ASSIGNMENT - 5

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Considering there is no noise,

1. $g_1 = f_1 + h_2 * f_2$ $\qquad g_2 = h_1 * f_1 + f_2$

$G_1 = F_1 + H_2 F_2$ $\qquad G_2 = H_1 F_1 + F_2$

$\Rightarrow F_1 = G_1 - H_2 F_2 \rightarrow (1)$

From (1)

$G_2 = H_1 (G_1 - H_2 F_2) + F_2$

$\Rightarrow G_2 = H_1 G_1 - H_1 H_2 F_2 + F_2$

$\Rightarrow F_2 = \dfrac{G_2 - H_1 G_1}{1 - H_1 H_2} \rightarrow (2)$

From (2)

$$F_1 = G_1 - H_2 \left(\frac{G_2 - H_1 G_1}{1 - H_1 H_2} \right)$$

$$= \frac{G_1 - G_1 \cancel{H_1} H_2 - H_2 G_2 + \cancel{H_1} G_1 H_2}{1 - H_1 H_2}$$

$$\Rightarrow F_1 = \frac{G_1 - H_2 G_2}{1 - H_1 H_2}$$

Hence, $f_1 = \mathcal{F}^{-1} \left(\dfrac{G_1(\mu) - H_2(\mu) G_2(\mu)}{1 - H_1(\mu) H_2(\mu)} \right)$

and $f_2 = \mathcal{F}^{-1} \left(\dfrac{G_2(\mu) - H_1(\mu) G_1(\mu)}{1 - H_1(\mu) H_2(\mu)} \right)$

$$H_1(\mu)\Big|_{\mu=0} = \int_{-\infty}^{\infty} h_1(x)\,dx = 1$$

$$H_2(\mu)\Big|_{\mu=0} = \int_{-\infty}^{\infty} h_2(x)\,dx = 1$$

As the blur kernels being low-pass filters.

For $\mu \neq 0$, $|H_1(\mu)| \leq 1$ & $|H_2(\mu)| \leq 1$

Hence $H_1(0) H_2(0) = 1$ making the denominator for $F_1 = 0$ & $F_2 = 0$.

Hence we won't be able to reconstruct the zero frequency / DC component perfectly.

To prevent, F_1 & F_2 blowing up to infinity, we can add a small constant.

$$\Rightarrow f_1 = \mathcal{F}^{-1}\left(\frac{G_1(\mu) - H_2(\mu)G_2(\mu)}{1 - H_1(\mu)H_2(\mu) + \varepsilon}\right)$$

and $$f_2 = \mathcal{F}^{-1}\left(\frac{G_2(\mu) - H_1(\mu)G_1(\mu)}{1 - H_1(\mu)H_2(\mu) + \varepsilon}\right)$$

also since $G_1(0)$, $G_2(0)$ are ~~in~~ finite quantities, ~~we won't have a blow up~~. we will have finite f_1, f_2.

Considering noise,

$g_1 = f_1 + h_2 * f_2 + \eta_1$ $\qquad$ $g_2 = h_1 * f_1 + f_2 + \eta_2$

Solving like before,

$$\Rightarrow F_1 = \frac{G_1 - N_1 - H_2(G_2 - N_2)}{1 - H_1H_2 + \varepsilon} = \frac{G_1 - H_2G_2}{1 - H_1H_2 + \varepsilon} + \frac{H_2N_2 - N_1}{1 - H_1H_2 + \varepsilon}$$

$$F_2 = \frac{G_2 - H_1G_1}{1 - H_1H_2 + \varepsilon} + \frac{H_1N_1 - N_2}{1 - H_1H_2 + \varepsilon}$$

At higher frequencies, no issues arise as denominator is not tending to zero.

At low frequencies, denominator $\to 0$, but still there will exist no issues because,

SNR ~~Relative noise~~ $\rightarrow$ $\dfrac{G_2 - H_1G_1}{H_1N_1 - N_2} \Big/ \dfrac{G_1 - H_2G_2}{H_2N_2 - N_1}$ is high

as N_1, N_2 are low.