Ex,

large-scale fading (shadowing) $S_{dB} = 10\log_{10} S \sim N(0, \sigma_s^2)$

M: $x = 10\log_{10} S \sim N(0, \sigma_s^2)$

calculate $S = 10^{\frac{x}{10}}$ → shadowing realisation

multiple realisations of shadowing

```
for i=1:1000
    x = randn(0, σs^2);
    S[i] = 10^(x/10);
end
```

plot(S)

$\sigma_s^2 = 8\,dB \qquad \sigma_s^2 = 10^{\frac{8}{10}}$

Ex2:

$h_i \sim CN(0,1)$

real imag

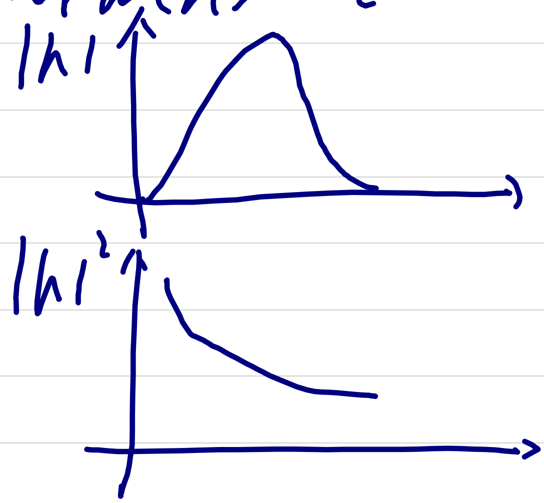
$h_i = h_{real} + i h_{imag}$

$CN(0,1)$ $N(0, \frac{1}{2})$ $N(0, \frac{1}{2})$

$= \frac{N(0,1)}{\sqrt{2}}$

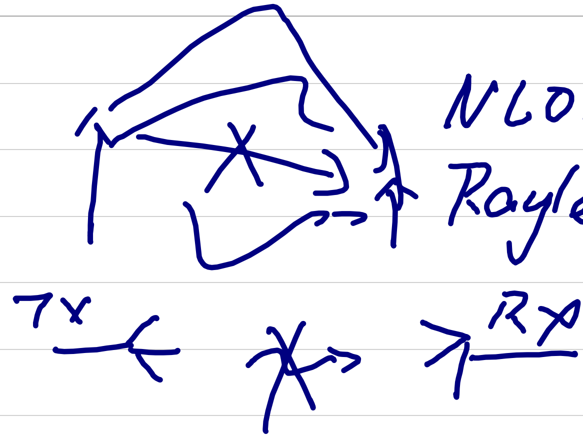
M:

```
for i = 1 : 10000
    hR = N(0,1)/sqrt(2) ~ 1/sqrt(2) * randn
    hI = N(0,1)/sqrt(2)
    h = hR + i hI
    norm[i] = abs(h)
    square abs.[i] = norm(h)^2
end
plot(norm)
plot(square abs)
```



① Ex2

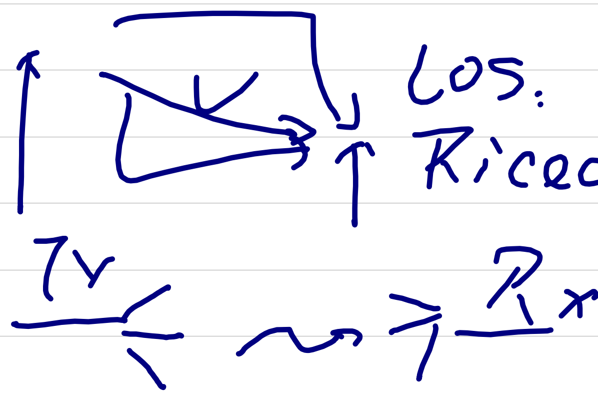
$h \sim CN(0,1)$



NLOS: Rayleigh

② Ex3

$h = h_{LOS} + h_{Rayleigh}$



LOS: Ricean

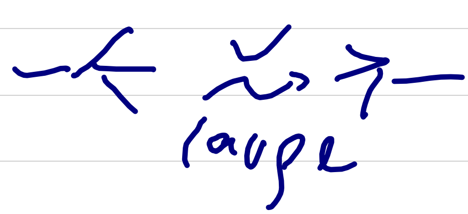
$$h = \sqrt{\frac{K}{K+1}}\,\bar{h} + \sqrt{\frac{1}{1+K}}\,\tilde{h}$$

K: how much energy is in LOS

Ricean factor

$K = 0 \Rightarrow h = \tilde{h}$ (Rayleigh)

