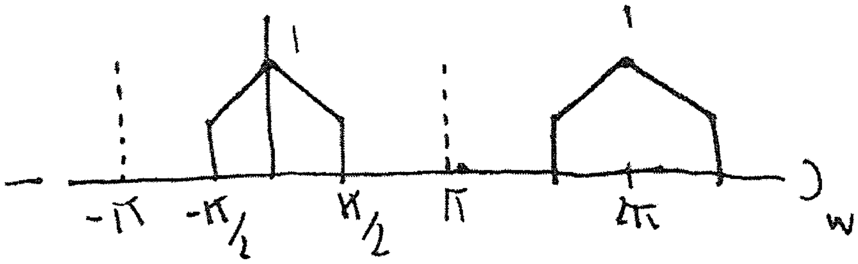

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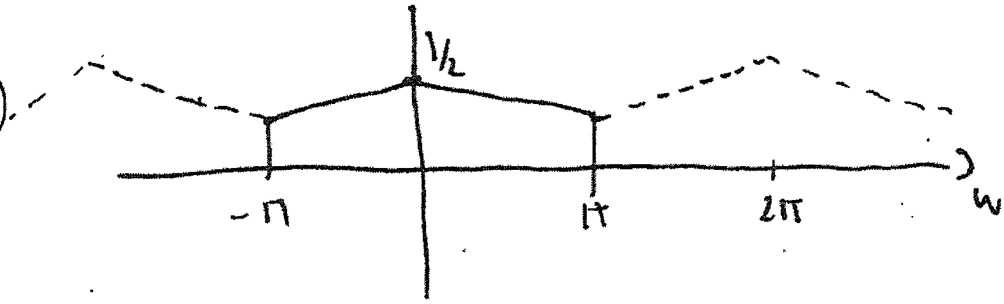

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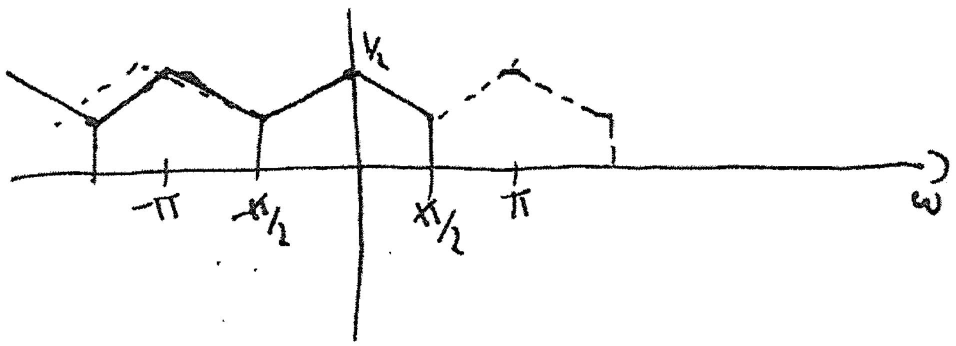

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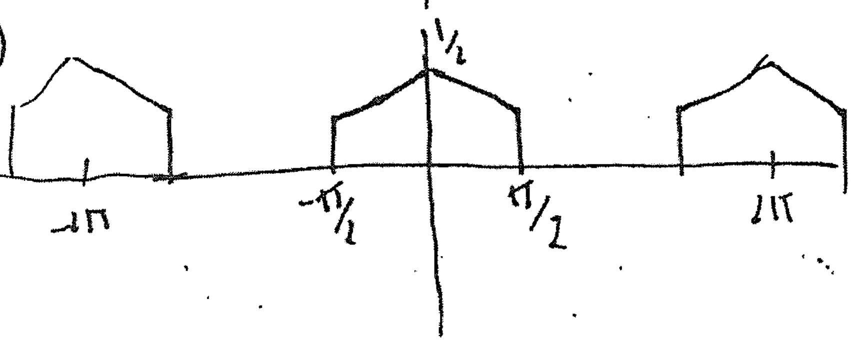

QUESTION 1.2

~~QUESTION~~

(a)

$$\hat{X}(z) = \frac{1}{2}\left[G(z^{1/2})H(z^{1/2}) + G(-z^{1/2})H(-z^{1/2})\right]X(z)$$

