Optical Communication

Notes Part B : Optical Fibre

4. The Cylindrical Dielectric Guide: Optical Fibre

In simplest form, an optical fibre is simply a cylindrical "thread" of glass surrounded by a larger cylinder of glass of lower index. Solving the fields in this geometry is much more difficult than in the planar case, but certain important features are the same:

- there may be one or more modes, depending on mainly the core width (diameter) and the index difference.

- where the core and cladding indices are n_c and n_o respectively, there will be an effective mode index $n' = \beta/k_o$, where for any mode: $n_o < n' < n_c$

If the transition from core to cladding is abrupt, as it was in the planar guide of part 3, we call this a step-index waveguide or fibre. Now, if the core radius is r_o, we define a normalised frequency parameter:

$$V = r_o k_o \sqrt{n_c^2 - n_o^2}$$

We have encountered the term $\sqrt{n_c^2 - n_o^2}$ for the slab waveguide as well; it is the numerical aperture (NA) of the fibre, and, like the NA of a lens, is a measure of its light-gathering power. A generalised cutoff condition is not straightforward to define, but the condition for single modedness is:

$$V < 2.405$$

Combining these equations we can write the condition for single-modedness as:

$$r_o < 0.383\lambda/\mathrm{NA}$$

which looks very similar to the equivalent condition for the slab waveguide from Part A, which is $d < 0.5\lambda/\mathrm{NA}$. Single-mode communications fibre typically uses Δn values ($\Delta n = n_c - n_o$) between 5×10^{-3} and 7×10^{-3}, giving NA values of about 0.12 - 0.15. If the fibre is designed to be single-moded beyond $\lambda = 1.2$ μm, the core radius r_o will then be from 3 to 4 μm.

The analytic solutions of the propagation constants for cylindrical fibre are also very complex, but we can see the general trends in figure 4.1; as in the slab guide, modes far from cut-off have effective indices near that of the core, while close to cut-off they are weakly confined, with most of the field in the cladding. The number of supported modes can be approximated, where $V \gg 2.4$, as $V^2/2$. Multi-mode step index fibre usually has Δn values similar to those for single mode, and core diameters of 50 and 62.5 μm are common standards. For the latter, taking NA = 0.12, the number of modes supported at 1.55 μm ≅ 1000.

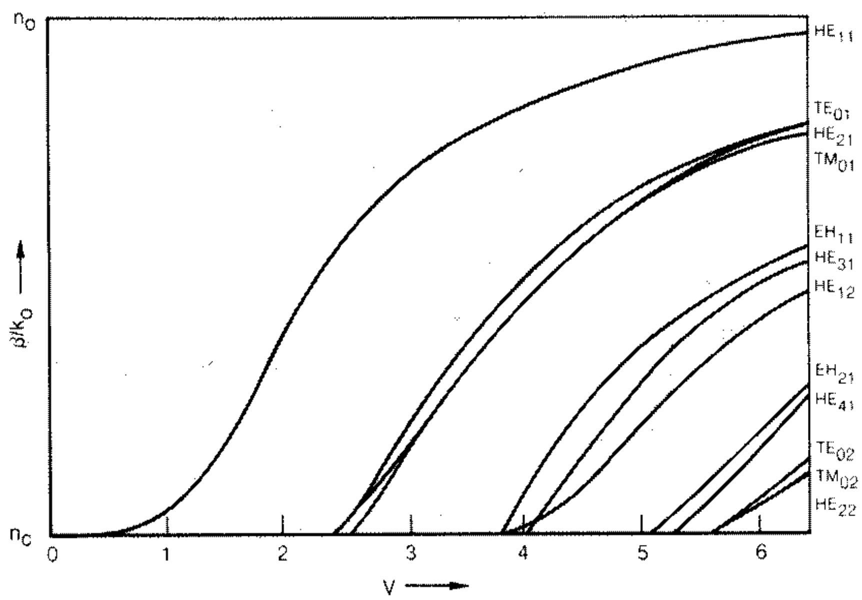

Figure 4.1 Effective index vs V for several lower-order modes in a step-index cylindrical guide (from P.K. Cheo, Fiber Optics & Optoelectronics, 2[nd] Ed., Prentice Hall, 1990). Here n_c is the cladding and n_o the core!