$K \to \infty \Rightarrow h \approx \bar{h} = e^{j\phi}$



fix K

for i=1:10000

$h_R = \frac{N(0,1)}{\sqrt{2}}$

$h_I = \frac{N(0,1)}{\sqrt{2}}$

$\tilde{h} = h_R + i h_I$ → Rayleigh

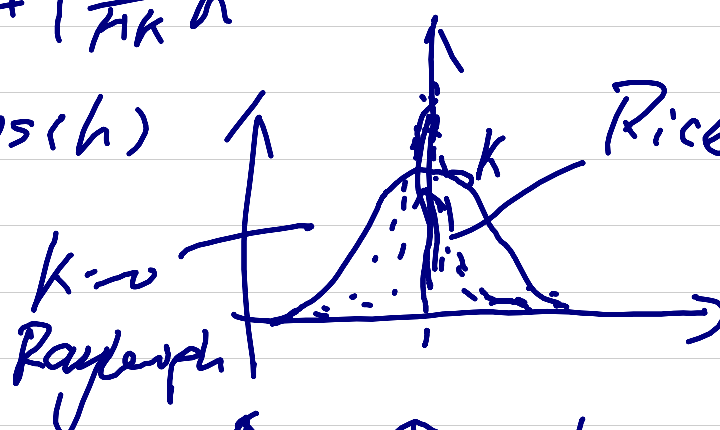
$\bar{h} = e^{j\phi} = 1$

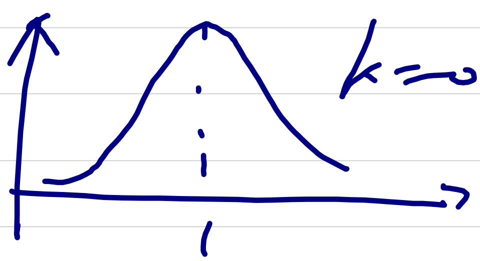
$h = \sqrt{\frac{K}{1+K}}\,\bar{h} + \sqrt{\frac{1}{1+K}}\,\tilde{h}$

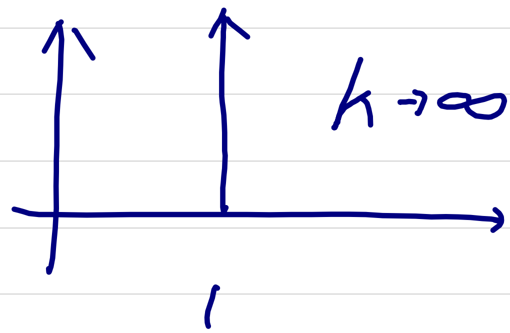
norm[i]=abs(h)

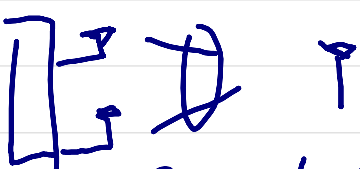
end

plot(norm)









correlated

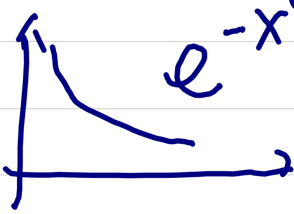
uncorrelated

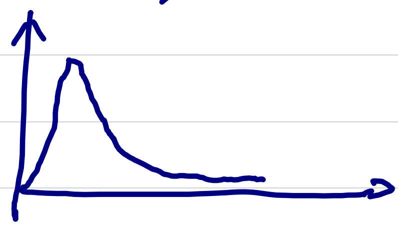
Ex4:

n complex normal RV

$$X_1 \cdot X_2 \cdots X_n \Rightarrow Y = |X_1|^2 + |X_2|^2 + \cdots + |X_n|^2$$

$n=1 \Rightarrow |X_1|^2$ (conly RV) $n=3$





$n=3$

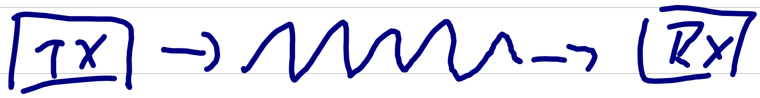
```
for i = 1:100
    h_R = N(0,1)/√2
    h_I = N(0,1)/√2
    h_1 = h_R + i h_I
    ⋮
    h_2 = h_R' + i h_I'

sum_nom_h[i] = abs(h_1)^2 + abs(h_2)^2
plot(sum_nom_h)
```

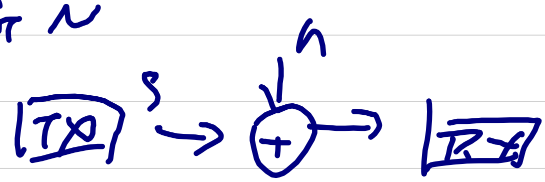
```
for i=1:100
  for j=1:n
    h = hR + j hz
    sum_norm.h[i] = sum_norm[i-1]
                    + abs(h)^2
```

Ex 5:

