(5) By Lagrange Multiplier Method,

$$E(f) = f^T C f - \lambda_1 (f^T f - 1) - \lambda_2 (f^T e)$$

$$\frac{d\,E(f)}{df} = 0 \Rightarrow \quad Cf - \lambda_1 f - \lambda_2 e = 0$$

$$\Rightarrow \quad e^t C f - \lambda_1 e^t f - \lambda_2 = 0.$$

$$\lambda_1 e^t f - \lambda_1 e^t f - \lambda_2 = 0$$

$$\Rightarrow \lambda_2 = 0.$$

$\Rightarrow \quad Cf = \lambda_1 f \Rightarrow f$ is eigenvector of C with eigenvalue λ_1

$f^T C f = \lambda_1$ & distinct eigen values

$\Rightarrow f$ corresponds to second highest eigenvalue.