The polarisation situation is much more complex as well. In the planar guide, modes are either TE or TM, and the m number can be thought of as the number of nulls (E=0) in the core. In a fibre, TE and TM modes are still those with the $\underline{E}$ and $\underline{H}$ fields perpendicular to the direction of propagation (and to each other), but we can also have mixed modes called EH or HE modes where neither $\underline{E}$ nor $\underline{H}$ are $\perp \underline{\beta}$. The nulls can be either radial or circumferential, so two indices are used, k and m. For the transverse modes, k=0, i.e. the modes are labelled TE_{om} and TM_{om}.

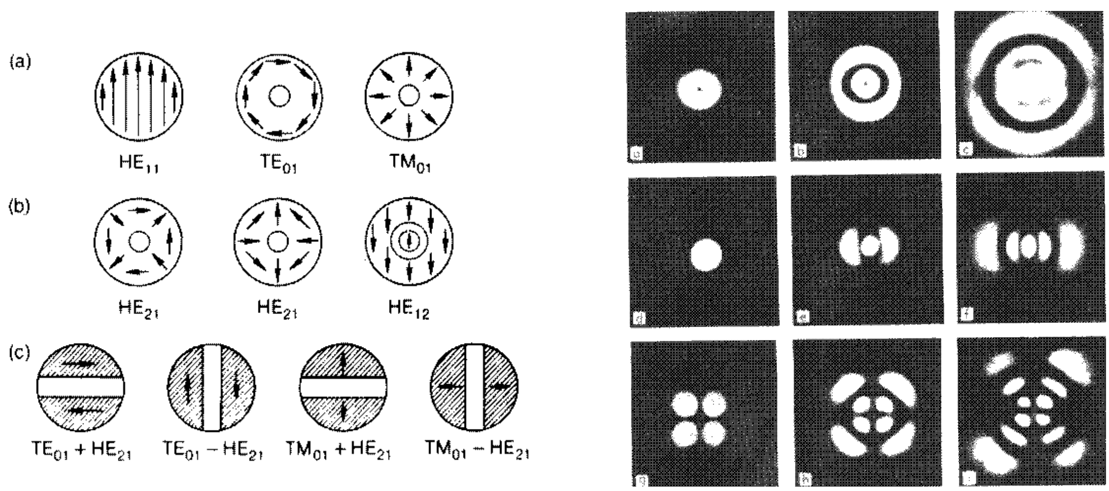

Figure 4.2 Shapes of lower-order fibre modes: Electric field (left) and intensity (right). (from Cheo).

For a highly multi-moded guide, modes with the same value of $q = k + 2m$ have approximately the same β values, and there are approximately 2q modes for each q value. The total number of q groups will be $Q = 2V/\pi$, and the total number of modes will thus be $M \cong V^2/2$, as stated above.

The variation in β values is of most significance because it causes dispersion: different parts of the signal in different modes do not travel at the same velocity, and thus the signal "smears out". The actual cause is more subtle, as what matters for signal propagation is group velocity, not phase velocity. For a guided mode, these are defined respectively as:

$v_g = d\omega/d\beta \qquad v_p = \omega/\beta$

In general, though, different modes have different phase and group velocities, and for the purposes of intermodal dispersion the approximation $v_g = v_p$ is reasonable. In this case all the modes have the same ω, as they all derive from the same light source, so the fractional variation in velocity is equal to the fractional variation in β. We can see from Fig. 4.1 that the effective index values for a highly multi-moded guide will be distributed through the range n_o to n_c, so the fibre index difference determines also the fractional variation in velocity in this case. We can define a group delay, τ_g, as the time taken by a signal to travel a length of fibre L, and this will be simply $\tau_g = L/v_g$. Approximating using $v_g = v_p$, and remembering that $\beta = n' k_o$ and $\omega/k_o = c$, we get:

$\tau_g = L\, n'/c, \quad \text{and} \quad \Delta\tau_g = L\, \Delta n'/c$

But we require that the spread in propagation times is less than the bit period, in order to limit inter-symbol interference, and this allows us to relate the bit-rate×length product BL to the index difference, in the case of highly multi-moded fibre (where the total variation in effective index is approximately equal to Δn):

$L\, \Delta n/c < 1/B \quad \text{so} \quad BL < c/\Delta n$

For an index difference of 5×10^{-3}, $BL < 60\times10^{6}$ km/s, e.g. maximum 10 Mbit/s for a 6 km cable. This is not particularly good compared to traditional co-ax copper cable, and orders of magnitude below the capacity of single mode fibre.