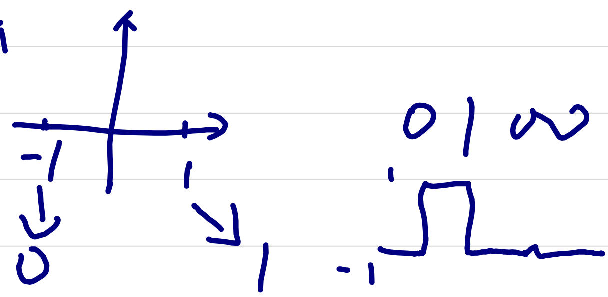
BPSK



① AWGN

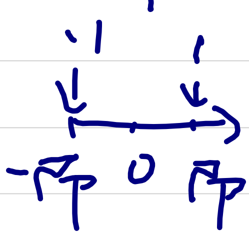


BPSK

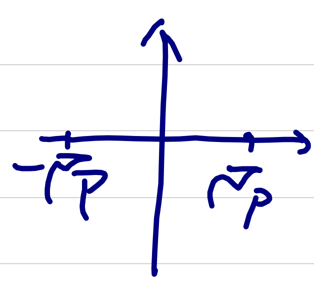


Power $= |s|^2$

$110100 \rightarrow \{1, 1, -1, 1, -1, -1\}$

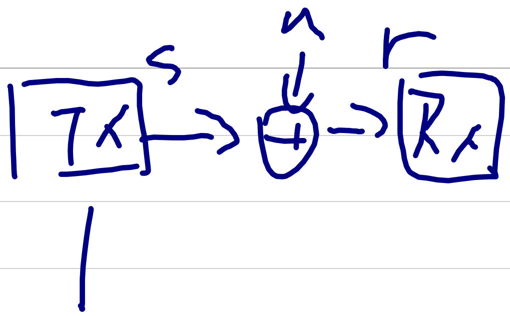


BPSK with power $P \Rightarrow$ symbols



$P = 10\text{dB} \Rightarrow 10 \Rightarrow 3..$

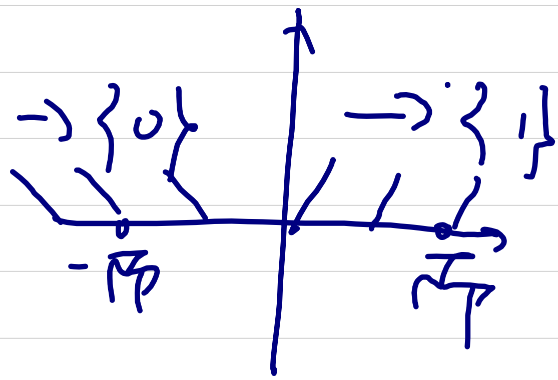
$P = 20\text{dB} \Rightarrow 100 \Rightarrow 10$



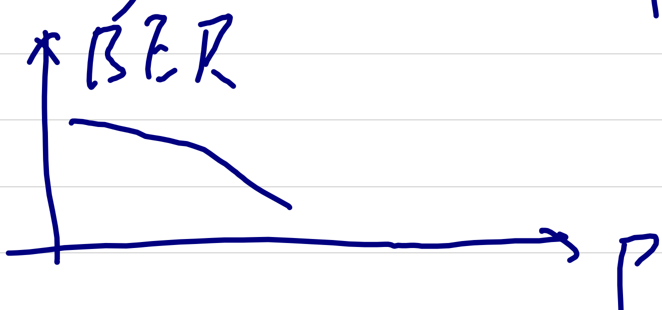
generate 10000 random bits

$0 \to (-1) \to -\sqrt{P}$

$1 \to \sqrt{P}$



$r = s + n \sim \mathcal{N}(0,1)$



$b_i \in \{0,1\} \mapsto s_i \in \{-\sqrt{P}, \sqrt{P}\}$

$\to \oplus \leftarrow n$

$\begin{cases} >0 \to 1 \\ <0 \to 0 \end{cases} \leftarrow Re\{y_i\} \leftarrow y_i = s_i + n$

Ex 5. 1)

BPSK/QPSK on AWGN channel

① BPSK

bit/data generation $\quad$ mapping $\quad$ power = SNR

$\underbrace{101\ldots011}_{1e^4} \rightarrow \boxed{-1 \quad +1} \rightarrow \boxed{\times\sqrt{P}}$ $\quad$ Tx

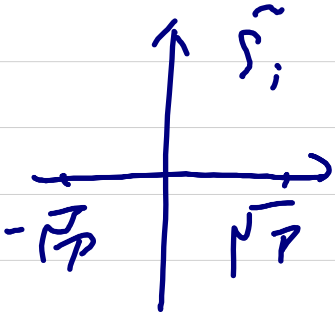
eg: $1 \rightarrow 1 \rightarrow \sqrt{P}$

$0 \rightarrow -1 \rightarrow -\sqrt{P}$

channel

$\rightarrow \boxed{s_i \oplus n_i} \quad$ CN $\rightarrow$

$n_i \sim CN(0,1)$



$\tilde{s}_i \rightarrow \begin{cases} 0. & Re\{\tilde{s}_i\} < 0 \\ 1. & Re\{\tilde{s}_i\} > 0 \end{cases}$

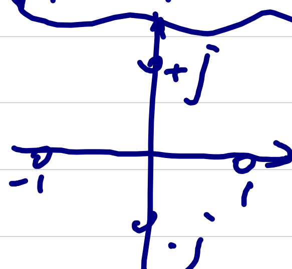
Rx

$$P_e = \frac{\#\text{error}}{\#\text{bits}}$$

② QPSK

Rx knows h_i

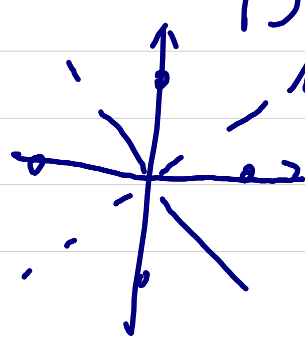
$\underbrace{00}_{1}\ \underbrace{10}_{-j}\ \underbrace{01}_{+j}\ \underbrace{00}_{1}\ \underbrace{10}_{-j} \rightarrow$