PR CONDITION:

$$G(z)H(z) + G(-z)H(-z) = 2 \quad \text{OR}$$

$$P(z) + P(-z) = 2$$

(b) $G(z) = a$ DOES NOT WORK

TRY $G(z) = az^{-1} + b + az$

$$P(z) = (z^{-2} + z^{-1} + 1 + z + z^{2})(az^{-1} + b + az) =$$

$$= az^{-3} + (a+b)z^{-2} + (2a+b)z^{-1} + (2a+b) + (2a+b)z + (a+b)z^{2} + az^{3}.$$

$$P(z) + P(-z) = 2 \Rightarrow \begin{cases} a+b = 0 \\ 2a+b = 1 \end{cases} \Rightarrow \begin{matrix} a = 1 \\ b = -1 \end{matrix}$$

$$G(z) = (z^{-1} - 1 + z)$$

(c) $\hat{X}(z) = 0 \iff P(z) + P(-z) = 0$

THIS IS ACHIEVED WHEN $G(t) = (1 - t^{-1})$

IN THIS CASE WE HAVE

$$P(z) = (1 + z + z^2 + z^3)(1 - t^{-1}) = z^3 - z^{-1}$$

THUS

$$P(z) + P(-z) = 0$$

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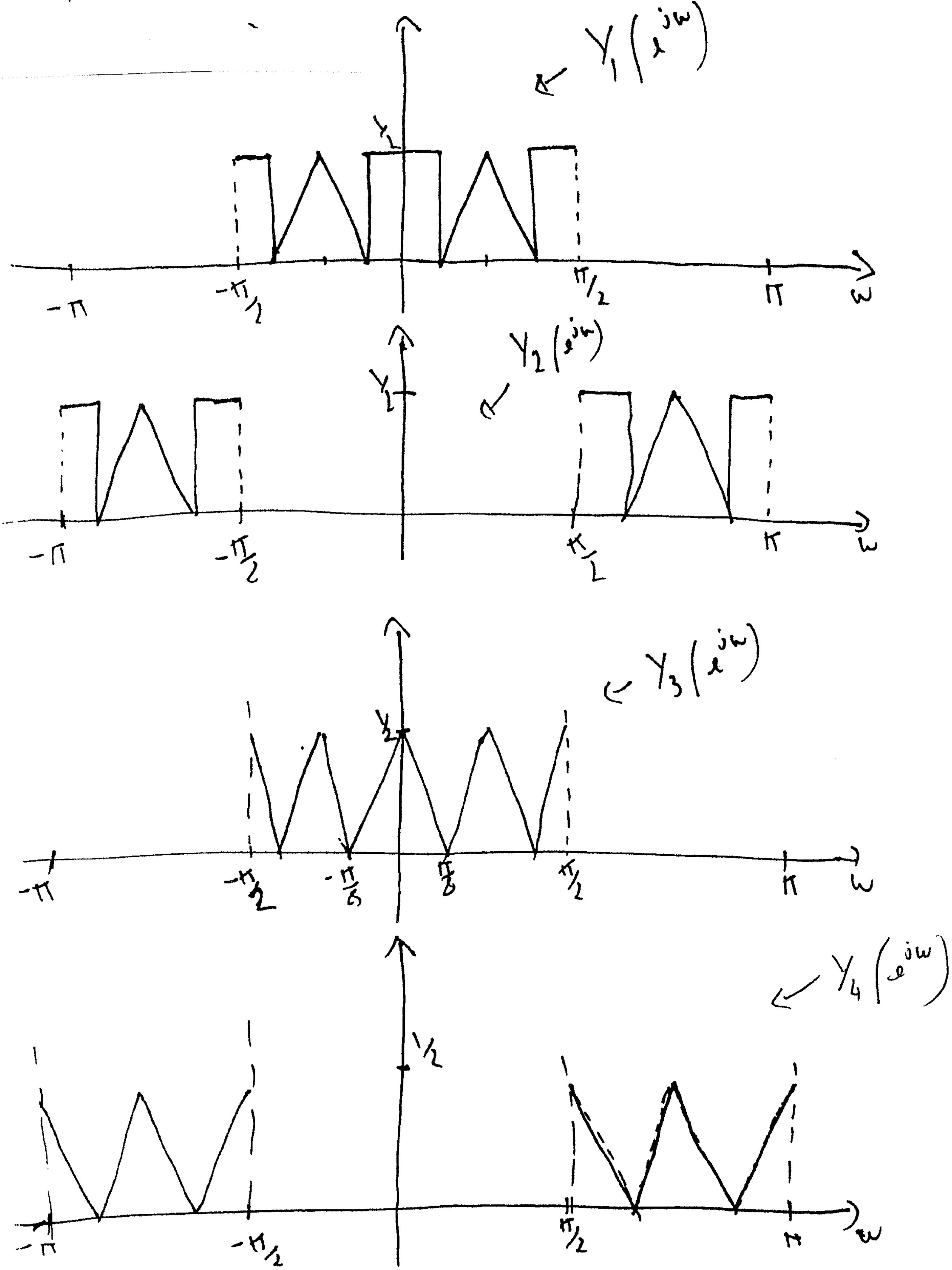
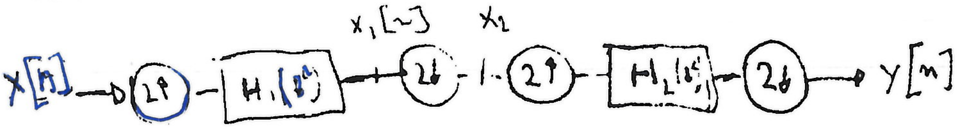
QUESTION 1.3


