Ex,

large-scale fading S_{dB} ~ $N(0, \sigma_s^2)$
↑shadowing $= 10 \log_{10} S$

M: $x = 10 \log_{10} S \sim N_x(0, \sigma_s^2)$

calculate $S = 10^{\frac{x}{10}}$ → shadowing realisation

multiple for $i = 1 : 1000$
realisations $x = \text{rand}_N(0, \sigma_s^2)$;
of
shadowing $S[i] = 10^{\frac{x}{10}}$;

end

$P(u \ t \ | \ s)$

$\sigma_s^2 = 8 \ dB$ $\sigma_s^2 = 10^{\frac{8}{10}}$

Ex2:

$h_i \sim CN(0,1)$

real imag

$h_i = h_{real} + i h_{imag}$

$CN(0,1)$ $N(0, \frac{1}{2})$ $N(0, \frac{i}{2})$

$= \dfrac{N(0,1)}{\sqrt{2}}$

M:

   for $i = 1 : 10000$

     $h_R = \dfrac{N(0,1)}{\sqrt{2}} \sim \dfrac{1}{\sqrt{2}} \times randn$

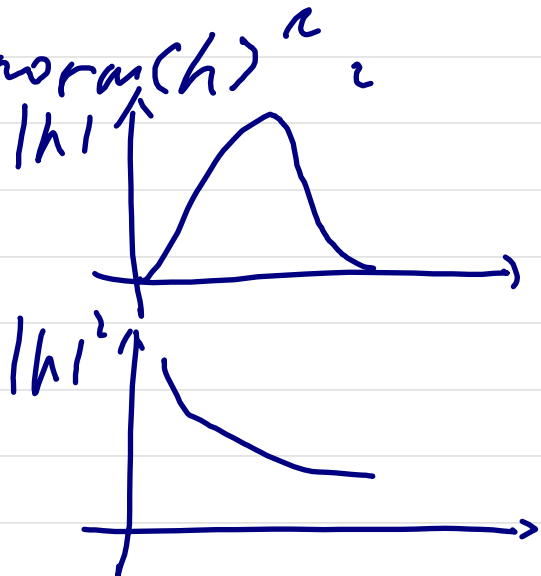 $h_I = \dfrac{N(0,1)}{\sqrt{2}}$

     $h = h_R + i h_I$

     $norm[i] = abs(h)$

     square abs $[i] = norm(h)^2$

   end

   plot (norm)

   plot (square abs)

① Ex2

$h \sim (N(0,1))$

NLOS:
Rayleigh

TX $\xleftarrow{}$ $\times$ $\rightarrow$ $\nearrow$ RX
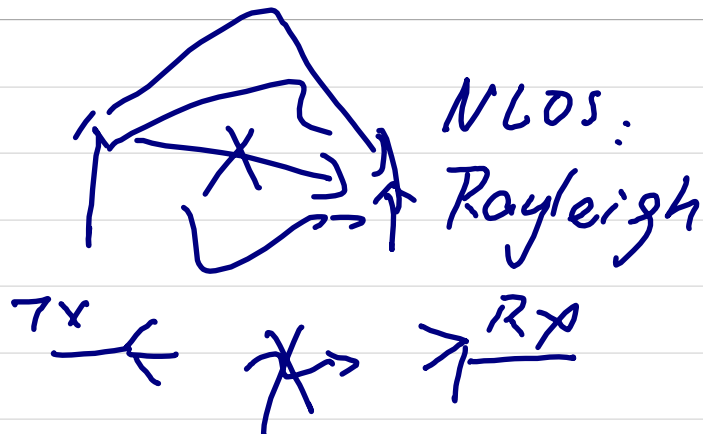
② Ex3

$h = h_{LOS} + h_{Rayleigh}$

LOS:
Ricean

TV $\xleftarrow{}$ $\sim$ $\nearrow$ RX

$$h = \sqrt{\frac{K}{K+1}} \bar{h} + \sqrt{\frac{1}{1+K}} \tilde{h}$$

k : how much $\overset{energy}{}$ is in LOS

Ricean factor

$k = 0 \implies h = \tilde{h}$ (Rayleigh)   TX $\xleftarrow{}$ $\times$ $\rightarrow$ $\nearrow$ RX

$k \to \infty \implies h \approx \bar{h} = e^{j\phi}$ $\xleftarrow{}$ $\sim$ $\nearrow$
                                                    larger

fix k
   for i = 1 : 10000
     $h_R = \dfrac{N(0,1)}{\sqrt{2}}$
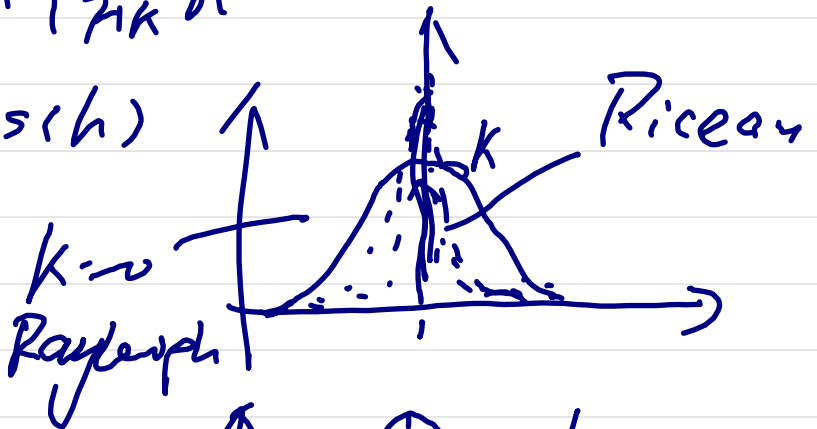 $h_I = \dfrac{N(0,1)}{\sqrt{2}}$
     $\tilde{h} = h_R + i h_I \rightarrow$ Rayleigh
     $\bar{h} = e^{j\phi} = 1$
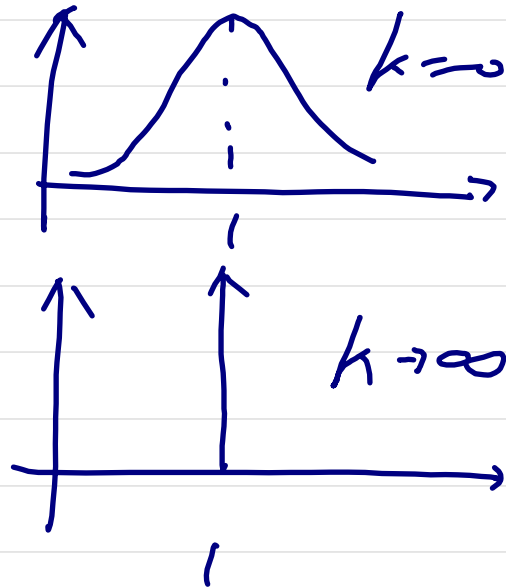 $h = \sqrt{\dfrac{k}{1+k}}\,\bar{h} + \sqrt{\dfrac{1}{1+k}}\,\tilde{h}$
     norm[i] = abs(h)
   end
   plot(norm)