$\rightarrow \boxed{\sqrt{P}}$

$CN(0,1)$

$\rightarrow \oplus \rightarrow$



ML decoding

$\tilde{s}_i = s_i * h_i$

$\min D_i$

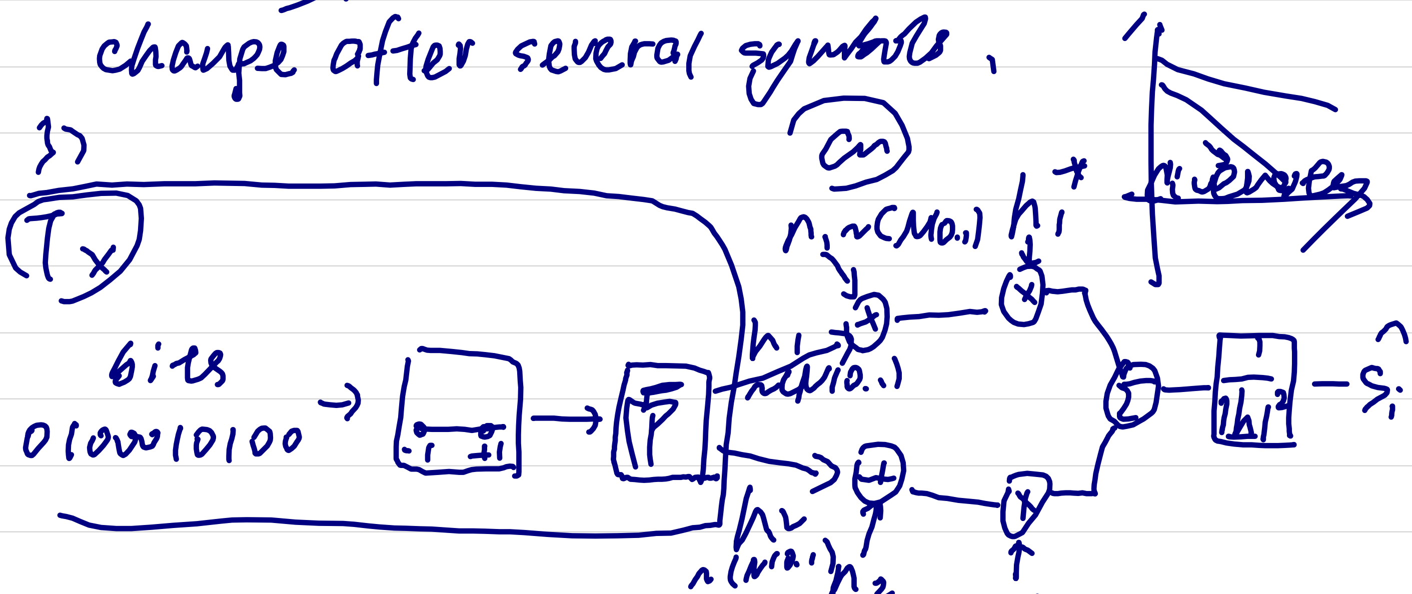
$$\hat{s}_i = \frac{\tilde{s}_i}{h_i}$$

$$P_e(\text{symbol}) = \frac{\#\text{ wrong symbol}}{\#\ T_x\text{ symbol}}$$

2) channel: $h_i s_i + CN(0,1)$. $h_i \sim CN(0,1)$

change after several symbols,

3)



6) MRT @ Rx not to do anything

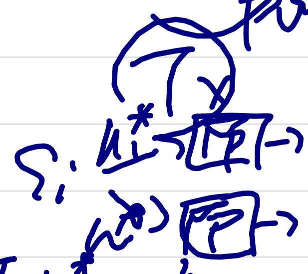
#noise = #receiver

MRT = MRC

(Tx need to know CN)

$h_1^* s_i$

$h_2^* s_i$



$$|\sqrt{P} h_1^* s_1|^2 + |\sqrt{P} h_2^* s_2|^2$$

$$= P(t)(|h_1|^2 + |h_2|^2)$$

$\shortparallel$

1

5)

$\underline{00}\,\underline{01}\,\underline{10}\,\underline{11}\,\underline{10}\,\underline{10}$
$S_6\ S_5\ S_4\ S_3\ S_2\ S_1$

2xsymbols each time

(S_2, S_1)

Alamouti: get diversity gain w/o CSI

$S_1, S_2 \to \begin{array}{c|c} -S_2^* & S_1 \\ S_1^* & S_2 \\ ② & ① \end{array}$ $\frac{1}{\sqrt{2}}$ h_1, h_2

$\to y^{①} = \frac{1}{\sqrt{2}}(h_1 S_1 + h_2 S_2) + n^{①}$

$y^{②} = \frac{1}{\sqrt{2}}(h_1(-S_2^*) + h_2 S_1^*) + n^{②}$

$[S_1\ S_2]\begin{bmatrix} -S_2^* \\ S_1^* \end{bmatrix}^* = 0$

orthogonal

$\Rightarrow y^{①} = \frac{1}{\sqrt{2}}(h_1 S_1 + h_2 S_2) + n^{①}$

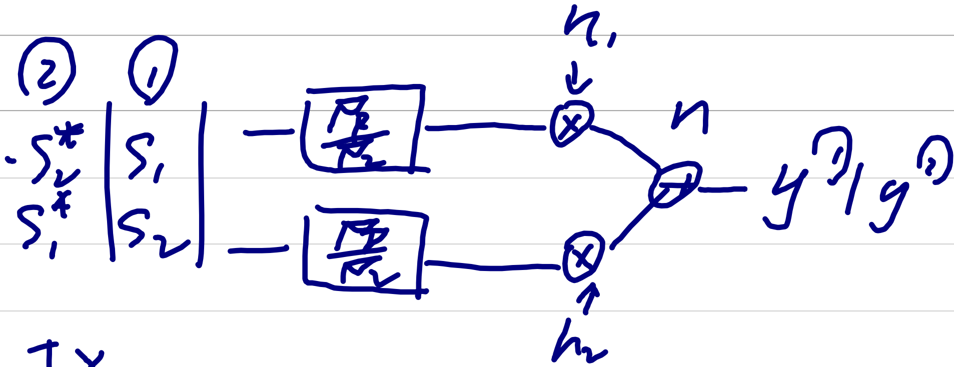
$y^{②*} = \frac{1}{\sqrt{2}}(-h_1^* S_2 + h_2^* S_1) + n^{②}$