An important way to reduce the variation in group velocity in multi-mode fibre is to gradually reduce the index from the centre of the core to the cladding. This is easily illustrated with an index diagram: taking a cross-sectional line through the fibre, we plot the index as a function of radial position. The distributions can then be depicted as in figure 4.3 below.

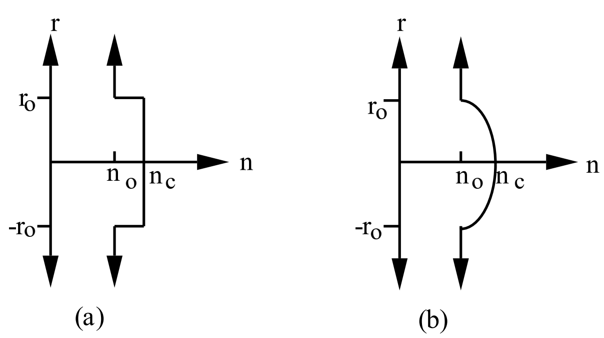

Figure 4.3 Index distributions for step-index and graded-index fibre.

This grading can have various mathematical forms. A convenient and popular one is the α-profile:

$$n(r) = n_c (1 - 2\Delta (r/r_o)^\alpha)^{1/2} \qquad r<r_o$$
$$= n_o \qquad r>r_o$$

where $\Delta = (n_c^2 - n_o^2)/2n_c^2 \cong (n_c - n_o)/n_c$
[Warning: Gowar uses n_o & n_c the other way round!]

Thus α=1 is a linear taper, α=2 quadratic, and $\alpha=\infty$ is step-index. Determining the phase and group velocities in graded index fibre analytically is complex, but it can be done in order to determine the pulse-broadening resulting from multi-path dispersion. If the signal spectral width is not considered, we find that the minimum dispersion is obtained for:

$$\alpha \cong 2 - \frac{12}{5}\Delta$$

It can also be shown that the pulse spreading is less in this optimum case than that for an equivalent step-index guide by a factor $\Delta/\sqrt{2}$. The reduction can be as much as 2 orders of magnitude! More extensive discussion of modes in cylindrical fibre can be found in, for example, Gowar, chapters 8 & 9, and Cheo, chapter 4.

5. Dispersion in Fibres

Dispersion can be defined as the variation of phase velocity (and consequently group velocity) with frequency (or wavelength). Inter-modal dispersion, as discussed in section 4, is not exactly dispersion in the sense described above, but we use the term because both contribute to the effect of pulse spreading due to variation in propagation time. We have considered in section 4 the effect of inter-modal dispersion on group delay time τ_g, but our approximation that the increase in pulse width is equal to the range of τ_g is crude, since the modes are not equally distributed through the velocity range. We can analyse the spread in propagation time more accurately using the *variance* σ of τ_g, according to:

$$\sigma_g^2 = \langle\tau_g^2\rangle - \langle\tau_g\rangle^2$$

where < > indicates taking a mean. Thus the overall pulse spreading is affected by both the total range of group velocities and by how the velocities are distributed within this range, the latter factor accounting for the greatly reduced dispersion in graded-index as compared to step-index fibre. [Note that for Gaussian distributions of the form $\exp[-(x/x_o)^2]$, the variance is just given by $\sigma = x_0/\sqrt{2}$.]