$k \to 0$
Rayleigh

Ricean

correlated
uncorrelated

$k \to 0$

$k \to \infty$

1

Ex 4:

n complex normal R V

$$X_1 . X_2 .. X_u \Rightarrow Y = |X_1|^2 + |X_2|^2 + .. + |X_u|^2$$
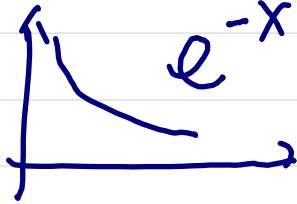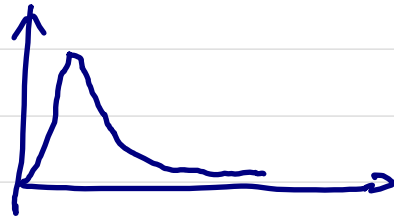
$n=1 = |X_1|^2$ (only 1 Rv) $n=3$

e^{-x}

$n=3$

for $l = 1 : 100$
  $h_R = \dfrac{N(0,1)}{\sqrt{2}}$
  $h_I = \dfrac{N(0,1)}{\sqrt{2}}$
  $h_1 = h_R + i h_1$
  $\vdots$
  $h_2 = h_R' + i h_2'$

sum_norm_h $[i] = abs(h_1)^2 2 + abs(h_2)^2 2$
plot (sum_non_h)

```
for i=1:low
   for j=1:n
      h=hR+j hz
      sum_norm.h[i] = sum_norm[i-1]
                    + abs(h)^2
```

Ex 5:

BPSK $\boxed{TX}$ → ∿∿∿ → $\boxed{RX}$

① AWGN

$\boxed{TX}$ $\xrightarrow{S}$ $\overset{\overset{n}{\downarrow}}{\oplus}$ → $\boxed{RX}$
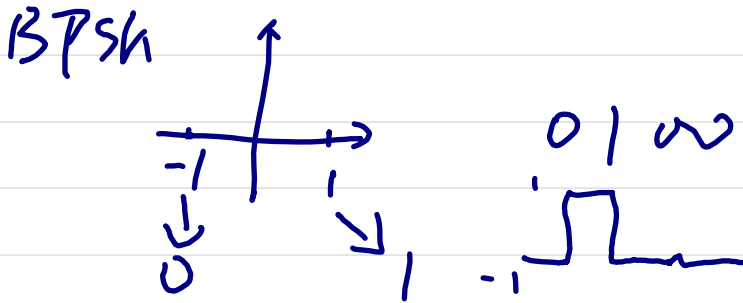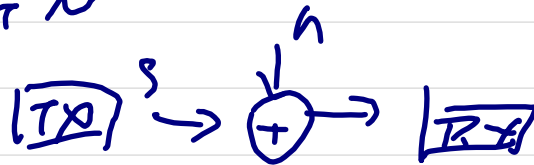
BPSK

$$\text{Power} = |S|^2$$

$110100 \rightarrow \{1, 1, -1, 1, -1, -1\}$
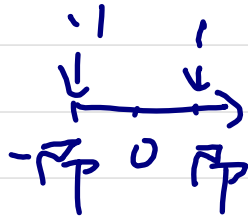
BPSK with power $P \Rightarrow$ symbols

$P = 10 \, dB \Rightarrow 10 \Rightarrow 3 ...$

$P = 20 \, dB \Rightarrow 100 \Rightarrow 10$

$$\boxed{TX} \xrightarrow{s} \overset{n}{\underset{\downarrow}{\oplus}} \xrightarrow{r} \boxed{RX}$$
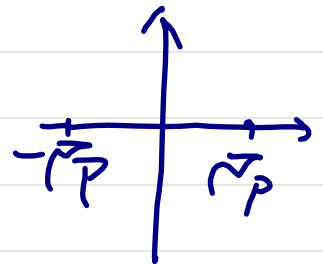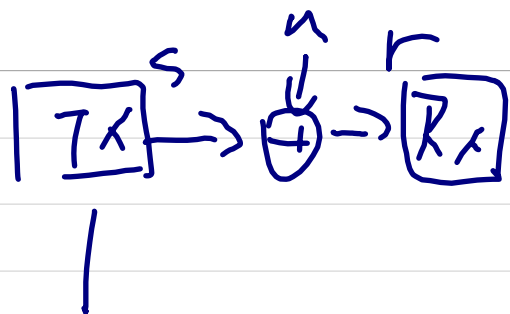
generate $1/w0$ random bits

$$0 \to (-1) \to -\sqrt{p}$$
$$1 \to \sqrt{p}$$

$$r = s + n \sim \mathcal{N}(0,1)$$

$$b_i \in \{0,1\} \longmapsto s_i \in \{-\sqrt{p}, \sqrt{p}\}$$

$$\oplus \leftarrow n$$

$$\begin{cases} >0 \to 1 \\ <0 \to 0 \end{cases} \leftarrow Re\{y_i\} \leftarrow y_i = s_i + n \leftarrow$$

Ex 5. 1)
BPSK/QPSK on AWGN channel

① BPSK

bit/data generation

$101 \ldots 011$ $\underbrace{}_{1e^4}$ $\rightarrow$ mapping $\boxed{\begin{array}{c} \cdot \quad \cdot \\ \hline -1 \quad +1 \end{array}} \rightarrow$ power·SNR $\boxed{\sqrt{P}}$ $\left(\! \begin{array}{c} ? \\ \end{array} \!\right)$
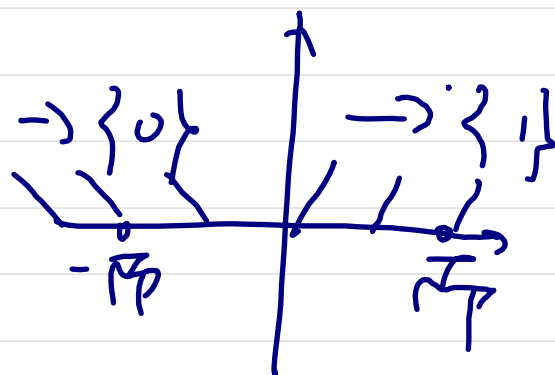
eg: $1 \rightarrow 1 \rightarrow \sqrt{P}$
$0 \rightarrow -1 \rightarrow \sqrt{P}$

channel

$\rightarrow \boxed{S_i \oplus} \; \textcircled{CN} \rightarrow$

$S_i \downarrow CN(0.1)$

$\uparrow \tilde{S}_i$ $\tilde{S}_i \rightarrow \begin{cases} 0. \; Re\{\tilde{S}_i\} < 0 \\ 1. \; Re\{\tilde{S}_i\} > 0 \end{cases}$ $\widehat{R_x}$

$\xrightarrow{\quad} -\sqrt{P} \;|\; \sqrt{P}$

$P_e = \dfrac{\#error}{\#bits}$

$\left(\overline{R_x \; knows \; h_i}\right)$

② QPSK

$\underset{1}{\overset{00}{\smile}} \; \underset{-j}{\overset{10}{\smile}} \; \underset{+j}{\overset{01}{\smile}} \; \underset{1}{\overset{00}{\smile}} \; \underset{-j}{\overset{10}{\smile}} \rightarrow$