$$\begin{bmatrix} y^{①} \\ y^{②*} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} n^{①} \\ n^{②} \end{bmatrix}$$

$$\hat{S}_1 = [h_1^*\ h_2]\begin{bmatrix} y^{①} \\ y^{②} \end{bmatrix} = \frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)S_1$$

$$\hat{S}_2 = [h_2^*\ -h_1]\begin{bmatrix} y^{①} \\ y^{②} \end{bmatrix} = \underbrace{\frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)}_{\text{diversity gain}} S_2$$

$\frac{1}{\sqrt{2}} \to 3\text{dB}$



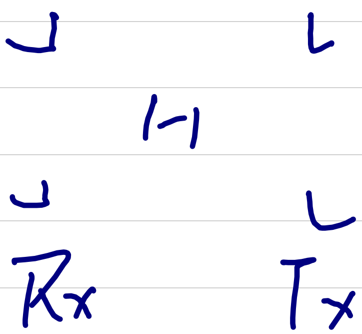
Tx

$$\Rightarrow \begin{cases} \hat{s}_1^{pre} = (h_1^* \;\; h_2)(y^{(1)}, y^{(2)}) \\ \hat{s}_2^{pre} = [h_2^* \;\; -h_1](y^{(1)}, y^{(2)}) \end{cases}$$

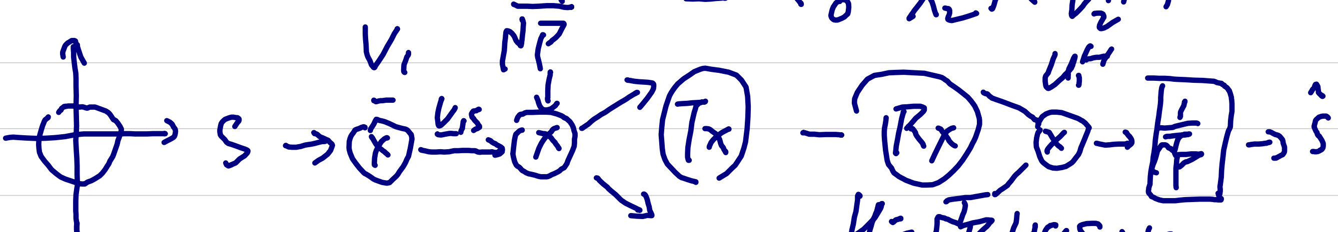
$$\Rightarrow \hat{s}_1 = \frac{\hat{s}_1^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

$$\hat{s}_2 = \frac{s_2^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

6. a) Dominant eigenmode transmission (DET)



$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \overset{SVD}{=} U\Lambda V^H = (\underline{u}_1|\underline{u}_2)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} \underline{v}_1^H \\ \underline{v}_2^H \end{pmatrix}$$



$\underline{y} = \sqrt{P} H \underline{u}_1 s + n$

$$\underline{y} = \sqrt{P} H \underline{v}_1 s + \underline{n}$$

$$\begin{array}{c} Rx_1 \\ Rx_2 \end{array}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \sqrt{P}\, U\Lambda \underline{V^H v_1} s + n$$

$$V^H v_1 = \begin{pmatrix} v_1^H \\ v_2^H \end{pmatrix} v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \sqrt{P}(\underline{u}_1|\underline{u}_2)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} s$$

$$= \sqrt{P}(\underline{u}_1|\underline{u}_2)\begin{pmatrix} \lambda_1 s \\ 0 \end{pmatrix} = \sqrt{P}\,\underline{u}_1 \lambda_1 s$$

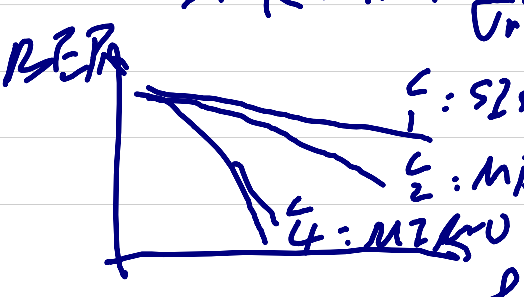
2x2 DET:
array gain = 4
div. gain = 4

2x2 Alamouti:
array gain = 2
(no CSIT: 3 dB loss)
d.v. gain = 4

Rx: $y = \sqrt{P} H \underline{u}_1 s + n$

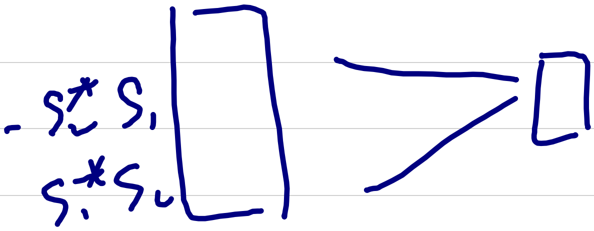
$$u_1^H y = \sqrt{P} H u_1^H u_1 \lambda_1 s + u_1^H n = \sqrt{P}\lambda_1 s + \underline{n}$$

$$SNR = |\lambda_1|^2 \frac{P}{\sigma_n^2 = 1} = \underline{\underline{|\lambda_1|^2 P}}$$