The next source of dispersion is material dispersion, which arises because the indices of the glass materials, and thus the light velocities, are (weakly) varying with wavelength. All signals have a finite spectral width $\Delta\lambda$; this may originate from the spectral width of the data signal, but in many cases in optical communications it will instead be dominated by the intrinsic $\Delta\lambda$ of the source, since achieving a narrow spectrum (compared to typical signal spectra) from a laser is very challenging. Since it is group velocity that we are concerned with, the pulse broadening is proportional to the <u>second</u> derivative of index, and we find that:

$$\Delta \tau_g = - \lambda \frac{L\Delta\lambda}{c} \frac{d^2n}{d\lambda^2} \quad \text{(derived in lectures)}$$

Thus the pulse spreading is proportional to the source spectral width $\Delta\lambda$. We can define a dispersion coefficient D = $|\lambda \,(d^2n/d\lambda^2)/c|$ (the modulus is taken because we do not care about the sign of $d^2n/d\lambda^2$), so that $\Delta\tau_g = L\cdot\Delta\lambda\cdot D$. It is convenient to express D in ps/nm·km, since the lengths and spectral widths are usually in km and nm respectively. Taking again our simplified requirement $\Delta\tau_g < 1/B$, we get a limit:

$BL < (D \cdot \Delta\lambda)^{-1}$

A reasonable example could be $\Delta\lambda = 1$ nm, D = 2 ps/nm·km, giving $BL < 0.5\times10^{12}$ km/s, which compares very favourably with the 60×10^6 km/s example from multi-mode fibre (10^4 higher!).

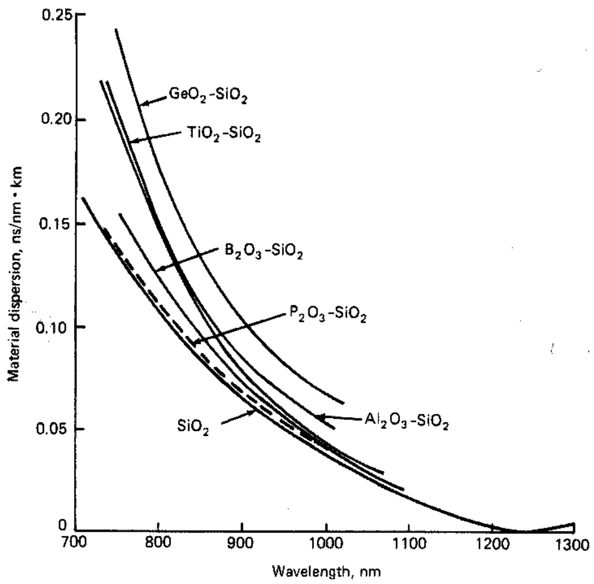

Figure 5.1 Material dispersion in silica fibre with various dopants. (From F.C. Allard, Fiber Optics Handbook, McGraw-Hill, 1990)

High quality fibre is generally made from silica (amorphous SiO_2), for which the dispersion goes from negative to positive in the near infra-red, and is in fact zero at about $\lambda = 1.3$ µm. This is the main reason for using this wavelength for long-distance, high bandwidth communications.

There is a second dispersive effect which is also due to the finite spectral width of the signal. In a particular mode, the group velocity is related to how much of the field is in the core and how much is in the cladding. For longer wavelengths, more of the field will be in the cladding, so β will clearly be less. This is purely a geometrical effect and is independent of the material dispersion, but like it, it causes a pulse broadening proportional to $\Delta\lambda$. We call this effect intra-modal or waveguide dispersion.

Waveguide dispersion can be altered by changing the index profile of the fibre, from the straight-forward step- or graded-index core, to concentric cores and other more complex geometries. Figure 5.2 illustrates this, and also how the spectrally-dependent dispersion is a sum of the material and waveguide effects. The combined effects of material and waveguide dispersion dominate for single-mode fibre. Note that although the combined material and waveguide dispersion will be exactly zero at a particular wavelength, dispersion is not eliminated because of the finite spectrum of the signal. In this case the pulse broadening will depend on the dispersion slope S = dD/dλ.