$CN(0.1)$
$\downarrow$
$\rightarrow \oplus \rightarrow$

$+j$
$\xrightarrow{\quad}$
$-j$
$\rightarrow \boxed{\sqrt{P}}$

$\sqrt{2N}$
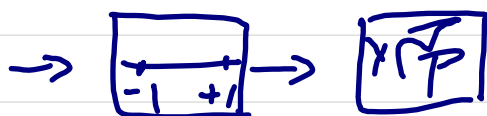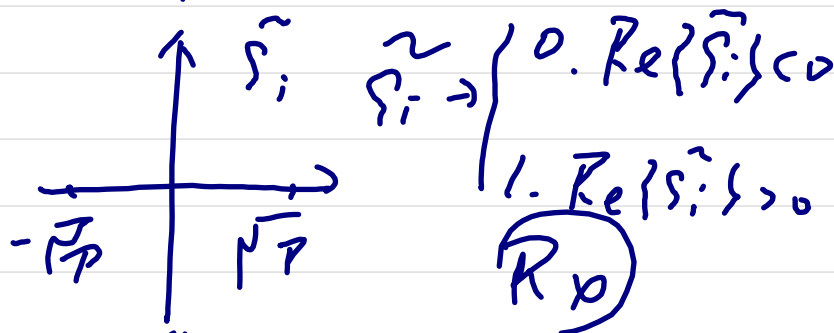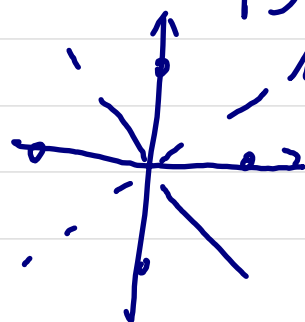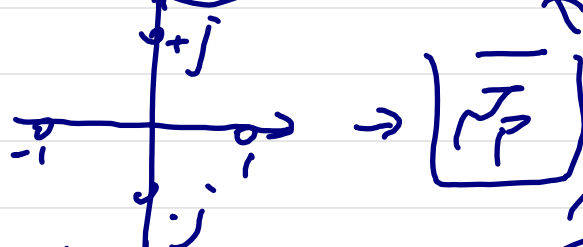
$\hat{S}_i = \dfrac{\tilde{S}_i}{h_i}$

ML decoding
$\tilde{S}_i = S_i + h_i$
$\min D_i$

$$P_e(\text{symbol}) = \frac{\# \text{ wrong symbols}}{\# Tx \text{ symbol}}$$

2) channel: $h_i s_i + CN(0,1)$. $h_i \sim CN(0,1)$
   change after several symbols.

3)

$n_1 \sim CN(0,1)$ h_i^+ receiver

$\boxed{Tx}$

bits
01001 0100 $\rightarrow$

$h_1 \sim CN(0,1)$

$n_2 \sim CN(0,1)$ h_2

h_2^+

$\frac{1}{|h|^2} - \hat{s}_i$

$\# \text{noise}$

4) MRT $\circlearrowright$ Rx not to do anything $\quad / = \# \text{receiver}$

bits
00100 $\rightarrow$ s_i

h_i^+

$\boxed{\frac{1}{|h_1^+ h_1|}}$

h_2^+

$\boxed{\frac{1}{|h_2^+ h_2|}}$

h_1

n

$\rightarrow s_i?$

h_2

power
normalize channel

MRT
$\equiv$ MRC
(Tx need
to know CN)
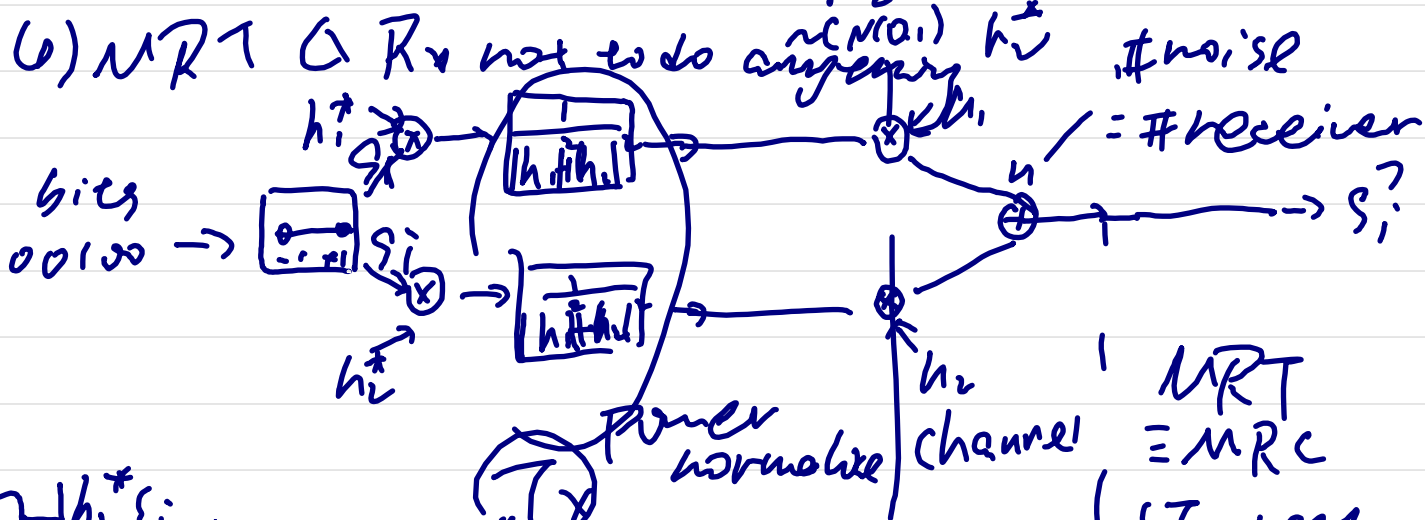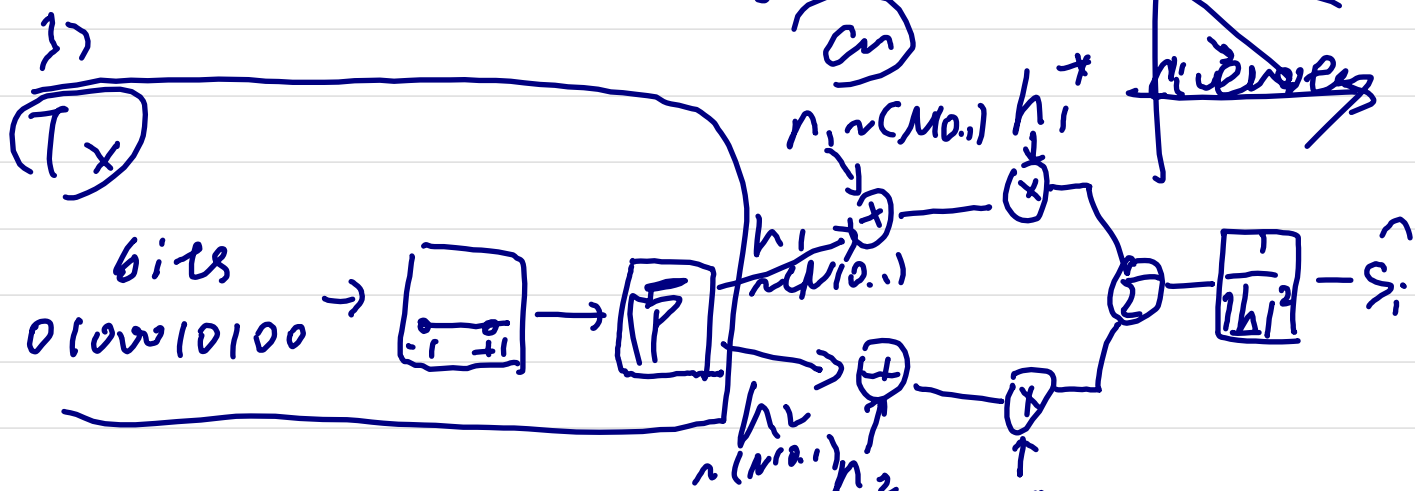
$\begin{bmatrix} h_1^* s_i \\ h_2^* s_i \end{bmatrix}$ 4 $\boxed{}$ $s_i \cdot h_1^* \boxed{\frac{1}{|P|}} \rightarrow$