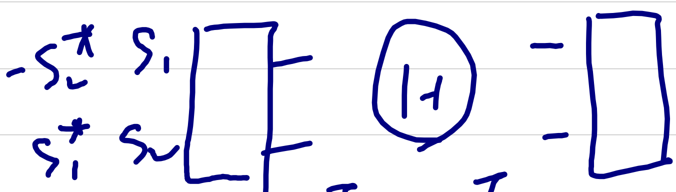
array gain = $|\lambda_1|^2$

b)



$$y(1) = h_1 s_1 + h_2 s_2 + n$$
$$y(2) = -h_1 s_2^* + h_2 s_1^* + n$$
$$\Rightarrow \underbrace{\begin{bmatrix} y(1) \\ y^*(2) \end{bmatrix}}_{y} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\hat{s}_1 = [h_1^* \;\; h_2]^T y \qquad \text{div. gain} = 2$$
$$\hat{s}_2 = [h_2^* \;\; -h_1]^T y$$



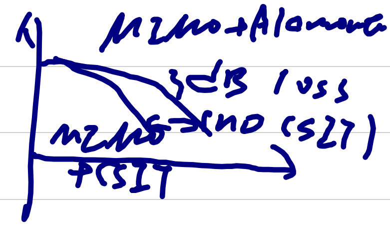
$$H = \begin{array}{c} \\ Rx_1 \\ Rx_2 \end{array} \begin{array}{c} \begin{array}{cc} Tx_1 & Tx_2 \end{array} \\ \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \end{array} = (\underline{h_1} \mid \underline{h_2})$$

$$\underline{y(1)} = \underline{h_1} s_1 + \underline{h_2} s_2 + \underline{n(1)}$$
$$\underline{y(2)} = -\underline{h_1} s_2^* + \underline{h_2} s_1^* + \underline{n(2)}$$
$$\Rightarrow \begin{bmatrix} \underline{y(1)} \\ \underline{y^*(2)} \end{bmatrix} = \begin{bmatrix} \underline{h_1} & \underline{h_2} \\ \underline{h_2}^* & -\underline{h_1}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \underline{n(1)} \\ \underline{n(2)} \end{bmatrix}$$

$$\hat{s}_1 = [\underline{h}_1^* \; \underline{h}_2^T] \, y$$

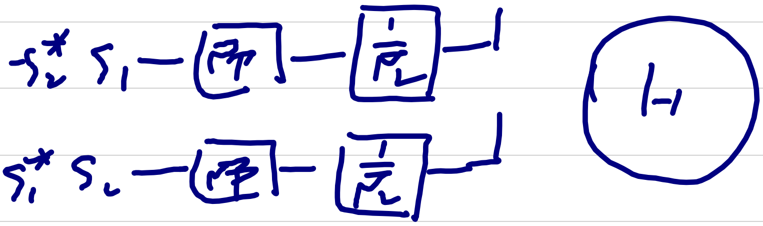
$$\hat{s}_2 = [\underline{h}_2^* \; -\underline{h}_1^T] \, y$$

$$\Rightarrow SNR = \frac{|h_1|^2 + |h_2|^2}{2} \Rightarrow$$



array gain = 1

diversity gain = 4



$y(1) \mid y(2)$

$\underline{2} \times 1 \quad \underline{2} \times 1$

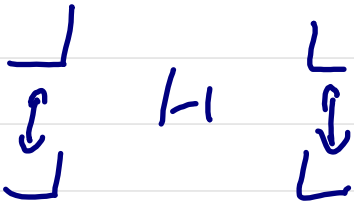
$$P_{tot} = \left(\frac{\sqrt{P}}{\sqrt{2}}\right)^2 \cdot 2 = P$$

$$\underline{y} = \begin{pmatrix} y(1) \\ \underline{y}(2)^* \end{pmatrix} \; 4 \times 1$$

$$\hat{s}_1 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_1^H \; \underline{h}_2^T] \, \underline{y}$$

$$\hat{s}_2 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_2^H \; -\underline{h}_1^T] \, \underline{y}$$

7.8



$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$

spacing $> \frac{\lambda}{2}$: h uncorrelated.

$< \frac{\lambda}{2}$: h correlated.

How correlation impact the performance?

Kronecker model — correlation at Rx. Tx (1.2).

$R_r = \begin{bmatrix} 1 & r^* \\ r & 1 \end{bmatrix}$ $R_t = \begin{bmatrix} 1 & t^* \\ t & 1 \end{bmatrix}$

$H = R_r^{\frac{1}{2}} H_w R_t^{\frac{1}{2}}$

no Rx correlation. only Tx correlation $\Rightarrow$ $R_r = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R_r^{\frac{1}{2}}$, $R_t = \begin{bmatrix} 1 & t \\ t & 1 \end{bmatrix}$

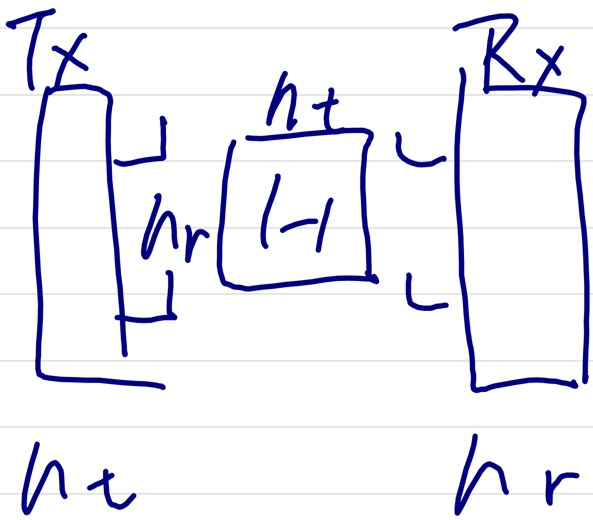
$H = H_w R_t^{\frac{1}{2}}$

task: repeat the previous exercise with channel $H = H_w R_t^{\frac{1}{2}}$ $(t = 0, 0.9)$