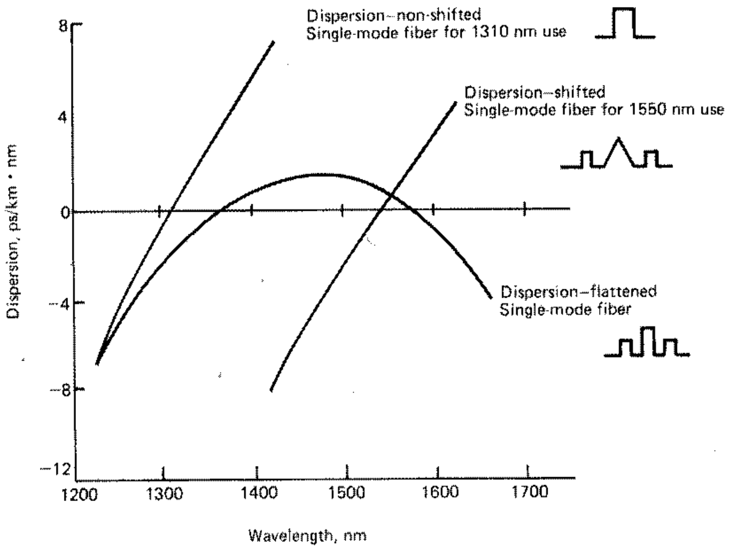

Figure 5.2 Spectral dispersion for various single-mode fibre profiles (from Allard)

Waveguide and material dispersion together comprise group velocity dispersion (GVD), and the dispersion parameter D normally includes both effects. The pulse broadening can be reasonably approximated using the variance in group delay, σ_g, and by expressing the spectral width of the signal as a variance, σ_λ. We can now define a "dispersion time":

$$\sigma_D = DL\,\sigma_\lambda$$

and can approximate the received pulse width variance σ in terms of the transmitted width σ_o according to

$$\sigma^2 = \sigma_D{}^2 + \sigma_o{}^2$$

An important difference between optical and electrical communication is that in the latter, oscillators can usually be considered to produce distinct frequencies, so that the modulated signal has a bandwidth entirely determined by the information bandwidth and the modulation format. In optics, however, the bandwidth of the source (LED or laser) is often significant compared to the information bandwidth, and may be much greater. Effectively this amounts to phase noise on the transmitted signal, but it also impacts on GVD. In the case where the inherent optical source spectral width $\sigma_{\lambda S}$ is much greater than the input signal spectral width, such as when an LED is used (see part C), σ_λ (and thus σ_D) will be independent of σ_o. However, where $\sigma_{\lambda S}$ does not dominate, the input signal bandwidth does come in, and this will be inversely proportional to the pulse width σ_o. In the case of so-called transform limited pulses, the pulse spectral width in the ω domain $\sigma_\omega = 0.5/\sigma_o$. In this case we obtain $\sigma^2 = \sigma_D{}^2 + (0.5/\sigma_\omega)^2$. Since $\omega = 2\pi c/\lambda$, and using the approximation $\sigma_\omega/\sigma_\lambda \cong \omega/\lambda$, we get $\sigma_\omega \cong 2\pi c/\sigma_\lambda$. , and so (expanding σ_D):

$$\sigma^2 = (DL)^2\sigma_\lambda^2 + \left(\frac{0.5\lambda^2}{2\pi c}\right)^2 \frac{1}{\sigma_\lambda{}^2}$$

The first term (the dispersive expansion of the pulse width in time) is proportional to the spectral width, while the second (the original transmitted pulse width) is inversely proportional to it. This implies that for each value DL there is an optimal transmitted pulse width that minimises the received pulse width.

Finally there is a weak dispersion due to the different polarisation directions. Typically the fibre cross-section has a slight asymmetry, particularly due to bending stresses, which causes variation in velocity with polarisation. The effect is only significant for signals in single-mode fibre with very low $\Delta\lambda$.

One way to eliminate polarisation dispersion is to launch only one polarisation by polarising the source, but then you must prevent this polarisation from mixing into others during propagation. Ironically, this is done by intentionally increasing the inherent dispersion by greatly increasing the cross-section asymmetry, so that the two polarisation directions defined by the structure do not interact. Various techniques exist, the simplest of which is elliptical core, but others such as bow-tie and stress rod geometries are used in which a linear stress is also introduced.

