Oct 13, 2018

Assignment - 4 CS 663 - Autumn 2018

Neharika Jali 160040101

Sucheta R 160040100

Swadha Sanghvi 160070037

Q6 $m <= n$ $A_{m \times n}$ $\underset{n \times n}{P = A^T A}$ $\underset{m \times m}{Q = AA^T}$

(a) $y^t P y = y^t A^T A y = (Ay)^T (Ay) = ||Ay|| \geq 0$

$\Rightarrow y^t P y \geq 0$

$z^t Q z = z^t A A^T z = (A^T z)^T (A^T z) = ||A^T z|| \geq 0$

$\Rightarrow z^t Q z \geq 0$.

Let v be an eigen vector of P with eigen value λ

$\Rightarrow Pv = \lambda v \Rightarrow v^T Pv = \lambda v^T v$

$= \lambda$

as $v^T P v \geq 0 \Rightarrow \lambda \geq 0$.

IIIly for $Qv = \lambda v \Rightarrow v^T Q v = \lambda v^T v$

$= \lambda$

as $v^T Q v \geq 0 \Rightarrow \lambda \geq 0$.

$\Rightarrow$ Eigen values of P & Q are non-negative

6. (b)

$$Pu = \lambda u$$

So $\lambda Au = APu$

$$= AA^TAu$$

$$\lambda Au = QAu$$

$\Rightarrow$ Au is an eigen vector of Q with eigenvalue λ.

$$Qv = \mu v$$

$$A^TQv = \mu A^Tv$$

$$A^TAA^Tv = \mu A^Tv \quad \Rightarrow \quad PA^Tv = \mu A^Tv$$

$\Rightarrow$ A^Tv is an eigen vector of P with eigenvalue μ.

$\underset{n\times n}{P} \; \underset{n\times y}{u}$ $\quad \underset{n\times y}{u}$. eigen vector is a column vector

$\Rightarrow$ $u_{n\times 1}$.

|||ly $v_{m\times 1}$.

(c)

$$Qv_i = \mu_i v_i$$

$\Rightarrow$ $AA^Tv_i = \mu_i v_i$

$\Rightarrow$ $A(A^Tv_i) = \mu_i v_i$

$\Rightarrow$ $A\left(\dfrac{A^Tv_i}{\|A^Tv_i\|}\right) = \dfrac{\mu_i v_i}{\|A^Tv_i\|}$

$\Rightarrow$ $Au_i = \dfrac{\mu_i v_i}{\|A^Tv_i\|}$

$\Rightarrow$ $Au_i = \gamma_i v_i$

$\gamma_i = \dfrac{\mu_i}{\|A^Tv_i\|} \geq 0$

because $\|A^Tv_i\| \geq 0$

and $\mu_i \geq 0$

from part (6)(a).

(a) $Au_i = \gamma_i v_i \quad \forall\, 1 \le i \le m$

$Au_i = 0 \quad \forall\, m+1 \le i \le n$

$\Rightarrow AV = U\Gamma$ where $V_{n \times n}$ is an orthonormal matrix

$\Gamma_{m \times n} \rightarrow$ diagonal matrix

max. m non-zero values.

$U_{m \times m}$ orthornormal matrix

$\Rightarrow A = U \Gamma V^T$

Values in Γ are non-negative as proved in part (b) c.

U & V are orthonormal because —

P, Q symmetric ; $Pu_1 = \lambda u_1$ & $Pu_2 = \lambda u_2$

$u_2^T P u_1 = \lambda_1 u_2^T u_1$

$(P^T u_2)^T u_1 = (Pu_2)^T u_1 = (\lambda_2 u_2)^T u_1 = \lambda_2 u_2^T u_1$

$\Rightarrow \lambda_2 u_2^T u_1 = \lambda_1 u_2^T u_1$

$\lambda_1 \neq \lambda_2 \Rightarrow u_2^T u_1 = 0.$