$E\{HH^H\} = R_r \otimes R_t$

$R_t^{\frac{1}{2}} \neq \begin{pmatrix} 1 & \sqrt{t} \\ \sqrt{t} & 1 \end{pmatrix}$

eusure at least 100 errors



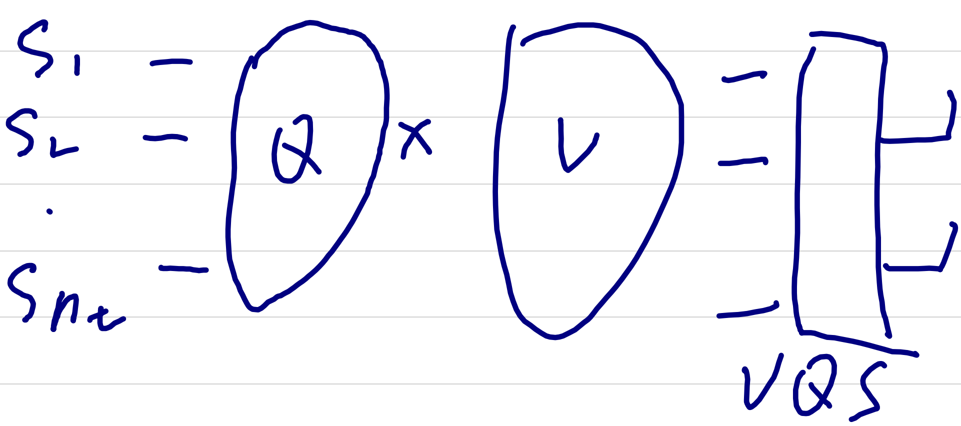
$$I(H,Q) = \log_2 \det(I + \rho H Q H^H)$$

$$C = \max_{Q} I(H,Q)$$

Q (obtained by WF)

$$\underset{n_r \times n_t}{H} = \underset{n_r \times n_r}{U} \overset{n_r \times n_t}{\Lambda} \underset{n_t \times n_t}{V^H}$$

$$Q = \begin{bmatrix} Q_1 & & \\ & \ddots & \\ & & Q_{n_t} \end{bmatrix} \quad Tr(Q) = P$$



$$\underline{J} = U\Lambda V^H V Q S = U \Lambda Q S$$

$$\underbrace{\underbrace{U^H}_{Tx} U}_{I} \Lambda \underbrace{V^H V}_{I} Q S + n = \Lambda Q S + n$$

$$\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{n_t} \end{pmatrix} \begin{pmatrix} q_1 & & \\ & \ddots & \\ & & q_{n_t} \end{pmatrix} \begin{pmatrix} s_1 \\ \vdots \\ s_{n_t} \end{pmatrix}$$

$$C = \sum_{i=1}^{n_t} \log_2 (1 + \lambda_i^2 q_i P) \qquad \begin{pmatrix} \lambda_1 q_1 s_1 \\ \lambda_2 q_2 s_2 \\ \vdots \\ \lambda_{n_t} q_{n_t} s_{n_t} \end{pmatrix}$$

$$H = U \Lambda V^H$$

$$H H^H = U \Lambda \underbrace{V^H V}_{I} \Lambda^H U^H = U \Lambda^2 U^H$$

$Tx \to V$

$Rx \to U^H$

$\boxed{2\times 2}$

$\boxed{2\times 4}$

$\boxed{4\times 2}$

WF

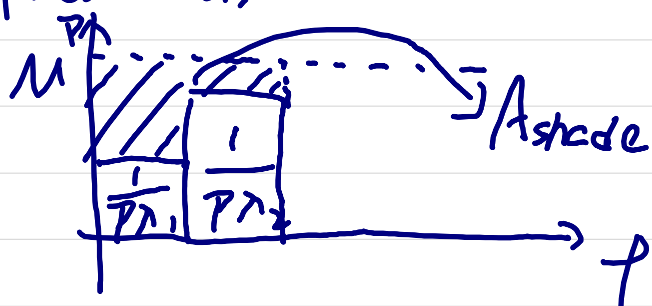
$$C = \log_2(1+\lambda_1^2 q_1) + \log(1+\lambda_2^2 q_2)$$

$$H = U\Lambda V^H \rightarrow (\lambda_1, \lambda_2)$$

$$q_1 + q_2 = P \Rightarrow \begin{cases} q_1 = x_1 P \\ q_2 = x_2 P \\ x_1 + x_2 = 1 \end{cases}$$

$N_s = \min(n_t, n_r) = 2$

#streams



$A_{\text{shade}} = 1$

$$\frac{SVD(H):\ 2}{SVD(H^H H):\ \text{no square}}$$

$$\left(\mu_i - \frac{1}{P\lambda_1^2}\right)^+ + \left(\mu_i - \frac{1}{P\lambda_2^2}\right)^+ = 1$$