6. Attenuation

The optical signal is not just distorted as it propagates in a fibre, it is also attenuated. In fact, for all but the highest bandwidths, it is attenuation rather than dispersion which limits the length of individual communication links, and thus determines repeater spacing. The attenuation is due to two types of effects: absorption of the light (conversion into heat), and scattering, which causes light to leak out of the core. If these effects are reasonably uniform, they cause a gradual and steady exponential drop in intensity with propagation distance z, according to:

$$I = I_o \exp(-\alpha z)$$

where α is the absorption (more correctly, attenuation) coefficient. If we describe the attenuation in dB, then we take:

$$10 \log \frac{I}{I_o} = - 10(\log e)\, \alpha z$$

Thus the attenuation in dB/km is given by 4.34α (for α in units km^{-1}). For high quality fibre, values lower than 0.2 dB/km can be obtained.

When a propagating field encounters an obstacle or other discontinuity, a portion of its energy is scattered into other directions. The distribution and strength of this scattering are complex functions of the nature of the scatterer, but a few basic characteristics can be identified. The most important is that scattering strength is highly dependent on the size of the object with respect to λ, and more specifically, that for scatterers with dimensions $<< \lambda$, the scattered intensity is roughly proportional to $(1/\lambda)^4$. The conclusions for fibre fabrication are that for low attenuation, large scatterers (cracks etc) must be completely eliminated, and that the very small scatterers (variations in the atomic structure, which cannot be eliminated in amorphous material), will have a greatly increasing effect with decreasing λ. This $1/\lambda^4$-dependent mechanism is called Rayleigh scattering, and indeed dominates fibre attenuation at short wavelengths. We can define a Rayleigh scattering absorption coefficient, $\alpha_R = C/\lambda^4$, where C is typically in the range 0.7 – 0.9 (dB/km)·μm^4. These values give 0.12 – 0.16 dB/km at 1.55 μm wavelength, which is near to the minimum attenuation achieved in practice.

There are three main classes of absorption. At long wavelengths, photons are absorbed by exciting mechanical vibrations of the atomic lattice, the quanta of which are called phonons, which are equivalent to heat. At high photon energies (short λ), light is absorbed by exciting transitions of the electrons in the glass to higher atomic states. Finally, impurities absorb photons of specific wavelengths through resonant effects. In fibre communications, the most important is the longitudinal vibration of the O-H bond which is present in any water left behind in the glass fabrication. The frequency of this vibration corresponds to a photon wavelength of 2.76 μm. However, there are also strong absorptions at the harmonics of the fundamental frequency, of which the first is very significant for communications, coming at λ = 1.383 μm. Clearly to make low-loss fibre, the water and other impurity content must be kept extremely low.

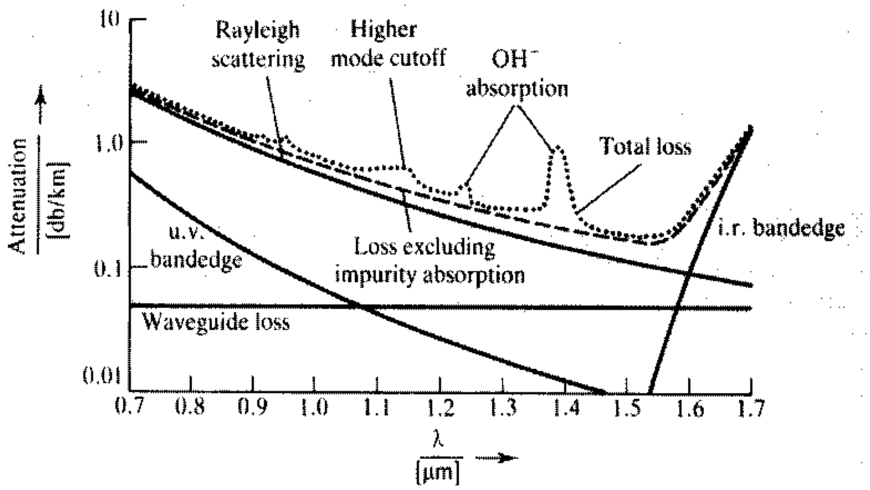

Figure 6.1 Propagation loss in silica fibre (from Gowar)

The overall attenuation due to scattering and absorption is shown in Fig. 6.1. The minimum is at $\lambda \cong 1.55$ μm, which makes this the other important "window" for long distance communications. Figure 6.2 shows more recent data, illustrating the significant reduction achieved in the OH peaks. The “C” band is the standard 1.5 μm band, while the “L” band refers to the extension of this band to longer wavelengths.

