Wavelets and Applications: Annihilation of Polynomials

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Annihilation of polynomials with proper highpass filters

Consider a filter of the form

$$H(z) = (1-z)^d Q(z), \tag{1}$$

where d is a positive integer and $Q(z)$ is a polynomial. We note that $H(z) = 0$ for $z = 1$. We also note that if $d > 1$ then

$$\frac{dH(z)}{dz} = d(1-z)^{d-1}Q(z) + (1-z)Q'(z).$$

Consequently $\frac{dH(z)}{dz} = 0$ for $z = 1$. Also remember that on the unit circle $z = e^{j\omega}$ therefore $z = 1$ when $\omega = 0$. This means that $H(z)$ is a high pass filter.

From the definition of the z-transform we have that:

$$H(z) = \sum_k h[k] z^{-k}.$$

Consequently,

$$H(1) = \sum_k h[k] = 0. \tag{2}$$

From the fact that

$$\frac{dH(1)}{dz} = 0$$

we also conclude that:

$$\sum_k k h[k] = 0. \tag{3}$$

Consider now a signal of the form $x[n] = an + b$ for some arbitrary constants a and b. This is a discrete-time linear polynomial and the claim is that for

$d \geq 2$, $H(z)$ annihilates that polynomial. The proof follows from expanding the convolution formula:

$$x[n]*h[n] = \sum_k h[k]x[n-k] = a\sum_k h[k](n-k)+b\sum_k h[k] = (an+b)\sum_k h[k]-a\sum_k kh[k] = 0,$$

where the last equality comes from (2) and (3). It is now natural to extend the above result to higher order polynomials and conclude that $H(z)$ in (1) annihilates polynomials of degrees up to $d-1$.