μ^* such that

$$\max\left\{\mu^* - \frac{1}{P\lambda_1^2}, 0\right\} + \max\left\{\mu^* - \frac{1}{P\lambda_2^2}, 0\right\} = 1$$

$$H = U \Sigma V^H$$

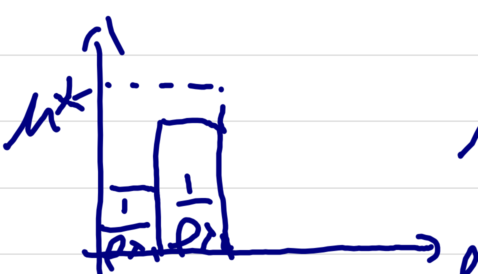
$$\Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

$$\downarrow$$

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \qquad q_1 + q_2 = 1$$

$$\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} \to Q \to V \to \overset{Tx}{\boxed{\vec{P}}}$$

$$C = \log_2(1 + Pq_1 \underbrace{\sigma_1^2}_{=\lambda_1}) + \log_2(1 + Pq_2 \underbrace{\sigma_2^2}_{=\lambda_2})$$



μ^* such that

$$\left\{\mu^* - \frac{1}{\rho\lambda_1}\right\}^+ + \left\{\mu^* - \frac{1}{\rho\lambda_2}\right\}^+ = 1$$

① generate matrix H

② SVD $H = U\Sigma V^H$ ③ $\Sigma = \begin{pmatrix}\sigma_1 & \\ & \sigma_2\end{pmatrix} \to \Lambda = \begin{pmatrix}\lambda_1 & \\ & \lambda_2\end{pmatrix}$

④ 

$$\left(\mu^* - \frac{1}{\rho\lambda_1}\right)^+ + \left(\mu^* - \frac{1}{\rho\lambda_2}\right)^+ = 1$$

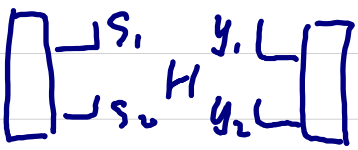
⑤ calculate $C = \log_2(1+\rho\lambda_1 s_1) + \log_2(1+\rho\lambda_2 s_2)$

$$C = \mathbb{E}\left(\log_2\det\left(I + \frac{\rho}{n_t}HH^H\right)\right)$$

avg. power

11.

2) ML receiver



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$\{\hat{s}_1 . \hat{s}_2\}$: all possible combinations

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = H \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} \quad \arg\min_{\hat{s}_1 . \hat{s}_2} \| y - \hat{y} \|^2$$

3) ZF

$$y = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{pmatrix} = H^{-1} y$$

4) MMSE

ZF $T = H^{-1}$

$$\tilde{s} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{bmatrix} = T \underline{y}$$

MMSE $T = \sqrt{\frac{2}{E_s}} H^{H} \left(H H^{H} + \frac{2}{\rho} I \right)^{-1}$

5) unordered

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = (h_1 | h_2) \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + n$$

$$= h_1 s_1 + h_2 s_2 + n$$

ZF-SIC: estimate one, remove one . . .

$$G = H^{-1} \quad G = \begin{pmatrix} G(1,:) \\ G(2,:) \end{pmatrix}$$

$$\tilde{s}_1 = G(1,:)\underline{y}$$

$$\tilde{s}_1 \to \hat{s}_1$$

$$\underline{\hat{y}} = \underline{y} - \underline{h}_1 s_1 = \underline{h}_2 s_2 + n \overset{MRC}{\Longrightarrow} \hat{s}_2 = \frac{\underline{h}_2^H}{\|\underline{h}_2\|} \hat{y}$$

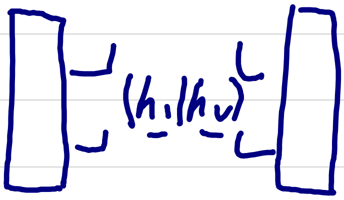
6) ordered

$$G = \begin{pmatrix} G(1,:) \\ G(2,:) \end{pmatrix}$$

Choose $G(i,:)$ such that $|G(i,:)|$ is min

$$\min\{|G(1,:)|, |G(2,:)|\}$$

7) Alamouti



$$\underline{y}(1) = H\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \underline{h}_1 s_1 + \underline{h}_2 s_2 + n_1$$

$$\underline{y}(2) = H\begin{pmatrix} -s_2^* \\ s_1^* \end{pmatrix} = -\underline{h}_1 s_2^* + \underline{h}_2 s_1^* + n_2$$

$$\begin{array}{c} \begin{pmatrix} -s_2^* & s_1 \\ s_1^* & s_2 \end{pmatrix} \\ \uparrow \quad\;\; \uparrow \\ 2 \quad\;\; 1 \end{array}$$

$$\begin{bmatrix} y(1) \\ y(2)^* \end{bmatrix} = \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \\ \underline{h}_2^* & -\underline{h}_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \underline{n}_1 \\ \underline{n}_2 \end{bmatrix}$$

$$\tilde{s}_1 = [\underline{h}_1^H \;\; \underline{h}_2^T] \begin{bmatrix} y(1) \\ \underline{y(2)^*} \end{bmatrix}$$

$$\tilde{s}_2 = [\underline{h}_2^H \;\; -\underline{h}_1^T] \begin{bmatrix} y(1) \\ y(2) \end{bmatrix}$$

12.

$$H = R_{Tx}^{i} H_W R_{Rx}^{i}$$