$\xrightarrow{n(s)} \boxed{P} \rightarrow$

$|\sqrt{P} h_1^* s_1|^2 + |\sqrt{P} h_2 s_2|^2$

$= P \left(\frac{E_s}{2}\right)(|h_1|^2 + |h_2|^2)$

5)

$\underline{00}\underline{01}\underline{10}\underline{11}\underline{10}\overline{10}$
$S_6\ S_5\ S_4\overline{S_3}\ \overline{S_2}\ \overline{S_1}$

2 × symbols
each time

(S_2, S_1)

Alamouti: get diversity gain w/o CSI

$S_1, S_2 \Rightarrow \begin{vmatrix} S_2^* & S_1 \\ S_1^* & S_2 \\ \textcircled{2} & \textcircled{1} \end{vmatrix} \boxed{\frac{1}{\sqrt{2}}} \overset{3}{\underset{P}{\rightrightarrows}} \overset{h_1}{\underset{h_2}{\searrow}} \oplus \rightarrow$

$y^{\textcircled{1}} = \frac{1}{\sqrt{2}}(h_1 S_1 + h_2 S_2) + n^{\textcircled{1}}$

$y^{\textcircled{2}} = \frac{1}{\sqrt{2}}(h_1(-S_2^*) + h_2 S_1^*) + n^{\textcircled{2}}$

$[S_1\ S_2]\begin{bmatrix} -S_2^* \\ S_1^* \end{bmatrix}^* = 0$

$\Rightarrow y^{\textcircled{1}} = \frac{1}{\sqrt{2}}(h_1 S_1 + h_2 S_2) + n^{\textcircled{1}}$

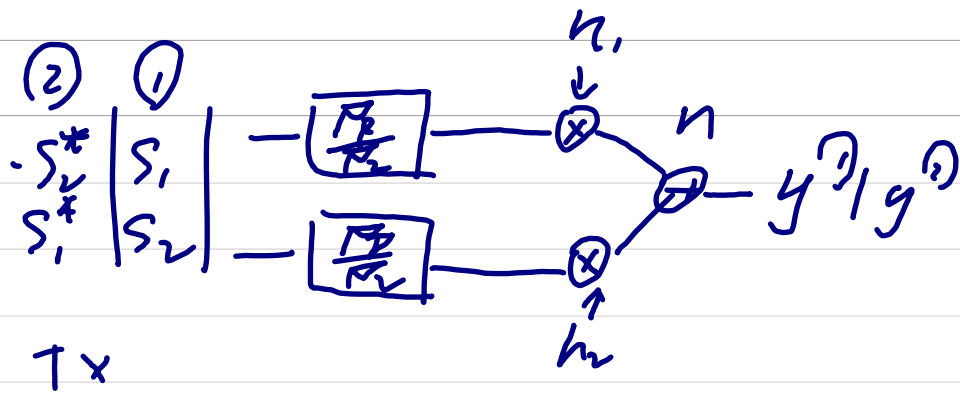$y^{\textcircled{2}*} = \frac{1}{\sqrt{2}}(-h_1^* S_2 + h_2^* S_1) + n^{\textcircled{2}}$

orthogonal

$$\begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}*} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} n^{\textcircled{1}} \\ n^{\textcircled{2}} \end{bmatrix}$$

$\hat{S_1} = [h_1^*\ h_2]\begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)S_1$

$\hat{S_2} = [h_2^*\ -h_1]\begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}(|h_1|^2 + |h_2|^2)S_2$

diversity gain
$\frac{1}{\sqrt{2}} \rightarrow 3dB$

② ①

$-S_2^* \,|\, S_1$

$S_1^* \,|\, S_2$

Tx

$\Rightarrow \begin{cases} \hat{S}_1^{pre} = (h_1^* \;\; h_2)\left(y^{①} \,|\, y^{②} \right) \\ \hat{S}_2^{pre} = [h_2^* \;\; -h_1)\left(y^{①} \,|\, y^{②} \right) \end{cases}$

$\Rightarrow \hat{S}_1 = \dfrac{\hat{S}_1^{pre} \times 2}{|h_1|^2 + |h_2|^2}$
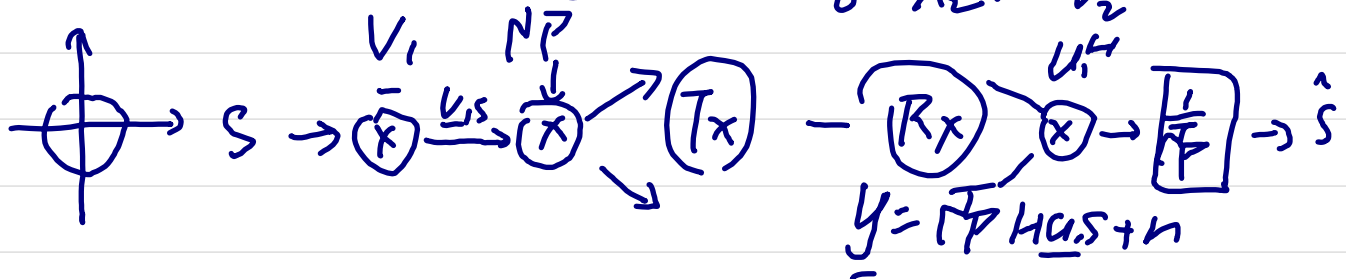
$\hat{S}_2 = \dfrac{\hat{S}_2^{pre} \times 2}{|h_1|^2 + |h_2|^2}$

6.ª Dominant eigenmode transmission (DET)

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

Rx   Tx $\xrightarrow{\text{SVD}} U\Lambda V^H$

$$= (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} V_1^H \\ V_2^H \end{pmatrix}$$

$$\underline{y} = \sqrt{P} H \underline{V}_1 s + \underline{n}$$

$$y = \sqrt{P} H \underline{u}_1 s + n$$

$$Rx_1 \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \sqrt{P} U\Lambda V^H V_1 s + n$$

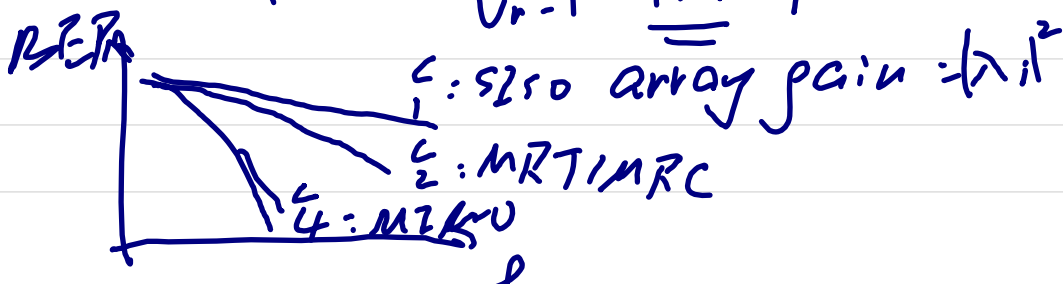$$= \frac{V_1^H}{V_2^H} v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \sqrt{P} (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} s$$

$$= \sqrt{P} (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 \\ 0 \end{pmatrix} s = \sqrt{P} \underline{u}_1 \lambda_1 s$$

