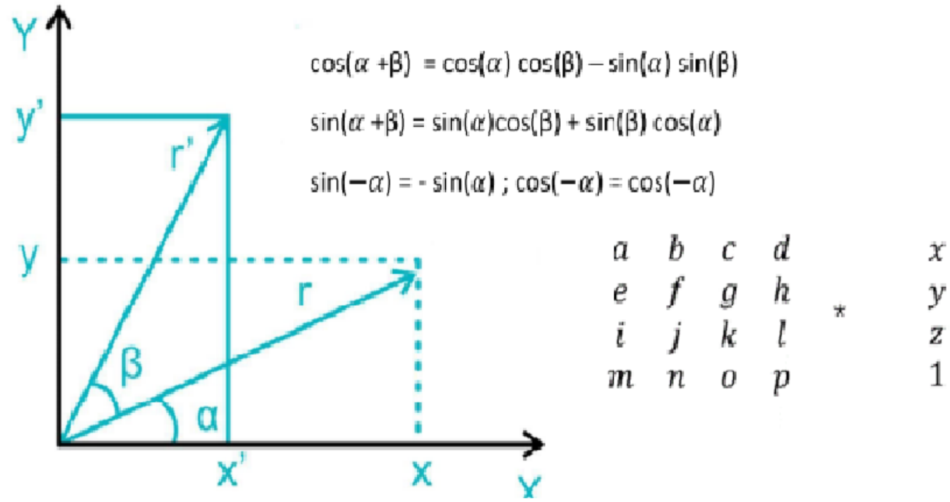
For a transformation: Rotation Matrix along z



$$x = rcos(\theta)$$

$$y = rsin(\theta)$$

$$x' = rcos(\theta + \phi)$$

$$y' = rsin(\theta + \phi)$$

$$z, w\ remain\ the\ same$$

$$x' = r[\cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)]$$

$$x' = rcos(\theta)\cos(\phi) - rsin(\theta)\sin(\phi)$$

$$but\ rcos(\theta) = x\ \&\&\ rsin(\theta) = y$$

$$\therefore x' = xcos(\phi) - ysin(\phi)$$

$$y' = r[\sin(\theta)\cos(\phi) + \sin(\phi)\cos(\theta)\ ]$$

$$y' = rsin(\theta)\cos(\phi) + rsin(\phi)\cos(\theta)$$

$$y' = ycos(\phi) + xsin\ (\phi)$$

We can represent the original co-ordinates of the system as a vector

$$initial\ point = [x \quad y \quad z \ \ w\ ]$$

Initial Co-ordinates $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$

$$Matrix\ M = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

$$Vector\ v = output\ vector = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} xcos(\phi) - ysin(\phi) \\ ycos(\phi) + xsin\ (\phi) \\ z \\ w \end{bmatrix}$$

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix}$$

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} xcos(\phi) - ysin(\phi) \\ ycos(\phi) + xsin\ (\phi) \\ z \\ w \end{pmatrix}$$

$$\begin{cases} ax + by + cz + dw = xcos(\phi) - ysin(\phi) \\ ex + fy + gz + hw = ycos(\phi) + xsin\ (\phi) \\ ix + jy + kz + lw = z \\ mx + ny + oz + pw = w \end{cases}$$

$$\begin{cases} \therefore a = \cos(\phi)\,, b = -\sin(\phi)\,, c = 0, d = 0 \\ e = \sin(\phi)\,, f = \cos(\phi)\,, g = \ 0, h = 0 \\ i = 0, j = 0, k = 1, l = 0 \\ m = 0, n = 0, O = 0, w = 1 \end{cases}$$

$$\therefore \begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 & 0 \\ \sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} == Rotation\ , whereby\ z\ remains\ the\ same$$

$$Matrix\ M = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 & 0 \\ \sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$