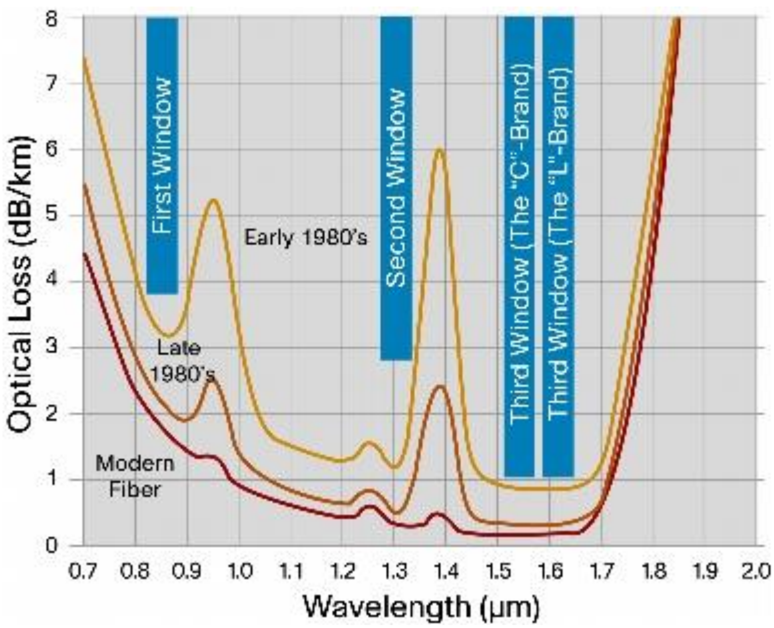

Figure 6.2 Propagation loss in silica fibre: improvements over time (from Cisco white paper).

Because the Rayleigh absorption varies little with type of glass, there is great interest in developing fibre from glasses other than silica having i.r. band-edges at longer wavelength, thus allowing even lower overall loss to be achieved. To date, such fibres have not achieved sufficient manufacturing quality or low cost.

Part B: Summary

Optical Fibre

Numerical aperture, criterion for single-modedness, typical values for these. Number of modes, mode geometries, polarisation considerations.

Pulse spreading due to intermodal dispersion. Step-index vs. graded index fibre.

Dispersion

Dependence of pulse width on group delay variance.

Material dispersion, dispersion coefficients, dispersion minima in silica glasses.

Waveguide dispersion, dispersion shifted fibre.

Attenuation

The attenuation coefficient α.

Rayleigh scattering, dependence on wavelength, typical values.

Ultraviolet and infra-red absorption.

Wavelength of minimum attenuation, typical values.

Part B: Problems

1) A step-index multi-mode fibre has a core diameter of 50 μm and cladding index of 1.45. If the intermodal dispersion is limited to 10 ns/km, what is the NA? What is the maximum bit-rate for a 10 km fibre length?

   Roughly how many modes are supported at 0.88 μm wavelength?

2) For the fibre above, find the maximum bit-rate B, limited by intermodal dispersion, as a function of fibre length, and compare this with the bit-rate limit imposed by signal-to-noise ratio. For the latter, assume a launched power of 1 mW, a required receiver power of 2×10^{-17} W per unit bandwidth Δf, and a fibre attenuation of 0.43 dB/km. You may use $\Delta f = B/2$. Over what length range is dispersion the more stringent limitation?

3) A single-mode fibre has a cut-off wavelength of 1 μm and $\Delta n = 0.005$. Find the core diameter, and estimate the effective index (using Fig. 4.1) at a wavelength of 1.3 μm if the cladding index is 1.47.

4) A Gaussian pulse of 100 ps width (FWHM) at 1.5 μm wavelength is launched into a SMF of dispersion coefficient D = 16 ps/km·nm. If the pulse spectrum is dominated by the modulation, find the pulse width (in time) after propagating 50 km.

5) A fibre is made from a material such that the attenuation minimum is at 1.7 μm wavelength, taking Rayleigh scattering and the i.r. edge into account. What minimum attenuation value would you expect this fibre to have?