2×2 DET:
array gain = 4
div. gain = 4
2×2 Alamouti:
array gain = 2
(h² rSI1: 3 dis 1955)
d.v. gain = 4

Rx: $y = \sqrt{P} H \underline{u}_1 s + n$

$$u_1^H y = \sqrt{P} H \underline{u}_1^H \underline{u}_1 \lambda_1 s + u_1^H n = \sqrt{P} \lambda_1 s + \underline{n}$$

$$SNR = |\lambda_1|^2 \frac{P}{\sigma_r^2 = 1} = |\lambda_1|^2 p$$

array gain $= |\lambda_1|^2$

① : SISO
② : MRT/MRC
④ : MIMO

b)
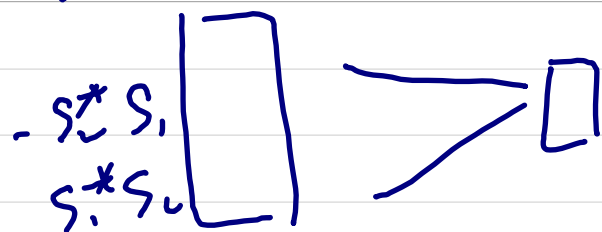
$$-S_2^* \ S_1 \quad \begin{bmatrix} \ \\ \ \end{bmatrix} \ \searrow \ \square$$
$$S_1^* \ S_2$$

$$y(1) = h_1 S_1 + h_2 S_2 + n$$
$$y(2) = -h_1 S_2^* + h_2 S_1^* + n$$

$$\Rightarrow \begin{bmatrix} y(1) \\ y^*(2) \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ h_2 \end{bmatrix}$$

$$\underbrace{}_{y}$$

$$\hat{S_1} = [h_1^* \ \ h_2^T] \, y \qquad \text{div. gain} = 2$$

$$\hat{S_2} = [h_2^* \ -h_1^T] \, y$$

$$-S_2^* \ S_1 \ \begin{bmatrix} \ \vdash \\ \ \vdash \end{bmatrix} \ \boxed{H} \ \begin{bmatrix} - \\ - \end{bmatrix}$$
$$S_1^* \ S_2$$
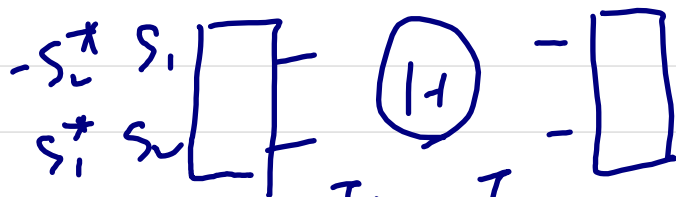
$$H = P_{x_1} \begin{pmatrix} \overset{T_{x_1}}{h_{11}} & \overset{T_{x_2}}{h_{12}} \\ P_{x_2} h_{21} & h_{22} \end{pmatrix} = (\underline{h_1} | \underline{h_2})$$

$$\underline{y(1)} = \underline{h_1} S_1 + \underline{h_2} S_2 + h(1)$$
$$\underline{y(2)} = -\underline{h_1} S_2^* + \underline{h_2} S_1^* + \underline{n_2}$$

$$\Rightarrow \begin{bmatrix} \underline{y(1)} \\ \underline{y^*(2)} \end{bmatrix} = \begin{bmatrix} \underline{h_1} & \underline{h_2} \\ \underline{h_2}^* & -\underline{h_1}^* \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} \underline{h(1)} \\ \underline{h_2} \end{bmatrix}$$
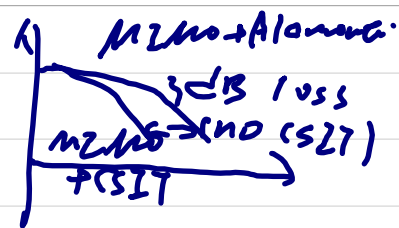
$$\hat{S}_1 = [h_1^* \; h_2^T] \, y$$

$$\hat{S}_2 = [h_2^* \; -h_1^T] \, y$$

$$\Rightarrow SNR = \frac{|h_1|^2 + |h_2|^2}{2} \to D$$

array gain = 1

diversity gain = 4

MIMO + Alamouti
3dB loss
MIMO → CSIno (SS27)
+CSIT

$$S_2^* \, S_1 - \boxed{\tfrac{P}{2}} - \boxed{\tfrac{1}{P_L}} - 1$$

$$S_1^* \, S_2 - \boxed{\tfrac{P}{2}} - \boxed{\tfrac{1}{P_L}} - 1$$

$$P_{tot} = \left(\tfrac{P}{2}\right)^2 \cdot 2 = P$$

$$\boxed{1, -1}$$

$$\begin{array}{c|c} y(1) & y(2) \\ \bar{2}x_1 & \bar{2}x_1 \end{array}$$

$$y = \begin{pmatrix} y(1) \\ y(2) \end{pmatrix} \quad 4 \times 1$$

$$\hat{S}_1 = \frac{P_L}{P_P} [h_1^H \; h_2^T] \, y$$

$$\hat{S}_2 = \frac{P_L}{P_P} [h_2^H \; -h_1^T] \, y$$

7.8

$$\begin{bmatrix} \\ \end{bmatrix} \quad \vdash\!\!\dashv \quad \begin{bmatrix} \\ \end{bmatrix} \quad H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

spacing $> \frac{\lambda}{2}$: h uncorrelated.

$\quad\quad\quad < \frac{\lambda}{2}$: h correlated.

How correlation impact the performance?

Kronecker model $\quad\quad\quad$ correlation at $Rx . Tx$.

$$R_r = \begin{bmatrix} 1 & (r^*) \\ r & 1 \end{bmatrix} \quad R_t = \begin{bmatrix} 1 & t^* \\ t & 1 \end{bmatrix} \quad (1.2)$$

$$H = R_r^{\frac{1}{2}} H_w R_t^{\frac{1}{2}}$$

no Rx correlation. $\quad\Rightarrow\; R_r = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R_r^{\frac{1}{2}}$
only Tx correlation $\quad\quad R_t = \begin{bmatrix} 1 & t^* \\ t & 1 \end{bmatrix}$

$$H = H_w R_t^{\frac{1}{2}}$$

task: repeat the previous exercise with
$\quad\quad$ channel $H = H_w R_t^{\frac{1}{2}}$ $(t = 0, 0.9)$

$$E\{ H H^H \} = R_r \otimes R_t$$
$$R_t^{\frac{1}{2}} \neq \begin{pmatrix} 1 & r_t^* \\ r_t & 1 \end{pmatrix}$$

euscune at least 100 errors
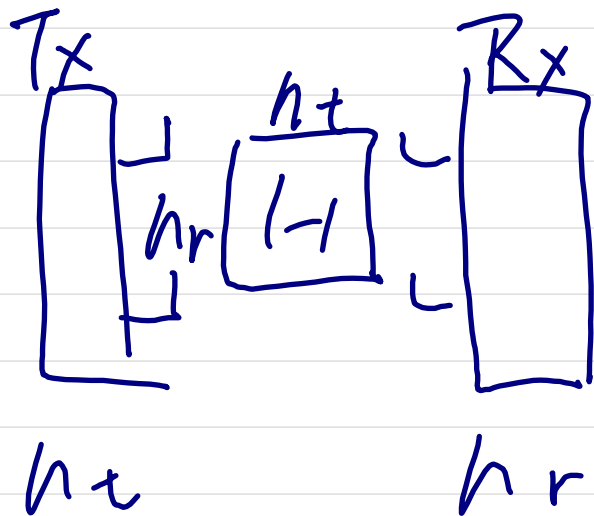
h_t

h_r

$I((H.Q) = log_2 det(I + \rho HQH^H)$

$C = max \ I((H.Q)$

Q (obtained by WF)