$Y_1(e^{jw})$
$1/2$
$-\pi$
$-\pi/2$
$\pi/2$
π
w
$Y_2(e^{jw})$
$1/2$
$-\pi$
$-\pi/2$
$\pi/2$
π
w
$Y_3(e^{jw})$
$1/2$
$-\pi$
$-\pi/2$
$-\pi/8$
$\pi/8$
$\pi/2$
π
w
$Y_4(e^{jw})$
$1/2$
$-\pi$
$-\pi/2$
$\pi/2$
π
w

EXERCISE 1.4

8

a.



$$X_1(z) = H_1(z^2)\, X(z^2)$$

$$X_2(z) = H_1(z)\, X(z)$$

$$Y(z) = H_2(z)\, H_1(z)\, X(z) \Rightarrow$$

$$H_{EQ}(z) = H_2(z)\, H_1(z)$$

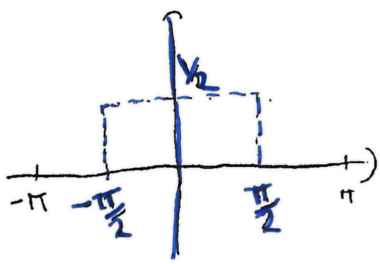
(b). (~~NEW EXAMPLE~~)

WHEN $M=2$

$$Y(z) = X(z)\,\frac{1}{2}\left(H(z^{1/2}) + H(-z^{1/2})\right)$$

SINCE $H(z)$ IS A LOW-PASS FILTER WITH CUT-OFF AT $\pi/4$

$$\frac{1}{2}\left(H(z^{1/2}) + H(-z^{1/2})\right) \Longleftrightarrow$$

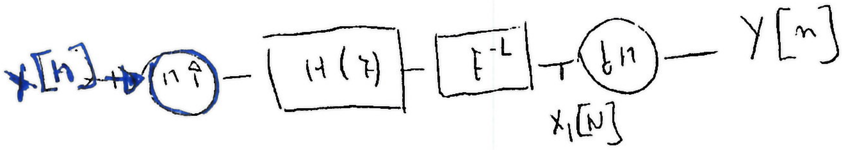


IS EQUIVALENT TO A LOW-PASS FILTER WITH CUT-OFF AT $\frac{\pi}{2}$ AND AMPLITUDE $1/2$

WHEN $M=4$

WE GET AN ALL PASS-FILTER WITH AMPLITUDE $1/4$

(C) (~~NEW EXAMPLE~~)



$$X_1(z) = H(z)\, z^{-L}\, X(z^M)$$

$$Y(z) = X(z) \cdot \frac{1}{M} \sum_{k=0}^{M-1} H\left(W_M^k z^{1/M}\right) z^{-L/M}\, W_M^{kL}$$

SINCE $H(z)$ IS IDEAL LOW PASS WITH CUT-OFF AT π/M, ALL THE ALIASED COMPONENT DON'T CONTRIBUTE AND $H(z^{1/M})$ IS AN ALL-PASS FILTER $\Rightarrow$

$$\frac{1}{M} \sum_{k=0}^{M-1} H\left(W_M^k z^{1/M}\right) z^{-L/M}\, W_M^{k} = \frac{1}{M} z^{-L/M}$$

THUS

$$H_{EQ}(z) = \frac{1}{M} z^{-L/M}$$