1.

Kraft's ineq: $\sum_{i=1}^{|X|} r^{-l_i} \leq 1$

$\sum r^{-l_i} = 4^{-1} \times 2 + 4^{-2} \times 7 = 0.9375 \leq 1$ ✓

To construct such a code we should make sure that any code is not a prefix of any other codeword in the code. We have

$\{0 . 1 . 20 . 21 . 22 . 30 . 31 . 32 . 33\}$ instantaneous code

$\{0.15 \, 0.5 \, 0.1 \, 0.1 \, 0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1\}$ prob

Expected length:

$E(L) = \sum P(l_i) l_i = 1 \times 0.15 \times 2 + 2 \times 0.1 \times 7$

2.

To check a codebook we first verify kraft's ineq:

if it satisfies $\Rightarrow$ there exists an instantaneous code.

if a code is <u>instantaneous</u>

(no codeword is a prefix of others)

$\downarrow$

uniquely decodable

(its extension must be non-singular)

$\downarrow$

non-singular $(x_1 \neq x_2 \Rightarrow C(x_1) \neq C(x_2))$

2.

a) $\{1, 01, 000, 001\}$

Kraft's ineq:

$$2^{-1} + 2^{-2} + 2^{-3} \times 2 \leq 1$$

inst. $\rightarrow$ uniquely decodable $\rightarrow$ non-singular

b) $\{0.10, 000, 100\}$

nonsingular (no codewords equal)? ✓

uniquely decodable (no ambiguity / different interpretation)? ✗

instantaneous (decode without ref. to future?) ✗

c) $\{01, 01, 110, 100\}$

$$\sum r^{-l_i} = 2^{-2} \times 2 + 2^{-3} \times 2 < 1$$

two codewords equal $\Rightarrow$ not nonsingular
↓
not u.d.
↓
not inst.

d) {0, 01, 011, 0111}

$$2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} < 1$$

non-singular ✓

u.d. ✓ (only one possible string. c^+ is non-singular)

inst. ✗ (0 is prefix of the others)

e) {10, 01, 0010, 0111}

non-singular ✓

u.d. ✓

inst. ✗ (01 is prefix of 0111)