$H = U \Lambda V^{H}$ $\quad h_r \times n_t$

$n_r \times n_t \quad h_r \times n_r \quad h_t \times n_t$

$Q = \begin{bmatrix} Q_1 & \\ & \ddots & \\ & & Q_{n_t} \end{bmatrix}$ $\quad Tr(Q)=P$

$\begin{matrix} S_1 & - \\ S_2 & - \\ \cdot & \\ S_{n_t} & - \end{matrix}$ $\quad \bigcirc Q \times \bigcirc V = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$

VQS

$$\underline{J} = U \Lambda V^H V Q S = U \Lambda Q S$$

$$\boxed{U^H} U \Lambda V^H V Q S + n = \Lambda Q S + n$$

$$\underbrace{\qquad}_{T_x \; I} \qquad \underbrace{\qquad}_{I}$$

$$\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{nt} \end{pmatrix} \begin{pmatrix} q_1 & & \\ & \ddots & \\ & & q_{nt} \end{pmatrix} \begin{pmatrix} S_1 \\ \vdots \\ S_{nt} \end{pmatrix}$$
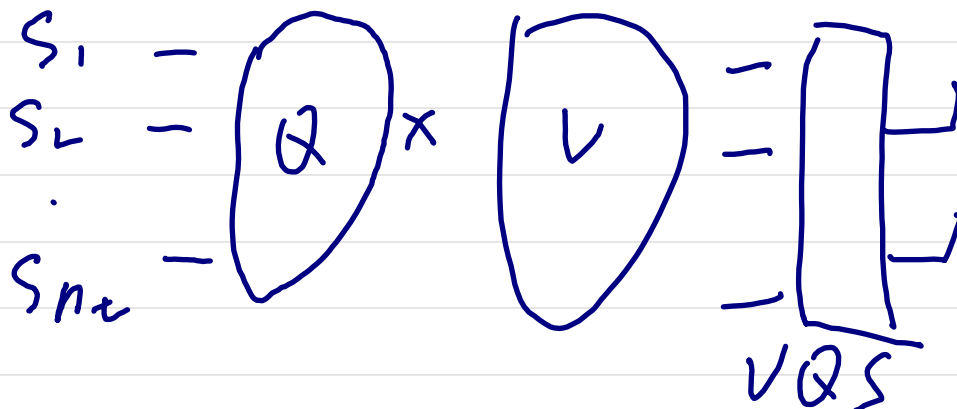
$$C = \sum_{i=1}^{nt} \log_2 \left(1 + \lambda_i^2 \, q_{i} \, P \right) \qquad \begin{pmatrix} \lambda_1 q_1 S_1 \\ \lambda_2 q_2 S_2 \\ \vdots \\ \lambda_{nt} q_{nt} S_{nt} \end{pmatrix}$$

$$H = U \Lambda U^H$$

$$H H^H = U \Lambda \underbrace{V^H V}_{I} \Lambda^H U^H = U \Lambda^2 U^H$$

$$\boxed{\begin{array}{l} T_x \to V \\ R_x \to U^H \end{array}}$$

$$\boxed{2\times 2}$$

$$\boxed{2\times 4}$$

$$\boxed{4\times 4}$$

$$C = \log_2(1 + \lambda_1 \widehat{q_1}) + \log(1 + \lambda_2^2 \widehat{q_2}) \quad \overset{w7}{\frown}$$

$$H = U \Lambda V^H \quad\downarrow$$

$$(\lambda_1 \quad \lambda_2)$$

$$q_1 + q_2 = 7$$

$$\Rightarrow \begin{cases} q_1 = x \cdot 7 \\ q_2 = x_2 7 \\ x_1 + x_2 = 1 \end{cases}$$

$$N_S = \min(n_t \cdot n_r)$$
$$= 2$$

Asteams

$$\exists A_{shade} = 1$$
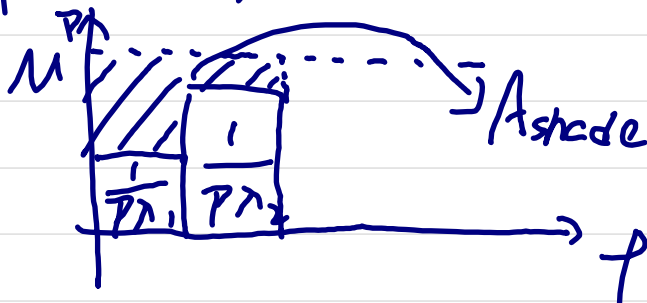
$$\frac{SVD(H):^2}{SVD(H^H H):} \; n\overset{\cdot}{o} \text{ square}$$

$$(\mu_i - \frac{1}{P\lambda_i^2})^+ + (\mu_i - \frac{1}{P\lambda_2^2})^+ = 1$$

μ^* such that
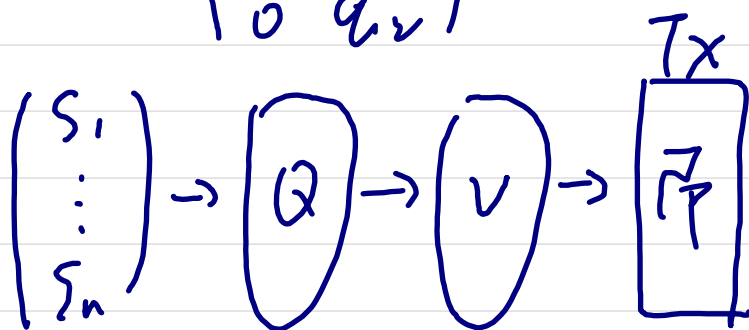
$$\max\{\mu^* - \frac{1}{P\lambda_i^2}, 0\} + \max\{\mu^* - \frac{1}{P\lambda_2^2}, 0\} = 1$$
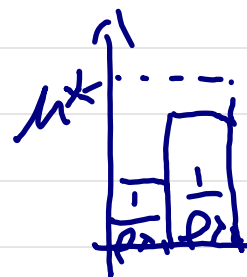
$$H = U \Sigma V^H$$

$$\Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

$$\downarrow$$

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \quad q_1 + q_2 = 1$$

$$\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} \rightarrow \boxed{Q} \rightarrow \boxed{V} \rightarrow \boxed{\vec{p}} \quad \boxed{\begin{array}{c} Tx \\ \vec{p} \end{array}}$$

$$C = \log_2(1 + P q_1 \sigma_1^2) + \log_2(1 + P q_2 \sigma_2^2)$$

$$\underset{\lambda_1}{\overset{\shortparallel}{}} \qquad \underset{\lambda_2}{\overset{\shortparallel}{}}$$

μ^* such that

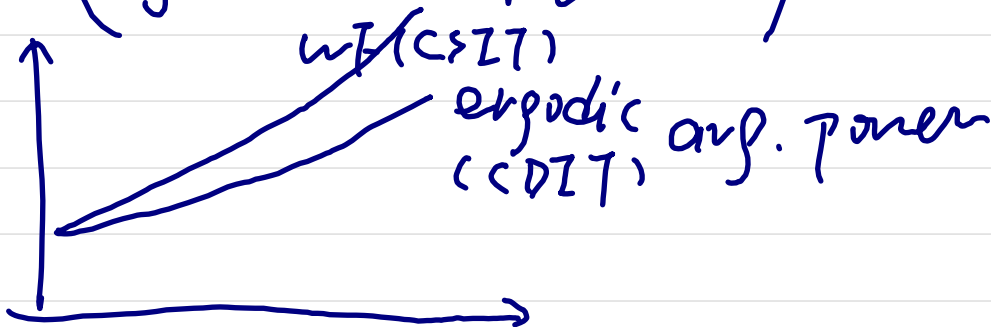$$\left\{ \mu^* - \frac{1}{P\lambda_1} \right\}^+ + \left\{ \mu^* - \frac{1}{P\lambda_2} \right\}^+ = 1$$

① generate matrix H

① SVD $\quad H = U\Sigma V^H$ ② $\Sigma = \begin{pmatrix} \sigma_1 & \\ & \sigma_2 \end{pmatrix} \Rightarrow \lambda = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$

③ $\left(\mu^* - \frac{1}{\rho\lambda_1}\right)^+ + \left(\mu^* - \frac{1}{\rho\lambda_2}\right)^+ = 1$

⑤ calculate $c = \log_2(1 + \rho\lambda_1 S_1) + \log_2(1 + \rho\lambda_2 S_2)$

$C = \mathbb{E}\left(\log_2 \det\left(I + \frac{\rho}{n_t}HH^H\right)\right)$

w/(CSIT)

ergodic avg. power
(CDIT)

11.

2) ML receiver

$$\left[\begin{array}{c}-S_1\\-S_2\end{array}H\begin{array}{c}y_1L\\y_2L\end{array}\right]$$

$$\begin{bmatrix}y_1\\y_2\end{bmatrix} = H\begin{bmatrix}S_1\\S_2\end{bmatrix} + \begin{bmatrix}n_1\\n_2\end{bmatrix}$$

$\{\hat{S_1}, \hat{S_2}\}$: all possible combinations

$$\begin{bmatrix}\hat{y_1}\\\hat{y_2}\end{bmatrix} = H\begin{bmatrix}\hat{S_1}\\\hat{S_2}\end{bmatrix} \quad \arg\min_{\hat{S_1}\cdot\hat{S_2}}\|y-\hat{y}\|^2$$

3) ZF

$$y = \begin{pmatrix}h_{11} & h_{12}\\h_{21} & h_{22}\end{pmatrix}\begin{pmatrix}S_1\\S_2\end{pmatrix} + \begin{pmatrix}h_1\\h_2\end{pmatrix}$$

$$\begin{pmatrix}\tilde{S_1}\\\tilde{S_2}\end{pmatrix} = H^{-1}y$$

4) MMSE

ZF $T = H^{-1}$

$$\tilde{S} = \begin{bmatrix}\tilde{S_1}\\\tilde{S_2}\end{bmatrix} = T \cdot y \qquad \text{MMSE } T = \sqrt{\frac{1}{E_s}}H^{H}\left(HH^{H} + \frac{1}{P}I\right)^{-1}$$

5) unordered

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = (\underline{h_1} | \underline{h_2}) \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + n$$

$$= \underline{h_1} s_1 + \underline{h_2} s_2 + n$$

$\approx [-s]c$; estimate one, remove one . . .

$$G = H^{-1} \qquad G = \begin{pmatrix} G(1,:) \\ G(2,:) \end{pmatrix}$$

$$\tilde{s}_1 = G(1,:) \underline{y}$$

$$\tilde{s}_1 \rightarrow \hat{s}_1$$

$$\underline{\hat{y}} = \underline{y} - \underline{h_1} s_1 = \underline{h_2} s_2 + n \overset{MRC}{\Longrightarrow} \hat{s}_2 = \frac{\underline{h_2}^H}{\|\underline{h_2}\|} \hat{y}$$

6) ordered

$$G = \begin{pmatrix} G(1,:) \\ G(2,:) \end{pmatrix}$$

Choose $G(i,:)$ such that $|G(i,:)|$ is min

$$\min\{|G(1,:)|, |G(2,:)|\}$$

7) Alamouti

$$\underline{y}(1) = H\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \underline{h}_1 s_1 + \underline{h}_2 s_2 + n_1$$

$$\underline{y}(2) = H\begin{pmatrix} -s_2^* \\ s_1^* \end{pmatrix} = -\underline{h}_1 s_2^* + \underline{h}_2 s_1^* + h_2$$

$$\begin{pmatrix} -s_2^* & s_1 \\ s_1^* & s_2 \end{pmatrix}$$
$$\begin{array}{cc} \uparrow & \uparrow \\ 2 & 1 \end{array}$$

$$\begin{bmatrix} y(1) \\ y(2)^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$\tilde{s}_1 = [\, \underline{h}_1^H \quad \underline{h}_2^T \,] \begin{bmatrix} y(1) \\ y(2)^* \end{bmatrix}$$

$$\tilde{s}_2 = [\, \underline{h}_2^H \quad -\underline{h}_1^T \,] \begin{bmatrix} y(1) \\ y(2) \end{bmatrix}$$

12.

$$H = R_{Tx}^{i} \, H_w \, R_{Rx}^{i}$$

13.   **CSI feedback**

Tx   Rx

$\underline{w}$ $

DET

$w = h^{H}$
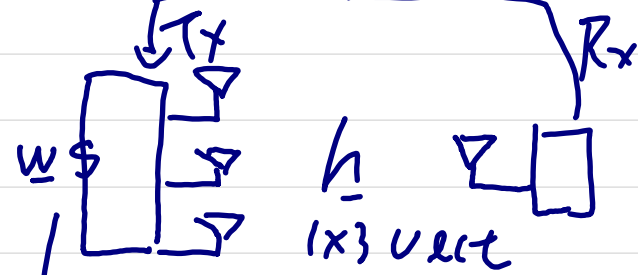
CSIT: how possible?

- Rx can estimate $\underline{h}$.
- Rx has to feedback $\underline{h}$ to Tx .
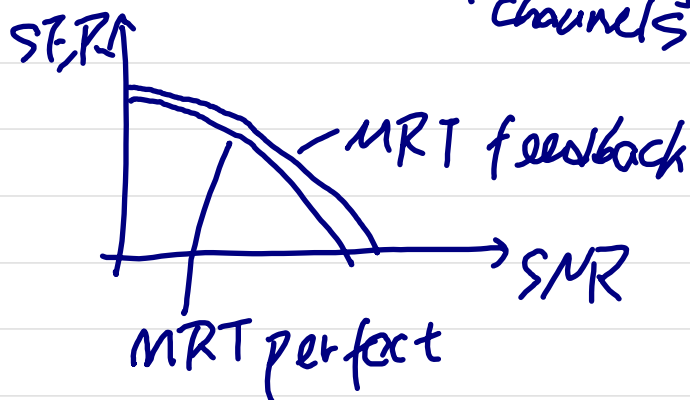
h

1×3 vect
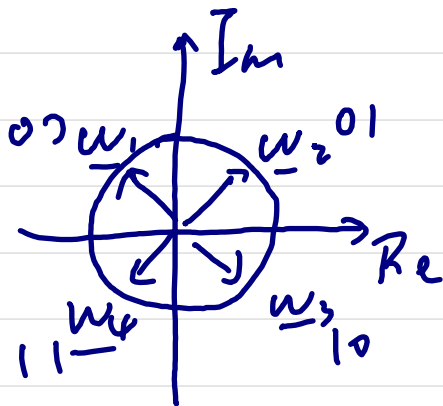
Rx can only feedback 2 bits

1. Rx estimate $\underline{h}$
2. Rx choose $\hat{\underline{h}}$

$$\underline{w} = \frac{\underline{h}^{H}}{|\underline{h}|}$$

quantised
precoding

$$W = \max_{i \in 1, 2 \cdots n_p} \| h_{w_i} \|^2$$

$\underline{W}_1 \cdots \underline{W}_{n_p}$ n_p precoding channels

SER

MRT feedback

SNR

MRT perfect

14.
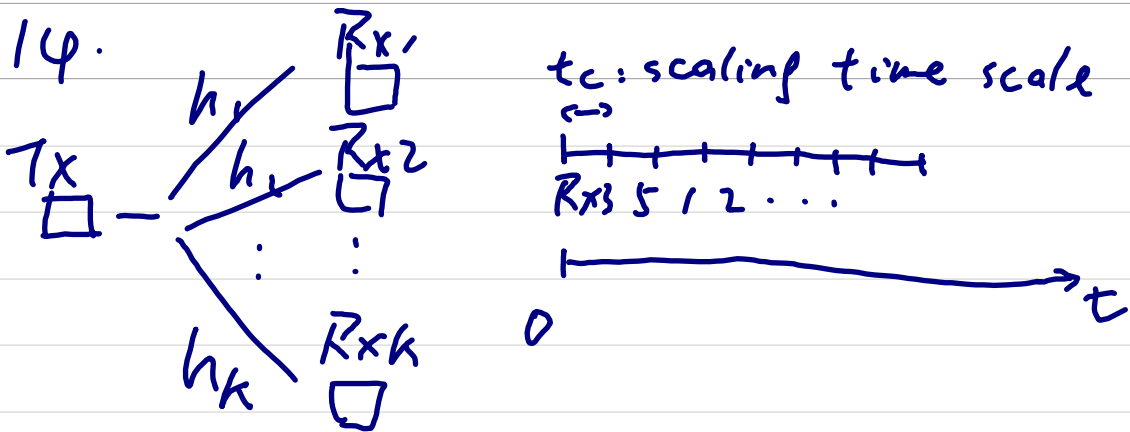
$q \in \{1.2..k\}$

$q^* = \underset{q \in \{1..k\}}{\text{argmax}} \ U_q$ — target func.

Exp, $q^* = \underset{q}{\text{argmax}} \ R_q$ rate $\sim [s]$

$q^{+\rho} = \underset{q}{\text{argmax}} \ V_q \dfrac{R_q}{\overline{R_q}}$ — long-term avg.

$\qquad\qquad$ QoS

proportional fair scheduling

Tx □

Rx1 □
Rx2 □
·
·
Rxk □

schedule for one user

t_c t_c t_c t_c → t
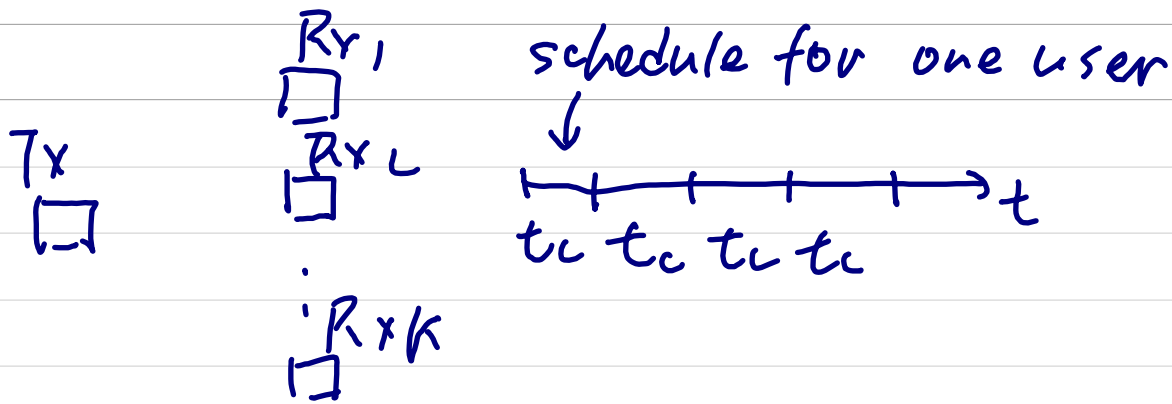
For each user in each slot

3 params:
$$\begin{cases} \gamma_q : QoS \\ R(t) : \text{current rate} \\ \overline{R(t)} : \text{long-term rate} \end{cases}$$

γ_q : given

$$R(t) = \log_2(1 + |h_q|^2 P)$$

$$\overline{R(t)} \begin{cases} (1 - \frac{1}{t_c})\overline{R_q(t-1)} + \frac{1}{t_c}R_q(t) & \text{if } q \text{ scheduled} \\ (1 - \frac{1}{t_c})\overline{R_q(t-1)} & \text{if } q \text{ not scheduled} \end{cases}$$

$$q_h^* = \arg\max_q \gamma_q \frac{R_q}{\overline{R_q}}$$

$$h_K(t) = \varepsilon h_K(t-1) + \sqrt{1-\varepsilon^2}\, n_K' \quad \begin{array}{l} \text{noise} \\ CN(0,1) \end{array}$$

for $t = 1 : 1e^5$

...

$\boxed{\begin{array}{l}\text{trade-off by slot length.}\\ t_i \to T : \text{may rate}\\ t_i \to 0 : \text{round ?Obin (may fair)}\\ \text{each user}\end{array}}$

① generate the channel of ~~each~~ user

$\quad q \in \{1 .. h\}$

$\quad h_q(t) = \varepsilon h_q(t-1) + \sqrt{1-\varepsilon^2}\, n$

② calculate the instantaneute rate
   and long-term rate of each user

$\quad R_q(t) = \log_2(1 + |h_q(t)|^2 p)$

$\quad \text{select } q^* = \underset{q}{\text{argmax}} \, \forall q \, \dfrac{R_q(t)}{\bar{R}_q(t-1)}$

$\quad$ update the long-term rate $\bar{R}_q(t)$,

in each slot t

  ① generate channel for each user

  ② calculate the inst. rate of user k

  $\quad R_q(t) = \log_2(1 + |h_q(t)|^2 p)$

  ③ choose user q that maximise

  $\quad q^* = \underset{q}{\text{argmax}} \, \forall q \, \dfrac{R_q(t)}{\bar{R}_q(t-1)}$ ⑤ iterate.

  ④ update long-term rate $\bar{R}_q(t)$